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WORLD-SHEET SYMMETRIES OF THE HETEROTIC STRING
IN $(10 + 496) + 16$ - DIMENSIONAL SUPERSPACE

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IN $(10 + 496) + 16$ - DIMENSIONAL SUPERSPACE

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ABSTRACT

A geometrical formulation of the heterotic string theory in superspace with $10 + 496$ bosonic and 16 fermionic coordinates is presented. Minimal constraints on this superspace are found which provide the local fermionic and two types of bosonic world-sheet symmetries of the heterotic string. The relation between this superspace and the anomaly-free $d = 10$ Einstein-Yang-Mills supergravity is discussed.

The existence of a self-dual world-sheet torsion, which is required by the world-sheet symmetries, explains the geometrical origin of the absence of right movers in the gauge degrees of freedom of the heterotic string. The action of the heterotic string in the full curved background has a particularly simple form :

$$S = \frac{1}{2} \int_W E_A{}^\mu * E^\mu{}_A - \int_{\Sigma_3} E_A{}^\mu \tilde{\pi}^\mu{}_A .$$

I. The heterotic string theory discovered by Gross, Harvey, Martinec and Rohm [1] is considered at present as a leading candidate for the role of a unified theory of all fundamental interactions. However, the existing formulations of the heterotic string theory are not quite satisfactory. The geometrical meaning of this theory is rather obscure and

there is no natural formulation of this theory in full curved superspace including Yang-Mills interactions.

The purpose of the present paper is to suggest a geometrical formulation of the heterotic string theory. The action of the heterotic string in the full curved superspace background, which will be suggested in the present paper, is a generalization of the point superparticle action of Brink and Schwarz [2], of covariant superstring action found by Green and Schwarz [3] and of curved space heterotic string action (with gauge degrees of freedom suppressed) found by Witten [4].

The essential feature of all these actions is the existence of the fermionic world-sheet symmetry found by Siegel [5] and of bosonic ones, found in [3]. In the flat superspace background these symmetries permit the choice of a light-cone gauge which contains only physical degrees of freedom of the string. A bosonic symmetry of another type was also discovered by Siegel [5]. He has considered the Lorentz-covariant action for a self-dual antisymmetric tensor gauge field and has shown that the Lagrange multiplier field is not dynamical.

We start by reformulating Witten's action [4] ^{*)} using the scale invariant supergeometry of Nilsson, Kallosh and Tollsten [8]. This will make it possible to eliminate $e^4 \Phi$ from the first term in action [4] and to suggest a generalization of the string action including gauge degrees of freedom.

Another important point is the following. Our action will be represented as a generalized σ -model, where the Wess-Zumino

^{*)} This action was generalized to $N=2$ type II B superstring in [9].

term is connected with the torsion of the curved superspace, similarly to what was found in the flat superspace by Henneaux, Mezincescu, Curtright and Zachos [9]. The basic point here is the possibility to reformulate the superspace theory [8] in such a way that there is no need to introduce an independent 3-form H : It is constructed from torsion as follows *)

$$H = \frac{1}{3!} E^A E^B E^C H_{CBA} = E^A \tilde{T}_{,A} = \frac{1}{2!} E^A E^B E^C T_{BC,A} \quad (1)$$

The fermionic world-sheet symmetry of this action constrains the background geometry, and as already noted by Witten [4] the corresponding constraints are weaker than that of the on-shell $N=1$ classical supergravity. However, in our formulation, since we do not start with 2-dimensional integral containing the B_{MN} - field [4], but with 3-dimensional one containing H_{ABC} (or $T_{BA,C}$), it appears that the form $H = E^A \tilde{T}_{,A}$ as distinct from $H = dB$ is not closed automatically. The world-sheet symmetry requires only $(A = a, \hat{a}, \alpha)$

$$Q_{ABC\delta} = Q_{ABC\hat{a}} = 0, \quad (2)$$

where the 4-form Q is

$$Q = dH = d(E_A T^A) \quad (3)$$

and puts no constraints on Q_{abcd} . This means that the Green-Schwarz mechanism of elimination of anomalies [11] which requires that

*) The analogous modification was realized in $d=11$ supergravity [10].

$$Q_{abcd} \sim \text{tr } R_{ab} R_{cd} \quad (4)$$

is already possible in our formulation, i.e. the constraints on background geometry allow the higher derivative terms (4).

