Lattice meson electric form factor using Wilson fermions

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Abstract

The electric form factor of the pseudoscalar meson (generic pion) is calculated in quenched lattice quantum chromodynamics with SU(2) colour using the Wilson formulation for fermions. Charge radii are calculated for different values of the hopping parameter. It is observed that heavier quarks have distributions of smaller radius. The results are compared with a previous calculation which used the staggered fermion scheme.

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1. INTRODUCTION

The methods of lattice quantum chromodynamics (QCD) are now well established and are being widely applied. However, the best way of dealing with fermion fields on the lattice is still not clear. The problem arises because locality, explicit chiral invariance of the action and suppression of species doubling cannot be imposed simultaneously. At present the Wilson scheme, which avoids species doubling, and the staggered formulation, which preserves a remnant of chiral symmetry, are commonly used in Euclidean lattice QCD simulations. Of course, in the continuum limit it is expected that the differences between these schemes are irrelevant. It is important, at finite lattice spacing, to demonstrate that the same physical results can be obtained with different lattice fermion formulations, or, at least, that the differences can be understood and ultimately controlled. To some extent this has already been done for mass spectrum calculations, for example by Billoire et al. Recently there has been some progress in the study of lattice hadron structure and it is natural to ask how these results depend on the way in which fermions were put on the lattice.

Hadron form factors, extracted from the vector current three-point function, are a useful probe of hadron structure. They can be calculated in lattice QCD in a (colour) gauge invariant way and, ultimately, are directly comparable to experimental results. In Ref. 10 it was shown how to calculate the electric form factor for the pseudo-Goldstone boson (a generic pion) on the lattice. A detailed study of the pseudoscalar meson electric form factor, and the charge radius extracted from it, was carried out as a function of quark mass with the physically reasonable result that heavier quarks have distributions of smaller radius. The
calculations of Ref. 11 were done using the staggered scheme for lattice fermions. In the present paper a similar study of the form factor is done using Wilson fermions. The calculation is done in a way (e.g., using the same gauge field configurations) that the results can be compared with the previous staggered fermion calculation.

In Sec. II expressions for the vector current three-point function and the meson electric form factor are derived. With Wilson fermions the derivation is much more straightforward than in the staggered case.12 Since, in the Wilson scheme, there is a Dirac field associated with every lattice site one can start with hadron interpolating fields that are local. In contrast, the staggered scheme requires construction of Dirac fields on hypercubes in the lattice. Considerable effort is then required to reduce the three-point function to a form involving effectively local operators.12 This leads in the staggered scheme to lattice matrix elements with contributions from states of different parity. In the Wilson scheme this problem does not occur. The final result of Sec. II is an expression for the electric form factor in terms of three-point and two-point hadron correlation functions.

Section III contains details of the numerical work and the results. The calculations were done for quenched approximation in a model with SU(2) colour (at \( \beta = 2.3 \)) and use gauge field configurations that were also used in Ref. 11. Five different values of the hopping parameter, from \( \kappa = 0.134 \) to \( \kappa = 0.158 \), were used. The electric form factor was calculated for a flavour-nonsinglet \( 0^- \) meson. The charge radius was calculated from the derivative of the form factor at zero momentum transfer. Form factors and charge radii are presented for a meson constructed from an equal mass quark-antiquark pair and also for the case of unequal mass quark and antiquark. The qualitative features of the results, e.g., the decrease in the radius of the quark distribution with increasing mass, are the same as observed with staggered fermions.

A more detailed comparison of the present results with those of Ref. 11 is given in Sec. IV. A superficial examination of charge radius versus mass in lattice units would suggest that Wilson and staggered fermions give different results. If the results are converted to physical units by fixing the length scale using pion and rho-meson masses, the charge radii become quite compatible. However, the length scales that emerge for the Wilson and staggered schemes come out to be quite different, indicating, perhaps, that one is still quite far from continuum physics.

Finally it should be noted that information about charge distributions can also be extracted from charge density correlations.13 This has been done by Wilcox13 for pseudoscalar mesons using the Wilson scheme. The results presented in this paper are compatible with those of Ref. 13.

