From Scale Invariance to Lorentz Symmetry

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It is shown that a unitary translationally invariant field theory in $1 + 1$ dimensions, satisfying isotropic scale invariance, standard assumptions about the spectrum of states and operators, and the requirement that signals propagate with finite velocity, possesses an infinite dimensional symmetry given by one or a product of several copies of conformal algebra. In particular, this implies the presence of one or several Lorentz groups acting on the operator algebra of the theory.

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Theories invariant under scaling transformations play an important role in the physics of fundamental interactions, condensed matter physics, and statistical mechanics. They appear at the fixed points of the renormalization group flows which describe the deep ultraviolet and infrared limits of the theory. We focus on the so-called “isotropic” scaling transformations when the time and all space coordinates scale in the same way

$$t \mapsto \lambda t, \quad \mathbf{x} \mapsto \lambda \mathbf{x}, \quad (1)$$

where λ is constant. This is the only possible form of scale invariance in relativistic field theories, where it is also believed to imply the larger conformal symmetry. This assertion has been rigorously proved in the case of two space-time dimensions, where the symmetry (1) combined with unitarity and some technical assumptions about the energy-momentum tensor (EMT), indeed, leads to conformal invariance [1,2]. Generalization of this theorem to higher space-time dimensions is a long standing problem, see [3,4] for recent progress.

A priori, the symmetry (1) can also appear in theories that are not relativistic. However, there is a large amount of theoretical evidence that the renormalization group fixed points characterized by (1) exhibit emergent Lorentz symmetry. This happens in the models of particle physics with broken Lorentz invariance [5–9], as well as the models of condensed matter systems [10–15], where an effective low-energy Lorentz symmetry (not to be confused with the Lorentz invariance of the fundamental forces) emerges from intrinsically nonrelativistic Hamiltonians. A partial explanation of this phenomenon is provided by the general observation that, in a unitary conformal field theory (CFT), most of perturbations violating Lorentz invariance are irrelevant, which implies that conformal fixed points are generically infrared stable [16,17]. However, this argument assumes the existence of a Lorentz invariant fixed point in the first place and does not tell anything about the criteria for this to happen.

In this Letter, I explore the relation between scale and Lorentz invariance in $1 + 1$ dimensions. I will show that the isotropic scale invariance [18] (1), supplemented by assumptions listed below, implies that the symmetries of the theory include one or several copies [20] of the full conformal algebra (which, of course, contains the Lorentz boosts). Namely, I am going to prove the following.

Theorem. 1.—Consider a local quantum field theory in $1 + 1$ dimensions obeying the symmetry (1). Lorentz invariance is not assumed. Instead, we postulate the following properties.

1. Translation invariance: The theory is invariant under translations in time and space which correspond to local conserved currents. It is convenient to combine these into the EMT [21] $T^\mu_\nu(t, x)$, whose conservation implies $\partial_\mu T^\mu_\nu = 0$. The energy and momentum

$$H \equiv -P_t = \int dx T^t_t(t, x), \quad P_x = - \int dx T^t_x(t, x),$$

commute with each other and define the evolution of local operators in time and space.

2. Positivity of energy: The Hilbert space of the theory is spanned by the eigenvectors of H and P_x ; the eigenvalues of H are non-negative with $H = 0$ only for the vacuum.

3. Unitarity: The theory possesses a Hilbert space with positive-definite norm. The EMT is Hermitian.

4. Existence of the dilatation current: The symmetry (1) corresponds to a current D^μ which is related to the EMT by [2,22] $D^\mu = \xi^\nu T^\mu_\nu + V^\mu$, where V^μ is a local operator. The dilatation charge $D = \int dx D^t(t, x)$ has the standard commutation relations with the energy and momentum

$$-i[D, P_\nu] = P_\nu. \quad (2)$$

5. Discrete, positive, diagonalizable spectrum of dimensions: The set of all local operators is spanned by operators ϕ_i transforming in the canonical way under dilatations

$$-i[D, \phi_i(\xi)] = \Delta_i \phi_i(\xi) + \xi^\mu \partial_\mu \phi_i(\xi), \quad (3)$$

where $\Delta_i \geq 0$ form a discrete set. We also assume that the number of operators of a given dimension Δ is finite [23].