For the introduction of gauge degrees of freedom in our approach with WZ term constructed from torsion it is natural to use the group manifold of the theory, i.e. the 496 bosonic coordinates, the tangent space torsion being the structure constants of $E_8 \times E_8$ (or $SO(32)$)*).

We have found that the preservation of fermionic world-sheet symmetry when the gauge degrees of freedom are included in the way described above puts the constraints not only on the supergeometry but also on the world-sheet geometry, where the self-dual torsion must appear. This explains the geometrical nature of the heterotic string constraints, the generalization of Siegel's transformations rules for the Lagrange multiplier [6] appearing as the standard two-bein transformation rules in the two-dimensional space with torsion.

2. Thus, our first step is to present the heterotic string action in the curved superspace background (gauge degrees of freedom suppressed) as follows:

$$S = \frac{1}{2} \int d^2\sigma \sqrt{-g} g^{mn} E_m^a E_n^b \gamma_{ab} - \int_{X_3} H, \quad (5)$$

where the world-sheet coordinates σ^m are (τ, σ) and $E_m^A = \partial_m Z^M E_M^A(z)$; $Z^M = (x^m, \theta^\mu)$; $A = (a, \alpha)$; $m, a = 0, 1, \dots, 9$; $\mu, \alpha = 1, \dots, 16$. The second term in the action, the so called WZ term is **)

*) See, however, [12] where the additional coordinates are two-dimensional fermions Ψ^I , vectors of $SO(32)$.

**) The reasonable definition of the integral over the third dimension as well as boundary conditions, providing the world-sheet symmetries, are contained in [13].

$$\int_{X_3} H = \frac{1}{3!} \int_{X_3} E^A E^B E^C H_{CBA} ,$$

the form $E^A = dZ^M E_M^A = d\sigma^i \partial_i Z^M E_M^A$ ($i=0,1,2$) being restricted to 3-manifold X_3 . The action (5) coincides with the action [4] for $H = dB$, for arbitrary H (5) generalizes it.

It is convenient to introduce the tangent space in two dimensions, i.e. to introduce the zweibeins $e_{\underline{a}}^{\underline{m}} \gamma_{\underline{a}\underline{b}}^{\underline{n}} = g^{\underline{m}\underline{n}}$, $\underline{a} = (0, 1)$. In the light-cone variables

$\sigma^{\pm} = \frac{1}{\sqrt{2}}(\sigma \pm \tau)$ we have $e_+^{\underline{m}}, e_-^{\underline{m}}, \gamma_{+-} = 1$. The action takes the form

$$S = \int d^2\sigma e E_+^{\underline{a}} E_-^{\underline{b}} \gamma_{\underline{a}\underline{b}} - \frac{1}{2} \int e^{\underline{a}\underline{b}} d\sigma_2 E_{\underline{a}}^A E_{\underline{b}}^B E_2^C H_{CBA} , \quad (6)$$

where

$$E^A = e^{\underline{a}} E_{\underline{a}}^A + d\sigma^2 E_2^A, E_{\pm}^A = e_{\pm}^{\underline{m}} E_{\underline{m}}^A, e^{\underline{a}} = d\sigma^{\underline{m}} e_{\underline{m}}^{\underline{a}}, \underline{a} = (+, -). \quad (7)$$

Under the fermionic local transformations [4,5] with a self-dual Majorana-Weyl spinor $K_{+\alpha}(\sigma^+, \sigma^-)$ as a parameter,

$$\delta E^{\alpha} \equiv \delta Z^M E_M^{\alpha} = \frac{1}{2} K_{+\beta} (\gamma^{\alpha})^{\beta\alpha} E_-^{\alpha}, \quad (8)$$

the variation of the action (6) takes the form [13]

$$\delta_E S = \int d^2\sigma e \delta E^{\alpha} \left\{ - (E_+^{\underline{a}} E_-^{\underline{b}} + E_+^{\underline{a}} E_+^{\underline{b}}) T_{B\alpha, \underline{a}} - E_+^{\underline{b}} E_-^{\underline{c}} H_{CB\alpha} + \int d\sigma_2 E_+^{\underline{c}} E_-^{\underline{b}} E_2^{\underline{d}} Q_{DBC\alpha} \right\}. \quad (9)$$