II. FORMALISM

In this section the relevant formulae for calculating the two-point and three-point functions are given. The Wilson scheme for lattice fermions is used. Two flavours of quarks (called u and d) are introduced. The Wilson fermion action is (suppressing colour and Dirac indices)

\[
S_F(u) = \bar{\psi} \Lambda \psi
\]

\[
= \sum_{x,\mu,\Gamma \in \{u,d\}} \kappa_F \left[ \bar{\psi}^f(x)(\gamma_{\mu}U_{\mu}(x))\psi^f(x+a_{\mu}) + \bar{\psi}^f(x+a_{\mu})(\gamma_{-\mu}U_{\mu}^\dagger(x))\psi^f(x) \right] - \sum_{\psi} \bar{\psi}(x)\psi(x),
\]

(1)
With $\rho = \frac{1}{\sqrt{2}}$, the form factor $I_{2k}$ is related to the charge density

$$[\langle \phi | \phi \rangle \langle 0 | \phi \rangle | \phi \rangle \langle 0 | \phi \rangle] \times$$

$$\langle z_1, \rho \rangle (z_2, \rho) \rho \langle z_3, \rho \rangle \rho \langle z_4, \rho \rangle \rho \times$$

$$\langle z_1, \rho \rangle \rho (z_2, \rho) \rho \langle z_3, \rho \rangle \rho \langle z_4, \rho \rangle \rho = \langle z_1, \rho \rangle \rho (z_2, \rho) \rho \langle z_3, \rho \rangle \rho \langle z_4, \rho \rangle \rho$$

(1)

when $0 < z_2 < z_3$.

In large time separations, with $\rho = \frac{1}{\sqrt{2}}$, the ground state is $\langle 1 | \rho \rangle = \rho (\frac{1}{\sqrt{2}}) \rho (\frac{1}{\sqrt{2}})$.

(2)

The charge density operator is

$$\langle (0)^0 | (0)^0 | \rho \rangle \rho \langle (0)^0 | (0)^0 | \rho \rangle = \langle (0)^0 | (0)^0 | \rho \rangle \rho \langle (0)^0 | (0)^0 | \rho \rangle$$

(2.1)

To get the pseudoscalar meson contribution to the form factor we calculate the transition matrix element between left and right vacuua. In (3.1), the pseudoscalar meson is a four-momentum variable, and in (3.2), it is used in the form factor function. Quark masses are not allowed to be\n
(3)

across the light cone. The four-momentum variables are therefore not allowed to\n
(4)

be calculated. The result is \( z \neq 0 \) or \( z \neq 0 \) for each flavor. The operator in (4)

(5)

makes sense with momentum $p$ at large time separation and is equal to the charge field action. The expression for the propagator of the pseudoscalar meson is

$$\langle (0)^0 | (0)^0 | \rho \rangle \rho \langle (0)^0 | (0)^0 | \rho \rangle = \langle (0)^0 | (0)^0 | \rho \rangle \rho \langle (0)^0 | (0)^0 | \rho \rangle$$

(6)

with $\langle (0)^0 | (0)^0 | \rho \rangle \rho \langle (0)^0 | (0)^0 | \rho \rangle = \langle (0)^0 | (0)^0 | \rho \rangle \rho \langle (0)^0 | (0)^0 | \rho \rangle$.

(7)

The correlation function

$$\langle x_1 \rho \rangle \rho (x_2, \rho) \rho \langle x_3, \rho \rangle \rho \langle x_4, \rho \rangle \rho = \langle x_1 \rho \rangle \rho (x_2, \rho) \rho \langle x_3, \rho \rangle \rho \langle x_4, \rho \rangle \rho$$

(8)

is used for the vector meson (quark-to-phonon) we use.

(9)

The local components in the Wilson scheme over staggered fermions is that a

(10)

can be described. The result is \( z \neq 0 \) or \( z \neq 0 \) for each flavor. Using the operator procedure a component vector current

$$\langle (0)^0 | (0)^0 | \rho \rangle \rho \langle (0)^0 | (0)^0 | \rho \rangle = \langle (0)^0 | (0)^0 | \rho \rangle \rho \langle (0)^0 | (0)^0 | \rho \rangle$$

(11)

the fermion action is invariant under global transformation

with $\zeta$ acting on color indices, is also satisfied.

