CERN-TH.4369/86

RU86/B/151

CAN ELEMENTARY SCALAR PARTICLES EXIST?

II. SCALAR ELECTRODYNAMICS

D.J.E. Callaway^{*)} and R. Petronzio

CERN -- Geneva

ABSTRACT

Strong evidence supports the notion that pure ϕ^4 field theory is a non-interacting, "trivial" theory. Should such a situation persist when gauge fields are present, the idea that elementary scalars can break symmetries spontaneously is called into question. Previously, we applied the Monte Carlo renormalization group to the SU(2)-Higgs model; here we consider the U(1)-Higgs model. No evidence for a non-trivial continuum theory was uncovered in either of these limits of the standard model of the weak interaction.

^{*)} Permanent address: Department of Physics, The Rockefeller University, 1230 York Avenue, New York, NY 10021-6399, U.S.A.

CERN-TH.4369/86

RU86/B/151

February 1986

1. - PROLEGOMENA

The concept of particle mass generation by spontaneous symmetry breaking¹⁾ has become one of the most important [and least understood²⁾] ideas in field theory. Typically, this symmetry breaking is produced by the introduction of a scalar field. This new field supposedly interacts with itself in such a fashion (via a ϕ^4 coupling) as to generate a vacuum state which lacks a symmetry of the Lagrangian. Gauge particles propagating in the asymmetric vacuum may then appear to be massive, while the underlying theory remains renormalizable.

The Higgs-Kibble mechanism in fact forms an essential part of the standard model³⁾ of the weak interaction. Nevertheless, a large body⁴⁾⁻⁶⁾ of evidence has been presented which suggests that pure ϕ^4 field theory is "trivial", that is, non-interacting in the infinite cut-off limit in four dimensions. (Of course, if the theory is non-interacting, its interactions cannot break the symmetry of the vacuum.) Recently, studies^{7),8)} of an SU(2) gauge field interacting with a fundamental representation scalar field have been made; the present work considers the interaction of a scalar field with a U(1) gauge theory. None of these studies (which are actually just limits of the standard model) uncovered any existence of a non-trivial theory. If the standard model itself is trivial, it is fair to question the whole idea of symmetry breaking by elementary scalars. There is an alternative and much more exciting possibility: if the standard model is [like the O(N) gauge model discussed below and in Ref. 7)] non-trivial only for a select range of parameters, the Higgs mass can be bounded or even calculated⁹⁾.

2. - THE MEANING OF TRIVIALITY

Why might the standard model be non-trivial even though the ϕ^4 sector seems to be? An understanding of why this possibility is realistic can be gained by a further examination of the idea of triviality. A simple example of a trivial theory is provided by the O(N) model, which also becomes NON-trivial when gauge fields are included. The pure O(N) theory can be defined¹⁰⁾ with an interaction Lagrange density

$$\mathcal{L}_I = - \frac{\lambda}{8N} (\varphi^2)^2 \quad (1)$$

The running coupling $\lambda(t)$ obeys an equation^{10),11)}

$$\frac{d\lambda(t)}{dt} = \frac{\lambda^2(t)}{16\pi^2} \quad (2)$$

in the large N limit. Here, $t \equiv \ln(Q/\mu)$ where Q is the running momentum scale and μ is the renormalization point. Equation (1) has the solution

$$\lambda(t) = \frac{\lambda_R}{1 - \lambda_R \cdot t / (16\pi^2)} \quad (3)$$

Note that $\lambda(t = 0) \equiv \lambda_R$ defines the renormalized coupling λ_R . This running coupling becomes infinite at a momentum scale $t = 16\pi^2/\lambda_R$, which is finite if λ_R is non-zero. Thus, the $O(N)$ model in the large N limit is inconsistent unless λ_R vanishes, i.e., unless it is a free field theory. The $O(N)$ model in the large N limit is therefore trivial. Note that the existence of this inconsistency (or "Landau ghost") in the interacting theory is essentially a non-perturbative phenomenon, for Eq. (3) holds for all λ in this large N limit. This $O(N)$ case can be contrasted to ordinary quantum electrodynamics, where the Landau ghost which appears in low orders of perturbation theory could be exorcised by a non-perturbative analysis (of the type performed here).

Interestingly enough, the addition of an $O(N)$ gauge field to the $O(N)$ model rescues it from triviality^{7),11)}. The renormalized gauge coupling g_R (rescaled by the factor $\sqrt{N}$ in the standard fashion) and quartic coupling λ_R satisfy the bound⁷⁾:

$$\lambda_R \leq \left(\frac{3 + \sqrt{6}}{2} \right) g_R^2 \quad (4)$$

provided that g_R is small enough. This is a particularly suggestive combination, for, to lowest order in the loop expansion,

$$\left(\frac{m_H}{m_W} \right)^2 = \frac{\lambda_R}{g_R^2} \leq \frac{3 + \sqrt{6}}{2} \quad (5)$$

Here, m_H (the "Higgs" mass) is the renormalized mass of the scalar field, and m_W (the "W" mass) is the mass of the $N-1$ massive gauge bosons. Thus, at least within the context of the large N limit of the $O(N)$ gauge model, the need for the existence of a non-trivial theory generates constraints (an upper bound on the mass of the "Higgs" boson). More speculative versions^{9),12),13)} of these ideas have led to upper bounds on the mass of the Higgs boson which is as low as 125 GeV¹²⁾.

