CERN-TH.4383/86

HIGHER "SPINS" IN ONE AND TWO SPACE-TIME DIMENSIONS

Anders K.H. Bengtsson

Physics Department, Queen Mary College,
London E1 4NS, United Kingdom

and

Ingemar Bengtsson

CERN - Geneva

ABSTRACT

A neutral massless matter field is coupled to gauge fields in such a way that the action is invariant under higher spin-like gauge transformations. Once "spin 4" is included, all even "spins" > 4 are required by gauge invariance. In one dimension, we construct the complete action. The model is consistent, but the "higher spin" fields have no dynamical consequences. In two dimensions, the constraint algebra is a graded extension of the Virasoro algebra.

The question whether massless fields of spin greater than two can interact with us in a consistent manner has been asked on and off for the past fifty years or so, but no definite conclusion has emerged. A few positive results exist; in particular, free actions for arbitrary spin have been constructed¹⁾, and so have their "string-like" generalizations²⁾. Cubic interaction terms for arbitrary spin have been constructed in the light front gauge³⁾, and covariantly for spin 3⁴⁾. Conserved currents exist for any spin⁵⁾. De Sitter space offers intriguing possibilities⁶⁾.

It is known, however, that a self-interacting higher spin theory [in the sense of Refs. 3) and 4)] does not exist. On the one hand, the existence of a non-trivial gauge algebra obeying the Jacobi identity has been ruled out for pure spin 3 and higher⁷⁾, and on the other, it has been shown by explicit calculation that the four-point coupling for pure spin 3 does not exist [see Ref. 8), where also the gauge algebra problem and the connection to string theory is discussed at some length]. The remaining possibility is then the old suggestion [see, e.g., Ref. 9)] that all higher spins together could form a consistent theory. In this letter we will, in fact, construct and solve two classically consistent higher spin theories involving all even higher spins co-operatively, but we succeed in doing so only by restricting ourselves to space-times with one or two dimensions - where neither spin nor self-interacting gauge fields exist - and it is up to the reader to decide whether this simplification is too drastic to be of relevance for the four-dimensional case.

To explain in which sense we talk about higher spins in one or two dimensions, we will begin with a brief description of what we want to do in four dimensions. There the free massless particles of arbitrary integer spin λ are described by fully symmetric double traceless fields with λ indices. The actions are invariant under the gauge transformations

$$\delta A_{\alpha_1 \dots \alpha_\lambda} = \partial_{(\alpha_1} \{ \alpha_2 \dots \alpha_\lambda \} \quad (1)$$

where the gauge parameter is traceless [other field representations exist¹⁰⁾, but we presume that all conclusions remain valid whichever representation one uses, although an admixture of lower spin fields, as in Ref. 2), might be useful]. The question that we want to answer is then whether Eq. (1) can be supplemented with non-linear terms in such a way that a non-Abelian Lie algebra results and, if so, whether one can find a non-linear extension of the free action which is invariant under the deformed transformations. For $\lambda = 1$ and 2, this procedure leads to the

Yang-Mills and the Einstein-Hilbert actions respectively, but - as mentioned above - for $\lambda > 2$ the answer is no. However, the failure occurs in a way which suggests that one might obtain a positive answer if one includes all higher spins (up to infinity) in the action. Unfortunately, this possibility is very hard to investigate with brute force, due to the length of the required calculations.

A variant of the above programme is to couple a matter field to a higher spin field. Thus, consider the action

$$S = \frac{1}{2} \int \partial_\alpha \varphi \partial_\alpha \varphi + g A_{\alpha_1, \dots, \alpha_\lambda} J_{\alpha_1, \dots, \alpha_\lambda} \quad (2)$$

where $J_{\alpha_1, \dots, \alpha_\lambda}$ is a current quadratic in the scalar field. Obviously, the current contains λ derivatives, so if we demand that S be invariant under (1), we must supplement it with a gauge transformation for ϕ , of the generic form

$$\delta \varphi = \partial^{\lambda-1} (\phi) \quad (3)$$

[The precise form of (3) is highly non-unique due to the possibility of field redefinitions.] For $\lambda = 2$, this is ordinary reparametrization invariance. One complication should be mentioned, namely that all odd spin fields have to carry a colour index³⁾, so that a neutral matter field - the case that we will consider - can interact in this way with even spins only.

