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HIDDEN STRING SYMMETRIES?

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ABSTRACT

The $E_{8(+8)}/SO(16)$ structure of $N = 1$ supergravity in $d = 11$ and the $SO(8,8)/SO(8) \times SO(8)$ structure of $N = 1$ supergravity in $d = 10$ conjectured previously are further conjectured to be sub-cosets of a $SO(d-2, d-2)/SO(d-2) \times SO(d-2)$ structure of the d -dimensional bosonic string. Kaluza-Klein techniques then yield the corresponding sub-cosets of the heterotic and Type II strings.

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1. N=1 supergravity in 11 dimensions

Some time ago ¹⁾⁻⁴⁾ we conjectured that the 128 bosonic degrees of freedom of N=1 supergravity in 11 dimensions, namely the metric g_{MN} (with 44 helicity states) and the antisymmetric tensor A_{MNP} (with 84 helicity states) are the Goldstone bosons associated with the coset

$$E_{8(+8)} / SO(16) \quad (1)$$

(As in Refs. 2)-4), we may use Table I to count the degrees of freedom.) In other words, there should exist a formulation of the theory with a global $E_{8(+8)}$ x local $SO(16)$ symmetry*. $E_{8(+8)}$ refers to the non-compact form with 120 compact and 128 non-compact generators. In spite of this, the coset (1) still describes a respectable theory since we are gauging the maximal compact subgroup ⁵⁾ $SO(16)$. This coset generalises the $SL(9,R)/SO(9)$ structure known to describe the 44 pure gravity degrees of freedom, $SO(9)$ being the transverse group in 11 dimensions ^{6),7),8)}. We shall not repeat all the reasons for making this conjecture**. They were based on the earlier discoveries of a hidden $E_{7(+7)}/SU(8)$ for N=8 supergravity in four dimensions ⁵⁾ and a hidden $E_{8(+8)}/SO(16)$ for N=16 supergravity in three dimensions ^{6),7),10)}.

Since $SO(16)$ is the transverse group in 18 dimensions, we conjectured further ^{1),2),3)} that there might correspond an 18 - dimensional formulation of the

* Possible extensions to the Kac-Moody algebra E_9 and the Lorentzian algebra E_{10} will not be discussed in this paper.

** Invariance of 11 - dimensional supergravity under the $SU(8)$ subgroup of $SO(16)$ has been proved in Ref. 9).

theory and that this may signal some connection with strings, an idea now all the more compelling in the light of the 18 - dimensional string theory proposed recently ^{11),12)}. We also pointed out several difficulties with this interpretation, however: (1) The proposed $SO(16)$ rotates particles of different spin into one another, in apparent conflict with the Coleman-Mandula theorem. (2) Both fermions and bosons belong to spinor representations of $SO(16)$, namely the inequivalent 128_L and 128_R^* , in apparent conflict with the spin-statistics theorem. (3) It is conventionally assumed that there is no supersymmetry in more than 11 dimensions. (4) The E_8 group appearing was the global non-compact $E_8(+8)$ as opposed to the local compact $E_8(-248)$ appearing in string theories ¹⁴⁾⁻¹⁷⁾. We shall attempt to resolve all these difficulties later in the paper. At the time, however, a more conservative approach to string applications was to truncate $N=1$ supergravity in 11 dimensions to $N=1$ supergravity in 10.

2. $N=1$ supergravity in 10 dimensions

Applying the same logic as before, we further conjectured ^{2),3)} that the 64 bosonic degrees of freedom of $N=1$ supergravity in 10 dimensions, namely the metric g_{MN} (35 helicity states), the antisymmetric tensor B_{MN} (28 helicity states) and the dilaton ϕ (1 helicity state) should correspond to some sub-coset of $E_8(+8)/SO(16)$ which, by truncation**, turns out to be

* Together, these form an irreducible representation of the supersymmetry algebra ¹³⁾.

** In the preprinted version of Ref. 2), we incorrectly identified this truncation with another coset of dimension 64, namely $E_7(-5)/SO(12) \times SU(2)$. However, this is not the appropriate one to describe $N=1$ supergravity in 10 dimensions, as was pointed out to me by Jean Thierry-Mieg. See Errata CERN-TH 4243/85.

$$\frac{so(8,8)}{so(8) \times so(8)} \quad (2)$$

This coset generalises the $SL(8,R)/SO(8)$ structure known to describe the 35 pure gravity degrees of freedom, $SO(8)$ being the transverse group in 10 dimensions. This idea was also based on the earlier discoveries of a hidden $SO(8,8)/SO(8) \times SO(8)$ for $N=8$ supergravity in three dimensions (18), (19).