It should be pointed out that it is NOT necessary for a theory to be asymptotically free in order to be non-trivial [though the $O(N)$ gauge model in the region given by Eq. (4) is asymptotically free in both λ and g^2]. It is only necessary to have a fixed point (i.e., a zero of the beta function) which is attractive as t increases. If this fixed point is located at the origin, then the theory is asymptotically free. Thus asymptotic freedom is a sufficient, but unnecessary, condition for triviality.

In fact, it seems to be true¹¹⁾ that a theory cannot be both asymptotically free in all couplings and still exhibit complete spontaneous symmetry breaking. For example, in the $O(N)$ gauge model, N must be greater than or equal to six in order for both the gauge and quartic couplings to be asymptotically free. However, for one vector representation scalar, the symmetry is only broken from $O(N)$ to $O(N-1)$. Thus, only for N equal to two is the symmetry broken completely, and for, e.g., $O(6)$ there remain ten massless non-Abelian gauge bosons corresponding to the generators of the remaining non-Abelian $O(5)$ subgroup. This seems to be¹¹⁾ the general pattern -- either there are a large number of massless gauge bosons, or else at least one coupling in the theory is not asymptotically free.

A class of exceptions to this general rule does exist; these are theories¹⁴⁾ which obey "eigenvalue" conditions, where every renormalized coupling in the theory is uniquely determined. A theory of this form could be correct, but it would exhibit "fine tuning" on the grandest scale possible.

Accordingly in general, it is necessary to consider the question of triviality in theories which are NOT asymptotically free in all couplings. A non-perturbative method must be applied to deal with this problem. We have used⁷⁾ the Monte Carlo renormalization group for this purpose. Specifically, a lattice version of the continuum model of interest is constructed, and the renormalization flows and fixed points under a comprehensive blocking transformation are studied. At any fixed points, the anomalous dimensions of the continuum theory (if one exists) associated with that point are calculated. If the theory is non-trivial (i.e., interacting), the scalar field anomalous dimensions must be non-zero. Possible eventual phenomenological results of such an analysis include upper bounds on the Higgs mass [as in the case of the $O(N)$ gauge model⁷⁾] or even calculable Higgs masses⁹⁾ [as in the case of the eigenvalue theories¹⁴⁾]. After

a brief discussion of the non-compact Abelian Higgs model, we determine its flow diagram, fixed points and critical exponents, and discuss our results in light of the above remarks.

3. - THE NON-COMPACT ABELIAN HIGGS MODEL

In order to perform an analysis of the type described here, it is necessary to formulate a lattice model of quantum electrodynamics. This can be done in many ways. We have chosen to use a non-compact version of the model [studied in, e.g., Ref. 15)] instead of the more traditional compact version¹⁶⁾⁻¹⁸⁾. The reason is that though the compact model has been well analyzed numerically in both its pure form¹⁸⁾ and when coupled to scalar fields^{19),20)}, it is certainly more complicated than the non-compact theory, and appears¹⁸⁾ to be plagued with numerical difficulties as well.

The second simplification is the fixing of the lattice Higgs field length to unity. Because the fixed- and variable-length models are in the same universality class, this should not affect our results. Additionally, in both the pure scalar⁶⁾ and SU(2)-Higgs models^{7),8)}, evidence supports this idea (the scalar self-coupling appears to be irrelevant in an appropriate renormalization scheme). The results of a preliminary calculation in which the scalar field magnitude is allowed to vary are also discussed below.

The fixed-length action is defined by¹⁵⁾

$$S = \frac{\beta}{2} \sum_{\text{Plaquettes}} (\Theta_{\text{plaq}})^2 - 2K \sum_{\langle ij \rangle} \cos(\varphi_i - \varphi_j + \Theta_{ij}) \quad (6)$$

where β is related to the gauge coupling g_0 by $\beta = g_0^{-2}$, K is the "hopping parameter", θ_{ij} and ϕ_i are real variables ranging from $-\infty$ to $+\infty$, and $\theta_{\text{plaq}} = \theta_{ij} + \theta_{jk} - \theta_{kl} - \theta_{li}$ is the path ordered sum of the four θ_{ij} defining an elementary plaquette. The $\{\phi_i\}$ are of course just the local phases of the scalar field, and $\langle ij \rangle$ refers to nearest-neighbour pairs, each summed exactly once.

This non-compact model for scalar QED can be thought of¹⁵⁾ as a limit of the compact model¹⁹⁾ with charge q and coupling $\tilde{\beta}$, in which $\tilde{\beta}$ and q increase without bound while the quantity $\beta \equiv \tilde{\beta}q^{-2}$ remains fixed. This relationship suggests the following limiting behaviour in the phase diagram.