In one and two dimensions, spin does not exist, but actions of the form (2) can be written down - as we will see - and Eq. (3) suggests that we call the gauge field a "spin λ " field if the concomitant transformation of the matter field contains $\lambda-1$ derivatives. This is in fact what we will do.

Let us start in one dimension, where there are no indices. Suppose we adopt the gauge transformation

$$\delta_\lambda \varphi(\tau) = \phi(\tau) \varphi^{(\lambda-1)} \quad (4)$$

where $\phi^{(\lambda-1)}$ means that ϕ has been differentiated $\lambda-1$ times. If two such transformations are commuted, we obviously obtain

$$[\delta_\lambda, \delta_\lambda] \varphi = \delta_\lambda \varphi + \delta_{\lambda+1} \varphi + \dots + \delta_{2\lambda-2} \varphi \quad (5)$$

So, as soon as $\lambda > 2$, these transformations do not close. However, if arbitrarily high λ 's are admitted, an infinite dimensional filtered Lie algebra indeed results. In this way, the introduction of arbitrarily high "spins" appears incapable. We are prepared to accept this, but what we cannot accept, for reasons mentioned earlier, is that both even and odd λ 's occur on the right-hand side of (5). This is easily remedied, however. It can be shown that the transformations

$$\delta_{2n} \varphi = (-1)^{n-1} \partial^{n-1} [\delta_{2n} \partial^n \varphi] \quad (6)$$

for arbitrary $n > 1$ or 2 form a closed associative algebra without any need for odd "spins" - the proof will be made obvious later - so it is the even "spin" gauge transformations that we will adopt.

As our free action, we take

$$S_0 = \frac{1}{2} \int d\tau \dot{\varphi} \dot{\varphi} \quad (7)$$

and the simplest gauge theory is obtained for $\lambda = 2$:

$$S_2 = \frac{1}{2} \int d\tau \dot{\varphi} \dot{\varphi} - A_2 \dot{\varphi} \dot{\varphi} \quad (8)$$

This is one-dimensional gravity and has been studied at length¹¹⁾. The action (8) is invariant under

$$\begin{aligned} \delta_2 \varphi &= \delta_2 \dot{\varphi} \\ \delta_2 A_2 &= \dot{\delta}_2 + \delta_2 \dot{A}_2 - \dot{\delta}_2 A_2 \end{aligned} \quad (9)$$

Let us now see how S_0 varies under (6) for an arbitrary $\lambda = 2n$. We find

$$\delta_{2n} S_0 \equiv \int \dot{\phi} (\delta_{2n} \dot{\phi}) = \frac{1}{2} \int \dot{\phi}_{2n} \phi^{(n)} \phi^{(n)} \quad (10)$$

This can be compensated by a cubic term in the action, viz.

$$S_{2n} = - \frac{1}{2} \int A_{2n} \phi^{(n)} \phi^{(n)} \quad (11)$$

if the gauge field transforms as

$$\delta_{2n} A_{2n} = \dot{\phi}_{2n} + \text{non-linear terms} \quad (12)$$

[for odd λ and a transformation of the generic form (3) there are no solutions]. Let us now specialize to spin 4. The variation of S_4 will produce the following quartic terms:

$$\delta_4 S_4 = \frac{1}{2} \int [\ddot{\phi}_4 A_4 - \dot{\phi}_4 \ddot{A}_4] \dot{\phi} \dot{\phi} + 3 [\dot{\phi}_4 \dot{A}_4 - \dot{\phi}_4 A_4] \dot{\phi} \dot{\phi} \quad (13)$$

We see that there is no hope of choosing the non-linear terms in (12) so that the action $S_0 + S_4$ is invariant. However, if we add a "spin 6" term to the action, the offending terms will cancel if we choose

$$\begin{aligned} \delta_4 A_4 &= \dot{\phi}_4 + \ddot{\phi}_4 A_4 - \dot{\phi}_4 \ddot{A}_4 \\ \delta_4 A_6 &= 3 \dot{\phi}_4 \dot{A}_4 - 3 \dot{\phi}_4 A_4 \end{aligned} \quad (14)$$

The form of $\delta_4 A_6$ is quite encouraging, since something similar for the odd spins 3 and 5 would be necessary to save the non-invariant four-point coupling for pure spin 3 in four dimensions, as discussed in Ref. 8).

