THREE-JET PRODUCTION IN pp COLLISIONS AT $\sqrt{s} = 63$ GeV
AND A DETERMINATION OF α_s

The Axial Field Spectrometer Collaboration

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ABSTRACT

We have studied pp collisions at the CERN ISR at $\sqrt{s} = 63$ GeV using a jet trigger. We demonstrate that a sample of well-separated triple high- p_T jet events can be described as due to the QCD bremsstrahlung process. The energy flow of these 3-jet events is well described by an independent fragmentation model, with a gluon-jet transverse width 33% greater than the quark-jet width. From the ratio of the 3- and 2-jet production cross-sections the strong coupling is determined to lowest order in QCD. We find $\alpha_s = 0.18 \pm 0.03$ (stat.) ± 0.04 (syst.). The error does not include the theoretical uncertainty in the fragmentation scheme.

We compare our results with measurements at the SPS Collider and discuss the different methods used.

1. INTRODUCTION

The use of large-aperture calorimeters in the study of hard processes involving quarks and gluons has proved to be a powerful technique through the unambiguous observation of 2-jet events at the Intersecting Storage Rings (ISR) and at the Super Proton Synchrotron (SPS) Collider [1]. Here we extend these studies to the observation of events with three high- p_T jets.

The $2 \rightarrow 2$ and $2 \rightarrow 3$ processes giving rise to the 2- and 3-jet events reflect different features of the partons. While the $2 \rightarrow 2$ process is due to the force between colliding partons, the $2 \rightarrow 3$ process involves the force field not only as a mediator but as something which can exist independently of the partons themselves. The rearrangement of the field during the hard scattering produces bremsstrahlung with a characteristic spectrum, and the manifestation of this as collimated jets gives information on the nature of the (force) field.

With the observation of 3-jet events we have encountered the gluons in three guises: as constituents of the proton, as mediators of the strong force, and finally as quasi-free excitations of the colour field.

In lowest-order QCD the strengths of the $2 \rightarrow 2$ and $2 \rightarrow 3$ processes are proportional to α_s^2 and α_s^3 respectively. Thus a measurement of the relative 3-jet yield allows a determination of the strong coupling α_s . The systematic errors from the uncertainties in the structure functions will largely cancel. This is also true for most experimental uncertainties, in particular the uncertainty on the absolute energy scale of the calorimeter and that on the luminosity.

Gluon bremsstrahlung was first observed in e^+e^- annihilations [2]. The present analysis is more difficult owing to the presence of many partons in the initial state. In particular, the energy of the partons participating in the hard sub-process has to be reconstructed from the detected particles. We have chosen to determine the sub-process energy with a jet algorithm, which reconstructs the parton momenta from two or three localized energy depositions. This requires a good representation in terms of a Monte Carlo model of the fragmentation of high- p_T gluons and quarks, and of the associated event, so that the measurements can be corrected for the imperfections of the jet algorithm and of the apparatus.

An important systematic error in our determination of the strong coupling comes from the theoretical uncertainty in the fragmentation model, which is used to translate the observed energy to the sub-process energy. It is not necessary to know precisely the *absolute* energy of the 2- and 3-parton final states, but to sample events at the *same* sub-process energy. Therefore one can allow the model to shift the sub-process energy, if this shift is *the same* for 2-jet and 3-jet events. But 2-jet and 3-jet events are potentially very different processes, e.g. 3-jet events as compared to 2-jet events could have a larger associated

particle production or have more colour strings produced. The exact theoretical framework for the description of these differences is not known so we are unable to estimate the resulting systematic error in our determination of the strong coupling. This is a very critical point, because the total jet E_T spectrum is steeply falling (see Fig. 5).

This problem cannot be circumvented by the use of a method of analysis which does not reconstruct the jets, but analyses the structure of the events in terms of energy-energy correlations [3]. In that case the sample would preferentially be selected by a global E_T trigger. This might, however, select 3-jet events at a slightly different sub-process E_T than the 2-jet events. The uncertainty in the representation of these effects in terms of a model would again lead to the problem mentioned above.

To reduce this uncertainty in the relative sub-process energy of 2- and 3-jet events, the structures of the events are modelled as accurately as possible. In particular we ensure that the width of the jets and the characteristics of the associated event are accurately tuned.

In Section 2 the apparatus and the trigger system are described. In Section 3 the jet algorithm is explained, 2- and 3-jet events are defined, and the corrected jet distributions are presented.

In Section 4 the effects of fragmentation and detector resolution are neglected for a while and the jet momenta found by the jet algorithms are identified with the parton momenta. Within this naïve jet model the data are compared with the lowest order $2 \rightarrow 2$ and $2 \rightarrow 3$ QCD matrix elements and the (uncorrected) strong coupling is deduced.

In Section 5 the fragmentation effects are studied. A model of independent jet fragmentation is tuned to our data, and we are thus able to define the corrections to the result of the naïve jet calculation in Section 4. In Section 6 we compare our determination of α_s with similar studies made at the CERN SPS Collider, and we analyse the different approaches taken. The summary and conclusions are given in Section 7.

2. APPARATUS AND DATA-TAKING

The data were taken with the Axial Field Spectrometer (AFS) at the CERN ISR. Proton-proton collisions at $\sqrt{s} = 63$ GeV were recorded for an integrated luminosity of 17000 nb^{-1} .