$K = 0$

In this limit, the action is quadratic in the θ_{ij} and therefore the path integral reduces to a product of Gaussian integrations. By a rescaling transformation the partition function Z can be calculated exactly (up to an overall factor) and the average plaquette is then given by

$$\begin{aligned} P &\equiv -\frac{2}{\partial\beta} \ln Z \\ &= \left\langle \frac{1}{2} \Theta_{\text{Pla}q}^2 \right\rangle \\ &= \frac{1}{4\beta} \end{aligned} \tag{7}$$

[Careful counting of the gauge degrees of freedom is a must to derive Eq. (7).] Hence there are no phase transitions for finite β when K is zero.

$\beta = 0$

When β equals zero, the model decouples to a product of finite integrals (this can best be seen in the unitary gauge, where all of the $\{\phi_i\}$ are set to zero). Again, there are no phase transitions.

$\beta \rightarrow \infty$

In this limit the model approaches an XY model in four dimensions, and thus presumably has a second-order phase transition at $2K = 0.30$. This value comes from an analysis^{20),21)} of a series in inverse powers of the co-ordination number ($q = 8$). The full phase structure of the theory is considered following the next section.

4. - DESCRIPTION OF THE PROCEDURE

How can the question of the triviality or non-triviality of a theory be answered in a sensible fashion? One way of approaching these questions is via the Monte Carlo renormalization group (MCRG). By dealing directly with the scaling properties of the various fields and operators under consideration, it allows an economical (albeit numerical) formulation of the problem. Additionally, by restricting the analysis to a few carefully chosen operators, the MCRG allows accurate and systematically improvable approximations. More direct methods⁵⁾ of studying triviality do not allow this simplification. The Monte Carlo renormalization group^{22),23),6),7)} is a combination of the ideas of Monte Carlo simulation²⁴⁾ with those of the real space renormalization group²⁵⁾. The system under consideration is divided up into "blocks" or collections of site variables and a smaller set of block variables is defined by averaging in some fashion over these site variables. The new block system is governed by a block renormalized action (defined by couplings $\{K'\}$) in the same fashion as the original site action (defined by couplings $\{K\}$) governed the site system. By studying the sequence of couplings $\{K\} \rightarrow \{K'\} \rightarrow \{K''\} \rightarrow \dots$ as repeated blockings are performed, the critical properties of the system can be determined. For a lattice gauge theory, these critical properties determine the continuum limit of the theory.

These ideas can be formulated in a more precise mathematical fashion as follows. Consider a system of site variables $\{\phi\}$ governed by a site action $S\{\phi\}$. A block renormalized action $S'\{\phi'\}$ is determined from the site action by use of the projection operator $P[\{\phi'\}, \{\phi\}]$:

$$\exp[-S'\{\phi'\}] = \text{Tr}_{\{\phi\}} P[\{\phi'\}, \{\phi\}] \cdot \exp[-S\{\phi\}] \quad (8)$$

where the trace (or functional integral) is only over the site variables $\{\phi\}$. The requirement that the renormalization group transformation preserve the partition function (vacuum-to-vacuum transition amplitude) of the system imposes the constraint

$$\text{Tr}_{\{\phi'\}} P[\{\phi'\}, \{\phi\}] = 1 \quad (9)$$

The projection operator is otherwise entirely arbitrary. The generalization of Eqs. (8) and (9) to a case where both spin and gauge fields are present is obvious.

As there are two sets of fields, (the $\{\theta_{ij}\}$ and the $\{\phi_i\}$) there are also two projection operators. It is convenient to choose these projection operators to be products of delta functions of the form

$$P[\{\varphi'\}, \{\varphi\}] = \prod_{\{\varphi'\}} \delta[\varphi'_i - f_i\{\varphi, \theta\}] \quad (10a)$$

$$P[\{\theta'\}, \{\theta\}] = \prod_{\{\theta'\}} \delta[\theta'_{ij} - g_{ij}\{\theta\}] \quad (10b)$$

First the original (site) system is considered to be divided into blocks which are two lattice units on a side. The lattice site nearest the origin in each block is then taken as the block lattice site. The f_i are then taken as the sum of the parallel transport of the four nearest $\{\phi_i\}$ in each block and the parallel transport of the next nearest neighbours. In this latter case, all 12 of the shortest paths are included. An overall normalization is included so that the block scalar field remains unimodular. Specifically

$$f_a = \arg \left\{ \sum_b \exp[i(\theta_{ab} + \varphi_b)] + \sum_b \sum_{c \neq b} \exp[i(\theta_{ab} + \theta_{bc} + \varphi_c)] \right\} \quad (11)$$

where the ϕ_b and ϕ_c are all within the block.