Having introduced spin 6, we are obliged to add even higher spins to cancel δS_6 . It should then be obvious - a proof will be given later - that the non-linear terms in Eq. (12) can always be chosen so that the action

$$S = \frac{1}{2} \int \dot{\phi} \dot{\phi} - \sum_{n=2}^{\infty} A_{2n} \phi^{(n)} \phi^{(n)} \quad (15)$$

is invariant under arbitrary even "higher spin" gauge transformations. Thus, the action (15) and the transformations (6) are a one-dimensional realization of the gauge algebra structure proposed in Ref. 8).

Let us now consider the consistency of our model; in particular, we want to know whether the A_{2n} 's really are gauge degrees of freedom. Varying (15) with respect to the Lagrange multipliers A_{2n} will produce a set of constraints which are satisfied if and only if

$$\ddot{\phi} = 0 \quad (16)$$

Given that, the equation one obtains upon varying ϕ is trivially satisfied. In particular, there is no equation for any of the A_{2n} 's, so that the latter are, in fact, true gauge fields and can be gauged away. This concludes the demonstration that our higher spin model is consistent, although it must be admitted that it is to some extent consistent by default, since it is equivalent to the free model.

Nevertheless, the model should be useful for developing some "geometric" understanding of the gauge symmetry. Let us first make the connection to the formalism of Ref. 8). First we rewrite the action (15) as (needless to say, we will be cavalier about surface terms)

$$S = \frac{1}{2} \sum_{n=1}^{\infty} \int (-1)^{n-1} \partial^{n-1} (A_{2n} \partial^n \phi) \partial \phi \quad (17)$$

where by A_2 we now understand $1+V_2$, with V_2 the "spin" 2 field, and a sign has been absorbed into the A_{2n} 's. We now define

$$\begin{aligned} D &= \sum_{n=1}^{\infty} (-1)^{n-1} \partial^{n-1} (A_{2n} \partial^n) = \\ &= A_2 \partial - \partial A_4 \partial^2 + (\partial^2 A_6 - A_4) \partial^3 + (\partial \partial A_6 - \partial^3 A_8) \partial^4 + \dots \end{aligned} \quad (18)$$

We see that, instead of odd spins as coefficients of $\partial^2, \partial^4, \dots$ we have derivatives of even spin fields, thus solving the colour problem (at least in one dimension), as conjectured in Ref. 8). The transformations (6) are collected into

$$\{ = \sum_{n=1}^{\infty} (-1)^{n-1} \partial^{n-1} (\{_{2n} \partial^n \quad (19)$$

The transformations are now given by

$$\begin{aligned} \delta_1 \varphi &= \{ \varphi \\ \delta_1 D &= [\{, D] \\ \delta_1 D\varphi &= \{ (D\varphi) \end{aligned} \quad (20)$$

Of course, these equations make sense only if $[\xi, D]$ is of the same form (18) as D itself. That this is so follows from the same argument that will be used later to prove that the transformations (6) close.

Now we can give a simple proof that the action is invariant, provided that the fields transform as implied by Eqs. (20). The action is

$$S = \frac{1}{2} \int d\tau \quad D\varphi \dot{\varphi} \quad (21)$$

so the variation becomes

$$\begin{aligned} \delta_1 S &= \frac{1}{2} \int [\{ (D\varphi) \partial\varphi + D\varphi \partial(\{ \varphi)] = \\ &= \frac{1}{2} \int [\{ (D\varphi) \partial\varphi - \partial(D\varphi) \{ \varphi] \end{aligned} \quad (22)$$

But this is zero since, using partial integration, we have

$$\int \partial f \{ g = \int \{ f \partial g \quad (23)$$

In much the same way, we can now couple a fermion to all odd spins. The relevant formulae in this case are

$$\begin{aligned} D &= i \sum_{n=1}^{\infty} (-1)^n \partial^{n-1} (A_{2n-1} \partial^{n-1} \\ \{ &= i \sum_{n=1}^{\infty} (-1)^n \partial^{n-1} (\{_{2n-1} \partial^{n-1} \end{aligned} \quad (24)$$

We take the transformations to be

$$\begin{aligned}\delta_i \psi &= i \psi \quad ; \quad \delta_i \bar{\psi} = i \bar{\psi} \\ \delta_i D &= [i, D]\end{aligned}\tag{25}$$

and the action is

$$S = \frac{1}{2} \int d\tau (\bar{D}\psi \psi - \bar{\psi} D\psi)$$

It is easily shown to be invariant. Thus our definition of D certainly leads to compact formulae. However, it is not quite clear what we should mean by "covariance" - our D , as defined in (18), covariantizes the transformations $\delta_\xi \phi = \xi \phi$, but it is not a derivative. This is also the case for ξ , i.e.,

$$\delta(fg) \equiv \delta f g + f \delta g = i f g + f i g \neq i(fg).\tag{26}$$

Hence a product of two functions, fg say, does not transform as $\xi(fg)$. How to deal with this situation is clearly a challenging problem.