The Axial Field Spectrometer (Fig. 1) consists of a cylindrical drift chamber and a box-like uranium/copper/scintillator calorimeter surrounding the interaction region [4]. The calorimeter covers 2π in azimuth and has a pseudo-rapidity coverage varying with azimuth from $|\eta| < 0.7$ to $|\eta| < 1.2$. In the

present analysis the drift chamber was used only to reconstruct the interaction vertex.

The energy resolution $\sigma(E)/E$ for electromagnetic and hadronic showers in the calorimeter was $18\%/E$ (GeV) and $42\%/E$ respectively at the time of data-taking.

The four calorimeter walls (see Fig. 1) are segmented in 48 rows with 16 towers on each. The trigger system [4] operates on 24 'jet sums'. A jet sum is defined as the sum of the energy deposited in six adjacent rows of towers (parallel to the beam), where the signals are weighted to correspond to the sum of the $E_T = \sqrt{p_T^2 + m^2}$ in the pp centre-of-mass system. In Fig. 1 the part of the calorimeter towers entering two of the 24 jet sums has been shaded. Adjacent jet sums overlap by four rows, and a jet sum covers on the average 45° of azimuth in the laboratory. In the centre-of-mass system, the coverage of the jet sums varies between 37° and 55° , as a result of the box-shape of the calorimeter and the centre-of-mass motion.

At the trigger stage, the jet sums are compared with three levels, which for the present data sample were set at 5.6 GeV, 9 GeV, and 13 GeV, respectively. The 1-jet trigger requires at least one jet sum above 13 GeV. The 2-jet trigger requires at least two jet sums above 9 GeV, and these jet sums should be separated from each other on each side in azimuth by at least one jet sum below 9 GeV. The 3-jet trigger requires three jet sums with E_T greater than 5.6 GeV, and there should be at least one jet sum between each jet pair with E_T below 5.6 GeV.

These three triggers provided a sample of a broad class of jet events on which the analysis is based.

3. DATA ANALYSIS

3.1 Energy clusters

In the off-line analysis the events were passed through the standard AFS cluster-finding procedure for the calorimeter [5]. The energy clusters correspond in most cases to the energy of individual particles, but in the core of jets with $E_T > 10$ GeV several particles often lead to just one energy cluster. In addition, the radioactivity of the uranium plates leads to spurious energy clusters of typically 100 MeV.

The events are required to have a vertex as reconstructed by the standard drift-chamber analysis program. Using this vertex, the momentum vectors corresponding to the clusters are calculated and transformed to the centre-of-mass system. A pion mass is used if the energy in the hadronic part of the cluster is greater than the energy in the electromagnetic part; otherwise zero mass is used.

In the further analysis only clusters with pseudorapidity $|\eta| < 1$ and $E_T \text{ (cms)} > 250 \text{ MeV}$ are used. This last requirement essentially removes the noise clusters.

3.2 The window jet algorithm

A jet-finding algorithm is applied to the clusters: a window of size $\Delta\phi \times \Delta\eta$ is defined which is moved in the ϕ - η plane until it contains maximum $\sum E_T$ ($\sum E_T$ is the scalar sum of the E_T 's of the clusters). A jet is defined by the clusters inside the window in its optimal position. Thus the 'jet E_T ' is the cluster $\sum E_T$ inside the window, and the jet momentum, which defines the jet axis, is given by the sum of the momenta of these clusters. The procedure is repeated on the remaining clusters to form the next jet and so forth.

The size of the jet window is chosen as a compromise between conflicting aims. One would like to use a large window in order to contain as much of the jet energy as possible, so that the reconstructed jet energy is relatively independent of the angular jet width. But a very large jet window can give rise to problems. This is illustrated by the extreme case where the events are selected by the sum of the E_T of all clusters in the calorimeter, i.e. in $\Delta\phi = 360^\circ$. Although jets dominate the event structure in such a trigger at $\sqrt{s} = 63 \text{ GeV}$ when more than about 30 GeV is emitted into $|\eta| < 1$ [1], the detailed event structure differs from that of a jet trigger. In particular, the energy carried by low- p_T particles is 2-3 times larger in a $\Delta\phi = 360^\circ$ trigger [1]. The origin of these low- p_T particles is not well understood, but at least some of them are due to particles produced in the beam jets.

Thus a larger jet window tends to favour events where a larger part of the energy is produced in soft processes. We have chosen a rather narrow jet window for the present analysis. This was in fact done already at the trigger stage, since the jet sum in the trigger has $\Delta\phi = 45^\circ$ on the average. This allows events to be collected with a lower trigger threshold giving a very large sample of high- p_T jet events ($\sim 300,000$), but requires a good understanding of the bias imposed on this jet sample.

The complete analysis is performed with three different choices of window size:

$$\begin{aligned} \Delta\phi \times \Delta\eta &= 35^\circ \times 0.80 \\ \Delta\phi \times \Delta\eta &= 45^\circ \times 0.90 \\ \Delta\phi \times \Delta\eta &= 60^\circ \times 1.05 \end{aligned} \tag{1}$$

This allows a check of the validity of the procedure, requiring the final value of α_s to be independent of the window size.

3.3 Two-jet, 3-jet, and 'star' 3-jet events

For each of the three window sizes a minimum total E_T of the jets, E_T^{cut} , is required, which is set to 22 GeV, 23 GeV, and 25 GeV, respectively, compatible with a good trigger efficiency. In particular, for the 60° window, the cut has to be far above the hardware threshold (e.g. $3 \times 5.6 \text{ GeV} = 16.8 \text{ GeV}$ for a symmetric 3-jet event), to be independent of the constraint from the 45° jet sums in the trigger.