In general, for the gauge field the blocking function has the form

$$g_{ij} = (\theta_{ia} + \theta_{ab} + \dots + \theta_{cj}) + \text{arbitrary gauge invariant function} \quad (12)$$

where the quantity in parentheses represents any path between block point i and block point j . We have chosen a form for the $\{g_{ij}\}$ which is linear in the $\{\theta_{ij}\}$. The point of this choice is that when K equals zero, the blocked action will always be quadratic in the $\{\theta'_{ij}\}$. Thus the most general form of the action when K vanishes is a sum of quadratic terms representing various closed loops. Note also that the restriction that the block link connecting two points have the same gauge transformation properties as any site path connecting these two points [an assumption implicit in Eq. (12)] does not allow an overall constant multiplier

for g_{ij} , thus eliminating many difficulties associated with linear projection operators²⁵⁾. The actual g_{ij} we use is the linear average of all the paths of the form shown in Fig. 1.

For a continuum limit of the theory to exist, there must be a fixed point of the renormalization group transformation. This fixed point will lie on a critical surface (which can subsume many continuum theories), and the triviality or non-triviality of ALL continuum theories on this surface can be determined by calculating the critical exponents (field theorists read "anomalous dimensions") at this fixed point. A non-trivial theory is an interacting renormalized theory with non-zero anomalous dimensions. All continuum theories in the domain of attraction of a fixed point are trivial or non-trivial depending only on the properties of the transformation at that one point.

In order to determine the fixed points and critical exponents of a theory, it is necessary to be able to calculate the action for the blocked system. The original formulation²²⁾ of the MCRG did not permit the block action (and hence the block renormalized couplings) to be extracted. Accordingly, we developed and repeatedly applied^{23),6)} the first practical general techniques for finding the block action. Our methods^{23),6)} are generally applicable and practical for systems with either continuous or discrete variables. [A totally different method was developed and applied to the Ising model (i.e., a discrete system) in Ref. 26)].

Our method⁶⁾ for continuous systems involved taking moments of the blocked variables, and was essentially equivalent to integration by parts. Some of the blocked variables were fixed. It was pointed out in Ref. 27) that in certain special cases (where the blocked action could be expanded in terms of known functions of a continuous variable) an improvement on this method could be made, and none of the blocked variables need be fixed. The idea⁶⁾ of integrating by parts to obtain moment equations can also be made more systematic²⁷⁾ and simplified if the quantity in square brackets either vanishes or is periodic at the limits of integration:

$$\int \mathcal{D}\theta' \frac{\partial}{\partial \theta'_{ij}} \left[\mathcal{O}\{\theta', \phi'\} \exp(-S'\{\theta', \phi'\}) \right] = 0 \quad (13)$$

for various functionals $\mathcal{O}\{\theta', \phi'\}$. The result is a set of simultaneous equations for the renormalized couplings. Specifically, we use the functionals

$$\begin{aligned}\sigma_1\{\theta'\} &\equiv \theta'_{\text{Plaq}} \\ \sigma_2\{\theta', \varphi'\} &= \sin(\varphi'_i - \varphi'_j + \theta'_{ij}) \equiv \sin y'\end{aligned}\tag{14}$$

and we have used only derivatives with respect to the $\{\theta'_{ij}\}$, although other equations can be derived by also differentiating with respect to the $\{\varphi'_i\}$.

If the blocked action is precisely of the form of Eq. (6), then the block couplings are given by

$$\beta' = \frac{\langle \cos y' \rangle \langle \theta'_{\text{Plaq}} \sin y' \rangle - 6 \langle \sin^2 y' \rangle}{(\langle \theta'_{\text{Plaq}} \sin y' \rangle)^2 - \langle \sin^2 y' \rangle \langle \theta'^2_{\text{Plaq}} \rangle}\tag{15a}$$

$$2K' = \frac{(6 - \beta' \langle \theta'^2_{\text{Plaq}} \rangle)}{\langle \theta'_{\text{Plaq}} \sin y' \rangle}\tag{15b}$$

where an average over the entire lattice is assumed.

Alternatively, an approximation can be made, and it can be assumed that the blocked system is well described by a (truncated) action of the form of Eq. (6), with effective couplings defined by Eqs. (14). This should be a good approximation provided that the blocking transformation is sufficiently comprehensive, i.e., that enough link and site fields are included in the averaging. The assumption is essentially that the fixed points of the system (and blocking transformation) under consideration are near to the surface where β and K are the only non-zero couplings. This assumption can be partially checked by calculating various expectation values in the blocked system and comparing them to a system of equivalent size, but governed entirely by the anticipated action. Another check is that the critical surfaces for the system should be reflected accurately

by the flow diagram. For the calculation discussed here, both checks were satisfied over the examined region of couplings to the 5-20% level.

It is important also to mention that an implicit assumption has been made. This is the assumption that hyperscaling holds for the non-compact Abelian Higgs model. "Hyperscaling" is essentially the notion that there is one important (correlation) length scale in the system at criticality, i.e., that the critical scaling properties of the system depend only upon this one length. The assumption of hyperscaling is built into the present formulation of the Monte Carlo renormalization group, and therefore cannot be checked within the context of this calculation.

