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**GALILEO'S EXPERIMENT ON FREE-FALLING BODIES
USING MODERN OPTICAL TECHNIQUES**

V. Cavasinni^{*)}, E. Iacopini^{**)}, E. Polacco^{***)}
and G. Stefanini

CERN, Geneva, Switzerland

ABSTRACT

We propose a method, based on well-known optical techniques, by which possible differences in the free-fall acceleration of bodies of different nuclear composition can be measured. The sensitivity of the method is almost a factor of 10 higher than in the experiment of Eötvös et al.

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^{*)} On leave of absence from Università di Pisa, Italy.

^{**) Permanent address: Scuola Normale Superiore di Pisa, and the INFN, Pisa, Italy.}

^{***)} Permanent address: Università di Pisa, and the INFN, Pisa, Italy.

1. INTRODUCTION

According to Galileo [1], the free-fall acceleration (g) of different bodies to the Earth is independent of the mass and the nature of each body. The uniqueness of g implies that the ratio between the gravitational and the inertial mass is identical for different bodies. This experimental fact is one of the basic principles of Einstein's general theory of relativity as well as of several alternative theories of gravitation.

After Galileo's pioneering experiment from the leaning tower of Pisa, the uniqueness of g was verified in 1922 at the 10^{-9} level of accuracy by the well-known Eötvös, Pekar and Fekete (EPF) experiment [2]. There are other more recent experiments, performed by Dicke et al. [3] and by Braginskii and Panov [4], which push the above-mentioned limit to 10^{-11} ; but, since they measure the gravitational attraction of the Sun on the sense bodies, they test only the universality of the long-range ($1/r^2$) term of the gravitational force.

There are various theoretical ideas—concerning, for instance, modified theories of gravity [5], low-mass axion exchange [6], two-gluon exchange [7], supergravity theories [8], and supersymmetry [9]—that lead quite naturally to a gravitational potential $V(r)$:

$$V(r) = (Gm/r)[1 + \alpha \exp(-\mu r)] , \quad (1)$$

where $1/\mu$ ranges between a few centimetres and some kilometres, and α might depend on the material of the sense body. Furthermore, possible departures from the Coulomb law were suggested by the results of experiments in the 10 cm [10] and 200 m [11] ranges.

Finally, a recent reanalysis of the original Eötvös data [12] has raised the question of a possible dependence of g on the binding energy per nucleon, characteristic of the body under study.

We think that a new high-accuracy experiment on the g uniqueness can shed light on the questions raised. For this purpose, we propose to make a direct measurement of differences of the order of $10^{-10}g$ or better in the free-fall acceleration of two bodies, by using a differential method which is based on a technique already developed to measure the absolute value of g [13] at the level of $10^{-8}g$.

The method is outlined in section 2 and the experimental apparatus is described in section 3. The data reduction and the sensitivity are discussed in section 4, whilst possible sources of systematic errors are considered in section 5.

2. THE EXPERIMENTAL METHOD

The method is based on a modified Michelson interferometer in which the two arms terminate in two free-falling cube-corner reflectors (see fig. 1). The two cube-corner reflectors R1 and R2 are fixed on a rigid disk DS made up of two half-disks of different materials. The two half-disks have the same radius but a thickness which is inversely proportional to their respective density; therefore, they have the same mass.

If the free-fall accelerations of the two materials differ by Δg , the disk experiences a torque T given by

$$T = (2/3\pi)Rm\Delta g , \quad (2)$$

where R is the disk radius and m is the mass of the whole disk. The angular acceleration of the disk will be

$$\dot{\omega} \equiv \partial\omega/\partial t = T/I = (4/3\pi)\Delta g/R , \quad (3)$$

where I is the moment of inertia. The optical path difference ΔL between the two arms of the Michelson interferometer will increase according to the equation

$$\Delta L(t) = \Delta L_0 + d[\omega_0 t + (1/2)\dot{\omega}t^2], \quad (4)$$

where ΔL_0 measures the optical path difference at $t = 0$, d is the distance between the vertices of the two cube-corner reflectors ($d \approx 2R$), ω_0 is the angular velocity of the disk at $t = 0$, and $\dot{\omega}$ is given by eq. (3).