6. Finite velocity of signal propagation: All commutators of local operators vanish outside a certain causal cone

$$[\phi_i(t, x), \phi_j(0, 0)] = 0, \quad \text{if } |x| > v_{\max}|t|. \quad (4)$$

Then the (improved) EMT of the theory decomposes into a sum of local dimension-2 tensors

$$T_\nu^\mu = \sum_{a=1}^N T_\nu^{(a)\mu}, \quad (5)$$

which are traceless, conserved, have linearly related off-diagonal components, and mutually commute

$$T_\mu^{(a)\mu} = 0, \quad \partial_\mu T_\nu^{(a)\mu} = 0, \quad (6a)$$

$$T_t^{(a)x} + v_a^2 T_x^{(a)t} = 0, \quad v_a^2 > 0, \quad (6b)$$

$$[T_\nu^{(a)\mu}, T_\rho^{(b)\lambda}] = 0 \quad \text{for } a \neq b. \quad (6c)$$

An immediate consequence of this result is the existence of N conserved currents

$$\begin{aligned} m^{(a)t} &= v_a^{-1} x T_t^{(a)t} + v_a t T_x^{(a)t}, \\ m^{(a)x} &= v_a^{-1} x T_t^{(a)x} + v_a t T_x^{(a)x}, \end{aligned}$$

generating Lorentz boosts with the “speeds of light” v_a . Moreover, $T_\nu^{(a)\mu}$ give rise to N copies of the conformal algebra $\mathcal{C}_a$, whose generators commute for $a \neq b$. Thus, the symmetry of the theory is $\mathcal{C}_1 \otimes \mathcal{C}_2 \otimes \dots \otimes \mathcal{C}_N$. In the case $N = 1$, we obtain a standard chiral CFT, while a parity symmetric CFT corresponds to $N = 2$, $v_1^2 = v_2^2$.

Note that if $N > 1$, the theory contains sectors which do not interact with each other. Indeed, consider the set $\{\Phi^{(a)}\}$ composed of all local operators that transform nontrivially under $\mathcal{C}_a$ and are singlets with respect to $\mathcal{C}_b$, $b \neq a$ (this set is not empty as it contains at least $T_\nu^{(a)\mu}$). The operators $P_\nu^{(a)} = -\int dx T_\nu^{(a)t}$ act as translations on the operators $\{\Phi^{(a)}\}$ without affecting the operators of the family $\{\Phi^{(b)}\}$. This implies that the operators of the a th family cannot appear in the operator product expansion of $\{\Phi^{(b)}\}$ and vice versa. Hence, the operator algebras $\{\Phi^{(a)}\}$ and $\{\Phi^{(b)}\}$ are completely decoupled and form two independent CFTs.

Proof of Theorem 1.—I now turn to the proof of (5) which consists of 3 steps.

Step 1: Let us make the EMT traceless by an appropriate improvement. The conservation of D^μ implies $T_\mu^\mu + \partial_\mu V^\mu = 0$. On the other hand, the EMT can always be supplemented by a total divergence

$$T_\nu^\mu \mapsto \hat{T}_\nu^\mu = T_\nu^\mu + \partial_\lambda Y_\nu^{\lambda\mu}, \quad (7)$$

where $Y_\nu^{\lambda\mu}$ is antisymmetric in its upper indices. The choice $Y_\nu^{\lambda\mu} = \varepsilon^{\lambda\mu} \varepsilon_{\nu\rho} V^\rho$, where $\varepsilon^{\lambda\mu}$, $\varepsilon_{\nu\rho}$ are antisymmetric symbols, $\varepsilon^{tx} = -\varepsilon_{tx} = 1$, yields $\hat{T}_\nu^\mu = 0$ as desired. In what follows, we omit the hats on the improved EMT.