5. - RESULTS

It remains to describe the technical details of the calculation. The analysis was performed in the unitary gauge ("setting all the $\{\phi_i\}$ to zero"). After the blocking, the $\{\phi_i\}$ were also gauge-rotated to zero by a procedure described in Ref. 7). The $\{\theta_{ij}\}$ were generated by the following procedure. First, the pure gauge part of the action was simulated using a Gaussian random number generator (this part of the action can be handled by an exact "heat bath" technique). The remaining part of the action (proportional to K) was then handled by accepting or rejecting the $\{\theta_{ij}\}$ generated in this fashion according to the algorithm of Metropolis, et al.^{28,24)}, applied only to this part of the action. The values of β' and K' were measured over 5000 iterations (following 500 equilibration iterations) on a 6^4 lattice. The machine used for the computation was the Livermore MFE Gray; for comparison, each point calculated in this fashion would have used roughly seven VAX hours.

The flow diagram resulting from the calculation is displayed in Fig. 2. The separatrix dividing the diagram into two regions (labelled "Higgs" and "Coulomb") corresponds to the phase boundary between these two regions. This phase boundary was also determined by hysteresis effects, and occurs typically at a value of K smaller (by about 20%) than that of the separatrix. Especially note the presence of the three fixed points, labelled F , G_1 and G_2 .

Consider first the fixed point G_2 . In the limit of large β , the model approaches a fixed-length ϕ^4 theory (which is an XY model). This theory should have^{19),21),29)} a second-order phase transition at $K^* = 0.15$ with a Gaussian exponent $\nu = \frac{1}{2}$. In the truncation scheme employed here,

$$\nu^{-1} = \frac{\ln \left(\frac{\partial k'}{\partial \kappa} \right) \big|_{\kappa^*, \beta^*}}{\ln 2} \quad (16a)$$

and the anomalous dimension γ_{ϕ^2} is thus³⁰⁾

$$\gamma_{\phi^2} = 2 - \nu^{-1} \quad (16b)$$

which vanishes for a Gaussian (trivial) theory. [The factor of $\ln 2$ in Eq. (16a) comes from the scale factor of the blocking.] From Fig. 3, it can be seen that a least-squares fit to the data (taken at $\beta = 5$) gives $K^* \approx 0.18$, $\nu = 0.37$, which is consistent with a Gaussian exponent.

The same analysis can be repeated at the point labelled "F" in Fig. 2. The result of a least-squares fit (see Fig. 4) yields:

$$\begin{aligned} \text{"F"} : \quad \beta^* &\approx 0.053 \\ \frac{\partial \beta'}{\partial \beta} \big|_{\beta^*} &\approx 12.4 \equiv y^d, \quad y = 3.6 \end{aligned} \quad (17)$$

These values are consistent with the expected result³¹⁾ $y = d = 4$ (in four dimensions) for a first-order transition. Note that the value obtained for the derivative (12.4) is much closer to the expected value (16) for a first-order (or "discontinuity") fixed point than to the value (4) expected for a Gaussian fixed point.

Finally, there remains the fixed point G_1 . This point is located roughly at $\beta^* = K^* = 0.3$ (the analysis described below was repeated for several other estimates of this fixed point with no appreciable changes). A line was defined by a parameter t such that along this line

$$\frac{\beta}{\beta^*} = \frac{\kappa}{\kappa^*} = t \quad (18)$$

and the results for a number of points were projected along this line using the relation

$$t'_{\text{effective}} \equiv \frac{\beta^* \beta' + \kappa^* \kappa'}{(\beta^*)^2 + (\kappa^*)^2} \quad (19)$$

The results are shown in Fig. 5, and yield the values (via a least-squares fit):

$$\begin{aligned} t^* &= 1.05 \\ \left. \frac{\partial t'}{\partial t} \right|_{t^*} &= 4.7 \\ \nu &= 0.45 \end{aligned} \quad (20)$$

which are again consistent with a Gaussian ($\nu = \frac{1}{2}$) fixed point. However, a small deviation from such a Gaussian value (indicating a non-trivial theory) cannot be entirely excluded by the present analysis. On the other hand, the value for the derivative $\partial t'/\partial t$ is considerably smaller than the value (16) expected if the phase transition were first order. Thus the evidence for the phase transition being second order is stronger than that for the fixed point being Gaussian.

6. - THE VARIABLE MAGNITUDE MODEL

A preliminary investigation was made of the variable-magnitude model. The action is given by [cf. Eq. (6)]

$$\begin{aligned} S &= -\frac{\beta}{2} \sum_{\text{Plaquettes}} (\Theta_{\text{plaq}})^2 + \sum_i V(\rho_i) \\ &\quad - 2\kappa \sum_{\langle ij \rangle} \rho_i \rho_j \cos(\varphi_i - \varphi_j + \Theta_{ij}); \\ V(\rho_i) &= \lambda (\rho_i^2 - 1)^2 \end{aligned} \quad (21)$$

where the $\{\rho_i\}$ range from zero to positive infinity, and the measure in the path integral is

$$\int \pi_i g_i d g_i \quad (22)$$

The blocking for the scalar field is a generalization of the method employed in Ref. 7). Equation (11) is replaced by

$$\begin{aligned} \varphi_a = \arg \left\{ \sum_b g_b \exp [i (\theta_{ab} + \varphi_b)] \right. \\ \left. + \sum_b \sum_{c \neq b} g_c \exp [i (\theta_{ab} + \theta_{bc} + \varphi_c)] \right\} \end{aligned} \quad (23a)$$

while

$$g'_a = \left\{ \frac{\sum_b g_b^2 + \sum_b \sum_{c \neq b} g_c^2}{\sum_b 1 + \sum_b \sum_{c \neq b} 1} \right\}^{1/2} \quad (23b)$$