A way to confront these issues directly is to try to add a mass term to the action. Let us recall how this goes for gravity, where one finds that the action

$$S_2^m = \frac{1}{2} \int \dot{\phi} \dot{\phi} - m^2 \phi \phi - A_2 \dot{\phi} \dot{\phi} - m^2 A_2' \phi \phi\tag{27}$$

is invariant under

$$\begin{aligned}\delta_2 \phi &= i_2 \dot{\phi} \\ \delta_2 A_2 &= \dot{i}_2 + i_2 \dot{A}_2 - \dot{i}_2 A_2 \\ \delta_2 A_2' &= \dot{i}_2 + i_2 A_2' + i_2 \dot{A}_2'\end{aligned}\tag{28}$$

Clearly the action makes no sense unless one can find a function so that $A_2' = f(A_2)$. Making a formal power series expansion of $f(A_2)$, one finds that Eq. (28) can be solved by

$$1 + A_2' = \frac{1}{1 - A_2} \quad (29)$$

This is the brute force method of adding a mass term. As an extra bonus, the introduction of a new variable $e = 1 + A_2'$ suggests itself, whereafter the action can be written

$$S = \frac{1}{2} \int e (D\varphi D\varphi - m^2 \varphi \varphi) \quad (30)$$

where $D\phi = e^{-1}\phi$ is a covariant derivative. In such a formalism, the addition of mass terms is straightforward, of course.

We have tried to apply the same procedure to our "higher spin" action, but unfortunately the resulting equations are very complicated, and we have not been able to ascertain whether they possess any solution.

For the moment we leave these speculations and turn to higher dimensions. As soon as the derivatives start carrying indices, and the gauge fields are required to be double traceless, the existence of an action analogous to (15) and invariant to all orders under higher spin transformations is not at all obvious. There is one more case which lends itself to easy treatment in a Hamiltonian framework, however, namely when the matter field lives in two dimensions and obeys $\square \phi = 0$. This is because such a field can be separated into left and right movers, so that the problem in a sense splits into two one-dimensional problems.

Let us recall how this works for matter coupled to "gravity"¹²⁾. The action

$$S_2 = \frac{1}{2} \int d^2x \partial_\alpha \varphi \partial_\alpha \varphi - A_{\alpha\beta} J_{\alpha\beta} \quad (31)$$

where

$$J_{\alpha\beta} = \partial_\alpha \varphi \partial_\beta \varphi - \frac{1}{2} \eta_{\alpha\beta} \partial_\gamma \varphi \partial_\gamma \varphi \quad (32)$$

is invariant under

$$\begin{aligned}\delta\varphi &= \xi^\alpha \partial_\alpha \varphi \\ \delta A_{\alpha\beta} &= \partial_{[\alpha} \xi_{\beta]} + \text{non-linear terms}.\end{aligned}\tag{33}$$

The current $J_{\alpha\beta}$ is traceless, so that variation with respect to $A_{\alpha\beta}$ yields two constraints only. In light front co-ordinates, they are

$$\begin{aligned}L_+^2(\tau, \sigma) &= \partial_+ \varphi \partial_+ \varphi \\ L_-^2(\tau, \sigma) &= \partial_- \varphi \partial_- \varphi\end{aligned}\tag{34}$$

Given the equation of motion $\partial_+ \partial_- \phi = 0$, the constraints obey

$$\partial_- L_+^2 = 0 \quad \partial_+ L_-^2 = 0.\tag{35}$$

In this sense the problem splits into two one-dimensional ones.