3. The bosonic string in d dimensions

The fields (g_{MN}, B_{MN}, ϕ) discussed above also correspond to the massless sector of the closed bosonic string, which exists in any dimension d for which the critical dimension formula (20)-(24)

$$d - 26 = \frac{k C_A}{2 + C_A} = k - r \quad (3)$$

is valid. Here we are considering a spontaneous compactification of the form

$$M_d = M_n \times G \quad (4)$$

where M_n is spacetime with zero cosmological constant and G is a non-abelian group manifold of dimension $k=d-n$ and rank r , and C_A is the second-order Casimir in the adjoint representation. In view of the indications that the bosonic string may even be the fundamental theory (16), (25)-(30), it is of interest to ask what the hidden symmetry might be in d dimensions.

The obvious generalisation of (2) which suggests itself is that the massless modes of the bosonic string are just the Goldstone bosons associated with the coset

$$\frac{SO(d-2, d-2)}{SO(d-2) \times SO(d-2)} \quad (5)$$

Let us see how far this conjecture may be justified. First let us use Table 1 to count the physical helicity states. The graviton g_{MN} has $d(d-3)/2$ which may always be assigned to $SL(d-2, R)/SO(d-2)$. As a check we note that

$$SO(d-2, d-2) \supset SL(d-2, R) \quad (6)$$

$$SO(d-2) \times SO(d-2) \supset SO(d-2) \quad (7)$$

so (5) is certainly consistent with pure gravity. The helicity states of B_{MN} number $(d-3)(d-2)/3$ and the dilaton has one, making a total of $(d-2)^2$. This is also the dimension of the coset (5) since $\dim SO(p, q)/SO(p) \times SO(q) = pq$.

There is an alternative "stringy" way to understand the counting, which also explains the $SO(d-2) \times SO(d-2)$ structure. The physical states of the closed bosonic string are obtained by acting on the vacuum with the creation operators $\alpha_M^{(-n)}$ and $\tilde{\alpha}_N^{(-n)}$ corresponding to left and right moving modes, respectively. In particular, the massless states are given by

$$\alpha_M^{(-1)} \tilde{\alpha}_N^{(-1)} | 0 \rangle \quad (8)$$

where, in the physical light-cone gauge, M and N range over 1 to $(d-2)$, hence we obtain $(d-2)^2$ massless states.

Moreover, one $SO(d-2)$ is associated with rotations in the left moving sector and the other $SO(d-2)$ with the right.

Since the number of massless states is preserved* by compactification from d to n on the group manifold G of dimension k , we expect the same coset description to apply except that the $(d-2)^2$ degrees of freedom are distributed as follows ²⁹⁾: $(n-2)^2$ for $g_{\mu\nu}$, $B_{\mu\nu}$ and ϕ ; $(n-2)k$ for the gauge bosons A_μ^i of G [left]; $(n-2)k$ for the gauge bosons B_μ^i of G [right] and k^2 for the scalars S^{ij} in the (adjoint, adjoint) representation of G [left] $\times$ G [right]. See Table 2. We shall also make use of the group decompositions

$$SO(d-2) [spacetime, left] \supset SO(n-2) [spacetime, left] \times SO(k) [internal, left] \quad (9)$$

$$SO(k) [internal, left] \supset G [left] \quad (10)$$

and those obtained by replacing left with right. The physical transverse spacetime group of the bosonic string is then given by the diagonal subgroup

$$SO(n-2) [spacetime, bosonic] \subset SO(n-2) [spacetime, left] \times SO(n-2) [spacetime, right] \quad (11)$$

In this paper, we shall primarily be interested in the following cases (i) $G = E_8 \times E_8$ for which $C_A = 60$, $k = 496$ and $r = 16$ in which case (3) yields $d = 506$ and $n = 10$. The relevant coset is then

$$\frac{SO(504, 504)}{SO(504) \times SO(504)} \quad (12)$$

* In a consistent ⁴⁾ Kaluza-Klein ansatz the number of massless states in n dimensions is less than or equal to the number in d dimensions ³¹⁾. The group manifold compactification of the bosonic string shares with the S^7 compactification of $d=11$ supergravity the feature of saturating this bound.

and (ii) $G = E_8$ for which $C_A = 60$, $k = 248$ and $r = 8$ in which case (3) yields $d = 266$ and $n = 18$. The relevant coset is then

$$\frac{SO(264, 264)}{SO(264) \times SO(264)} \quad (13)$$

The first will be associated with superstrings in 10 dimensions, both heterotic and Type II, while the second will be associated with the 18 - dimensional string and 11 - dimensional supergravity*.

