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VANISHING SCALAR MASSES IN NO-SCALE SUPERGRAVITY

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ABSTRACT

We study radiative corrections to the effective scalar potential in no-scale supergravity models with local supersymmetry spontaneously broken by a gravitino mass. We give a simple proof that gauge non-singlet scalar fields acquire no supersymmetry breaking masses at the one-loop level, and give a general argument extending this result to all orders of perturbation theory in the effective low-energy theory, proving also that no trilinear soft supersymmetry breaking terms in the effective potential are generated. These results are applicable to the four-dimensional no-scale supergravity theory obtained from the superstring after compactification, and support suggestions that the dominant source of global supersymmetry breaking in this theory may be the gaugino mass.

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There is a special class of $N = 1$, $d = 4$ supergravity models in which the effective scalar potential is flat at the tree level^{1),2)}, apart from superpotential terms, and there are no supersymmetry-breaking scalar masses m_0 even though local supersymmetry is spontaneously broken by a gravitino mass $m_{3/2}$. These so-called no-scale supergravity models^{3),2)} have a maximally symmetric $SU(n+1,1)/SU(n+1) \times U(1)$ Kähler manifold structure²⁾. They have been advocated^{3),2)} as a possible framework for the dynamical generation of the gauge hierarchy $m_W \ll m_P$. More recently, a variant of these no-scale models has emerged as the low energy four-dimensional limit of the superstring after compactification on a Calabi-Yau manifold^{4),5)}. The absence of a global supersymmetry-breaking scalar mass m_0 at the tree level²⁾ leads to the hope that a small value of m_0 (and hence m_W) $\ll m_P$ could be generated by radiative corrections. In order to implement this prospective solution^{3),2)} to the gauge hierarchy problem, one must understand the corrections to the flatness of the tree-level scalar potentials. Non-perturbative effects⁶⁾ in the superstring might lift this degeneracy^{5),7)}. Higher order terms in the topological expansion of the functional integral for the superstring also might destroy the flatness of the effective scalar potential, but we are not yet able to calculate them. However, we can calculate loop corrections⁸⁾⁻¹⁰⁾ in the four-dimensional no-scale supergravity theory, and the results are of more general interest even if the high energy limit of no-scale supergravity is different from the superstring.

In this paper we first give general arguments why loop corrections to the no-scale supergravity potential can never generate non-vanishing supersymmetry-breaking masses m_{0i} for the gauge non-singlet scalar fields ϕ^i , both in the simple case with a trivial superpotential $W(\phi^i)$, and then in the case where $\ln|W(\phi^i)| \neq 0$. Secondly, we give an explicit elementary derivation of the absence^{8),9)} of m_{0i} at the one-loop level, if $\ln|W(\phi^i)| = 0$, and indicate how it can be extended to all loops if $\ln|W(\phi^i)| = 0$. Finally, we extend these results to one or more loops even if $\ln|W(\phi^i)| \neq 0$, and also prove that trilinear soft supersymmetry breaking terms vanish to all orders.

We start by reminding the reader of the canonical form²⁾ for the Kähler potential for no-scale supergravity models with many fields (z, ϕ^i) :

$$G = -3 \ln(z + z^* - \sum_{i=1}^n \phi^i \phi_i^*) + \ln|W(\phi)|^2 \quad (1)$$

where z is necessarily a gauge-singlet field, while the ϕ^i are in general gauge non-singlets. For the moment we set $\ln|W(\phi)| = 0$. The Kähler structure of this model is that of a non-compact non-linear $SU(n+1,1)/SU(n+1) \times U(1)$ σ -model. The

SU(n+1) symmetry is not explicit in the formulation (1), but can be exhibited²⁾ if we rewrite the Kähler potential in the maximally symmetric form

$$G = -3 \ln \left(1 - \frac{1}{3} \sum_{i=0}^n y^i y_i^* \right) + \ln |W(y^0)|^2 \quad (2)$$

where we have made the field redefinition²⁾

$$z = \frac{1}{2} \frac{1 - \sqrt{\frac{1}{3}} y^0}{1 + \sqrt{\frac{1}{3}} y^0}, \quad \phi^i = \frac{\sqrt{\frac{1}{3}} y^i}{1 + \sqrt{\frac{1}{3}} y^0} : i=1,2,\dots,n \quad (3)$$