Since in eqs. (5), (6) the 3-form H a priori is not supposed to be equal to dB , we observe that the necessary conditions for the possibility to compensate (9) by the zweibein transformations [4,5, 13]

$$\delta_e S = \int d^2\sigma e (\delta e_+^{\underline{m}}) e_{\underline{m}+} E_-^{\underline{a}} E_-^{\underline{b}} \gamma_{\underline{a}\underline{b}}, \delta e_-^{\underline{m}} = 0 \quad (10)$$

are the following

$$Q_{ABC} \delta = 0, \quad (11)$$

$$T_{\alpha\beta,\gamma} = H_{\alpha\beta\gamma} = (\gamma_\beta)_{\alpha\gamma} T_\beta^\gamma, \quad (12)$$

$$T_{\beta\alpha,\gamma} + T_{\gamma\alpha,\beta} + H_{\beta\gamma\alpha} = 2(\gamma_\beta)_{\alpha\gamma} T_\alpha^\gamma. \quad (13)$$

All these conditions are satisfied in the scale-invariant superspace [8] corresponding to the on-shell $d = 10$ $N = 1$ supergravity. However, as already explained above, eqs. (11)-(13) contain also the possibility to describe the anomaly-free theory.

3. Our purpose in what follows is to find the description of the superspace [10, 14] without introduction of an independent 3-form H into the theory, but by constructing it from the geometrical objects E^A and Ω_B^A . We start with the redefinition of the off-shell superspace [14].

$$\begin{aligned} \Omega_B^A &= \Omega_B^A + \Delta_B^A, \quad T'^A = T^A + E^B \Delta_B^A, \\ R_B^A &= R_B^A + (d\Delta + \Omega\Delta + \Delta\Omega + \Delta\Delta)_B^A, \end{aligned} \quad (14)$$

i.e. we make a simultaneous modification of connections, torsions and curvatures. We would like to get, according to (1), (12), (13),

$$\begin{aligned} H_{\alpha\beta\gamma} &= T'_{\alpha\beta,\gamma} = T_{\alpha\beta,\gamma}, \\ H_{\beta\gamma\alpha} &= T'_{\alpha\beta,\gamma} - T'_{\alpha\gamma,\beta}, \\ T'_{\alpha\gamma,\beta} &= (\gamma_\alpha)_{\gamma\beta} T_\beta^\gamma, \\ H_{\alpha\beta\gamma} &= T'_{\beta\alpha,\gamma} - T'_{\gamma\alpha,\beta} + T'_{\beta\gamma,\alpha}. \end{aligned} \quad (15)$$

This would correspond exactly to eq. (1). Note that T^α is irrelevant since due to the Majorana-Weyl properties $T_{,\alpha} = 0$ and

$$E^A \tilde{T}_{,A} = E^a \tilde{T}_{,a} = \frac{1}{2} E^a E^B E^C T_{BC,a} \quad (16)$$

The relation of the modified theory with constraints (15) to the original version [14] is as follows ^{x)}. Since the Bianchi identity $DT^A + E^B R_B^A = 0$ is satisfied [14], for the primed quantities we have

$$D'T'^A + E^B R_B'^A = (DE^B - T^B) \Delta_B^A, \quad (17)$$

where

$$DE^A - T^A = X^A, \quad DX^A = 0, \quad (18)$$

and

$$D'(T'^A - X^A) + E^B R_B'^A = 0. \quad (19)$$

Eq. (19) shows that the fact, that the solution of Bianchi identity $DT^A + E^B R_B^A = 0$ was already found in [14], guarantees that the primed quantities (with modified constraints) satisfy the corresponding identities (19). In the simplest case that the covariantly constant form X^A does not exist, we get the normal relation between T'^A and $R_B'^A$; if it exists (which is unlikely), the role of new torsion is played by $(T' - X)^A$.

Thus we conclude that the structure of modified superspace (14), (15) gives us the possibility to introduce the 3-form H as in eq. (1), using the necessary connections Δ_{α}^a , $\Delta_{\alpha c}^b$ in eqs. (14).

Until now we have tried to convince the reader that from the point of view of superspace it is equivalent to use either H or $E^a \tilde{T}_{,a}$. However the next part of the paper will show immediately that from the point of view of the string theory the second choice is more natural.

x) The detailed investigation of this problem is now in progress.