4. Superstrings in 10 dimensions

We now describe the recipe for obtaining the heterotic string in 10 dimensions. Although we are following^{25),28)} the group manifold approach we may borrow some of the group theoretical ideas of torus compactification^{26),12)} from 26 dimensions, since the d - dimensional string compactified on G of dimension k is equivalent to the 26 - dimensional string compactified on T^r where the rank r and dimension k are related by (3). First we decompose the 504 right-moving modes under

$$SO(504)[\text{spacetime, right}] \supset SO(9)[\text{spacetime, right}] \times G[\text{right}] \quad (14)$$

$$504 \rightarrow (8_v, 1) + (1, \text{adjoint}) \quad (15)$$

* Rank 24 choices like $G = E_8 \times E_8 \times E_8$ for which $n=2$ will not be discussed.

and then under

$$G[\text{right}] \supset SO(8)[\text{internal}, \text{right}] \quad (16)$$

To account for fermions, the physical transverse spacetime group of the heterotic string is then given by the diagonal subgroup

$$SO(8)[\text{spacetime}, \text{heterotic}] \subset$$

$$SO(8)[\text{spacetime}, \text{bosonic}] \times SO(8)[\text{internal}, \text{right}] \quad (17)$$

In this way some of the states transforming as tensor representations of $SO(8)[\text{spacetime}, \text{bosonic}]$ now transform as spinor representations of $SO(8)[\text{spacetime}, \text{heterotic}]$. To account for supersymmetry, moreover, the embedding (16) must be regular and such that the adjoint of $G[\text{right}]$ contain both the 8_v and 8_s representations of $SO(8)[\text{internal}, \text{right}]^*$. The massless states of the heterotic string are then obtained by retaining just the 8_s . Together with the 8_v of $SO(8)[\text{spacetime}, \text{right}]$ in (15), the counting would yield a coset based on $SO(504, 16)$. However, since we do not expect to put bosons and fermions into the same

* This rules out $Spin(32)/Z_2$ as a candidate for $G[\text{right}]$. The $Spin(32)/Z_2$ string is obtained in Ref. 12) by choosing $G[\text{right}] = E_8 \times E_8$ and $G[\text{left}] = Spin(32)/Z_2$. On the group manifold however, $G[\text{left}] = G[\text{right}]$ and $Spin(32)/Z_2$ cannot be obtained from $d=506$. See Ref. 32)

multiplet without invoking some coset structure based on supergroups*, we must reduce the coset yet further to one which describes only the bosonic sector namely

$$\frac{SO(504, 8)}{SO(504) \times SO(8)} \quad (18)$$

Thus the heterotic string falls into the general category of cosets

$$\frac{SO(p, q)}{SO(p) \times SO(q)} \quad (19)$$

where p refers to the left-moving modes, and q to the right moving modes. This coset has dimension pq which in the present case is 4032 made up of the graviton (35), the antisymmetric tensor (28), the dilaton (1) and the $E_8 \times E_8$ gauge bosons (8.496).

The Type II strings are obtained by repeating the above procedure for both right and left sectors, retaining the $(8_c, 8_s)$ of $SO(8)$ [internal, left] $\times$ $SO(8)$ [internal, right] in the case of Type IIA and the $(8_s, 8_s)$ in the case of Type IIB. Once again ignoring fermions, we obtain

$$\frac{SO(8, 8)}{SO(8) \times SO(8)} \times \frac{SO(8, 8)}{SO(8) \times SO(8)} \quad (20)$$

* This is an interesting possibility but will not be pursued here.

The first factor corresponds to N=1 supergravity in 10 dimensions already discussed in Section 2 and the second to the extra 64 degrees of freedom provided by the fields $S^{I'J}$ or S^{IJ} where I' refers to 8_c and I to 8_s . These are bosons under $SO(8)$ [spacetime, Type II] transforming as $8_v + 56_v$ and $1 + 28 + 35_s$ respectively.