The light intensity on the two photodetectors D1 and D2 (see fig. 1), neglecting all noise contributions, is given by

$$J_1 = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi(t), \quad (5)$$

$$2J_2 = I_1 + I_2 - 2\sqrt{I_1 I_2} \cos \phi(t), \quad (6)$$

where I_1 and I_2 are the light intensities present in the two interferometer arms, and $\phi(t)$ is the phase difference between the two optical paths, which is given by the equation

$$\phi(t) = (2\pi/\lambda)\Delta L(t) = \phi_0 + (2\pi d/\lambda)[\omega_0 t + (1/2)\dot{\omega}t^2]. \quad (7)$$

If $\Delta g = 0$, the time dependence of the phase difference $\phi(t)$ is linear, i.e. the light intensity on the two photodetectors D1 and D2 varies sinusoidally with a frequency given by

$$f = d\omega_0/\lambda. \quad (8)$$

If the free-fall accelerations of the two materials which constitute the disk differ by Δg , the frequency modulation f of the light intensity detected by D1 and D2 will shift linearly during the measurement at the constant rate

$$\partial f/\partial t = (2/3\pi)(d/R)(\Delta g/\lambda), \quad (9)$$

which contributes to the phase difference $\phi(t)$ with the term

$$\Delta\phi = \pi(d/\lambda)\dot{\omega}t^2 = (4/3)(d/R)(\Delta g/\lambda)t^2. \quad (10)$$

The t -squared dependence of $\Delta\phi$ is precisely the signature for a possible violation of the g uniqueness.

Other contributions from local gravity gradients and from non-gravitational forces can be estimated by using a homogeneous disk. Moreover, a cross-check against large systematic errors can easily be performed by repeating the experiment with three disks of composition A-B, B-C, and A-C.

3. EXPERIMENTAL APPARATUS

The experimental apparatus is shown schematically in fig. 1. The light source is a commercially available, frequency stabilized He-Ne laser ($\lambda = 632.8$ nm), providing an output power of about 1 mW. The laser light intensity is monitored by the photodetector D0 via the beam splitter BS0. The two arms of the interferometer start at the beam splitter BS1. The first arm is defined by the mirrors M0 and M1 and the cube-corner reflector R1, whereas the second arm is defined by the mirror M2

and the cube-corner reflector R2. The system of mirrors M0, M1, and M2, together with the compensator C, allows for an easy equalization of the optical paths in the two arms.

The beam splitter BS1, the mirrors M0, M1, and M2, and the cube-corner reflectors R1 and R2 are installed in a stainless-steel cylindrical vessel, 7 m in length and 200 mm in diameter. This vessel is positioned with the axis aligned with the vertical direction and is evacuated down to 10^{-6} Torr.

The disk DS (diameter ≈ 100 mm, mass ≈ 1.5 kg), with the two cube-corner reflectors fixed on the symmetry axis, rests initially on a small stand at the top of the vessel. The stand can slide along vertical rails and is driven by a transmission gear coupled to an external servomotor. The basic mechanical outline is similar to that used for absolute gravity-meters such as the one described in ref. [13].

A measurement starts by accelerating the stand downwards by 3g for 10 ms; then by letting the whole stand fall with acceleration g for 1 s (a drop of ≈ 5 m) controlled by the servomotor. In this way the disk will undergo free fall, separated from the stand by a distance of ≈ 1 mm. To stop the disk the stand is decelerated by 3g until it stops (≈ 1.7 m). The gear is also used to pull the stand, with the disk on it, to the top of the vessel for further measurements. When the stand is in the top position, an angular rotation ω_0 (≈ 10 mrad/s) is given to the disk by an externally controlled gear. This is in order to eliminate the sign ambiguity in the phase difference $\phi(t)$, as obtained from the light intensities, eqs. (5) and (6). According to eq. (8), the rotation will also produce a light modulation frequency in the 1.5 kHz range.