An event is accepted as a 2-jet event if the two largest jets have axes with $|\eta| < 0.30$ and their total jet E_T is larger than E_T^{cut} .

An event is accepted as a 3-jet event if the three largest jets have axes with $|\eta| < 0.40$ and their total jet E_T is larger than E_T^{cut} .

One-, 2-, and 3-jet triggers are allowed to enter the 2-jet and the 3-jet samples, and the same event is allowed to be both a 2- and a 3-jet event. An event with four or more high- p_T jets is allowed to be a 2-jet or a 3-jet event.

It is also required that the 2-jet events have total $\vec{p}_T$ of the two jets (and for 3-jet events of the three jets) less than approximately 2.5 times the energy resolution. Since the energy resolution is $\sigma(E) \simeq 0.4/E$ (E in GeV), this requirement becomes

$$|\vec{p}_T| < \sqrt{E_{T_{\text{jets}}}} \quad (2)$$

This ensures a reasonable momentum balance in the transverse plane, and removes cosmic-ray events, background from beam halo, and events where poor energy measurements have led to a large transverse-momentum imbalance. The influence on the value of the strong coupling as deduced from the sample is negligible.

Since the jet axes are close to 90° in polar angle, the analysis is restricted to the transverse plane. A study of the angular distribution of jets with respect to the beam axis is not attempted here, since our acceptance for jets covers only a narrow range of polar angles.

The jet system is given a transverse boost to obtain zero transverse momentum. For a 3-jet event the scaled p_T 's of the jets are defined:

$$\begin{aligned} x_1 &= 2p_{T_1} / \sum p_T \\ x_2 &= 2p_{T_2} / \sum p_T \\ x_3 &= 2p_{T_3} / \sum p_T \\ x_1 &\geq x_2 \geq x_3 \end{aligned} \quad (3)$$

$$x_1 + x_2 + x_3 = 2 .$$

Given the total transverse jet energy, the structure of a 3-jet event (in the transverse plane) is described by two dimensionless numbers. We use x_1 and x_2 (the 'Dalitz plot'), or x_3 and ω , where ω is the angle between the second and the third jet in the transverse plane, given by

$$\cos \omega = 1 - 2(x_2 + x_3 - 1)/(x_2 x_3) . \quad (4)$$

3.4 Data

As an example we present data obtained with the medium window size ($\Delta\phi, \Delta\eta$) = (45°, 0.9). Figure 2a gives the 'Dalitz plot' of 3-jet events with $x_3 \geq 0.25$ and $\omega \geq 40^\circ$. For comparison, Fig. 3 shows Monte Carlo generated distributions, described later.

Figure 2b gives a survey of the geometry of 3-jet events, in particular the variation of the variables x_3 and ω in the $x_1 x_2$ plane. We define a class of so-called 'star' 3-jet events, with $x_3 \geq 0.40$ and $\omega \geq 60^\circ$. Figure 4 shows two examples of star 3-jet events. Note that jets are generally resolved into several clusters owing to the excellent granularity of the calorimeter.

The data sample contains 6001 2-jet events and 426 star 3-jet events. Figure 5 gives the uncorrected distributions of the 2-jet and the 3-jet E_T for the two samples, and their ratio.

For the quantitative study of the 3-jet cross-section, two slices of the 'Dalitz plot' are selected. For sufficiently large momentum of the third jet, $x_3 \geq 0.40$, the angle ω between the second and third jets is studied, and for sufficiently large inter-jet angle, $\omega \geq 80^\circ$, the distribution of the fractional momentum x_3 of the third jet is studied. These distributions are presented in Fig. 6 as the number of 3-jet events normalized to the number of 2-jet events, in both cases having $\sqrt{s} E_T^{\text{jets}} \geq 23$ GeV. The data have been corrected for trigger and off-line filter inefficiencies. The combined efficiencies are 75-80% for 3-jet events and 99% for 2-jet events. In this way the samples are defined by the jet algorithm alone.

The data in Fig. 6 have also been corrected for the finite size of the rapidity interval for jets. This means that we give cross-sections for jets at zero rapidity. The inclusive rapidity distributions for jets are given in Fig. 7, and the procedure for extrapolation to the transverse configuration is described in Appendix A.

4. ANALYSIS OF PARTON FINAL STATES

The principal idea of this study is to perform a simple analysis of 2- and 3-jet final states within the framework of lowest-order QCD, corrected for effects of fragmentation and detector resolution. In this section we expose the essential features of the analysis, postponing the detailed corrections to the next section. The momenta delivered by the jet algorithm are taken at face value and are identified with the momenta of the produced partons.

In the next section it will be demonstrated that the star 3-jet events are not just fluctuations of the fragmentation of two high- p_T jets or contamination from beam jets. In anticipation of this result, Fig. 3a gives the 'Dalitz plot' for the full Monte Carlo sample with multi-jets included. Figure 3b is obtained by excluding the multi-jet events from the event generator but still reconstructing 3-jet events on the simulated events. It is seen that *the previously defined region of star 3-jet events cannot be populated by the 2-jet event generator*. Thus the attribute 'star' describes not only the topology, but also the quality of star 3-jet events.