We see that the Kähler structure is explicitly SU(n+1) symmetric in Eq. (2), while the superpotential explicitly breaks SU(n+1) $\rightarrow$ SU(n). It is well known²⁾ that the tree level effective scalar potential

$$V = e^G \left(G_i (G^{-1})^i_j G^j - 3 \right) \quad (4)$$

obtained from (1) or (2) is flat, but the representation (2) shows that the SU(n+1) invariance of this flatness is accidental. The full SU(n+1) invariance is not reflected in all the theory generated by (2), and cannot be expected to hold for the effective potential when loops are calculated. Instead we expect the linearly realized symmetry SU(n+1) to be broken to SU(n) and the non-linearly realized SU(n+1,1) $\rightarrow$ SU(n,1). However, we do not expect any further breaking of the symmetry by radiative corrections. Radiative corrections in non-linear chiral Lagrangians for pion dynamics with a potential $V(\pi) = 0$ and hence $m_\pi = 0$ [cf. $V(\phi^i) = 0$, $m_{0i} = 0$] do not generate such a potential: it is only explicit chiral symmetry breaking which can generate $m_\pi \neq 0$. In the immortal words of Howard Georgi: "A Goldstone boson is a Goldstone boson is a Goldstone boson"¹¹⁾.

What happens if we now introduce into (1) a non-trivial superpotential for the gauge non-singlet fields ϕ^i : $W(\phi^i) \equiv \omega \hat{W}(\phi^i)$ where $\hat{W}$ is conventionally normalized and ω is an overall scaling factor? We expect that the effective action for non-singlet scalar fields will now take the form

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{KE} + [V \propto \omega^{n+2}] + [m_0^2 |\phi|^2?] \quad (5)$$

where the questioned term in the second bracket is the possible supersymmetry-breaking scalar mass under discussion. We want to know whether such a term

$$m_0 = m_0(m_p, \langle z \rangle, \Lambda) \quad (6)$$

can be generated by radiative corrections, where we have exhibited the possible dependences of m_0 , namely those on the Planck mass, the expectation value of the z field, and the cut-off Λ introduced in calculating the radiative corrections. Suppose that such a value of m_0 is generated, and consider the theory in the domain of small field values $|\phi^i| \ll m_0$. In this domain the tree-level theory looks like a theory without a superpotential $W(\phi^i)$, in which case we have previously argued that $m_0 = 0$. It is difficult to believe that radiative corrections for $\ln|W(\phi^i)| \neq 0$ will generate such a drastic violation of supersymmetry as occurs when $|m_0| \gg |\phi^i|$.

The above arguments are suggestive, but they should be supplemented by more precise arguments based on the explicit forms of one- and higher-loop radiative corrections, which we now do. It is well known that the one-loop radiative corrections give

$$\delta V_{1\text{-loop}} = \frac{1}{64\pi^2} \left\{ \Lambda^2 \text{Str } \mathcal{M}^2 + \text{Str } \mathcal{M}^4 \ln \frac{\mathcal{M}^2}{\Lambda^2} + \Lambda^4 \text{Str } \ln \left(1 + \frac{\mathcal{M}^2}{\Lambda^2} \right) - \text{Str } \mathcal{M}^4 \ln \left(1 + \frac{\mathcal{M}^2}{\Lambda^2} \right) \right\} \quad (7)$$

where Λ is a cutoff. The supertraces appearing in Eq. (7) are defined by

$$\text{Str } F(\mathcal{M}^2) = \text{Tr } F(M_S^2) - 2 \text{Tr } F(M_F^2) - 4 F(m_{3/2}^2) + 2 F(4m_{3/2}^2) \quad (8)$$

where M_S^2 and $M_F^2 (\equiv m_F^+ m_F)$ are the spin-0 bosonic and spin- $\frac{1}{2}$ fermionic mass squared matrices needed to calculate the effective potential. The penultimate term in Eq. (8) is the contribution of the gravitino whose mass is $m_{3/2} = e^{G/2}$, and the last term is the contribution of the ghost associated with the gauge fixing condition $\gamma_\mu \phi^\mu = 0$.

