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BROKEN N = 2 SUPER YANG-MILLS THEORIES
WITH A FINITE VACUUM ENERGY

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ABSTRACT

We show that an explicit symmetry breaking of the N = 2 ultra-violet finite super Yang-Mills theories through the addition of soft mass terms does not affect the finiteness of the vacuum energy above one loop. One-loop vacuum energy is finite provided the mass terms satisfy the sum rules

$$\sum_J (2J+1)(-1)^{2J} m_J^{2K} = 0 \quad (K = 0, 1, 2).$$

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Supersymmetry has the most remarkable property of softening the ultra-violet divergences encountered in formulating field theories of elementary particles. In particular, the $N = 4$ super Yang-Mills theory and a wide class of $N = 2$ super Yang-Mills theories have been shown to be ultra-violet convergent to all orders^{1), 2), *}. Furthermore, it is possible to add certain terms to the theory which explicitly break the supersymmetry but do not destroy the finiteness of the Green functions (in the special gauge)^{4), 5)}. In Refs. 6), we have shown that an explicit supersymmetry breaking of the $N = 4$ super Yang-Mills theory through the addition of soft mass terms does not disturb the finiteness of the vacuum energy to any order of perturbation theory provided the mass terms satisfy the additional sum rules

$$\sum_{J=0, 1/2} (2J+1) (-1)^{2J} m_J^4 = 0$$

Here we show that an explicit supersymmetry breaking of the $N = 2$ ultra-violet finite super Yang-Mills theories through the addition of soft mass terms does not disturb the finiteness of the vacuum energy above one loop. One-loop vacuum energy is finite provided the mass terms satisfy the sum rules

$$\sum_J (2J+1) (-1)^{2J} m_J^{2K} = 0, \quad K = 0, 1, 2 \quad (1)$$

An arbitrary $N = 2$ supersymmetric Yang-Mills theory can be described in terms of $N = 1$ superfields using the action²⁾

$$\begin{aligned} S = & \int d^4x d^2\theta \left[\bar{\varphi}^i (\exp(gV))_{ij} \varphi_j + \right. \\ & \left. \bar{X}_{a6} (\exp(gV^6))_b^a X_6^b + Y_{a6} (\exp(-gV^6))_b^a \bar{Y}_6^b \right] + \\ & \left(\frac{1}{64} \frac{t^2}{C_2(G)} \int d^4x d^2\theta W^a W_a + h.c. \right) + \\ & g \left(\int d^4x d^2\theta \varphi_i R_i^{6a} X_6^b Y_{a6} + h.c. \right) \end{aligned} \quad (2)$$

+ gauge-breaking + ghost + source terms.

*) Recently, a wide class of the $N = 1$ super Yang-Mills theories³⁾ has been found to be finite.

Here φ_i , X_6^b , Y_{b6} are chiral superfields transforming according to the adjoint R^6 and $\bar{R}^6$ representations of the gauge group respectively. The matrices R_{ib}^{6a} , where $i = 1, \dots, \dim G$ and $a, b = 1, \dots, \dim R^6$, are the matrix representations R^6 . We take $W_\alpha = \bar{D}^2 [\exp(-gV) D_\alpha \exp(gV)]$, where $V = V_k T_k$ and $(T_k)_{ij} = -f_{kij}$. The group invariants $C_2(G)$, $C_2(R^6)$ and $T(R^6)$ are defined in the usual fashion, i.e.,

$$\begin{aligned} f_{ijk} f_{elk} &= C_2(G) \delta_{ie}, \\ R_{ib}^{6a} R_{ea}^{6b} &= T(R^6) \delta_{ie}, \\ R_{ib}^{6a} R_{ic}^{6b} &= C_2(R^6) \delta_c^a \end{aligned}$$

The fields X_6 and Y_6 together describe an $N = 2$ hypermultiplet. The hypermultiplet is coupled to an $N = 2$ supersymmetric Yang-Mills multiplet (SYM). Such a theory is always finite above one loop and is finite at one loop provided²⁾

$$C_2(G) = \sum_6 T(R^6)$$

The special case with only one representation, the adjoint, reproduces the fact that $N = 4$ SYM is finite.