4.1 Bremsstrahlung in QCD

The qualitative features of gluon bremsstrahlung are given by the following approximation to the probability of emitting a gluon of energy E at angle w from a scattered quark:

$$\frac{dP^{q+qg}}{dw dE} \simeq \frac{8\alpha_s}{3\pi} \frac{1}{Ew} . \quad (5)$$

The formula is valid when w is small and the gluon energy E is small compared to the momentum transfer to the scattered quark.

The general calculation of the lowest order $2 \rightarrow 2$ and $2 \rightarrow 3$ QCD processes has been performed by several groups [6-9]. We follow the notation of Gottschalk, Monsay and Sivers [7], and introduce the dimensionless quantities K and $F(x_1, x_2)$ which are related to the spin and colour summed/averaged lowest-order matrix elements $M_{12 \rightarrow ab}$ and $M_{12 \rightarrow abc}$ for the $2 \rightarrow 2$ and $2 \rightarrow 3$ processes in the transverse configuration

$$(4\pi\alpha_s)^2 K = N_s \bar{\Gamma} |M_{12 \rightarrow ab}|^2 \quad (6)$$

$$(4\pi\alpha_s)^3 F(x_1, x_2) = N_s \bar{\Gamma} |M_{12 \rightarrow abc}|^2 s .$$

N_s is a statistical factor, $N_s = 1/2$ for $gg \rightarrow gg$, $N_s = 1/6$ for $gg \rightarrow ggg$, etc., and s is the invariant-mass-squared of the parton system.

Inserting the matrix elements in the parton model gives the following simple formula for the yield of 3-jet events relative to 2-jet events (assuming one parton species per proton):

$$\frac{d\sigma^{3\text{jets}}(E_T > E_T^{\text{cut}})}{dx_1 dx_2 d\eta_1 d\eta_2 d\eta_3} \bigg|_{\eta_i=0} \bigg/ \frac{d\sigma^{2\text{jets}}(E_T > E_T^{\text{cut}})}{d\eta_1 d\eta_2} \bigg|_{\eta_i=0} = \frac{1}{\pi} \frac{x_1}{\sin \omega} \frac{\alpha_s}{4\pi} \frac{F(x_1, x_2)}{K} . \quad (7)$$

The factor $x_1/\sin \omega$ describes the variation of the phase-space volume per unit area of the 'Dalitz plot'. The ratio $F(x_1, x_2)/K$ expresses the ratio of the 3-jet and the 2-jet cross-sections.

The ratio F^{sym}/K , where $F^{\text{sym}} = F(2/3, 2/3)$ is the 3-jet matrix element in the symmetric star configuration, is listed in Table 1. We observe that for qq scattering, the 3-jet/2-jet ratio F^{sym}/K , is approximately twice as large as for e^+e^- collisions, owing to the presence of the initial state as a source of bremsstrahlung. For gg collisions the ratio is approximately twice the ratio for qq collisions, reminiscent of the larger 'colour coupling' of the gluon.

The right-hand side of Eq. (7) is studied in Fig. 8 for the sub-processes of e^+e^- , qq, qg, and gg collisions, using $\alpha_s = 0.20$ in the same two regions of the 'Dalitz plot' as in Fig. 6. With no structure in the matrix element, i.e. $F(x_1, x_2)/K = \text{constant}$, one obtains the 'phase-space' curve. The deviation of the QCD expressions from phase-space behaviour is small in the region of almost symmetric star 3-jet events. But as the inter-jet angle or the third jet momentum approaches zero, the infrared and the collinear divergences become important.

4.2 The parton mixture

A Monte Carlo simulation (see Section 5) shows that the average Feynman x of the incoming partons in our sample of jet events is $\langle x \rangle = 0.475$. Using the Duke-Owens structure functions [10] with $Q^2 = \Gamma(E_T^{\text{jets}})^2/3$ and $\Lambda_{\text{QCD}} = 0.2 \text{ GeV}$ one expects that at this $\langle x \rangle$ the 2-jet events are 54% qq, 40% qg, and 6% gg. However, the gluon structure function is still very uncertain, and in previous studies [5] we have found indications that it is softer than the one given by Duke and Owens. We have multiplied the Duke-Owens gluon structure function by a factor of 0.45.

The uncertainty of the gluon structure function leads to a systematic error on α_s which is estimated to be 6%. This is the decrease of α_s resulting from the use of the original Duke-Owens structure functions instead of the modified set used here.

With our downscaled gluon structure function the composition of scattered partons at $x = 0.475$ and zero rapidity is 70% qq, 28% qg, and 2% gg. These numbers represent the parton luminosities weighted by the lowest-order $2 \rightarrow 2$ matrix elements, so the generalization of expression (7) above is obtained by the substitution:

$$\frac{F}{K} \rightarrow 0.70 \left(\frac{F}{K} \right)_{qq} + 0.28 \left(\frac{F}{K} \right)_{qg} + 0.02 \left(\frac{F}{K} \right)_{gg} . \quad (8)$$

For qq and qq' ($q' \neq q$) collisions the ratio $F(x_1, x_2)/K$ is nearly the same. The subprocesses $qg \rightarrow qq'\bar{q}$, $gg \rightarrow q\bar{q}$, and $gg \rightarrow q\bar{q}g$ are neglected here -- they contribute less than 3%.

4.3 Description of the data by QCD

The distribution of 3-jet events is now parametrized with the QCD expressions. The only free parameter is α_s , which is determined from the total number of star 3-jet events, that is the region $\omega > 60^\circ$ in Fig. 6a.