We consider first the simple model with $\ln|W(\phi^i)| = 0$, for which the only non-vanishing mass at the tree level is $m_{3/2} = e^{G/2}$. Substituting this into (7) and (8), we find

$$V_{1\text{-loop}} = \delta V_1 = f_1(G) \quad (9)$$

It is now an easy matter to check that

$$\frac{\partial V_{1\text{-loop}}}{\partial z} = G_z f_1' , \quad \frac{\partial V_{1\text{-loop}}}{\partial \phi^i} = G_i f_1' \quad (10a)$$

and

$$\frac{\partial^2 V_{1\text{-loop}}}{\partial \phi^i \partial \phi_j^*} = G_i^j f_1' + G_i G_j^j f_1'' , \quad \frac{\partial^2 V}{\partial \phi^i \partial \phi_j^*} = G_{ij} f_1' + G_i G_j f_1'' \quad (10b)$$

where

$$G_z = \frac{-3}{(z+z^* - \phi^i \phi_i^*)} , \quad G_i = \frac{3\phi_i^*}{(z+z^* - \phi^i \phi_i^*)} , \quad G_j^j = \frac{3\phi_j^j}{(z+z^* - \phi^i \phi_i^*)} \quad (11a)$$

and

$$G_{ij} = \frac{-3\phi_i^* \phi_j^*}{(z+z^* - \phi^i \phi_i^*)^2} , \quad G_i^j = \frac{-3\delta_i^j}{(z+z^* - \phi^i \phi_i^*)} + \frac{3\phi_i^* \phi_j^j}{(z+z^* - \phi^i \phi_i^*)^2} \quad (11b)$$

Substituting (11) into (10b), we discover

$$\frac{\partial^2 V_{1\text{-loop}}}{\partial \phi^i \partial \phi_j^*} = 0 + O(\phi^2) , \quad \frac{\partial^2 V_{1\text{-loop}}}{\partial \phi^i \partial \phi_j^*} = -\frac{3\delta_i^j}{(z+z^*)} f_1' + O(\phi^2) \quad (12)$$

However, when we minimize the effective potential $V_{1\text{-loop}}$ with respect to z , Eq. (10a) tells us that $f_1' = 0$, and then Eq. (12) tells us that

$$\frac{\partial^2 V}{\partial \phi^i \partial \phi_j^*} = O(\phi^2) , \quad \frac{\partial^2 V}{\partial \phi^i \partial \phi_j^*} = O(\phi^2) \Rightarrow m_{0i}^2 = 0 \quad (13)$$

at the one-loop level.

This result is not new^{8),9)}. Remarkably, however, it can be extended to any loop order, even in the presence of a non-trivial superpotential $W(\phi^i)$. First we observe that at any order in perturbation theory $\delta V_{n\text{-loop}}$ can be written in the form

$$\delta V_{n\text{-loop}} = f_n \left(\left\{ \text{Str}(\mathcal{M}^2)^p (\ln \mathcal{M}^2)^q : p, q \geq 0 \right\} \right) \quad (14)$$

where $\mathcal{M}$ is the tree-level mass matrix. Then we remember that in the simple case with trivial $W(\phi^i)$, the only non-zero mass at the tree level is $m_{3/2}$ (8), so that

$$\delta V_{n\text{-loop}} = \delta V_n = f_n(G) \quad (15)$$

We can then repeat the analysis of Eqs. (10) and (13) to deduce that $m_{o_i}^2 = 0$ at any loop level, if $\ln|W(\phi^i)| = 0$.

The extension to $\ln|W(\phi^i)| \neq 0$ is conceptually simple, but algebraically involved. In order to facilitate the discussion we work out the case of a single observable field ϕ : the results can easily be generalized to the interesting case with many ϕ^i . We introduce the notation

$$G_0 = -3 \ln(z + z^* - \phi^i \phi_i^*) \quad , \quad F = \ln W \quad (16)$$

for the different pieces of the full Kähler potential (1):