5. Strings in 18 dimensions

Once again we may follow a heterotic route and decompose the 264 right-moving modes under

$$SO(264) [spacetime, right] \supset SO(8) [spacetime, right] \times E_8 [right] \quad (21)$$

as

$$264 \rightarrow (16, 1) + (1, 248) \quad (22)$$

and then under

$$E_8 [right] \rightarrow SO(16) [internal, right] \quad (23)$$

as

$$248 \rightarrow 120 + 128_R \quad (24)$$

identifying the $SO(16)$ [spacetime, heterotic] as in (17); or else a Type II route by repeating the above procedure for both sectors with either $(128_L, 128_R)$ for Type IIA or $(128_R, 128_R)$ for Type IIB. It is the Type IIA route which corresponds to 11 - dimensional supergravity since we recognise the 128_L which describes the gravitino in 11 - dimensions and 128_R which describes the graviton and

antisymmetric tensor. The global non-compact $E_{8(+8)}$ now makes its appearance as a subgroup of the numerator in the coset (13)

$$\begin{aligned} \text{global } E_{8(+8)} &\subset \text{global } SO(120, 128) \\ &\subset \text{global } SO(264, 264) \end{aligned} \quad (25)$$

and is to be contrasted with the local compact $E_{8(-248)}$ which makes its appearance as a subgroup of the denominator

$$\begin{aligned} \text{local } E_{8(-248)} &\subset \text{local } SO(248) \\ &\subset \text{local } SO(264) \end{aligned} \quad (26)$$

Note, however, that the analogy with the 10 - dimensional string breaks down since there is no supersymmetry in 18 dimensions ¹¹⁾ and the $SO(16)$ transverse group cannot be extended to the full ¹²⁾ Lorentz group $SO(1,17)$. Indeed, we can now confront the three remaining problems raised in Section I.

6. Spontaneous breaking of spacetime symmetries

The key is spontaneous symmetry breaking. The problem of (1) the Coleman-Mandula theorem (2) the spin-statistics theorem, and (3) supersymmetry, all disappear if we assume that the spacetime vacuum state is not given by the 18 dimensional transverse group $SO(16)$ but only by its 11 dimensional subgroup $SO(9)$

$$SO(16) \supset SO(9) \quad (27)$$

where the embedding is the non-regular one defined by $16 \rightarrow 16$. Under (27), we have the decompositions

$$128_L \rightarrow 128 \quad (28)$$

which describes the gravitino in 11 dimensions, and

$$128_R \rightarrow 44 + 84 \quad (29)$$

which describes the graviton (44) and antisymmetric tensor (84). This 11 - dimensional vacuum is now compatible with supersymmetry and with the extension of the transverse $SO(9)$ to the full Lorentz group $SO(1,10)$. Since the transverse group of the S-matrix is now only $SO(9)$ rather than $SO(16)$, moreover, there is no conflict with the Coleman-Mandula theorem nor with the spin statistics theorem since the $44 + 84$ of (29) are now tensor rather than spinor representations and $SO(9)$ does not put the graviton and antisymmetric tensor into the same multiplet. We have already seen a similar phenomenon with the Type II strings in Section 4. The $S^{I'J}$ or S^{IJ} transform as spinors under $SO(8)$ [spacetime, left] $\times SO(8)$ [spacetime, right] but there is no conflict with spin-statistics since they transform as tensors under the diagonal symmetry $SO(8)$ [spacetime, Type II], which is the symmetry of the vacuum.*

The picture which emerges is the following. The fundamental theory is the d - dimensional bosonic string with equations invariant under the big symmetry implied by the coset (5). As with all G/H σ - models, we cannot expect the vacuum to have a symmetry bigger than H (at least classically), but even an unbroken H vacuum

* NB The 35_s is, of course, a tensor representation of $SO(8)$ even though the subscript s stands for spinor!

symmetry would in general raise problems with Coleman-Mandula, spin-statistics, supersymmetry or Lorentz invariance. Thus it appears then the string equations only admit vacuum solutions for which H is broken down to some smaller symmetry and for which these problems do not arise. The $M_d = M_n \times G$ is one example of this for which the vacuum symmetry is Poincaré [bosonic] $\times G$ [left] $\times G$ [right]. However, this vacuum is unstable having a tachyon and no supersymmetry. Indeed it has no fermions at all. Presumably there exist other vacua with unbroken symmetries Poincaré [heterotic] $\times G$ [left] and Poincaré [Type II] in 10 dimensions and Poincaré [N=1 supergravity] in 11 dimensions, which have no tachyon, do have fermions and are supersymmetric. Presumably also there are vacua with no tachyon and fermions but which are not supersymmetric*. It remains a challenge to show that these vacua really are solutions of the bosonic string equations. If 11 - dimensional supergravity should turn out to be just a branch of string theory, it would leave one feeling like the man who discovers he has been speaking prose all his life without realizing it.