(12)

where $\zeta$ acts on Dirac indices. For SU(2) the relation is

$$\zeta = 5 \zeta \lambda \rho \rho \lambda \rho \rho$$

(13)

and satisfies the relation 1 in

In numerical calculations, the fermion matrix is diagonal in flavor parameter. The correlation of the Wilson tensor will be put equal to 1

(14)

where $\lambda$ is the unit vector in the $\lambda$-direction and $\rho$ is the hopping

(15)
matrix element by
\[ \langle O^-, \hat{p} | \rho(0) | O^-, \hat{p}^1 \rangle = \frac{\langle E_{q+\mathbf{p}} \rangle}{2E_{\hat{p}} E_{\hat{p}^1}} \tilde{F}(q). \] (14)

For numerical calculations it is convenient to take the ratio\(^{18}\)
\[ R = \left\{ \frac{A(O_q, t_x, t_z) A(O_{\hat{q}}, t_x, t_z)}{g_0(0, t_x) g_0(\hat{q}, t_x)} \right\}^{1/2}, \] (15a)
\[ \frac{(E_q + M_0)}{2E_q M_0} \tilde{F}(q). \] (15b)

The three-point function is calculated as the derivative of a two-point function with the charge operator acting as a source.\(^{19}\) This does not differ in any essential detail from the staggered fermion calculation. The details can be found in Ref. 12.

III. RESULTS

Numerical calculations were done in a model for QCD using only SU(2) colour. The lattice was 10\times 20 \times 10 \times 16 sites in size with the charge operators carrying momentum in the 2-direction. Eighteen gauge field configurations were used. These were prepared in quenched approximation using the heat bath Monte-Carlo method\(^{20}\) with the Wilson gauge field action at \( \beta = 2.3 \). The gauge field Monte Carlo was done on a 10\times 16 lattice which was doubled in the 2-direction.

Quark propagators were calculated for one initial space-time point in each gauge field configuration using the conjugate-gradient method.\(^{21}\) Iterations were carried out on the propagators until the maximum change in the two-point function \( g_0(0, t_x) \) at any time point was less than 0.025\% in four iterations. The absolute value squared of the residual vectors was less than \( 10^{-12} \) to \( 10^{-11} \). The calculations used about 20 h on a two-pipe Cyber 205 using half-precision arithmetic.

The same statistical analysis was used here as for the previous study with staggered fermions.\(^{11}\) For the two-point functions the statistical error at each time step was obtained using the whole sample of eighteen configurations. The error in the meson mass was determined from the error matrix of the least-squares fit of an exponential to the zero-momentum two-point function. Covariances between the different factors in the ratio \( R \) [Eq. (15a)] were included in the statistical error of the form factor. Again all configurations were included in a single sum.

Pseudoscalar and vector meson masses for equal mass quark and antiquark are shown in Fig. 1 as a function of \( \kappa^{-1} \), the inverse of the hopping parameter. Calculations were done at five values of \( \kappa \) (from 0.134 to 0.158) and the meson masses span about the same range in lattice units as in the calculation with staggered fermions. The solid lines in Fig. 1 are fits used for extrapolation. They will be discussed in the next section.

Figure 2 shows a typical result for the ratio \( R \) of Eq. (15a) versus the time \( t_x \) of the meson annihilation operator. The charge density is placed at \( t_z = 4 \). With the time boundary conditions used here, a net charge is present only between meson creation \( (t = 0) \) and annihilation \( (t = t_x) \). When \( t_x < t_z \), \( R \) is essentially zero.