Since the constraints contain all the effects of "gravity", they are what we should generalize to "higher spins". First, however, we will motivate what we are doing by considering the action

$$S_4 = \frac{1}{2} \int d^2x \partial_\alpha \varphi \partial_\alpha \varphi - A_{\alpha\beta\gamma\delta} J_{\alpha\beta\gamma\delta}\tag{36}$$

We demand that $A_{\alpha\beta\gamma\delta}$ is a double traceless "spin 4" field, and that the action is invariant to cubic order under

$$\begin{aligned}\delta\varphi &= \xi_{\alpha\beta\gamma} \partial_\alpha \partial_\beta \partial_\gamma \varphi + \partial_\sigma \xi_{\alpha\beta\gamma} \partial_\alpha \partial_\beta \varphi \\ \delta A_{\alpha\beta\gamma\delta} &= \partial_{[\alpha} \xi_{\beta\gamma\delta]}\end{aligned}\tag{37}$$

where $\xi_{\alpha\beta\gamma}$ is traceless. Moreover, the constraints $J_{\alpha\beta\gamma\delta}$ should have a property analogous to Eq. (35). It is non-trivial to satisfy these restrictions, but a solution exists. It is

$$\begin{aligned} J_{\alpha\beta\gamma\delta} = & -\frac{2}{3} \partial_\alpha \partial_\beta \varphi \partial_\gamma \partial_\delta \varphi + \frac{1}{3} \eta_{\alpha\beta} \partial_\epsilon \partial_\gamma \varphi \partial_\epsilon \partial_\delta \varphi + \\ & + \frac{1}{12} \eta_{\alpha\beta} \eta_{\gamma\delta} \Box \varphi \Box \varphi - \frac{1}{6} \eta_{\alpha\beta} \eta_{\gamma\delta} \partial_\epsilon \partial_\gamma \varphi \partial_\epsilon \partial_\delta \varphi \end{aligned} \quad (38)$$

The constraints become (up to terms which vanish when $\Box \phi = 0$)

$$\begin{aligned} L_+^4(\tau, \sigma) &= \partial_+^2 \varphi \partial_+^2 \varphi \\ L_-^4(\tau, \sigma) &= \partial_-^2 \varphi \partial_-^2 \varphi \end{aligned} \quad (39)$$

We have not considered the action any further. Instead, we switch to a Hamiltonian language, using light front dynamics to deal with the higher derivatives. This means that x^- is treated as "time", while the fundamental Poisson brackets are

$$[\varphi(x^+), \varphi(\gamma^+)] = -\frac{1}{2\partial_+} \delta(x^+ - \gamma^+) \quad (40)$$

Then we can work out the algebra of the constraints L_+^4 . As expected, the algebra does not close unless we introduce a "spin 6" constraint as well, namely

$$L_+^6(\tau, \sigma) = \partial_+^3 \varphi \partial_+^3 \varphi \quad (41)$$

In general, we define the "spin $2n$ " constraint

$$L_{+f}^{2n} = \int f \partial_+^n \varphi \partial_+^n \varphi \quad (42)$$

where f is some function. A short calculation gives

$$[L_{+f}^4, L_{+g}^4] = L_{+(\ddot{f}g - f\ddot{g})}^4 + L_{+(3f\dot{g} - 3\dot{f}g)}^6 \quad (43)$$

Taking further Poisson brackets will, of course, bring in the full set of constraints given in Eq. (42). A long calculation gives

$$[L_{+f}^{2a}, L_{+g}^{2b}] = -2(-1)^b \sum_{k=0}^{a+b-1} \binom{a+b-1}{k} X_k \quad (44)$$

where

$$X_k = \sum_{i=0}^q L_{+(-1)^{q+i}}^{2(a+q-i)} \left[\frac{1}{2} \left(\frac{q-1+i}{q-i} \right) + \left(\frac{q-1+i}{q-1-i} \right) \right] \partial^{2i} (f g^{(a+b-1-k)}) \quad (45)$$

if $b-a+k = 2q$ and

$$X_k = \sum_{i=0}^q L_{+(-1)^{q+i+1}}^{2(a+q-i)} \left[\frac{1}{2} \left(\frac{q+i}{q-i} \right) + \left(\frac{q+i}{q-1-i} \right) \right] \partial^{2i+1} (f g^{(a+b-1-k)}) \quad (46)$$

if $b-a+k = 2q+1$. This is a generalization of the Virasoro algebra¹³⁾, to which it reduces if $a=b=1$. Again we see how closure of the algebra requires the introduction of all even "spins"; it is associative by construction. The gauge algebra problem is thereby solved in two dimensions.