4. Using the arguments given above we suggest to write the string action as follows.

$$S = \int d^2\sigma e E_+^a E_-^b \eta_{ab} - \frac{1}{2} \int_{x_3} E^A E^B E^C T_{BC,A} \quad (20)$$

Let us introduce the form $E^A = d\sigma^M \partial_M Z^A E_M^A = dZ^M E_M^A = e^a E_a^A$ restricted to the world-sheet W ($M, a = +, -$), and the dual form

$$*E^a = e^b \varepsilon_{ba} E^{\theta a} \quad (21)$$

In these terms the action takes a simple form

$$S = \frac{1}{2} \int_W E_a * E^a - \int_{x_3} E_a \tilde{T}^a, \quad (22)$$

where the "twisted" torsion $\tilde{T}^a$ is defined as

$$\tilde{T}^a = \frac{1}{2} E^B E^C T_{BC}^a \quad (23)$$

The variation of the action (20), (22) under the transformations $\delta E^A = \delta Z^M E_M^A$ now takes the following form ($Q_{ABC} = 0$ is assumed)

$$\delta S = -2 \int d^2\sigma e \delta E^a \left\{ E_+^a E_-^b T_{[B\alpha],a} + E_-^a E_+^b T_{(B\alpha),a} \right\} \quad (24)$$

[], () means antisymmetric and symmetric combinations with weight one. Eq. (24) then reduces to

$$\delta S = -2 \int d^2\sigma e \delta E^a \left\{ E_+^a E_-^b T_{b\alpha,a} + E_-^a E_+^b T_{\beta\alpha,a} \right\} \quad (25)$$

If

$$T_{b\alpha,a} = (\gamma_b)_{\alpha\beta} T_a^\beta, \quad T_{\beta\alpha,a} = (\gamma_a)_{\alpha\gamma} T_\beta^\gamma \quad (26)$$

we get

$$\delta_E S = -2 \int d^2\sigma e \delta E^a E_-^b (\gamma_b)_{\alpha\beta} T_+^\beta \quad (27)$$

where

$$T_+^\beta \equiv E_+^A T_A^\beta \quad (28)$$

For fermionic k-symmetry transformations (8) eq. (27)

becomes

$$\delta_E S = -\kappa_{+\beta} (\gamma_\alpha \gamma_\beta)^\beta T_+^\alpha E_-^a E_-^b \gamma_{ab} = -\kappa_{+\beta} T_+^\alpha E_-^a E_-^b \gamma_{ab} = 0. \quad (29)$$

This transformation can be compensated by the zweibein gauge transformation (10) of the form

$$\delta e_+^m e_{m+} = \kappa_{+\beta} T_+^\beta, \quad \delta e_-^m = 0. \quad (30)$$

Eq. (27) shows also that the bosonic type gauge transformations

$$[2,13] \quad \delta E^\alpha = \gamma_- T_+^\alpha \quad (31)$$

with the anti-self-dual vector $\gamma_a = -\varepsilon_{ab} \gamma^b$ as a parameter, is the symmetry of the action (20), (22).

Now everything is prepared for the construction of the action of the heterotic string in the full background.

5. The action which defines the consistent coupling of the heterotic string to background supergravity and Yang-Mills fields appears to be the simple generalisation of eq. (22):

$$S = \frac{1}{2} \int_W E_A * E^A - \int_{X_3} E_A \tilde{T}^A, \quad (32)$$

where the form $E^A = dZ^M E_M^A$ in the first term is restricted to the world-sheet W , i.e. $E^A = d\sigma^m \partial_m Z^M E_M^A$,

($m=0,1$) and in the second term it is restricted to X_3 ,

i.e. $E^A = d\sigma^i \partial_i Z^M E_M^A$ ($i=0,1,2$). The following notations are used here and in what follows: $Z^M = (X^m, X^{\hat{m}}, \theta^A)$;

$A = (a, \hat{a}, \alpha)$; $\hat{m}, \hat{a} = 10, \dots, 505$; $\mathcal{M} = (m, \hat{m})$;

$\mathcal{A} = (a, \hat{a})$; $\mathcal{M}, \mathcal{A} = 0, 1, \dots, 9, 10, \dots, 505$.

Thus $Z^M(\sigma)$ are the coordinates of space with 10+496 bosonic and 16 fermionic coordinates, and we have introduced the script letters $\mathcal{M} = m, \hat{m}; \mathcal{A} = a, \hat{a}$ for all bosonic coordinates.

There is nothing new in the definition of WZ term in the action (32) as compared with (22) except that we have now summation over $\mathcal{A}$ instead of a and over $\mathcal{M}$ instead of m .