$$G = G_0 + F + F^\dagger \quad (17)$$

so that G_0 denotes the $SU(n+1,1)/SU(n) \times U(1)$ metric piece. For the all-order proof we make two technical assumptions about the form of the effective potential V and the superpotential W . The first is that the effective potential V only depends on supertraces $\text{Str } f_a(\mathcal{M}^2)$ of some functions $f_a(\mathcal{M}^2)$, where the supertrace of any function was defined in Eq. (8). The subscript a simply labels the different functional forms of the functions f_a . For example, at the one-loop level in Eq. (7) four such functions are needed, including $f_1(\mathcal{M}^2) \equiv \mathcal{M}^2$ and $f_2(\mathcal{M}^2) \equiv \mathcal{M}^4 \ln \mathcal{M}^2/\Lambda^2$. In higher loops, additional functions such as $(\mathcal{M}^2)^p (\ln \mathcal{M}^2/\Lambda^2)^q$ will also appear. Therefore, this assumption is just what one expects naïvely from perturbation theory. Our second assumption is that $\partial W/\partial \phi$ and $\partial^2 W/\partial \phi^2$ both vanish at $\phi = 0$. Both these conditions are trivially satisfied for the trilinear superpotential terms found in most theories.

The 4×4 scalar mass matrix squared M_S^2 and the 2×2 fermion mass matrix squared $M_F^2 = m_F^\dagger m_F$ depend on G_0 , W and W^* , the derivatives $W^k \equiv \partial^k W/\partial \phi^k$ for $k = 1, 2, 3$ and their complex conjugates $\bar{W}_k \equiv (W^k)^*$. Note that W and the W^k depend only on ϕ , and not on ϕ^* , while W^* and the $\bar{W}_k$ depend only on ϕ^* , and not on ϕ . We do not present here the precise forms of M_S^2 and M_F^2 but only note the following

properties which they pass on to V.

(i) Dependence on $W^1(W_1)$ is through the products $\alpha_1 \equiv W^1 W_1$, $\bar{\alpha}_2 \equiv W_2 W^1$ ($\alpha_2 \equiv W^2 W_1$) and $\bar{\alpha}_3 \equiv W_3 W^1$ ($\alpha_3 \equiv W^3 W_1$), except for a dependence on $\bar{x} \equiv W^1 W^*$ ($x \equiv W_1 W$) in the off-diagonal 12(21) elements of M_F^2 .

(ii) Dependence on W^2 and W_2 is mainly through $\alpha_2(\bar{\alpha}_2)$. However, there is also a dependence via $\sigma \equiv W_2 W^2$ in the 22 and 44 elements of M_S^2 as well as in the 22 element of M_F^2 .

(iii) Dependence on $W^3(W_3)$ is always through $\alpha_3(\bar{\alpha}_3)$, which only appear in the 42(24) elements of M_S^2 .

These three properties are vital for our all-order proof.

We first consider the first derivative of the effective potential V. Using the above properties of the effective potential V we have

$$\frac{\partial V}{\partial \phi} = \frac{\partial V}{\partial G_0} \frac{\partial G_0}{\partial \phi} + \sum_{k=0}^3 \frac{\partial V}{\partial W^k} W^{k+1} \quad (18)$$

where W^0 denotes the superpotential W. As $W^{1,2} = 0$ at $\phi = 0$, and $\partial G_0 / \partial \phi|_{\phi=0} = 0$ also, it follows that

$$\left. \frac{\partial V}{\partial \phi} \right|_{\phi=0} = \left(\frac{\partial V}{\partial W^2} W^3 + \frac{\partial V}{\partial W^3} W^4 \right) \Big|_{\phi=0} \quad (19)$$

Property (ii) above tells us that

$$\frac{\partial V}{\partial W^2} = \frac{\partial V}{\partial \alpha_2} W_1 + \frac{\partial V}{\partial \sigma} W_2 \quad (20)$$

which therefore vanishes at $\phi = 0$. Property (iii) above tells us that

$$\frac{\partial V}{\partial W^3} = \frac{\partial V}{\partial \alpha_3} W_1 \quad (21)$$

which also vanishes at $\phi = 0$. Hence

$$\left. \frac{\partial V}{\partial \phi} \right|_{\phi=0} = 0 \quad (22)$$

and the earlier assumptions that $W_{1,2} = 0$ at $\phi = 0$ guarantee that $\phi = 0$ is an extremum of the effective potential for the low energy observable fields ϕ at any loop order. It also follows from (20) and (21) that

$$\left. \frac{\partial}{\partial \phi} \left(\frac{\partial V}{\partial W^2} \right) \right|_{\phi=0} = 0, \quad \left. \frac{\partial}{\partial \phi} \left(\frac{\partial V}{\partial W^3} \right) \right|_{\phi=0} = 0 \quad (23)$$

and the fact that $W_{1,2}$ do not depend on ϕ , but only on ϕ^* , tells us that

$$\left. \frac{\partial}{\partial \phi^*} \left(\frac{\partial V}{\partial W^3} \right) \right|_{\phi=0} \quad (24)$$

also. Equations (23) and (24) will be useful in what follows.