As has been shown in Ref. 5), one can add an explicit $N = 1$ supersymmetry breaking term which does not affect the finiteness of the Green functions (in a specific gauge).

Let us consider the following term⁵⁾

$$\Delta S = \int d^4x [\Delta \mathcal{L}_1 + \Delta \mathcal{L}_2 + \Delta \mathcal{L}_3] \quad (3)$$

Here

$$\Delta \mathcal{L}_1 = -\frac{m}{2} \int d^2\theta \varphi_i \varphi_i - \mu_6 \int d^2\theta X_6^a Y_{a6} + h.c., \quad (4)$$

$$\Delta \mathcal{L}_2 = \int d^2\theta (\underline{L}_1 \varphi_i \varphi_i + \underline{L}_{26} X_6^a Y_{a6}) + h.c. = \quad (5)$$

$$L_1 A_i A_i + L_{26} x_6^a y_{a6} + h.c., \quad (5)$$

cont.

$$\underline{L}_1 = L_1 \theta^2, \quad \underline{L}_{26} = L_{26} \theta^2,$$

$$\begin{aligned} \Delta \mathcal{L}_3 = \int d^4 \theta [\underline{U}_1 \bar{\varphi}^i (\exp(gV))_{ij} \varphi_j + \\ \underline{U}_{26} \bar{X}_{a6} (\exp(gV^6))_b^a X_6^b + \underline{U}_{36} Y_{a6} \\ (\exp(-gV^6))_b^a \bar{Y}_6^b] = U_1 A^i A_i + U_{26} x_{a6} x_6^a \\ + U_{36} y_{a6} y_6^a, \quad \underline{U}_1 = \theta^2 \bar{\theta}^2 U_1, \quad \underline{U}_{26} = \theta^2 \bar{\theta}^2 U_{26}, \end{aligned} \quad (6)$$

where $U_1 + U_{26} + U_{36} = 0$ for each representation R^6 . Note that the Lagrangian $\Delta \mathcal{L}_1$ preserves the $N = 1$ supersymmetry. The radiative corrections to the effective action have the form⁷⁾

$$\Delta S = \int d^4 x d^4 \theta F(\dots) \quad (7)$$

Using the formula (7) and the standard counting rules, we obtain that the ultra-violet divergences of the vacuum energy are represented in the form

$$\begin{aligned} \Delta E = \int d^4 \theta C_i \underline{U}_i + \int d^4 \theta C_{ij} \underline{U}_i \bar{D}^2 D^2 \underline{U}_j \\ + \int K_{ii} \bar{\underline{L}}_i \underline{L}_i d^4 \theta + \int d^4 \theta P_{ij} m_j^2 \underline{U}_i, \end{aligned} \quad (8)$$

where

$$\begin{aligned} \underline{U}_i &= (\underline{U}_1, \underline{U}_{26}, \underline{U}_{36}), \\ \underline{L}_i &= (\underline{L}_1, \underline{L}_{26}), \\ m_i &= (m, \mu_6) \end{aligned}$$

and $C_i, C_{ij}, K_{ii}, P_{ij}$ are ultra-violet divergent coefficients. Note that possible ultra-violet divergent terms $\int \underline{U}_i D^2 \underline{L}_j d^4 \theta$ are equal to zero due to the R-

invariance of the action (2).