The renormalized potential $V'(\rho_i)$ is determined from an (in principle) infinite set of equations generated by the identities

$$\int (\pi_i g_i d g_i) \frac{1}{g_j} \frac{d}{d g_j} [(g_j)^n e^{-S}] = 0 \quad (24)$$

which hold if n is greater than zero. The result of a preliminary investigation is that, much like the single-component model⁷⁾, the theory flows into the fixed-length model Eq. (6) very rapidly. However, a further investigation with other blocking schemes⁵⁾ should be performed.

7. - CONCLUSIONS AND DISCUSSION

Our results are consistent with the notion that in the fixed-scalar-magnitude limit of non-compact scalar QED, the Higgs-Coulomb phase transition is governed (in our blocking scheme) by three fixed points. One, labelled "F" in

Fig. 2, seems to be a first-order transition (or "discontinuity") fixed point, and the other two, labelled G_1 and G_2 , are evidently trivial Gaussian fixed points. If this interpretation is correct, there is no non-trivial continuum limit to this model. This scenario is supported (but by no means proved) by the evidence given here.

Assuming the validity of our picture, the Higgs-Coulomb phase transition in this model would be entirely second-order, with the exception of the first-order point F. The only other numerical analysis⁵⁾ of non-compact scalar QED was performed using the variable-magnitude model [Eq. (21)] for small scalar self-coupling λ . For small λ , this transition was seen to be of first order, consistent²⁹⁾ with the Coleman-Weinberg mechanism³²⁾. However, the authors could not eliminate the possibility that the transition became second-order in the fixed-length (large λ) limit.

Experience with related models, in fact, suggests that such a scenario³³⁾ (the Higgs-Coulomb transition changing order as λ increases) is quite plausible. [The situation is well reviewed in Ref. 34).] In the case of compact scalar QED, the first-order nature of the transition for small λ was first noted by Munehisa and later by others¹⁹⁾. Recent studies³⁵⁾, however, find a second-order Coulomb-Higgs phase transition in the fixed-length limit, with critical exponent $\nu \approx 0.56$. (compare the Gaussian value $\nu = \frac{1}{2}$). A phase transition which changes its order in this fashion is typically associated with tricritical behaviour, a common phenomenon¹⁸⁾ in the compact U(1) theory. A change in the order of the Higgs transition may also occur in the SU(2)-Higgs model³⁵⁾⁻³⁷⁾ as well as in a lattice superconductor^{38),39),33)}.

In conclusion, therefore, we have performed a Monte Carlo renormalization group analysis of an Abelian Higgs model with one fundamental representation scalar. No evidence for a non-trivial continuum limit was found. Several important assumptions were made (hyperscaling and universality), and the main errors are expected to arise from truncation. Further studies on this model should certainly be made. As the results obtained here have no obvious generalization to more complicated theories [for example, a new fixed point might appear if more scalar generations were included³⁸⁾] we are currently extending this analysis to the full standard model.

ACKNOWLEDGEMENTS

This study would not have been possible without the use of the Livermore MFE Cray. The authors gratefully acknowledge the assistance of M.A.B. Bég and P.K. Williams in obtaining this computer time.

This work was supported in part by the U.S. Department of Energy under contract No. DE-AC02-81ER40033B.000.