We next consider the second derivatives of V which give the mass terms for the low energy fields ϕ at any extremum, starting with $\partial^2 V / \partial \phi \partial \phi^*$. Using the facts that $\partial G_0 / \partial \phi = W^{1,2} = 0$ at $\phi = 0$, and Eq. (23), it is easy to verify that

$$\left. \frac{\partial^2 V}{\partial \phi \partial \phi^*} \right|_{\phi=0} = \left[\frac{\partial V}{\partial G_0} \frac{\partial^2 G_0}{\partial \phi \partial \phi^*} + \left(\frac{\partial}{\partial \phi^*} \frac{\partial V}{\partial W^2} \right) W^3 \right] \Big|_{\phi=0} \quad (25)$$

which we can rewrite as

$$\left. \frac{\partial^2 V}{\partial \phi \partial \phi^*} \right|_{\phi=0} = \left. \frac{\partial V}{\partial G_0} \right|_{\phi=0} \left(3 \exp \frac{G_0}{3} \right) \Big|_{\phi=0} + \left(\frac{\partial}{\partial \phi^*} \frac{\partial V}{\partial W^2} \right) W^3 \Big|_{\phi=0} \quad (26)$$

We now show that the second term in (26) vanishes because of global supersymmetry. Using $W_{1,2} = 0$ at $\phi = 0$, Eq. (20) tells us that

$$\left(\frac{\partial}{\partial \phi^*} \frac{\partial V}{\partial W^2} \right) \Big|_{\phi=0} = \left(\frac{\partial V}{\partial \sigma} W_3 \right) \Big|_{\phi=0} \quad (27)$$

so the second term in (26) vanishes iff $\partial V / \partial \sigma = 0$ at $\phi = 0$, since in general $W_3 = 0$ at $\phi = 0$. Denoting $S_a \equiv \text{Str } f_a$ we have

$$\frac{\partial V}{\partial \sigma} = \sum_a \frac{\partial V}{\partial S_a} \frac{\partial S_a}{\partial \sigma} \quad (28)$$

and property (ii) above tells us that

$$\begin{aligned}
 \frac{\partial S_a}{\partial \sigma} &= \frac{\partial}{\partial \sigma} \text{Tr} f_a(M_S^2) - 2 \frac{\partial}{\partial \sigma} \text{Tr} f_a(M_F^2) \\
 &= \text{Tr} \left[\phi_a(M_S^2) \frac{\partial M_S^2}{\partial \sigma} \right] - 2 \text{Tr} \left[\phi_a(M_F^2) \frac{\partial M_F^2}{\partial \sigma} \right]
 \end{aligned} \tag{29}$$

where $\phi_a(x) \equiv df_a(x)/dx$. The forms of the mass matrices are such that

$$\frac{\partial M_S^2}{\partial \sigma} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & C & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C \end{pmatrix}, \quad \frac{\partial M_F^2}{\partial \sigma} = \begin{pmatrix} 0 & 0 \\ 0 & C \end{pmatrix} \tag{30}$$

where the non-zero quantities C are equal because of global supersymmetry. We now use the facts that $M_S^2(\phi = 0) = 0$ and $M_F^2(\phi = 0) = \begin{pmatrix} 4e^{G_0} & 0 \\ 0 & 0 \end{pmatrix} \times |W|^2$ and Eqs. (30) substituted into Eq. (29) to deduce that

$$\begin{aligned}
 \frac{\partial S_a}{\partial \sigma} &= \phi_a(0) \cdot C + \phi_a(0) \cdot C - 2 \left(\phi_a(4e^{G_0}) |W|^2 \cdot 0 + \phi_a(0) \cdot C \right) \\
 &= 0
 \end{aligned} \tag{31}$$