The photodetectors D0, D1, and D2 are PIN photodiodes used in photovoltaic mode (quantum efficiency 0.2 A/W), followed by a low-noise current-to-voltage amplifier. The photodiode signals are sampled at a frequency of 50 kHz by a 14-bit ADC. The measurements are stored in a fast buffer memory, which is read by a microcomputer at the end of each data-taking for off-line analysis.

4. SENSITIVITY OF THE APPARATUS

The diode amplifier signals during the free fall can be expressed as

$$\begin{aligned} S_1(t) &= c_1 + s_1 \cos [\phi(t)] + r_1(t), \\ S_2(t) &= c_2 + s_2 \cos [\phi(t)] + r_2(t), \end{aligned} \quad (11)$$

where $c_{1,2} \propto I_1 + I_2$, $s_{1,2} \propto 2\sqrt{I_1 I_2}$ and $r_{1,2}$ account for noise contributions which might be partially correlated in the two arms.

During the fall time T (≈ 1 s) a number N ($\approx 50,000$) of samples of these signals is accumulated. The analysis consists in fitting the sampled signals with eqs. (5) and (6), using the least-squares method.

The sensitivity of the apparatus, taking into account the random noise contributions in a single interferometer arm can be expressed in the form

$$(\Delta g/g)_{\text{sens}} = 9\sqrt{5/2} \sigma (R/d)(\lambda/gT^2)(1/\sqrt{N}), \quad (12)$$

where σ is the r.m.s. noise deviation on a single sample, normalized to the peak cosine amplitude.

The major noise sources of which we are aware are as follows:

a) Fluctuations of the laser beam intensity J

The type of laser considered here gives typically $\Delta J/J \approx 10^{-6}/\sqrt{\text{Hz}}$. With a photodiode amplifier bandwidth equal to half the sampling frequency and a measuring time of one second, the sensitivity limit contributed by the laser noise is

$$(\Delta g/g)_{\text{laser}} \approx 5 \times 10^{-13}. \quad (13)$$

b) Electronic sampling uncertainty

The quantization noise for a 14-bit ADC is given by

$$\sigma_{\text{sampling}} = 1/[2^{13} \times \sqrt{12}] \sim 3.5 \times 10^{-5}, \quad (14)$$

which, according to eq. (12), will contribute by

$$(\Delta g/g)_{\text{sampling}} \approx 10^{-13}. \quad (15)$$

c) Vibrational noise

It is to be expected that the major source of noise will be vibrations of those elements of the interferometer which are fixed to the vessel.

At this stage and without any preliminary test, it is quite difficult to give a quantitative estimation for the level of vibrational noise in the frequency range (1.5 kHz) in which we are interested. However, a rough estimate of the interferometer requirements can be derived by calculating the noise level corresponding to the desired sensitivity.

According to eq. (12), a $\Delta g/g \sim 10^{-10}$ corresponds to

$$\sigma \sim 2 \times 10^{-4}/\sqrt{\text{Hz}} \quad (16)$$

or $\langle \Delta L \rangle \sim 2 \times 10^{-9} \text{ cm}/\sqrt{\text{Hz}}$, well within the capabilities of a standard Michelson interferometer.

5. SYSTEMATIC ERRORS

The major sources of systematic error are the following.

a) Laser pressure

The two cube-corner reflectors experience a radiation pressure due to the laser light. The force on each cube-corner reflector is given by

$$F_{\text{rad}} = 2P/c \quad (17)$$

where P is the light power impinging on it. The two radiation forces would balance if BS1 were a perfect 50% beam splitter and if the two cube-corner reflectors were placed at the same distance from the centre of the disk. The radiation force is to be compared directly with the one due to Δg itself, which is

$$F_g = (m/2)\Delta g \approx 10^{-9} \text{ N}. \quad (18)$$

Therefore radiation pressure effects could become important only for laser powers of the order of 1 W, together with strongly unbalanced split beams.

b) Laser wavelength stability

According to eq. (7), if the laser wavelength drifts, i.e.