A further improvement casts the scaling transformation of the EMT into the canonical form (3) with $\Delta = 2$. To see this, note that the commutation relations (2) imply

$$-i[D, T_\nu^\mu] = 2T_\nu^\mu + \xi^\lambda \partial_\lambda T_\nu^\mu + A_\nu^\mu, \quad (8)$$

where the local tensor A_ν^μ is such that the integral $\int d\xi^\lambda \varepsilon_{\lambda\mu} A_\nu^\mu$ vanishes for any contour interpolating between the spatial infinities. Then, it must have the form $A_\nu^\mu = \varepsilon^{\mu\lambda} \partial_\lambda B_\nu$ for some local B_ν . The trace-free property of the EMT implies

$$\varepsilon^{\mu\lambda} \partial_\lambda B_\mu = 0. \quad (9)$$

Now, we expand B_ν in the complete basis defined in (3)

$$B_\nu = \sum_i \beta_{\nu,i} \phi_i. \quad (10)$$

One notices that operators with dimension $\Delta_i = 1$ cannot appear in this sum. Indeed, expanding the EMT as $T_\nu^\mu = \sum_i \gamma_{\nu,i}^\mu \phi_i$ and substituting into (8), we obtain

$$\sum_i (\Delta_i - 2) \gamma_{\nu,i}^\mu \phi_i = \sum_i \beta_{\nu,i} \varepsilon^{\mu\lambda} \partial_\lambda \phi_i.$$

Clearly, the lhs does not contain operators of dimension two. On the other hand, the derivative of a dimension one operator would produce a term of dimension two on the rhs, which would lead to contradiction. Then, a redefinition of the form (7) with

$$Y_\nu^{\lambda\mu} = \varepsilon^{\lambda\mu} \sum_i (\Delta_i - 1) \beta_{\nu,i} \phi_i, \quad (11)$$

brings the commutator of the EMT with D to the desired form. Note that the new EMT remains traceless: substituting (10) into (9), we obtain $\sum_i \varepsilon^{\mu\lambda} \beta_{\nu,i} \partial_\lambda \phi_i = 0$. The cancellation must hold within each subspace of operators with a given dimension, which translates into the trace-free property of the improvement term (11).

Step 2: This is the key step in the proof. We will demonstrate the following property of the Fourier transformed EMT [24]:

$$\begin{aligned} \tilde{T}_\nu^\mu(\omega, k) &\equiv \int dt dx e^{-i\omega t + ikx} T_\nu^\mu(t, x) = 0 \\ \text{for } |k/\omega| &< v_{\max}^{-1}. \end{aligned} \quad (12)$$

First, we show that the Fourier components (12) annihilate the vacuum

$$\tilde{T}_\nu^\mu(\omega, k)|0\rangle = 0 \quad \text{for } |k/\omega| < v_{\max}^{-1}. \quad (13)$$

Consider the expectation value of the commutator of two identical EMT components (no summation over the repeated indices)

$$\begin{aligned} F_{0\nu}^\mu(\xi) &\equiv \langle 0|[T_\nu^\mu(0), T_\nu^\mu(\xi)]|0\rangle \\ &= \int d\omega dk e^{i\omega t - ikx} \rho_\nu^\mu(\omega, k) - \text{H.c.}, \end{aligned} \quad (14)$$

where the spectral density is defined as usual

$$\langle 0|\tilde{T}_\nu^\mu(\omega', k')\tilde{T}_\nu^\mu(\omega, k)|0\rangle = (2\pi)^4 \delta(\omega' + \omega) \delta(k' + k) \rho_\nu^\mu(\omega, k). \quad (15)$$

The positivity of energy and scale invariance fix the form of the spectral density, $\rho_\nu^\mu(\omega, k) = \theta(\omega) \omega^2 \hat{\rho}_\nu^\mu(k/\omega)$. The conservation and the trace-free property of the EMT yield the relations between the spectral functions corresponding to the different components

$$\hat{\rho}_x^t(x) = x^2 \hat{\rho}_x^x(x) = x^2 \hat{\rho}_t^t(x) = x^4 \hat{\rho}_t^x(x). \quad (16)$$

Changing the integration variable in (14) we obtain

$$\begin{aligned} F_0(t, x) &= \int d\omega dx \theta(\omega) \omega^3 \hat{\rho}(x) e^{i\omega(t - \kappa x)} - \text{H.c.} \\ &= 2\pi i \int dx \hat{\rho}(x) \delta'''(t - \kappa x) \\ &= \frac{2\pi i}{x^3 |x|} \hat{\rho}'''(t/x), \end{aligned} \quad (17)$$

where, for clarity, we suppressed the indices μ, ν . Then, (4) implies that $\hat{\rho}'''(x)$ vanishes if $|x| < v_{\max}^{-1}$, and hence, $\hat{\rho}(x)$ is, at most, quadratic in its argument. Comparing to (16), we conclude that the spectral densities of all EMT components vanish at $|k/\omega| < v_{\max}^{-1}$, implying (13) by unitarity.