The curves in Fig. 6 labelled 'QCD parton level' are calculated with this α_s . For each window size a separate value of α_s is determined. These couplings are denoted $\alpha_s(\text{uncorr.})$ and are listed in Table 2. They are uncorrected as they are determined directly from the jets delivered by the algorithm. They rely on a direct interpretation of the jet momenta found by the algorithm as the momenta of the QCD quanta.

The distribution of the inter-jet angle for the experimental data in Fig. 6a should not be considered for angles smaller than the width of the window, so the sensible region is $\omega > 45^\circ$. The variation of the angle of the third jet is obtained here for a rather constant energy of this jet, so that any systematic energy shift of the jet algorithm does not change the angular distribution, which is close to the true parton distribution. The agreement with 'QCD at the parton level' is fairly good.

The x_3 distribution of the experimental data in Fig. 6b deviates from 'QCD at the parton level' at low x_3 . However, we do not expect that our jet algorithm reconstructs the jet energy correctly for arbitrarily low x_3 . Therefore, this plot serves to point out the region where the naïve description in terms of perturbative QCD breaks down, namely below $x_3 = 0.40$, corresponding to observed jet $E_T \simeq 5$ GeV.

In this section we have used a naïve jet model and performed a 'Dalitz analysis' of 3-jet production and determined the strong coupling.

The momentum distribution of the third jet deviates from QCD at low momenta, and the uncorrected strong coupling depends on the size of the jet

window (see Table 2). Therefore a study of the distortions due to fragmentation and beam jets is required; this study is the subject of the next section.

5. FRAGMENTATION EFFECTS

The hard processes of quarks and gluons are well understood in terms of a fundamental theory, perturbative QCD. In contrast, the fragmentation of the scattered partons into hadrons and the associated particle production from the beam remnants are poorly understood.

A good representation of these processes is necessary in order to correct and interpret the results of the naïve jet analysis in the previous section.

In this section a description is given of how we have adjusted the Monte Carlo program ISAJET [11] to our data, and how we interpret the ISAJET events as final states of the $2 \rightarrow 2$ and $2 \rightarrow 3$ QCD processes. The purpose is to correct the values of α_s found in the previous section.

5.1 The parton dynamics

The structure functions used are those of Duke and Owens [10] with $\Lambda_{\text{QCD}} = 0.2 \text{ GeV}$ and $Q^2 = 2stu/(s^2 + t^2 + u^2)$, but as discussed in Section 4.2, we have multiplied the gluon structure function with a factor of 0.45.

The hard processes are the lowest-order QCD $2 \rightarrow 2$ scatterings. The scattered partons are allowed to branch into more partons forming a cascade of final-state partons, according to the Fox-Wolfram algorithm [12] with a minimum invariant mass of a parton branching set to 6 GeV. The cascade produces jet-widening and multi-jets.

The Fox-Wolfram algorithm is an approximation to the exact QCD formulae for gluon bremsstrahlung. The approximation is good when the gluon bremsstrahlung is nearly collinear with the scattered partons. For wide-angle 3-jet production the collinear approximation is poor, and furthermore the initial state as a source of bremsstrahlung is not taken into account. Therefore, the model is used only for jet acceptance calculations, and not as the theoretical input for the matrix elements for 3-jet production, where the exact lowest-order matrix elements introduced in Section 4.1 are used.

5.2 Independent fragmentation scheme

In 3-jet studies in e^+e^- annihilations the fragmentation model used was found to influence the determination of α_s [13]. Independent fragmentation models were found to lead to smaller values of α_s than the string model, and among the independent fragmentation schemes there was also some disagreement, partly due to the method of implementing momentum conservation. In the Feynman-Field fragmentation of a parton the mass of the jet is greater than the mass of

the parton. The fragmentation scheme conserves energy, and this leads to a loss of momentum. For a 2-jet or a perfect star 3-jet these momentum losses cancel on the average, but for asymmetric 3-jet events the outcome of the independent fragmentation produces a system of hadrons which, on the average, move in the opposite direction to the gluon jet. This momentum non-conservation can be eliminated in two ways. The jet system as a whole can be boosted to rest, or the momenta of the hadrons in the individual jets can be rescaled, so that the momentum vector of the jet is proportional to the momentum vector of the original parton. The latter approach produces more gluon jet energy than the first and therefore leads to a smaller α_s .

Our model follows ISAJET and uses independent fragmentation of the partons produced by the cascade, and the beam remnants. The momentum of the central jet system is conserved by a *boost* such that it is at rest in the rest frame of the colliding partons. (After the boost all momenta in the central jet system are rescaled to conserve the energy of the system, and finally the system is transformed to the pp rest frame.)

5.3 Fragmentation of quarks and gluons

The final-state partons are fragmented according to the Feynman-Field fragmentation scheme [14], as used in ISAJET, but we have adjusted the parameters to fit our data as summarized in Table 3.

The longitudinal fragmentation of quarks was changed from ISAJET to agree with our previous measurements [5]. The gluon jets are fragmented slightly softer than the quark jets.

In our model the transverse jet width for quarks is greater than in ISAJET, and we also found it necessary to use a different transverse width for quark and gluons jets: The gluon jets produced by the model are 33% wider than the quark jets. This unusual feature of independent fragmentation was introduced on the basis of studies of the jet width as measured in the calorimeter. From our model (see below) the 2-jet events are expected to consist mostly of quark jets, and our star 3-jet events mostly of two quark jets and one gluon jet, which often carries the smallest momentum. A comparison of the width of the smallest jet in a 3-jet event and of jets in 2-jet events, taking into account the different E_T of the jets, is therefore sensitive to the differences between the widths of quark and gluon jets.