According to the standard counting rules, the coefficients C_i are quadratically divergent, while the other coefficients C_{ij} , K_{ii} , P_{ij} have logarithmic divergences. The coefficients C_i , C_{ij} , K_{ii} , P_{ij} are expressed in terms of the Green functions of $N = 2$ SYM. For instance:

$$C_i = - \int \langle A^i(x) A_i(x) \rangle_{N=2} d^4x, \quad (9)$$

$$K_{ii} = -i \int \langle A_i(x) A_i(x) (A_i(y) A_i(y))^+ \rangle_{N=2} d^4x d^4y \quad (10)$$

Our main observation is that due to the finiteness of $N = 2$ SYM, there are no ultra-violet subdivergences in the calculation of the coefficients C_i , C_{ij} , K_{ii} and P_{ij} . Therefore, the structure of the ultra-violet divergences coincides with the corresponding one-loop structure up to the coefficient, namely

$$C_i \sim \Lambda^2 C_i(g) \int d^4x, \quad (11a)$$

$$C_{ij} \sim \ln \Lambda^2 \bar{C}_{ij}(g) \int d^4x, \quad (11b)$$

$$K_{ii} \sim \ln \Lambda^2 \bar{K}_{ii}(g) \int d^4x, \quad (11c)$$

$$P_{ij} \sim \ln \Lambda^2 \bar{P}_{ij}(g) \int d^4x, \quad (11d)$$

where Λ is a cut-off parameter. One can also check that fact by using the renormalization group arguments. For instance, let us consider the coefficient K_{11} . The anomalous dimension of the operator $\varphi_i(x, \theta) \varphi_i(x, \theta)$ is equal to zero⁵⁾. The renormalization group equation for the corresponding D-function which is free from trivial ultra-violet divergences is

$$\left(\mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} + 2f(g) \right) \mathcal{D} = 0, \quad (12)$$

$$\mathcal{D}(p^2, \dots) = p^2 \frac{dK}{dp^2}, \quad (13)$$

$$K = \int \langle A_i(x) A_i(x) \rangle,$$

$$(A_i(0) A_i(0))^+ \rangle_{N=2} e^{ipx} d^4x \quad (14)$$

In our case $\beta(g) = \gamma(g) = 0$. As a consequence of a trivial dimensional analysis, we obtain that $\mathcal{D}(p^2, g)$ does not depend on p^2

$$\mathcal{D}(p^2, g) = \mathcal{D}(g) \quad (15)$$

Using the formulae (10) and (12)-(15), we reproduce the formula (11c). Note that we can also eliminate a possible dependence of the ultra-violet divergent coefficients (11a)-(11d) on the coupling constant g by the finite renormalization of the parameters $\underline{U}_i$, $\underline{L}_i$, m_i and the superfields ϕ_i , X_6^b , Y_6^b

$$\begin{aligned} \underline{U}_i &\rightarrow Z_{ij}^u(g) \underline{U}_j, \\ \underline{L}_i &\rightarrow Z_{ij}^L(g) \underline{L}_j, \\ \varphi_i &\rightarrow Z^{\varphi}(g) \varphi_i, \\ X_6^b &\rightarrow Z_{66'}^b(g) X_{6'}^b, \\ Y_6^b &\rightarrow Z_{66'}^{\bar{b}}(g) Y_{6'}^b, \\ m_i &\rightarrow Z_{ij}^m(g) m_j \end{aligned} \quad (16)$$

Therefore, in our renormalization scheme ultra-violet divergences of the vacuum energy arise only at the one-loop level. The expression for the one-loop vacuum energy density has the well-known form

$$\varepsilon = \frac{E}{\int d^4x} = \frac{1}{2(2\pi)^3} \sum_J \int \frac{(-1)^{2J} (2J+1)}{\sqrt{\vec{p}^2 + m_J^2}} d^3\vec{p} \quad (17)$$

Here m_J is a mass of a particle with a spin J . The integral (17) is ultra-violet finite and it is equal to

$$\varepsilon = \frac{1}{64\pi^2} \sum_J (-1)^{2J} (2J+1) m_J^4 \ln m_J^2 \quad (18)$$

provided the masses satisfy the sum rules

$$\sum_J (2J+1) (-1)^{2J} m_J^{2K} = 0, \quad K = 0, 1, 2 \quad (19)$$

Note that the sum rule (19) with $K = 0$ is a trivial consequence of the supersymmetry.

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