To extend Eq. (13) to any state in the Hilbert space, let us replace (14) by the expectation value in a thermal ensemble with temperature T (again, no summation over the repeated indices)

$$\begin{aligned} F_{T\nu}^\mu(\xi) &\equiv \langle [T_\nu^\mu(0), T_\nu^\mu(\xi)] \rangle_T = \int d\omega dk e^{i\omega t - ikx} \rho_{T\nu}^\mu(\omega, k) \\ &\quad - \text{H.c.}, \end{aligned} \quad (18)$$

where the thermal spectral density $\rho_{T\nu}^\mu(\omega, k)$ is defined by the relation (15) with the vacuum correlator on the lhs replaced by the thermal average. In a unitary theory, it is positive definite and vanishes if and only if $\tilde{T}_\nu^\mu(\omega, k) = 0$. Because of scale invariance, the spectral densities entering (18) have the form [25] $\rho_T(\omega, k) = \omega^2 \hat{\rho}_1(\omega/T, k/\omega)$. The dependence of $\hat{\rho}_1$ on the first argument prevents us from

explicitly taking the integral over frequency, so we have to take a different route. Let us subtract the vacuum value of the commutator from its thermal average

$$\begin{aligned} \mathcal{D}(t, x; T) &\equiv F_T(t, x; T) - F_0(t, x) \\ &= T^4 \int d\hat{\omega} d\hat{k} \hat{\omega}^2 e^{-iT x (\hat{k} - \hat{\omega} t/x)} \\ &\quad \times [(1 - e^{-\hat{\omega}}) \hat{\rho}_1(\hat{\omega}, \hat{k}/\hat{\omega}) - \epsilon(\hat{\omega}) \hat{\rho}(\hat{k}/\hat{\omega})], \end{aligned} \quad (19)$$

where $\epsilon(\hat{\omega})$ is the sign function. In deriving (19), we have rescaled the integration variables and used the standard connection between the thermal spectral densities for positive and negative frequencies [26], $\rho_T(-\omega, -k) = e^{-\omega/T} \rho_T(\omega, k)$. We want to consider (19) as a function of the variables x and $\theta \equiv t/x$. Because of (4), this function vanishes for $|\theta| < v_{\max}^{-1}$ and nonzero x at any T . However, the behavior at $x = 0$ is subtle: $\mathcal{D}$ is a distribution in the two-dimensional space (t, x) and its restriction to a line $t = \theta x$ must be appropriately specified. In the appendix, we show that

$$x^3 \mathcal{D}(\theta x, x; T) = 0 \quad \text{for } |\theta| < v_{\max}^{-1}, \quad (20)$$

in the well-defined sense of one-dimensional distributions in the variable x . Then, multiplying (19) by x^3 and taking the integral over x , we obtain

$$\frac{\partial^3}{\partial \theta^3} \int_0^\infty \frac{d\hat{\omega}}{\hat{\omega}} (1 - e^{-\hat{\omega}}) \hat{\rho}_1(\hat{\omega}, \theta) = 0, \quad \text{if } |\theta| < v_{\max}^{-1}, \quad (21)$$

where we have used that the integrand is an even function of $\hat{\omega}$ and, as proved above, the vacuum spectral density vanishes for these values of θ . Next, we use the relations (16) which remain valid in the thermal bath. Combined with (21), they imply $\rho_{T\nu}^\mu(\omega, k) = 0$ for $|k/\omega| < v_{\max}^{-1}$, and (12) follows by unitarity.