In contrast to what was found with staggered fermions the ratio \( R \) is not completely time independent. This should have been anticipated since it was known already from mass calculations in the Wilson scheme that a large time interval is needed before one sees pure single exponential fall-off of the two-point function. The time dependence in \( R \) introduces some arbitrariness in the calculation of the form factor.
mass was found in the staggered fermion calculation (see Fig. 3). At

disappearance does not affect the determination with decays. The light quark

observered. However, in the Wilson case the radius of the heavy quark

result in a mass with unpaired mass quark the charge distribution

transferred to (a) and (b) are shown in Fig. 6. The charge radial extracted from

the light quark. The radius for one unit of momentum

factors were calculated with the change of the respective lattice spacing, 0.125, 0.128

was increased to 0.130 and adjusting the other to 0.125. From

are not determined in mass, a calculation was done fixing one of the

Finally, we consider the situation where quarks of different flavors

using the custo operator.

factor agreement, within statistical errors, with the form factor obtained

when the extended operator is used. However, in both cases the form

the two-point and three-point correlators are calculated in

The calculation was done at \( x = 0, \) with the operator 0 extracted

\begin{equation}
\langle (0) | (0) \rangle \langle (2) | (2) \rangle = \langle (n, x) | (0) \rangle \langle (0, x) | (2) \rangle = \langle (n, x) | (2, x) \rangle
\end{equation}

was used and the form factor was extracted from the three-point function

\begin{equation}
\langle x | \rho(x) | \rho(y) \rangle + \langle \rho(x) | \rho(y) | \rho(z) \rangle + \langle \rho(y) | \rho(z) | \rho(x) \rangle = \langle (n, x) | (0) \rangle
\end{equation}

The extended operator

versions, by using an extended composite operator to annihilate the form

\begin{equation}
\langle (0) | (0) \rangle = 2m_0 \beta Z
\end{equation}

momentum transfer squared \( \beta Z \) at

}\end{equation}

case of the heavy quark. In Fig. 7, where \( x \) and \( y \) are plotted versus

reconstruction of vector dominance leads by the same mechanism as observed

same as those utilizing the staggered scheme. In Sec. 6 A more quantitative

-1, use two extremely small Wilson fermion and the

seever that the parameter \( Y \), which is defined in Eq. (5), are plotted versus

In Fig. 8, In a previous study of the meson form factor it was op-

on the modulus of the charge radii are shown

value of \( \beta Z \) with \( \rho(x) \) which is defined in Eq. (5), and then using the relation

root mean square radius \( \rho \) is defined as the square of the charge moment.

as expected the form factor increases as the quark mass increases. The

function of \( x \), larger values of \( x \) are plotted in Fig. 9, a

the form factor, at \( n = 0 \), \( m = 0 \), and also plotted in Fig. 9. The form factor

\begin{equation}
\langle (0) | \rho(x) | (0) \rangle \langle (0) | \rho(y) | (0) \rangle = 2m_0 \beta Z
\end{equation}

\begin{equation}
\langle (0) | (0) \rangle = 2m_0 \beta Z
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\end{equation}
Ref. 11). This probably indicates that, for the values of \( x \) used here, the ratio of effective quark masses was not as extreme as it was for the staggered fermion calculation.

IV. COMPARISON OF RESULTS WITH WILSON AND STAGGERED FERMIONS

In the continuum limit the Wilson and staggered formulations for lattice fermions should contain the same physics. At finite lattice spacing the chiral properties of the fermions are obviously different so it is clearly interesting to compare the pion structure that emerges with the two schemes. To enable readers to make this comparison in the way they feel is most appropriate a complete enumeration of the results, including those of Ref. 11, is given in Table I.

In lattice units the pseudoscalar mass range covered is about the same in the Wilson and staggered calculations. A superficial examination of Table I clearly shows that the form factors and charge radii are different. However, the different mass splitting between pseudoscalar and vector states observed in the Wilson and staggered schemes shows that different ranges of effective quark mass are being covered in these calculations. Therefore, a direct comparison of the results in lattice units is probably not meaningful.

For a more detailed comparison masses and radii are converted into physical units. The usual procedure for fixing the scale is used here. Masses are extrapolated to the light quark region, that is, to where a realistic pion and rho-meson splitting is obtained and the scale (i.e., the lattice spacing \( a \)) is set by fitting the rho-meson mass.