REFERENCES

- 1) P.W. Higgs, Phys. Lett. 12 (1964) 132; Phys. Rev. Lett. 13 (1964) 508;
Phys. Rev. 145 (1966) 1156;
P.W. Anderson, Phys. Rev. 130 (1963) 439;
F. Englert and R. Brout, Phys. Rev. Lett. 13 (1964) 321;
G.S. Guralnik, C.R. Hagen and T.W.B. Kibble, Phys. Rev. Lett. 13 (1964) 585.
- 2) D.J.E. Callaway and D.J. Maloof, Phys. Rev. D27 (1983) 406;
D.J.E. Callaway, Phys. Rev. D27 (1983) 2974.
- 3) S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
A. Salam, in Elementary Particle Theory: Relativistic Groups and Analyticity, (Nobel Symposium No. 8), ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.
- 4) K.G. Wilson, Phys. Rev. B4 (1971) 3184;
K.G. Wilson and J. Kogut, Physics Reports 12C (1974) 75;
R. Schrader, Phys. Rev. B14 (1976) 172;
G.A. Baker and S. Krinsky, J. Math. Phys. 18 (1977) 590.
G.A. Baker and J. Kincaid, Phys. Rev. Lett. 42 (1979) 1431.
C.M. Bender, F. Cooper, G.S. Guralnik, R. Roskies and D.H. Sharp, Phys. Rev. D23 (1981) 2976; D23 (1981) 2999;
M. Aizenmann, Phys. Rev. Lett. 47 (1981) 1; Commun. Math. Phys. 86 (1982) 1;
J. Fröhlich, Nucl. Phys. B200 [FS4] (1982) 281;
A.D. Sokal, Ann. Inst. Henri Poincaré 37 (1982) 317;
P. Hasenfratz, in Recent Developments in Quantum Field Theory, eds. J. Amjorn, B.J. Durhuus and J.L. Petersen (Elsevier, 1985).
- 5) B. Freedman, P. Smolensky and D. Weingarten, Phys. Lett. 113B (1982) 481;
C. Aragão de Carvalho, C.S. Carraciolo and J. Fröhlich, Nucl. Phys. B21 [FS7] (1983) 209;
C.B. Lang, Phys. Lett. 155B (1985) 399; Nucl. Phys. B265 [FS15] (1986) 630;
I.A. Fox and I.G. Halliday, Phys. Lett. 159B (1985) 148.
- 6) D.J.E. Callaway and R. Petronzio, Nucl. Phys. B240 [FS12] (1984) 577.
- 7) D.J.E. Callaway and R. Petronzio, Nucl. Phys. B267 (1986) 253.
- 8) I. Montvay, Phys. Lett. 150B (1985) 441; DESY preprint 85-005 (1985);
V.P. Gerdt et al., Nucl. Phys. B265 [FS15] (1986) 187; Zeit. f. Physik C29 (1985) 363.
- 9) D.J.E. Callaway, Nucl. Phys. B233 (1984) 189.
- 10) L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320;
S. Coleman, R. Jackiw and H.D. Politzer, Phys. Rev. D10 (1974) 2491.
- 11) D.J. Gross and F. Wilczek, Phys. Rev. D8 (1973) 3633;
T.P. Cheng, E. Eichten and L.-F. Li, Phys. Rev. D9 (1974) 2259.
- 12) M.A.B. Bég, C. Panagiotakopoulos and A. Sirlin, Phys. Rev. Lett. 52 (1984) 883;
R. Dashen and H. Neuberger, Phys. Rev. Lett. 50 (1983) 1897.

- 13) N. Cabibbo, L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B158 (1979) 295;
 L. Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B136 (1978) 115;
 K.S. Babu and E. Ma, Phys. Rev. D31 (1985) 2861; Zeit. f. Physik C29 (1985) 41;
 E. Ma, Phys. Rev. D31 (1985) 1143;
 A. Bovier and D. Wyler, Zurich ETH preprint (1984).
- 14) N.P. Chang, Phys. Rev. D10 (1974) 2706;
 E. Ma, Phys. Rev. D11 (1975) 322.
- 15) W.D. Toussaint and R.L. Sugar, Phys. Rev. D32 (1985) 2061.
- 16) K.G. Wilson, Phys. Rev. D10 (197) 2245;
 J. Kogut and L. Susskind, Phys. Rev. 11 (1975) 395;
 A.M. Polyakov, Nucl. Phys. B120 (1977) 429;
 T. Banks, R. Myerson and J. Kogut, Nucl. Phys. B129 (1977) 493;
 R. Savit, Phys. Rev. Lett. 39 (1977) 55;
 K. Osterwalder and E. Seiler, Ann. Phys. (N.Y.) 110 (1978) 440.
- 17) A.H. Guth, Phys. Rev. D21 (1980) 2291.
- 18) M. Creutz, L. Jacobs and C. Rebbi, Phys. Rev. D20 (1979) 1915; Phys. Rev. Lett. D20 (1979) 1915; Phys. Rev. Lett. 42 (1979) 1390;
 B. Lautrup and M. Nauenberg, Phys. Lett. 95B (1980) 63;
 T.A. De Grand and D. Toussaint, Phys. Rev. D27 (1980) 2478;
 K.J.M. Moriarty, Phys. Rev. D25 (1982) 2185;
 G. Bhanot, Nucl. Phys. B205 [FS5] (1982) 168;
 J. Jersak, T. Neuhaus and P.M. Zerwas, Phys. Lett. 133B (1983) 103; Nucl. Phys. B251 [FS13] (1985) 299;
 R. Gupta, M.A. Novotny and R. Cordery, Northeastern University preprint NUB No.2654 (1985).
- 19) K.C. Bowler et al., Phys. Lett. 104B (1981) 481;
 D.J.E. Callaway and L.J. Carson, Phys. Rev. D25 (1982) 531;
 J. Ranft, J. Kripfganz and G. Ranft, Phys. Rev. D28 (1983) 360;
 T. Muehisa and Y. Muehisa, Nucl. Phys. B215 [FS7] (1983) 508;
 Y. Muehisa, Phys. Rev. D30 (1984) 1310;
 G. Koutsoumbas, Phys. Lett. 140B (1984) 379;
 V.P. Gerdt et al., Dubna preprint E2-83-758 (1983);
 K. Jansen et al., Phys. Lett. 155B (1985) 268;
 D. Espriu and J.F. Wheeler, Nucl. Phys. B258 (1985) 101.
- 20) J.F.F. Labastida, E. Sanchez-Velasco, R.E. Shrock and P. Wills, Nucl. Phys. B264 (1986) 393.
- 21) P.R. Gerber and M.E. Fisher, Phys. Rev. B10 (1974) 4697.
- 22) S.K. Ma, Phys. Rev. Lett. 37 (1976) 461;
 R.H. Swendsen, Phys. Rev. Lett. 42 (1979) 859; Phys. Rev. B20 (1979) 2080;
 Real Space Renormalization, Topics in Current Physics, ed. Th.W. Burkhardt and J.M.J. van Leeuwen, (Springer, Berlin, 1983), Vol. 30;
 Phase Transitions, Cargèse 1980, ed. M. Lévy, J.C. Le Guillou and J. Zinn-Justin (Plenum Press, New York, 1982).
- 23) D.J.E. Callaway and R. Petronzio, Phys. Lett. 139B (1984) 189; 145B (1984) 381; 148B (1984) 445; 149B (1984) 175.