The definition of the dual form $*E^a$ is given by eq. (21), and the definition of the dual form $*E^{\hat{a}}$ introduces into the theory the new two-bein form $v^a = d\sigma^m v_m^a$, so that

$$*E^{\hat{a}} = v^a \varepsilon_{a\beta} E^{\beta\hat{a}} \quad (33)$$

and the form E^A restricted to W is

$$E^A = d\sigma^m \partial_m Z^M E_M^A = e^a E_a^A \delta_a^A + \quad (34)$$

$$+ e^a E_a^\alpha \delta_\alpha^A + v^a E_a^{\hat{a}} \delta_{\hat{a}}^A.$$

The difference between the already existing in the world-sheet zweibein form e^a and the new one v^a is the self-dual form λ^a .

$$v^a = e^a + \lambda^a, \quad \lambda^a = \varepsilon^{a\beta} \lambda_\beta, \quad (35)$$

which defines a world-sheet self-dual torsion form

$$Dv^a = D\lambda^a = t^a, \quad De^a = 0, \quad t^a = \varepsilon^{a\beta} t_\beta, \quad (36)$$

the structure group of the tangent space being the Lorentz group $SO(1,1)$.

It will be clear later that: i) without introducing the new zweibein we are unable to preserve the fermionic local symmetry (30) and that ii) the requirement that this new zweibein does not bring into the theory the non-physical degree of freedom leads to the appearance of a new type of bosonic world-sheet symmetry which takes place only when the world-sheet

tensor form t^a is self-dual. So we simply start with a self-dual form λ^a .

The useful properties of the zweibeins are the following

$$\begin{aligned} e_m^a e_a^n &= \delta_m^n, \quad e_a^m e_m^n \gamma^{ab} = g^{mn}, \quad v_a^m = e_a^m - \lambda_a^m, \\ v_m^a v_a^n &= \delta_m^n, \quad v_a^m v_m^n \gamma^{ab} = g^{mn} - \lambda^{mn}, \quad \lambda^{mn} = e_a^m \lambda_a^n + \lambda_a^m e_a^n, \\ \varepsilon_{ab} v^a v^b &= \varepsilon_{ab} e^a e^b \rightarrow e = \det e_m^a = v = \det v_m^a. \end{aligned} \quad (37)$$

The tensor $\lambda_{ab} = \lambda_a^m e_{mb} + e_a^m \lambda_{mb}$ is twice self-dual, symmetric and traceless.

6. We consider first the symmetries of the target space.

The general covariant transformations of the superspace Z^M with 506 bosonic and 16 fermionic coordinates, under which the action of the heterotic string (32) is invariant, are

$$\begin{aligned} \delta Z^M &= \xi^M = (\xi^\mu, \xi^M), \quad \delta \partial_m Z^M = \partial_m Z^N \partial_N \xi^M, \\ \delta E_M^A &= -\partial_M \xi^N E_N^A. \end{aligned} \quad (38)$$

The parameter of transformations $\xi^M(Z)$ has 506 bosonic and 16 fermionic components and is an arbitrary function of all Z^M .

The structure group of the tangent space characterized by E^A, Ω_B^A , is defined as follows: The connection takes value in the algebra of the Lorentz group for a, b and α, β and of the group $\hat{G} = E_8 \times E_8$ (or $SO(32)$) for $\hat{a}, \hat{b}$.

Thus the action of the heterotic string (32) is invariant under the transformations of the structure group $SO(1,9) \times E_8 \times E_8$ (or $SO(1,9) \times SO(32)$) and this symmetry is connected with parameters $\Lambda^{[ab]}(Z)$, $\Lambda_{\alpha\beta}^{\alpha}(Z) = \frac{1}{4}(\gamma_{\alpha\beta}^{\alpha})^{\alpha} \Lambda^{\alpha\beta}$ and $\Lambda_{[\hat{a}\hat{b}]} = \int \hat{a} \hat{b} \hat{c} \Lambda^{\hat{c}}$, where

f_{abc} are the structure constants of the group $\hat{G}$. The vielbein form E^A and the torsion form $T^A = DE^A$, as well as the "twisted" torsion form $\hat{T}^A$ transform covariantly under the action of the structure group.