With staggered fermions the meson masses are extrapolated by fitting with the formula

\[
\cosh M_3 a = A + B a
\]

as was done by Billoire et al.\(^{23}\) The results are shown in Fig. 8. For the pseudoscalar meson (pion)

\[
\cosh M_0 a = 1.013(6) + 3.35(4) a
\]

which agrees very well with Ref. 23. The vector meson mass fit is

\[
\cosh M_4 a = 1.50(6) + 3.4(2) a
\]

which is slightly different from that of Billoire et al.\(^{23}\) The end result is a lattice spacing for the staggered calculation of \( a = 0.242 \pm 0.02 \) fm.

For Wilson the pseudoscalar mass is extrapolated (see Fig. 1) using

\[
(M_0 a)^2 = A_0 / \kappa + B_0
\]

with the fitted values \( A_0 = 1.56(4), B_0 = -9.6(2) \). This gives \( \kappa_c = 0.162 \) as the value where the pion mass vanishes and chiral symmetry, in the Nambu-Goldstone mode, is restored.\(^2\) The vector meson mass looks completely linear in \( \kappa^{-1} \) and was extrapolated with

\[
M_4 a = \frac{A_1}{\kappa} + B_1
\]

The fitted values are \( A_1 = 0.66(2), B_1 = -3.4(1) \). The lattice scale inferred from these values is \( aW = 0.15 \pm 0.005 \) fm.

The charge radius \( r_{rms} \) versus \( M_0 \), now with lattice spacing removed, is shown in Fig. 9. It is encouraging that at smaller meson masses the Wilson and staggered formulations yield compatible results. At larger masses where one gets further away from the Goldstone (or pseudo-Goldstone) behaviour of the pseudoscalar meson, there is no reason why the two schemes should give the same meson structures. For completeness the experimental value\(^{24}\) for the pion charge radius is also shown in Fig. 9 although it is premature to take this comparison very seriously.
A. "Similar

of both will still remain open to the continuation
it is possible, therefore, that the discrepancy in values is due to the
resulting will be found for the W and Z mass.
when calculated using the measured values, which when compared to physical values, with the will still be

of the calculated and measured values. The charge radial distribution, which are comparable.

ACKNOWLEDGMENTS

Continuation: potential
continuation possibility, the one is still far from settling
rather different between the two, and the agreement
integer mass, the measured results from the experiment, to make the comparison in physical units,
when converted to physical units, with the radial strength calculated. The charge radial distribution, which are comparable.

"Similar

A. "Similar

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of the calculated and measured values. The charge radial distribution, which are comparable.
9B. Velikson and D. Weingarten, Nucl. Phys. B249, 433 (1985);
15W. Wilcox, University of Kentucky preprint.
24B. Velikson and D. Weingarten, Nucl. Phys. B249, 433 (1985);
30W. Wilcox, University of Kentucky preprint.
35If only one state contributes this removes the time boundary effects and time dependence completely. This ratio can still be used to remove the time dependence even if periodic boundary conditions are used in time.
36In this case it is necessary to place the operator whose matrix element is being calculated symmetrically about the time origin.
The mean square radius of a mean with an equal mass quadrupole is also shown.

\[ r_{\text{avg}} = \sqrt{\frac{1}{N} \sum \left( \left( x - \mu \right)^4 \right)} \]

The mean square radius of a mean with an equal mass quadrupole is also shown.

\[ r_{\text{avg}} = \sqrt{\frac{1}{N} \sum \left( \left( y - \mu \right)^2 \right)} \]

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The mean square radius of a mean with an equal mass quadrupole is also shown.

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The mean square radius of a mean with an equal mass quadrupole is also shown.

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The mean square radius of a mean with an equal mass quadrupole is also shown.

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The mean square radius of a mean with an equal mass quadrupole is also shown.

\[ r_{\text{avg}} = \sqrt{\frac{1}{N} \sum \left( \left( w - \mu \right)^2 \right)} \]
8. Pseudoscalar (●) and vector (■) meson masses using staggered fermions versus the bare quark mass. The solid lines are fits described in the text.

9. The root mean square radius calculated using Wilson (●) and staggered (■) fermions versus pseudoscalar meson mass $M_\rho$. The experimental value of the pion charge radius is also shown (○).