The comparison of the jet widths in the data and the model is presented in Fig. 9, giving the angular jet profile for jets in 2-jet events, for the two largest jets in star 3-jet events, and for the smallest jet in star 3-jet events. We find good agreement between the data and the Monte Carlo calculations.

It is important that the jet width is well represented in the model, because it is used to correct the jet algorithm for the jet energy falling outside our relatively narrow jet window.

One should add that recent studies of QCD on a lattice have shown [15] that the string attached to a gluon has a larger string tension than the string attached to a quark. This implies a larger (non-perturbative) transverse momentum width of the gluon jet [16], which justifies our model from the theoretical point of view.

Within the Monte Carlo model calculation one can determine the probability that a given jet is a gluon jet. The parton type of an observed jet is taken as the type of the final-state parton having its momentum vector closest to the observed jet. The probabilities for a jet to be a gluon in the three classes studied in Fig. 9 are 8%, 28%, and 63%, respectively. The average transverse energies of the original partons were also calculated and are given in Fig. 9.

We have also monitored the types of the two primary scattered partons in the Monte Carlo events. 'Untriggered' 2-jet events with primary partons inside $|\eta| < 0.3$ (i.e. events selected with $|\vec{E}_T|$ of the primary partons of 30 GeV) are found to be 67% qq, 30% qg, and 3% gg. The values for the events reconstructed as 2-jet events and star 3-jet events are given in Table 4. Notice that the wider gluon jet in our model leads to a sizeable 'trigger bias' of the reconstructed events towards a quark-jet enrichment.

Finally Table 4 gives the set of final-state parton types.

5.4 The beam jets

The fragmentation of the beam jets is of great concern because their contribution to the observed 'jets' leads to a systematic distortion of the jet energy. It is important not only to have the correct average beam jet activity in the central region, but also to have the correct fluctuations. The total transverse energy in the model from the beam jets in the central region ($|\eta| < 0.9$) is denoted by $\tilde{E}_T^b$ and its distribution is parametrized as

$$dn/d\tilde{E}_T = A \tilde{E}_T^b \exp(-c \tilde{E}_T) . \quad (9)$$

For a given value of $\langle \tilde{E}_T \rangle$ the parameter b determines the shape of the distribution, and is given by

$$b + 1 = \langle \tilde{E}_T \rangle^2 / \sigma(\tilde{E}_T)^2 . \quad (10)$$

KNO scaling implies shape invariance, i.e. b independent of $\sqrt{s}$. Minimum-bias events are well described by $b = 0.8 \pm 0.1$, but b is not necessarily the same

for an event associated with a high- p_T reaction. In the model a constant b is used, and $\langle \tilde{E}_T \rangle$ is taken as a constant times the $\langle \tilde{E}_T \rangle$ in a minimum-bias event at the reduced scattering energy ($\sqrt{s} - E^{\text{jets}}$).

We have determined $\langle \tilde{E}_T \rangle$ and b in our model by adjusting parameters in ISAJET and comparing with the 2-jet experimental data in the following way: for a 2-jet event we define a spectator region with $\Delta\phi = 90^\circ \pm 45^\circ$ from the largest jet and $|\eta| < 0.7$, and then study the mean and the dispersion of the spectator E_T in this region of the calorimeter. Large fluctuations do, of course, occur in these regions owing to multi-jet production, so 5% of the 2-jet events, which have an additional jet of $E_T > 3$ GeV anywhere in the calorimeter, are excluded from this investigation. The comparison of the E_T spectrum in the data and the Monte Carlo calculation is shown in Fig. 10. The agreement was obtained with $\langle \tilde{E}_T \rangle = 3.1$ GeV at $\sqrt{s}(\text{reduced}) = 33$ GeV, and $b = 0.75$, close to the value of $b = 0.80$ of minimum-bias events.

Finally, the transverse momentum of the incoming partons in the model was adjusted and a good fit of the distribution of the total p_T of 2- and star 3-jet systems was obtained by boosting the jet systems to a momentum with Gaussian width 1.60 GeV/c in each transverse component. The beam jets receive the recoil.

5.5 Background to 3-jet events

The first application of the Monte Carlo model is to investigate possible backgrounds to 3-jet production. Figures 3a and b are the 'Dalitz plots' of the Monte Carlo events with and without the multi-jet production included in the event generator. We observe, as mentioned in Section 4, that the region of star 3-jet events is practically empty in a pure 2-jet model.

Some events are produced by the model outside the star 3-jet region. In Fig. 3b we can distinguish two classes of fake 3-jet events. Firstly, a jet in a 2-jet event is sometimes split, leading to a fake 3-jet event with small w . Secondly, the beam jet particles and stray particles from high- p_T jets can form a spurious third jet at low x_3 .

We have also estimated the background from double parton scattering where three of the four jets simulate a 3-jet event. Four-jet events are observed, for example, in total transverse energy triggered data, but the definition of the star 3-jet sample is such that the contamination from double parton scattering is negligible. A simple calculation yields that before the cut on total transverse momentum of the star 3-jet system, the sample contains at most 1% double parton scatterings. The total jet p_T cut reduces this limit further.

5.6 Comparison of model and experimental data

For a quantitative test of the model we study the distribution of the angle between the third and the second jet, and the third-jet momentum distribution, that is the w and the x_3 plots.