- 24) K. Binder, in Phase Transitions and Critical Phenomena, ed. C. Domb and M.S. Green, (Academic, New York, 1976), Vol. 5B.
- 25) K.G. Wilson, Rev. Mod. Phys. 47 (1975) 773;
Th. Niemeijer and J.M.J. van Leeuwen, in Phase Transitions and Critical Phenomena, op. cit., Vol. 6.
- 26) R.H. Swendsen, Phys. Rev. Lett. 52 (1984) 1165.
- 27) M. Falcioni et al., Nucl. Phys. B265 [FS15] (1986) 187.
- 28) N. Metropolis et al., J. Chem. Phys. 21 (1953) 1087.
- 29) M. Peskin, Ann. Phys. (N.Y.) 112 (1978) 122.
- 30) See, e.g.,
D. Amit, Field Theory, The Renormalization Group and Critical Phenomena, (Mc Graw-Hill, New York, 1978), Sec. 8.8;
E. Brézin, J.C. Le Guillou and J. Zinn-Justin, in Phase Trnsitions and Critical Phenomena, Vol. VI, ed. C. Domb and M.S. Green (Academic, N.Y., 1976);
S. Weinberg, Phys. Rev. D8 (1973) 3497;
L. Di Castro et al., Ann. Phys. (N.Y.) 87 (1974) 327.
- 31) B. Nienhuis and M. Nauenberg, Phys. Rev. Lett. 35 (1975) 477.
- 32) S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888.
- 33) H. Kleinert, Nuovo Cimento Lett. 35 (1982) 405; Phys. Lett. 128B (1983) 69;
San Diego preprint (1985);
- 34) J. Jersak, Aachen preprint PITHA 85/25 (1985), Review Talk given at the Workshop on Lattice Gauge Theory, Wuppertal, November 1985.
- 35) K. Jansen et al., Nucl. Phys. B265 [FS15] (1986) 187.
- 36) C.B. Lang, C. Rebbi and M. Virasoro, Phys. Lett. 104B (1981) 294;
M. Tomiya and T. Hattori, Phys. Lett. 104B (1984) 370.
- 37) J. Jersak et al., Phys. Rev. D32 (1985) 2761;
V.P. Gerdt, et al., Nucl. Phys. B265 [FS15] (1986) 145; Zeit. f. Phys. C29 (1985) 363;
W. Langguth and I. Montvay, Phys. Lett. 165B (1985) 135;
I. Montvay, Phys. Lett. 150B (1985) 441; DESY preprint 85-005 (1985);
H.G. Evertz et al., Aachen preprint PITHA 85/23 (1985).
- 38) B.I. Halperin, T.C. Lubensky and S.-K. Ma, Phys. Rev. Lett. 32 (1974) 292;
C. Dasgupta and B.I. Halperin, Phys. Rev. Lett. 47 (1981) 1556.
- 39) I.D. Lawrie, J. Phys. C15 (1982) L879;
J. Bartholomew, Phys. Rev. B28 (1983) 5378;
G. Bhanot and B.A. Freedman, Nucl. Phys. B190 [FS3] (1981) 357;
Y. Munehisa, Yamanishi preprint 85-01 (1985).

FIGURE CAPTIONS

Fig. 1: The linear average of all paths of this type gives the function g_{ij} defined in Eq. (1^ob).

Fig. 2: Schematic plot of the flow diagram for the non-compact Abelian Higgs model under our blocking transformation.

Fig. 3: Plot of K' versus K at the value $\beta = 5$ (approximating the fixed point G_2).

Fig. 4: Plot of β' versus β at $K = 1$ (approximating the fixed point F).

Fig. 5: Plot of t' versus t (approximating the fixed point G_1).

- Figure 1 -

- Figure 2 -

- Figure 3 -

- Figure 4 -

- Figure 5 -