Going back to Eq. (28), we now see that $\partial V / \partial \sigma = 0$ at $\phi = 0$ and hence Eq. (26) becomes

$$\left. \frac{\partial^2 V}{\partial \phi \partial \phi^*} \right|_{\phi=0} = \left. \frac{\partial V}{\partial G_0} \right|_{\phi=0} \left(3e^{G_0/3} \right)_{\phi=0} \tag{32}$$

Since the only dependence of V on z is also through G_0 , it is clear that

$$\frac{\partial V}{\partial z} = \frac{\partial V}{\partial G_0} \frac{\partial G_0}{\partial z} = \frac{\partial V}{\partial G_0} \left(-3e^{G_0/3} \right) \tag{33}$$

so that

$$\left. \frac{\partial^2 V}{\partial \phi \partial \phi^*} \right|_{\phi=0} = - \left. \frac{\partial V}{\partial z} \right|_{\phi=0} \tag{34}$$

This means that in any order of perturbation theory the $\partial^2 V / \partial \phi \partial \phi^*$ term in the mass squared matrix for the scalar fields vanishes when we minimize the effective

potential with respect to z^*).

We now turn to the $\partial^2 V / \partial \phi \partial \phi$ terms in the effective scalar mass squared matrix, and show that they also vanish at $\phi = 0$. Using the facts that $\partial G_0 / \partial \phi$, $\partial^2 G_0 / \partial \phi^2$ and $W_{1,2}$ all vanish at $\phi = 0$, we see that

$$\left. \frac{\partial^2 V}{\partial \phi^2} \right|_{\phi=0} = \left[\sum_{k=1}^3 \frac{\partial V}{\partial W^k} W_{k+2} + \sum_{k=2,3} \frac{\partial}{\partial \phi} \frac{\partial V}{\partial W^k} W_{k+1} \right]_{\phi=0} \quad (35)$$

Using Eqs. (20) and (21) which mean that $\partial V / \partial W^2|_{\phi=0} = \partial V / \partial W^3|_{\phi=0} = 0$, and the results (23), we are left with only one remaining term in Eq. (35):

$$\left. \frac{\partial^2 V}{\partial \phi^2} \right|_{\phi=0} = \left(\frac{\partial V}{\partial W^1} W^3 \right) \Big|_{\phi=0} \quad (36)$$

Using now the property (i) introduced earlier, we have

$$\left. \frac{\partial V}{\partial W^1} \right|_{\phi=0} = \left(\frac{\partial V}{\partial \alpha_1} W_1 + \frac{\partial V}{\partial \alpha_2} W_2 + \frac{\partial V}{\partial \alpha} W^* + \frac{\partial V}{\partial \alpha_3} W_3 \right) \Big|_{\phi=0} \quad (37)$$

Because W_1 and $W_2 = 0$ at $\phi = 0$, the first two terms in (37) vanish, so we are left with

$$\left. \frac{\partial V}{\partial W^1} \right|_{\phi=0} = \left(\frac{\partial V}{\partial \alpha} W^* + \frac{\partial V}{\partial \alpha_3} W_3 \right) \Big|_{\phi=0} \quad (38)$$

The first of these two terms can be written as

$$\frac{\partial V}{\partial \alpha} = \sum_a \frac{\partial V}{\partial S_a} \frac{\partial S_a}{\partial \alpha} = \sum_a \frac{\partial V}{\partial S_a} \frac{\partial S_a}{\partial x} f_a(M^2) \quad (39)$$

Using again the property (i), we know that we need only retain the M_F^2 term in (39):

*) We remind the reader that the possibility suggested in Ref. 8) that the one-loop radiative corrections to the effective potential may fix the value of z has been excluded by Ref. 10). However, non-perturbative effects may be able to fix z , as discussed in Ref. 7).

$$\frac{\partial V}{\partial x} = \sum_a \frac{\partial V}{\partial s_a} \left(-2 \frac{\partial}{\partial x} \text{Tr} f_a(M_F^2) \right) = \sum_a \frac{\partial V}{\partial s_a} \left(-2 \text{Tr} \phi_a(M_F^2) \frac{\partial M_F^2}{\partial x} \right) \quad (40)$$