Firstly, the Monte Carlo model at the parton level was compared with the exact QCD matrix elements. Apart from an overall factor we found surprisingly good agreement. This was not expected *a priori*, since the Monte Carlo has only the approximate matrix elements for wide-angle 3-jet production. This agreement implies that our model implements the structures of the exact lowest-order matrix elements up to a constant factor.

On Fig. 6 the shaded bands show the outcome of the Monte Carlo. The Monte Carlo bands have been multiplied by 1.45, and are then in excellent agreement with the experimental data. This means that the structure of the 3-jet matrix elements agrees with the data, but that the overall abundance of 3-jet events relative to 2-jet events in our Monte Carlo model is a factor of 1.45 too low.

The model gives a good description of the yield of events at low x_3 , where QCD at the parton level falls much below the data. This extra yield at low x_3 cannot be accounted for as fluctuating 2-jet events (the dashed curve). They are interpreted as fluctuations of 3-jet events with even lower x_3 , which conspire with contamination from beam jets to produce a third jet observed at higher x_3 .

5.7 Classification of parton final states

To correct the α_s found using the jet algorithm we will study how the 3-jet/2-jet ratio is distorted in the combined process of jet fragmentation, detector simulation, and jet reconstruction.

To do so we need to know the 3-jet/2-jet ratio for our model 'at the parton level', and this turns out to be a subtle point. The model starts from the 2-parton events, and by means of the parton cascade it generates events with two, three, or more partons--some examples are given in Fig. 11a. Which of these events should be counted as 2-jet events and which as 3-jet events, and what is the kinematics of the two or three jets?

In our extraction of the 2-jet cross-section in Ref. [5], we used a similar Monte Carlo model, and defined that part of the 2-jet events cascade into several partons as given by ISAJET's final-state cascade. Thus the parton cascade was regarded not as a generator of multi-jets but rather as a part of the fragmentation of a scattered parton.

This definition of a 2-jet event is maintained here, so that all the events produced by the Monte Carlo are identified as 2-jet events. The kinematics of the two jets is given by the two primary scattered partons.

Likewise, the 3-jet sample is formed by the events with at least three partons after the cascade. With more than three partons produced, the cascade beyond the 3-parton state is regarded as 'internal fragmentation' of the three jets from the three partons. When both primary scattered partons cascade, the primary parton with the smallest virtual mass is defined as one jet, and the cascade of the other primary parton defines the other two. With these conventions we obtain a reasonable jet-widening for jets in 2- and 3-jet events. The classification is exemplified in Fig. 11b.

We expect that these conventions are the best interpretation of the lowest-order matrix elements. This belief is based on analogy with e^+e^- annihilation into hadrons at $\sqrt{s} = 30$ GeV. In that process, the lowest order $O(\alpha_s^0)$ cross-section increases by 5% when $O(\alpha_s)$ corrections are included, but the jet topology changes considerably. With an invariant mass cut-off at 5 GeV, 30% of the events include a radiated gluon [17]. Thus the 3-jet events produced by final-state bremsstrahlung do not increase the total cross-section significantly, but rather take away part of the 2-jet cross-section. This principle is maintained by interpreting the 'final-state cascade' as described above.

It is possible to define 2- and 3-jet events differently. In Appendix B we describe an alternative scheme, which leads to a 36% larger value for the lowest-order α_s .

5.8 The corrected α_s

Using the 'final-state cascade' definition of a jet, the star 3-jet abundance relative to the 2-jet abundance for the Monte Carlo events at the parton level is calculated. This is done at $x = 0.475$, the mean Feynman x of the simulated events. The same rapidity intervals as for the data are used, and the cross-sections are extrapolated to the transverse configuration as described in Appendix A.

Next the relative star 3-jet abundance in the detector-simulated, jet-reconstructed Monte Carlo events is calculated, using the usual rapidity intervals and extrapolation.

A correction factor is then defined as the relative star 3-jet abundance 'at the parton level', divided by the relative star 3-jet abundance for the fully simulated and reconstructed events. This factor is given in Table 2 for each window size, together with the corrected values of α_s , which are practically independent of the window size, a necessary condition for the validity of our determination. The final value is

$$\alpha_s = 0.18 \pm 0.03 \text{ (stat.)} . \quad (11)$$

Note that the three determinations of α_s in Table 2 with the three window sizes are not statistically independent.

5.9 Evaluation of the model

In this section we present further evidence for the validity of our model for the calculation of the correction factor, and discuss the systematic error on α_s .

To test whether the fragmentation scheme is a good description of our data, the energy flow for the star 3-jet events in the transverse plane is studied in Fig. 12. The model is in good agreement with the experimental data. This plot is well-suited to examining the behaviour in the three inter-jet regions. The regions at $\phi \simeq 190^\circ$ and $\phi \simeq 310^\circ$ are adjacent to the smallest jet, which is often a gluon jet, so this region is sensitive to the softest particles in the gluon jet. The JADE group [18] has studied the analogous distributions for e^+e^- annihilations and interpreted an excess in these regions relative to the region at $\phi \simeq 60^\circ$ as a string fragmentation effect. We observe that our independent fragmentation model is able to describe the relative yields in these regions.