$$k = \partial \lambda / \partial t \neq 0, \quad (19)$$

the linear time-dependent term

$$\phi(t) = \phi_0 + (2\pi d/\lambda)\omega_0 t \quad (20)$$

will generate a time-squared dependence given by

$$\Delta\phi = -(2\pi d/\lambda)(k/\lambda)\omega_0 t^2, \quad (21)$$

to be compared with the phase difference due to Δg given by eq. (10). However, since in a frequency-stabilized laser one has, in one second,

$$\Delta\lambda/\lambda \leq 10^{-10}, \quad (22)$$

the rate k of eq. (19) is such that

$$k/\lambda \leq 10^{-10} \text{ s}^{-1}, \quad (23)$$

corresponding to

$$\Delta g/g \approx 2.5 \times 10^{-14}. \quad (24)$$

c) Disk mechanics

Two bodies placed at different heights fall with a fractional acceleration difference

$$\Delta g/g \approx 2h/R_E, \quad (25)$$

where h is the height difference and R_E is the Earth's radius.

In our case, the axial symmetry of the disk eliminates all the contributions to the torque due to possible multipole terms in the g expansion and to the Coriolis force. The two cube-corner reflectors (mass of each ~ 50 g) break the axial symmetry; however, by adding the proper tiny amount of mass on each half-disk, we are confident that the symmetry can be restored at least at the 1‰ level, which corresponds to a $\Delta g/g \sim 3 \times 10^{-11}$.

d) Interferometer arms misalignment

A misalignment ϵ of the wave-fronts in the two interferometer arms will introduce, in the optical path difference ΔL given by eq. (4), the term

$$\Delta L_{\text{mis}} = (1/2) g t^2 [(1/\cos \epsilon) - 1] \approx (1/4) g \epsilon^2 t^2. \quad (26)$$

The corresponding error in Δg is given by

$$(\Delta g/g)_{\text{mis}} \approx (3\pi/8)(R/d)\epsilon^2, \quad (27)$$

so that in order to reach the required sensitivity we must have $\epsilon \leq 10^{-5}$, a value which can be obtained with standard optical techniques.

The stability of such an adjustment can easily be tested by using a homogeneous disk.

e) Residual gas pressure

The residual gas pressure could produce a torque on the free-falling disk, because of the different thicknesses of the half-disks. The air resistance at $v = 10$ m/s and ambient pressure contributes with a $\Delta g/g \sim 10^{-1}$, which means that a residual pressure of 10^{-6} Torr is enough to ensure that, also at the maximum disk velocity, the force acting upon it is well below the force due to Δg , given by eq. (18).

Moreover, this effect can easily be overcome by making the external shape of the two half-disks identical.

f) Sampling

The clock-sampling frequency must be stable at a level of 10^{-10} s for the one second measuring time. This is not a problem since the present rubidium masers have short-term stabilities of one part in 10^{12} .

g) Electric and magnetic field effects

The material used for the disks is non-magnetic metals. These should be covered by a homogeneous conducting layer to be sure that the whole disk is at the same electrical potential. The disk should be electrically grounded before each measurement.

Suitable shields can be installed against the electric- and magnetic-field gradients present in a normal laboratory environment.

6. CONCLUSIONS

We have omitted a detailed discussion of the devices for controlling the fall of the stand and the disk, which could be, probably, adapted from those used in gravity-meters.

On the other hand, most of the effects that limit the sensitivity of the absolute g-meters, in particular those connected with the interferometer, become negligible in a differential measurement of this type.

We conclude that, to our present understanding, the proposed measurement on the g uniqueness is quite feasible at the 10^{-10} level of accuracy.

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 ...‘Simplicio: Ma chi posasse la maggiore sopra la minore? Salviati: Le accrescerebbe peso, quando il suo moto fusse più tardo, ritarderebbe in parte la velocità della maggiore, tal che il loro composto si muoverebbe men veloce, essendo maggiore dell'altra; che è contro il vostro assunto. Concludiamo per ciò, che i mobili grandi e i piccoli ancora, essendo della medesima gravità in spezie, si muovono con pari velocità.
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Figure caption

Fig. 1 : Schematic diagram of the experimental apparatus.

Fig. 1