At this point, the assertion made about Eq. (6) is trivial to prove, since such transformations are generated by our L^{2n} 's. Closure of the transformations (24) can be proved similarly. Our proposal for a "higher spin" theory in two dimensions is now defined by the equation of motion $\partial_+ \partial_- \phi = 0$, together with the infinite set of constraints (42) (and similarly for L_-^{2n}). The model is almost, but not quite, trivial. All the constraints are satisfied if

$$\partial_+^2 \phi = \partial_-^2 \phi = 0 \quad (47)$$

(so that the "higher spin" invariances do have some dynamical effects, unlike the case in one dimension).

The results reported here may not mean very much for the four-dimensional case, but at the least they show that there is nothing a priori absurd about transformations with higher derivatives in them. Moreover, the two-dimensional construction could be interesting in itself. Recall that the action for a string is given by Eqs. (31) and (32), if an extra index on the ϕ field is understood, so that we have D independent fields taking values in a D -dimensional Minkowski space. The negative sign in front of one of the scalars in the action makes all the difference. The question whether a "higher spin string" could be constructed

as well obviously suggests itself, but our limited experience with higher derivatives prevents us from saying very much about the dynamics allowed by the constraints (42).

When the model is quantized, the algebra (44)-(46) will develop central charges due to normal ordering. It is still consistent provided that the concomitant BRS charge is nilpotent. In the conventional string calculation¹⁴⁾ one has two parameters at one's disposal (the dimension D and a normal ordering constant), and nilpotency of the BRS charge yields two conditions, so that there is never any danger of inconsistency, although the dimension is fixed at the unpalatable value 26. The string suggested here could conceivably yield another D , but it could very easily be inconsistent as well, since nilpotency of its BRS charge will yield an infinite set of conditions, which may be incompatible. It is perhaps worth looking into, though.

ACKNOWLEDGEMENT

We thank André Neveu for his interest.

REFERENCES

- 1) C. Fronsdal, Phys. Rev. D18 (1978) 3624;
J. Fang and C. Fronsdal, Phys. Rev. D18 (1978) 3630;
T. Curtright, Phys. Lett. 85B (1979) 219.
- 2) I. Bengtsson, CERN preprint TH.4334/85 (1985).
- 3) A.K.H. Bengtsson, I. Bengtsson and L. Brink, Nucl. Phys. B227 (1983) 3141.
- 4) F.A. Berends, G.J.H. Burgers and H. van Dam, Z. Phys. C24 (1984) 247;
I. Bengtsson, CERN preprint TH.4097/85 (1985).
- 5) B. de Wit and D.Z. Freedman, Phys. Rev. D21 (1980) 358;
S. Deser, Brandeis preprint BRX-TH 167 (1984);
F.A. Berends, G.J.H. Burgers and H. van Dam, Leiden preprint (1985).
- 6) M. Flato and C. Fronsdal, Phys. Lett. 97B (1980) 236.
- 7) A.K.H. Bengtsson, Phys. Rev. D32 (1985) 2031;
F.A. Berends, G.J.H. Burgers and H. van Dam, Nucl. Phys. B260 (1985) 295;
G.J.H. Burgers, Ph.D. Thesis, Leiden (1985).
- 8) A.K.H. Bengtsson and I. Bengtsson, Queen Mary College preprint 86-16 (1986).
- 9) C. Fronsdal, in "Supergravity", eds. P. van Nieuwenhuizen and D.Z. Freedman (North Holland, 1979).
- 10) M.A. Vasil'ev, Sov. J. Nucl. Phys. 32 (1980) 439;
C. Aragone and H. La Roche, Nuovo Cimento 72A (1982) 149.
- 11) L. Brink, P. Di Vecchia and P. Howe, Nucl. Phys. B118 (1977) 7;
C.A.P. Galvao and C. Teitelboim, J. Math. Phys. 21 (1980) 1863.
- 12) L. Brink, P. Di Vecchia and P. Howe, Phys. Lett. 65B (1976) 471;
S. Deser and B. Zumino, Phys. Lett. 65B (1976) 369.
- 13) M. Virasoro, Phys. Rev. D1 (1970) 2933.
- 14) M. Kato and K. Ogawa, Nucl. Phys. B212 (1983) 443;
S. Hwang, Phys. Rev. D28 (1983) 2614.