Thus we see that just as in the usual (x^m, θ^M) superspace, where the action of the coordinate transformation group does not distinguish between x^m and θ^M and the parameter of the transformations F^M is an arbitrary function of Z^M , in our case now F^M is an arbitrary function of all types of $Z^M = (x^m, \hat{x}^{\hat{m}}, \theta^M)$. However the action of the structure group $SO(1,9)$ in the usual case definitely distinguishes between E^a and E^α , T^a and T^α , they are independent, which makes it possible to put different constraints on the vector and spinor components of torsion. This property is a consequence of the fact that $\Omega_\alpha^a = \Omega_a^\alpha = 0$. Since now the structure group is a direct product of $SO(1,9)$ and $E_8 \times E_8$, $\Omega_{\hat{a}}^a = \Omega_a^\alpha = \Omega_\alpha^{\hat{a}} = \Omega_{\hat{a}}^\alpha = 0$ and the tangent space objects $E^a, E^{\hat{a}}, E^\alpha, \hat{T}^a, T^{\hat{a}}, T^\alpha$ transform independently, so that the different constraints on torsion components with a or $\hat{a}$ or α can be taken. This means that the structure group as distinct from the coordinate group differentiate between 10 bosonic, 496 bosonic and 16 fermionic components of the tangent space, which is the reason of our notation $(10+496)+16$ for the corresponding space.

It becomes clear now that the second term in the action (32), which contains the tangent space torsion $T_{[BC,A]}$, distinguishes between 10 bosonic coordinates with $T_{[\hat{c},a]}$ and 496 bosonic coordinate with $T_{[\hat{c},\hat{a}]}$. In the last case

we have a compact space since $T[\hat{a}\hat{b}, \hat{c}]$ are the structure constants. The 10-dimensional part $T_{[ab,c]}$ in general has no constant part, in which case all 10 dimensions are non-compact. However, the theory contains also the possibility (with the breakdown of $SO(1,9)$ group to $SO(1,3) \times SO(6)$) that the 6-dimensional part of $T_{[ab,c]}$ acquires a constant part and only 4 dimensions remain non-compact.

7. The world-sheet symmetries of the heterotic string action (32) are now under investigation. There are three types of them: the fermionic local symmetry and two types of bosonic ones. The second one was absent in eq.(22). First we perform the variation of Z^M resulting in some $\delta E^\alpha = \delta Z^M E_M^\alpha$, $\delta E^{\hat{a}} = \delta E^{\hat{a}} = 0$. The variation of the action (32) takes the form which is a straightforward generalization of eq. (24), Q_{ABCS} is assumed to vanish, $A=a, \hat{a}, \alpha$.

$$\delta_E S = -2 \int d^2\sigma e \delta E^\alpha (E_+^A E_-^B T_{[B\alpha], A} + E_-^A E_+^B T_{(B\alpha), A}). \quad (39)$$

One should not forget the new definition of E_+^A which means that $E_+^a = e_+^m E_m^a$, $E_+^{\hat{a}} = v_+^m E_-^{\hat{a}} = (e_+^m - \lambda_+^m) E_-^{\hat{a}}$. The terms which now survive in (39) are

$$\delta S = -2 \int d^2\sigma e \delta E^\alpha (E_+^A E_-^B T_{B\alpha, A} + E_-^A E_+^B T_{B\alpha, A}). \quad (40)$$

If in addition to eq. (26) we have

$$T_{\alpha\beta, a} = T_{\hat{\alpha}\hat{\beta}, A} = 0, \quad T_{\beta\alpha, A} = (\gamma_\beta)_\alpha T_A^\beta, \quad (41)$$

then the variation of the action (40) takes the form coinciding exactly with eq. (27) where the definition of T_+^β in (28) now includes the summation over the full tangent space

$A=(\alpha,\hat{\alpha},\alpha)$. Under the fermionic k-symmetry transformations (8) the variation of (32) becomes exactly as in eq. (29), and we come to the crucial point that to be able to compensate this variation we were obliged to introduce the new type of zweibein. Then under the transformations (30) supplemented by

$$\delta v_+^m = \delta v_-^m = 0. \quad (42)$$

we find that k-symmetry exists.

The first type bosonic symmetry (31) remains the symmetry of the action (32), $T_+^\beta = E_+^A T_A^\beta$.