7. Implications for four dimensions

As mentioned in Section I, coset structures of the kind discussed in the present paper have already appeared in four - dimensional supergravity theories. They simply correspond to the non-linear σ - model which

* It is not unreasonable to suppose that the new 10 - dimensional $O(16) \times O(16)$ strings (L Alvarez-Gaumé, private communication) are bosonic string vacua of this kind.

describes the scalar fields, eg $E_{7(+7)}/SU(8)$ for $N=8$ and $[SO(k,6)/SO(k) \times SO(6)] \times [SU(1,1)/U(1)]$ for $N=4$ coupled to k $N=4$ vector multiplets. In these cases, of course, both the G and H in G/H correspond to internal symmetries. However, it is not difficult to understand their spacetime origin. Consider, for example, the $SL(d-2, R)/SO(d)$ structure of pure gravity. The $SL(d-2, R)$ invariance of the lightcone gravitational variables ⁸⁾

$$\delta g_{MN} = \Lambda_P{}^Q x^P \partial_Q g_{MN} + \Lambda_M{}^P g_{PN} + \Lambda_N{}^P g_{MP} \quad (30)$$

where $M=1 \dots (d-2)$, is clearly a spacetime symmetry. But if one considers a compactification on the torus T^k for which the extra dimensional components of g_{MN} are independent of the extra coordinates, then an internal $SL(k, R)$

$$\delta g_{mn} = \Lambda_m{}^p g_{pn} + \Lambda_n{}^p g_{mp} \quad (31)$$

where $m=1 \dots k$, is contained within the larger spacetime symmetry. Thus the four dimensional scalars belong to an $SL(k, R)/SO(k)$ σ - model*. The number of scalars simply correspond to the number of parameters of the torus T^k . When we allow for the other bosonic fields, this global symmetry becomes even larger eg the $E_{7(+7)}/SU(8)$ comes from T^7 compactification of $N=1$ supergravity in 11 dimensions ⁵⁾, and an $[SO(6,6)/SO(6) \times SO(6)] \times [SU(1,1)/U(1)]$ comes from T^6 compactification of $N=1$ supergravity in 10 dimensions ¹⁸⁾. The above interpretation is in

* Actually $GL(k, R)/SO(k)$ owing to an extra rescaling degree of freedom ⁵⁾.

fact a general phenomenon of Ricci-flat compactifications 33)-35), 2), 3). The scalars reside in the $g_{mn}(x,y)$ components of the Kaluza-Klein metric, where x^μ refers to spacetime and y^m to the extra dimensions, whereas the parameters reside in the ground-state metric $\hat{g}_{mn}(y)$. But since

$$\hat{g}_{mn}(y) = \langle g_{mn}(x,y) \rangle \quad (32)$$

we see that the parameters are just the vevs of the scalars coming from the metric. Moreover, since these parameters are completely arbitrary so are the vevs ie we have a flat potential 33)-35), 2), 3). (See Ref. 36) for the phenomenological virtues of flat potentials.) For example, compactification of 11 - dimensional supergravity on $K3 \times T^3$ involves 33) 134 parameters corresponding to the 134 scalars of N=4 supergravity coupled to 22 N=4 vector multiplets and hence* to the coset 37), 2), 3).

$$\frac{SO(22,6)}{SO(22) \times SO(6)} \times \frac{SU(1,1)}{U(1)} \quad (33)$$

Cosets of the form

$$\frac{SO(26-n, 10-n)}{SO(26-n) \times SO(10-n)} \quad (34)$$

* This symmetry is harder to explain since it is not a subcoset of $E_{8(8)}/SO(16)$. On the other hand the massless sector is not a consistent truncation See Refs. 4), 31).

have recently been considered by Narain ³⁸⁾ in his compactification into tori of $(10-n)$ and $(26-n)$ dimensions of the right moving superstring and left moving bosonic string sectors respectively*. There is obviously some overlap between his work and ours but the precise connection is not obvious. By considering torus compactification the values of p and q in Narain's $SO(p,q)$ are determined by the rank r of the group, whereas by considering group-manifold compactification the values of p and q in our $SO(p,q)$ are determined by the dimension k of the group. Thus in the maximal case of rank 22 eg $SO(44)$ he obtains an $SO(6,22)$ whereas we obtain from (3) the much bigger $SO(946,946)$. Our bigger cosets are essential, of course, if we are to explain groups like $E_{8(+8)} \subset SO(120,128)$.

