Jürgen Jost

Riemannian Geometry and Geometric Analysis

Fifth Edition

Springer

Contents

1	Foundational Material	1
1.1	Manifolds and Differentiable Manifolds	1
1.2	Tangent Spaces	6
1.3	Submanifolds	10
1.4	Riemannian Metrics	13
1.5	Existence of Geodesics on Compact Manifolds	28
1.6	The Heat Flow and the Existence of Geodesics	31
1.7	Existence of Geodesics on Complete Manifolds	34
1.8	Vector Bundles	37
1.9	Integral Curves of Vector Fields. Lie Algebras	47
1.10	Lie Groups	56
1.11	Spin Structures	62
	Exercises for Chapter 1	83
2	De Rham Cohomology and Harmonic Differential Forms	87
2.1	The Laplace Operator	87
2.2	Representing Cohomology Classes by Harmonic Forms	96
2.3	Generalizations	104
2.4	The Heat Flow and Harmonic Forms	105
	Exercises for Chapter 2	110
3	Parallel Transport, Connections, and Covariant Derivatives	113
3.1	Connections in Vector Bundles	113
3.2	Metric Connections. The Yang-Mills Functional	124
3.3	The Levi-Civita Connection	140
3.4	Connections for Spin Structures and the Dirac Operator	155
3.5	The Bochner Method	162
3.6	The Geometry of Submanifolds. Minimal Submanifolds	164
	Exercises for Chapter 3	176
4	Geodesics and Jacobi Fields	179
4.1	1st and 2nd Variation of Arc Length and Energy	179
4.2	Jacobi Fields	185

4.3	Conjugate Points and Distance Minimizing Geodesics	193
4.4	Riemannian Manifolds of Constant Curvature	201
4.5	The Rauch Comparison Theorems and Other Jacobi Field Estimates	203
4.6	Geometric Applications of Jacobi Field Estimates	208
4.7	Approximate Fundamental Solutions and Representation Formulae	213
4.8	The Geometry of Manifolds of Nonpositive Sectional Curvature	215
	Exercises for Chapter 4	232
A	Short Survey on Curvature and Topology	235
5	Symmetric Spaces and Kähler Manifolds	243
5.1	Complex Projective Space	243
5.2	Kähler Manifolds	249
5.3	The Geometry of Symmetric Spaces	259
5.4	Some Results about the Structure of Symmetric Spaces	270
5.5	The Space $Sl(n, \mathbb{R})/SO(n, \mathbb{R})$	277
5.6	Symmetric Spaces of Noncompact Type	294
	Exercises for Chapter 5	299
6	Morse Theory and Floer Homology	301
6.1	Preliminaries: Aims of Morse Theory	301
6.2	The Palais-Smale Condition, Existence of Saddle Points	306
6.3	Local Analysis	308
6.4	Limits of Trajectories of the Gradient Flow	324
6.5	Floer Condition, Transversality and $\mathbb{Z}_2$ -Cohomology	332
6.6	Orientations and $\mathbb{Z}$ -homology	338
6.7	Homotopies	342
6.8	Graph flows	346
6.9	Orientations	350
6.10	The Morse Inequalities	366
6.11	The Palais-Smale Condition and the Existence of Closed Geodesics	377
	Exercises for Chapter 6	390
7	Harmonic Maps between Riemannian Manifolds	393
7.1	Definitions	393
7.2	Formulae for Harmonic Maps. The Bochner Technique	400
7.3	The Energy Integral and Weakly Harmonic Maps	412
7.4	Higher Regularity	422
7.5	Existence of Harmonic Maps for Nonpositive Curvature	433
7.6	Regularity of Harmonic Maps for Nonpositive Curvature	440
7.7	Harmonic Map Uniqueness and Applications	459
	Exercises for Chapter 7	466
8	Harmonic maps from Riemann surfaces	469
8.1	Twodimensional Harmonic Mappings	469
8.2	The Existence of Harmonic Maps in Two Dimensions	483

8.3	Regularity Results	504
	Exercises for Chapter 8	517
9	Variational Problems from Quantum Field Theory	521
9.1	The Ginzburg-Landau Functional	521
9.2	The Seiberg-Witten Functional	529
9.3	Dirac-harmonic Maps	536
	Exercises for Chapter 9	543
A	Linear Elliptic Partial Differential Equations	545
A.1	Sobolev Spaces	545
A.2	Linear Elliptic Equations	549
A.3	Linear Parabolic Equations	553
B	Fundamental Groups and Covering Spaces	557
	Bibliography	560
	Index	576