Step 3: We now construct a sequence of local dimension-two operators $\Phi_n(\xi)$, such that

$$\partial_t \Phi_n = \partial_x \Phi_{n+1}, \quad (22a)$$

$$\Phi_1 = -T_x^t, \quad \Phi_2 = T_x^x = -T_t^t, \quad \Phi_3 = T_t^x. \quad (22b)$$

Consider the integral of the EMT component T_t^x over the space. Its average over a normalizable state is

$$\langle \alpha | \int dx T_t^x(t, x) | \alpha \rangle = \int \frac{d\omega}{2\pi} e^{i\omega t} \langle \alpha | \tilde{T}_t^x(\omega, k=0) | \alpha \rangle.$$

Because of (12), the matrix element on the rhs is proportional [27] to $\delta(\omega)$ implying that $\int dx T_t^x(t, x)$ is time independent. Hence, there exists a local operator Φ_4 such

that $\partial_t T_t^x = \partial_x \Phi_4$. Its Fourier transform is $\tilde{\Phi}_4(\omega, k) = (\omega/k)T_t^x(\omega, k)$, and, thus, also satisfies the property (12). Then, by applying to Φ_4 the same reasoning as above, one concludes that there must exist another operator Φ_5 such that $\partial_t \Phi_4 = \partial_x \Phi_5$ and so on.

According to assumption 5, the number N of linearly independent operators in the sequence Φ_n is finite. Inside the linear envelope of the operators Φ_n , $n \leq N$, we can define a map $\Phi \mapsto \tilde{\Phi}$, where $\partial_t \Phi = \partial_x \tilde{\Phi}$. It is straightforward to show that this map is Hermitian with respect to the scalar product

$$\langle \Phi, \Psi \rangle = \int d\xi_1 d\xi_2 f^*(\xi_1) f(\xi_2) \langle 0 | \Phi^+(\xi_1) \Psi(\xi_2) | 0 \rangle,$$

where $f(\xi)$ is an arbitrary test function introduced to make the integral finite. Therefore, it can be diagonalized by a linear transformation

$$\Phi_n = \sum_a c_{na} \Psi_a, \quad \partial_t \Psi_a = v_a \partial_x \Psi_a, \quad (23)$$

where Ψ_a are local operators and the eigenvalues v_a are real. This yields the decomposition (5) of the EMT with [28]

$$T_x^{(a)t} = -c_{1a} \Psi_a, \quad T_x^{(a)x} = -T_t^{(a)t} = c_{2a} \Psi_a, \\ T_t^{(a)x} = c_{3a} \Psi_a. \quad (24)$$

Equations (22b) and (23) imply the relations between the coefficients $c_{n+1,a} = v_a c_{na}$ which yield the properties (6c) and (6a).

The property (6b) will follow if we prove that the operators Ψ_a , Ψ_b corresponding to different eigenvalues $v_a \neq v_b$ commute at the coincident time, which we choose to be $t = 0$. Indeed, this will imply that the time evolution of the operators for different a is completely independent, being given by the Hamiltonians $H^{(a)} \propto \int dx \Psi_a$, and thus, the vanishing of the commutator is preserved for a nonzero temporal separation. Locality fixes the most general form of the equal-time commutator

$$[\Psi_a(0, x), \Psi_b(0, 0)] = \sum_{i=0}^3 Q_i(0, x) \delta^{(i)}(x), \quad (25)$$

where $\delta^{(i)}(x)$ is the i th derivative of the δ function and the local operators $Q_i(t, x)$ have dimensions $3 - i$. The sum terminates because we have assumed the spectrum of dimensions to be non-negative.