But $\phi_a(M_F^2)$ is diagonal at $\phi = 0$, since M_F^2 is, whereas property (i) tells us that $\partial M_F^2 / \partial x$ is off-diagonal. Hence

$$\text{Tr} \left(\phi_a(M_F^2) \frac{\partial M_F^2}{\partial x} \right) \Big|_{\phi=0} = 0 \quad \Rightarrow \quad \frac{\partial V}{\partial x} \Big|_{\phi=0} = 0 \quad (41)$$

In a similar way, we find

$$\frac{\partial V}{\partial \bar{\alpha}_3} \Big|_{\phi=0} = \sum_a \frac{\partial V}{\partial s_a} \frac{\partial}{\partial \bar{\alpha}_3} \text{Str} f_a(M^2) \quad (42)$$

which becomes, using property (iii),

$$\frac{\partial V}{\partial \bar{\alpha}_3} \Big|_{\phi=0} = \sum_a \frac{\partial V}{\partial s_a} \text{Tr} \left(\phi_a(M_S^2) \frac{\partial M_S^2}{\partial \bar{\alpha}_3} \right) = 0 \quad (43)$$

because $\bar{\alpha}_3$ only appears in an off-diagonal element of M_S^2 . Combining (41) and (43), we see that $\partial V / \partial W^1 \Big|_{\phi=0}$ (38) vanishes, and hence also $\partial^2 V / \partial \phi^2 \Big|_{\phi=0}$ (36). Thus all components of the scalar mass squared matrix vanish at $\phi = 0$, and it is clear that a similar result holds if we include any number of gauge non-singlet observable fields ϕ^i .

There is a simple argument which indicates that trilinear soft supersymmetry breaking terms vanish in any order of perturbation theory. For if they did not, the standard renormalization group equations³⁾ connecting soft supersymmetry breaking mass parameters would then generate non-zero scalar masses in higher orders, which we have argued above do not appear. To prove that trilinear soft supersymmetry breaking terms do indeed vanish in all orders of perturbation theory, we must also show that $\partial^3 V / \partial \phi^3 = 0$ at $\phi = 0$. Looking at the expression

$$\begin{aligned}
 \frac{\partial^3 V}{\partial \phi^3} = & \left(\frac{\partial^3 V}{\partial \phi^2 \partial \phi_0} \right) \frac{\partial \phi_0}{\partial \phi} + 2 \left(\frac{\partial^2 V}{\partial \phi \partial \phi_0} \right) \frac{\partial^2 \phi_0}{\partial \phi^2} + \left(\frac{\partial V}{\partial \phi_0} \right) \frac{\partial^3 \phi_0}{\partial \phi^3} \\
 & + \sum_{k=0}^3 \frac{\partial^3 V}{\partial \phi^2 \partial W^k} W^{k+1} + 2 \sum_{k=0}^3 \frac{\partial^2 V}{\partial \phi \partial W^k} W^{k+2} \\
 & + \sum_{k=0}^3 \frac{\partial V}{\partial W^k} W^{k+3}
 \end{aligned} \tag{44}$$

we see that many of these terms vanish thanks to the vanishing at $\phi = 0$ of many expressions discussed previously in this paper. Others can be shown to vanish by simple extensions of the previous arguments which we do not discuss in detail here.

The low energy sector (z, ϕ^i) that we have considered in this paper is almost identical to that obtained from a truncation of the superstring⁴⁾, which presumably corresponds⁵⁾ to the low-energy limit of the tree-level topologically trivial sector of the superstring when compactified on a Calabi-Yau manifold. One difference is that the effective four-dimensional supergravity theory obtained from the superstring contains another gauge singlet field S ⁴⁾. However, it is easy to check that including the S field does not change our arguments that $m_{0i}^2 = 0$. A more important difference is that the low-energy limit of the superstring contains a non-trivial kinetic term for the gauge superfields⁴⁾, which may lead to a non-zero gaugino mass $m_{\frac{1}{2}}$ in some order¹²⁾, once $m_{3/2} \neq 0$. We have not included gauge supermultiplet terms in our supertraces (8), nor in the rest of our analysis. However, it is well known that if a gaugino mass arises in n -loop order, radiative corrections will generate scalar masses in $(n+1)$ -loop order²⁾. The results of this paper suggest that scalar masses cannot arise without the prior appearance of gaugino masses^{12), 2)}. Hence we expect the dominant source of global supersymmetry breaking in the low-energy theory obtained from the superstring to be the gaugino mass.

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