The second type of bosonic gauge symmetry is connected with the transformations

$$\delta E^{\hat{a}} \equiv \delta Z^M E_M^{\hat{a}} = \zeta_+ E_-^{\hat{a}}, \quad (43)$$

where $\zeta_+(\sigma)$ is the only non-vanishing component of the self-dual vector $\zeta_a(\sigma) = \varepsilon_{ab} \zeta^b(\sigma)$. The variation of the action (32) under these transformations is equal

$$\delta_E S = \int d^2\sigma e \delta E^{\hat{a}} \left\{ (-2\nabla_+ E_-^{\hat{a}} + t_{-+}^a E_a^{\hat{a}} - 2E_+^A E_-^B T_{BA}^{\hat{a}}) + (d\sigma_2 E_+^B E_-^C Q_{DCB\hat{a}}) \right\}. \quad (44)$$

The necessary conditions for $\delta_E S$ in eqs. (43), (44) to be compensated by

$$\delta_v S = (\delta v_+^m) v_{m+} E_-^{\hat{a}} E_-^{\hat{b}} \gamma_{\hat{a}\hat{b}}, \quad \delta e_+^m = \delta e_-^m = \delta v_-^m = 0, \quad (45)$$

are of two kinds. The first one concerns the world-sheet torsion and takes the form

$$t^+ = 0. \quad (46)$$

This condition is satisfied in our formulation according to eq. (36). The second one concerns the superspace torsion:

$$Q_{DCB\hat{a}} = 0, \quad Q = d(E_{\mathcal{A}} \tilde{T}^{\mathcal{A}}), \quad (47)$$

$$T_{\alpha\hat{a},\mathcal{A}} = T_{a\hat{a},\mathcal{A}} = 0. \quad (48)$$

The necessary variation of σ_+^m is the standard gauge variation of the zweibein

$$(\delta\sigma_+^m)\sigma_{m+} = -\nabla_+\zeta_+ - \zeta_-^+ t_{-+,+}. \quad (49)$$

Thus all types of the world-sheet symmetries of the action (32) of the heterotic string in the general background are exhibited. They require the presence of the self-dual form λ^{α} . This explains the constraints excluding the right movers from the gauge degrees of freedom of the heterotic string in the flat background [3], or, alternatively, the existence of the Lagrange multiplier. It seems that just this property of the heterotic string was one of the reasons for the authors of this theory to call its formulation "somewhat awkward and contrived" [3]. Now this property of the heterotic string becomes a natural consequence of the geometrical structure of the theory. If instead we would use the two-dimensional geometry based only on one zweibein $e_{\pm}^m$, then the action (32)

$$S = \int d^2\sigma e E_+^{\mathcal{A}} E_-^{\mathcal{B}} \gamma_{\mathcal{AB}} + WZ$$

would take the form

$$S = \int d^2\sigma e (\Pi_+^{\mathcal{A}} \Pi_-^{\mathcal{B}} \lambda_{\mathcal{AB}} - \frac{1}{2} \lambda_{++} \Pi_-^{\hat{a}} \Pi_-^{\hat{b}} \gamma_{\hat{a}\hat{b}}) + WZ,$$

where $\Pi_{\pm}^{\mathcal{A}} = e_{\pm}^m \partial_m Z^{\mathcal{M}} E_{\mathcal{M}}^{\mathcal{A}}$, and the non-geometrical Lagrange multiplier λ_{++} with a somewhat mysterious transformation rule, generalising that of [6] for curved background, would appear. In fact, it corresponds exactly to our transformation rule (49), the only difference is that in (49) we use $\nabla_+ \sigma \equiv \sigma_+^m \nabla_m = (e_+^m - \lambda_+^m) \nabla_m$ instead of $\nabla_+^e \equiv e_+^m \nabla_m$, as in [6].

8. Let us discuss now the supergeometry of the $(10+496) + 16$ dimensional superspace, described above in which the heterotic string can live. The corresponding constraints are:

$$Q_{ABC\delta} = Q_{ABC\hat{a}} = T_{\alpha\beta, \hat{a}} = T_{\hat{e}\alpha, \hat{A}} = T_{\hat{e}\beta, \hat{A}} = 0$$

$$T_{\beta\alpha, \alpha} = (\gamma_{\alpha})_{\alpha\gamma} T_{\beta}^{\gamma}, \quad T_{\hat{e}\alpha, \hat{A}} = (\gamma_{\hat{e}})_{\alpha\gamma} T_{\hat{A}}^{\gamma} \quad (50)$$