Now, consider the Jacobi identity

$$-i[P_t, [\Psi_a, \Psi_b]] = -i[\Psi_a, [P_t, \Psi_b]] + i[\Psi_b, [P_t, \Psi_a]].$$

From (25), one reads the lhs

$$\sum_{i=0}^3 \dot{Q}_i(0, x) \delta^{(i)}(x). \quad (26)$$

While on the rhs, upon substituting the time derivatives of $\Psi_{a,b}$ by their space derivatives using (23), we find a term proportional to the fourth derivative of the δ function, $(v_a - v_b)Q_3(0, x)\delta^{(4)}(x)$. This must vanish implying $Q_3 = 0$. But, then, the contribution with $\delta^{(3)}$ disappears from (26), and requiring that it also vanishes on the rhs, we obtain $Q_2 = 0$. Continuing this argument, we conclude that the commutator vanishes altogether. ■

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Appendix: Analysis of $\mathcal{D}(t, x; T)$.—Scale invariance and continuity in temperature imply the relations

$$\mathcal{D}(\lambda x, \lambda t; T/\lambda) = \lambda^{-4} \mathcal{D}(x, t; T), \quad (A1a)$$

$$\lim_{T \rightarrow 0} \mathcal{D}(t, x; T) = 0. \quad (A1b)$$

These must be understood in the sense of two-dimensional distributions. We define the object

$$\mathcal{D}(\theta x, x; T) = \int dt \delta(t - \theta x) \mathcal{D}(t, x; T) \\ = \lim_{\lambda \rightarrow \infty} \int dt \lambda \varphi(\lambda(t - \theta x)) \mathcal{D}(t, x; T), \quad (A2)$$

where we have regularized the last expression by approximating the δ function with a sequence of smooth functions of compact support. Consider the convolution of (A2) with an arbitrary test function weighted by x^3

$$\int dx x^3 \mathcal{D}(\theta x, x; T) f(x) \\ = \lim_{\lambda \rightarrow \infty} \int dx dt x^3 \varphi(t - \theta x) f(x/\lambda) \mathcal{D}(t, x; T/\lambda). \quad (A3)$$

Vanishing of $\mathcal{D}$ at $|x| > v_{\max}|t|$ implies that, if $|\theta| < v_{\max}^{-1}$, the function $f(x/\lambda)$ in the integrand can be replaced by $f_1(x; \lambda) \equiv \varphi_1(x) f(x/\lambda)$, where $\varphi_1(x)$ is a localized test function that does not depend on λ and equals one in a certain interval around $x = 0$ [this interval is determined by the overlap of the causal cone with the support of $\varphi(t - \theta x)$]. The sequence $f_1(x; \lambda)$ has a well-defined limit $\varphi_1(x) f(0)$ at $\lambda \rightarrow \infty$. Then, (A1a) implies that (A3) vanishes leading to (20).

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- [20] An elementary example of the latter situation is provided by a theory of several massless fields—scalars or fermions—propagating with different velocities.
- [21] The reader should not be confused by the relativistic notations: though we use the space-time indices μ, ν , etc., that take values t, x , we cannot raise or lower them due to the absence of a Lorentz metric. We will, sometime, combine the time and space coordinates into a single vector ξ^μ with $\xi^t = t$, $\xi^x = x$. Summation over repeated indices is assumed unless stated otherwise.
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- [23] In fact, it is sufficient to make this assumption only for the level $\Delta = 2$ containing the (improved) EMT.
- [24] This is trivially valid for a $(1+1)$ -dimensional relativistic theory where the EMT decomposes into the left- and right-moving parts, $T_\nu^\mu(t, x) = T_\nu^{(l)\mu}(t+x) + T_\nu^{(r)\mu}(t-x)$, and hence, its Fourier transform is localized on the light cone.
- [25] We again omit the indices μ, ν .
- [26] Note that $\rho_T(\omega, k)$ is related to the imaginary part of the retarded Green's function in the thermal state: $2\text{Im}G_T^{\text{ret}}(\omega, k) = (1 - e^{-\omega/T})\rho_T(\omega, k)$.
- [27] It cannot contain derivatives of the δ function as in the vicinity of $k = 0$ the matrix element $\langle \alpha | \tilde{T}_t^x(\omega, k) | \alpha \rangle$ is a product of a positive distribution $\langle \alpha | \tilde{T}_t^t(\omega, k) | \alpha \rangle$ and a bounded function $(\omega/k)\Theta(v_{\text{max}} - |\omega/k|)$.
- [28] If one imposes invariance under spatial parity, it is possible to show that the eigenvalues v_a come in sign-symmetric pairs $\pm v_a$. Then, the partial EMTs (24) can be grouped into EMTs of parity-symmetric subsystems.