In Narain's analysis modular invariance requires

$$p - q = \text{mod } 8 \quad (35)$$

and that is also the case for all cosets considered in this paper. It is amusing to note, in this connection,

* In four dimensions, curiously enough, we obtain the same $SO(22,6)$ as in (33). As far as I know, there is no obvious connection. The 22 in (33) comes from the 22 harmonic 2-forms on $K3$. However the intersection matrix of these 2-forms on $K3$ contains the Cartan matrix of $E_8 \times E_8$, so who knows? I am grateful to R. Coquereaux and M. Atiyah for bringing this to my attention. See Ref. 39). for a layman's introduction.

that $N=4$ supergravity coupled to 6 mod 8 $N=4$ vector multiplets is just what is required ⁴⁰⁾ to avoid the no-go theorems ⁴¹⁾ for auxiliary fields*. Perhaps this can also be traced back to the bosonic string where, of course, the algebra always closes off-shell.

Phenomenologically it is compactifications with one unbroken supersymmetry in four dimensions which are the most phenomenologically interesting, and T^{22}/G compactifications with this property have recently been discussed in Ref. 42). Moreover, in contrast to those obtained by Calabi-Yau ³⁵⁾ compactification ^{4),31),43)}, the Kaluza-Klein ansatz for the massless sector is likely to be consistent. Apart from the fact that this would place an upper bound on the number of families ³¹⁾, it might also permit a determination of the Kähler manifold describing the massless scalars. For example, the 54 spin -0 superpartners of the 27 quarks and leptons of an E_6 family might be assigned to the dimension 54 coset ^{2,3)}

$$E_7 / E_6 \times U(1) \quad (36)$$

thus realizing the idea, albeit in a somewhat different context, that the quarks and leptons are superpartners of the Goldstone bosons ⁴⁴⁾ associated with exceptional coset spaces ⁴⁵⁾.

A comprehensive review of coset manifolds in four - dimensional supergravity is given in Ref.46).

* I am grateful to Paul Townsend for pointing this out.

8. Conclusions

Since the proof of the hidden symmetries in supergravity was notoriously difficult, the same may be true of the hidden string symmetries conjectured in this paper. The $SL(d-2, R)$ symmetry of pure gravity g_{MN} is just a global subgroup of the general coordinate group. We expect the $SO(d-2, d-2)$ symmetry of the (g_{MN}, B_{MN}, Φ) system to emerge as a global subgroup of the combined general coordinate transformations and abelian gauge transformations of the antisymmetric tensor. Moreover, this might correspond to an extension of ordinary general relativity to a theory based on Einstein's non-symmetric metric ⁴⁷⁾. This will be discussed elsewhere ⁴⁸⁾.

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d-bein	e_M^A	$d(d-3)/2$
Gravitino	ψ_M	$2^\alpha(d-3)$
Vector	A_M	$(d-2)$
Spinor	χ	2^α
Scalar	ϕ	1

ANTISYMMETRIC TENSOR GAUGE FIELDS

A_{MNP}	$(d-4)(d-3)(d-2)/6$
A_{MN}	$(d-3)(d-2)/2$
A_M	$(d-2)$
A	1

Table 1: Degrees of freedom in d dimensions. For Dirac spinors $\alpha = d/2$ if d is even and $\alpha = (d-1)/2$ if d is odd. Divide by two for Majorana spinors, by two for Weyl and by four for Majorana-Weyl. Similarly divide by two for self-dual antisymmetric tensor field strengths.

d dimensions	n dimensions	degrees of freedom
g_{MN}	$g_{\mu\nu}$	$n(n-3)/2$
	$A_\mu^i + B_\mu^i$	$k(n-2)$
	$s^{(ij)}$	$k(k+1)/2$
B_{MN}	$B_{\mu\nu}$	$(n-2)(n-3)/2$
	$A_\mu^i - B_\mu^i$	$k(n-2)$
	$s^{[ij]}$	$k(k-1)/2$
ϕ	ϕ	1
TOTAL		<hr/> $(d-2)^2$

Table 2: Degrees of freedom of the n-dimensional massless states resulting from compactification of the d-dimensional bosonic string on a group manifold of dimension $k = d-n$.

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