The exact and detailed investigations of this superspace and of its relation to the anomaly-free (due to $Q_{abcd} \neq 0$) supergravity coupled to the Yang-Mills theory is the matter of a future investigation. However if for the moment we will strengthen these constraints and require $Q_{ABCD} = 0$, then the qualitative analysis shows that the superspace under consideration is equivalent to the scale-invariant 10+16-dimensional superspace [8] of the coupled Einstein-Yang-Mills system. We note that our T_A^{β} consists of $\{\delta_{\alpha}^{\beta}, (\gamma_{\alpha})^{\beta\gamma} \lambda_{\gamma}, \chi_{\hat{a}}^{\beta}\}$. Eq. $Q=dH=0$ in the space $(x^m, x^{\hat{m}}, \theta^{\mu})$ corresponds to the equation $dH \sim b_{\tau} FF$ in the space (x^m, θ^{μ}) since now the r.h.s. of this eq. arises from $T_{[AB}^{\hat{a}} H_{\hat{a}CD]} \sim F_{[AB}^{\hat{a}} F_{CD]}^{\hat{a}}$, since $T_{AB, \hat{a}} \sim H_{AB\hat{a}} \sim F_{AB, \hat{a}}$ in eq. $dH=0$ in the big space. This property in the bosonic sector of the theory was already noted in [15]. We note, however, that in our formulation supersymmetry is never broken.

We note also that the constraints on the heterotic string background obtained above do not distinguish between $B_{M_1 M_2}$ [8] or $B_{M_1 \dots M_6}$ [13] versions of the superspace since in the last case the transformation of the superspace of the type (14) is also possible, which leads to eq. (1).

If we are willing to use the Cartan-Weyl basis for the gauge group $E_8 \times E_8$, we may separate from 496 $X^{\hat{m}}$ the 16

X^I ($I=1, \dots, 16$) , corresponding to the abelian generators
 [3] and 480 remaining coordinates will define the harmonic
 space of the type $\frac{E_8 \times E_8}{[U(1)]^{16}}$ [16].

In conclusion we would like to emphasize that the theory
 of the heterotic string is very complicated, it has a tremendous-
 ly large symmetry (see Table I), and it is encouraging that its
 action (32) is so simple.

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Table I. Symmetries of the heterotic string action

$$S = \frac{1}{2} \int_W E_A^* E^A - \int_{X_3} E_A \tilde{T}^A$$

Parameters	Transformations
$\xi^M(z)$ Coordinate transform. in target space	$\delta z^M = \xi^M$; $\delta E_M^A = -\partial_M \xi^N E_N^A$
$\Lambda_B^A(z)$ Structure group of target space	$\delta E^A = E^B \Lambda_B^A$; $\delta \Omega_B^A = (-d\Lambda - \Lambda\Omega + \Omega\Lambda)_B^A$
$\kappa_{a\alpha}(\sigma) = \varepsilon_{a\beta} \kappa_{\alpha}^{\beta}(\sigma)$ Fermionic world-sheet symmetry	$\delta z^M = \frac{1}{2} \kappa_{+\beta} (\gamma^a)^{\beta\alpha} E_{\alpha}^M E_{-}^a$; $(\delta e_{+}^m) e_{m+} = \kappa_{+\beta} T_{+}^{\beta}$; $\delta e_{-}^m = \delta \sigma_{+}^m = \delta \sigma_{-}^m = 0$
$\eta_a(\sigma) = \varepsilon_{a\beta} \eta^{\beta}(\sigma)$ Bosonic world-sheet symmetry	$\delta z^M = \eta_{-} T_{+}^a E_{\alpha}^M$
$\zeta_a(\sigma) = \varepsilon_{a\beta} \zeta^{\beta}(\sigma)$ Bosonic world-sheet symmetry	$\delta z^M = \zeta_{+} E_{-}^a E_a^M$; $(\delta \sigma_{+}^m) \sigma_{m+} =$ $= -\nabla_{+} \zeta_{+} - \zeta_{-} t_{+,+}$; $\delta \sigma_{-}^m = \delta e_{+}^m = \delta e_{-}^m = 0$
$\xi^m(\sigma)$ Coordinate transform. of the world-sheet	$\delta \sigma^m = \xi^m$; $\delta E_m^A = -\partial_m \xi^n E_n^A$
$\Lambda_{\beta}^a(\sigma) = \varepsilon_{\beta}^a \Lambda(\sigma)$ Structure group of the world-sheet	$\delta e^a = e^{\beta} \Lambda_{\beta}^a$; $\delta \sigma^a = \sigma^{\beta} \Lambda_{\beta}^a$; $\delta \Omega_{\beta}^a = (-d\Lambda - \Lambda\Omega + \Omega\Lambda)_{\beta}^a$
$f(\sigma)$ Conformal group of the world-sheet	$\delta e_a^m = f e_a^m$; $\delta \sigma_a^m = f \sigma_a^m$

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