If the 3-jet events produced in hadron collisions are fragmented according to the string model [19] the bremsstrahlung gluon jets will stretch their strings sometimes towards the scattered quarks and sometimes forward towards the proton remnants. Therefore the string effects in the transverse plane will be weaker than the string effect in the event plane of 3-jet events produced in e^+e^- annihilations. In addition, the inter-jet regions are also populated by particles from strings not attached to the high- p_T jets but to the beam remnants. So although the data are in agreement with our independent fragmentation model, string effects of the expected order of magnitude are not excluded by the data.

We conclude that our fragmentation model gives an excellent fit to the energy flow in 2- and 3-jet events. After this test we believe that the systematic error in the correction factor due to imperfections in the tuning of the model is minimized. We emphasize, however, that the value of the correction factor is quite sensitive to this fine-tuning. From such studies we attribute a systematic error of 16% to the uncertainty of the parameters (e.g. the jet width) in our model. For example, if the $\langle \tilde{E}_T \rangle$ of the beam jets (see Section 5.4) is increased by 10%, a 9% lower value of α_s is deduced. The correction for trigger efficiency leads to a systematic error of 10% and the uncertainty of the gluon structure function contributes 6% (see Section 4.2). The total systematic error on α_s is then 20%, which does not include the theoretical uncertainty inherent in our particular choice of fragmentation scheme. Indeed the variety of fragmentation schemes proposed for hadron collisions is greater than for e^+e^-

annihilations, owing to the additional beam jets. These can be either independent of the high- p_T jet system or integrated, for example through strings.

6. COMPARISON WITH OTHER EXPERIMENTS

6.1 Definition of K factors

Our measurement of α_s is based on the exact lowest-order QCD formulae for the $2 \rightarrow 2$ and $2 \rightarrow 3$ processes, and on the fragmentation scheme defined in Section 5. We have demonstrated the necessity of specifying the classification of parton final states into 2-jet and 3-jet events (see Section 5.7 and Appendix B). This classification also enters into the definition of the K factors, expressing the higher-order corrections to the lowest-order (LO) QCD formulae:

$$\sigma^n = K_n \sigma_{LO}^n, \quad (12)$$

where n is the number of jets. In addition to the classification scheme, the K factors also depend on the jet kinematics, e.g. the jet p_T , and on the parton types.

The value α_s (LO) extracted in this paper is related to the true, or higher-order (HO), strong coupling α_s (HO) by

$$\alpha_s \text{ (LO)} = \alpha_s \text{ (HO)} K_3/K_2. \quad (13)$$

6.2 Comparison with e^+e^- experiments

The 2-jet and 3-jet events observed in the present study have a subprocess energy of approximately 30 GeV, which is close to the c.m. energy $\sqrt{s} = 29$ GeV of the e^+e^- annihilations observed by the TPC collaboration working at SLAC [20]. We take this experiment as representative of the many measurements of α_s in e^+e^- annihilations. They find different values of α_s for various choices of the fragmentation scheme. The string fragmentation scheme yields $\alpha_s = 0.183 \pm 0.010$. For an independent fragmentation scheme very similar to ours, they find $\alpha_s = 0.147 \pm 0.015$ with equal quark and gluon jet width, and $\alpha_s = 0.135 \pm 0.015$ for 50% wider gluon jets. (The errors include statistical and systematic contributions.) The latter value is relevant for comparison with the value found by AFS and consistency requires that the ratio of the K factors be

$$K_3/K_2 = \alpha_s \text{ (AFS)}/\alpha_s \text{ (TPC)} = 1.3 \pm 0.25 \pm 0.30. \quad (14)$$

Here we have made a comparison with the value α_s (AFS) obtained using the 'final-state cascade' classification (see Section 5.7); thus this scheme results indeed in a value of K_3/K_2 close to one.

6.3 Comparison with SPS Collider data

In this section we compare our determination of α_s with similar analyses performed by the UA1 [21] and UA2 [22] Collaborations working at the SPS Collider. These experiments study $p\bar{p}$ collisions at other values of $\sqrt{s}$ and average $\langle E_T \rangle$ of the jet systems, which implies that the dominant parton species in the sub-processes are different. Also the methods and parton classifications are different. It therefore requires a detailed discussion to compare the results. The analyses are summarized in Table 5.

We have presented two interpretations of the lowest-order QCD matrix elements, given as the two classification schemes. The first scheme was described in Section 5.7 and is labelled '*final-state cascade jet*'. Here a jet is defined as a final-state Fox-Wolfram cascade of partons. We expect that this scheme gives a value of the strong coupling close to the 'real' α_s .

The alternative classification scheme is described in Appendix B, and is labelled '*resolution of partons*'. In this scheme two partons are counted as different jets if their invariant mass is greater than some cut-off value which is set at 6 GeV. The *inclusive* 2-jet and 3-jet cross-sections can then be defined and identified with the lowest-order QCD matrix elements. This gives a larger value of the strong coupling which is denoted by α_s^{res} in Table 5.

In the AFS the very high $\langle E_T^{\text{jets}} \rangle$ relative to the $\sqrt{s}$ implies that the dominant sub-process is $q\bar{q}$ scattering, with some $q\bar{q}$ admixture. The UA2 Collaboration selects events with $\langle E_T^{\text{jets}} \rangle > 70$ GeV in $p\bar{p}$ collisions at $\sqrt{s} = 630$ GeV: thus the dominant scattering is $g\bar{g}$ with some $q\bar{q}$ and $q\bar{q}$ admixture. The jets are defined according to the scheme '*resolution of partons*', but instead of inclusive cross-sections they use *exclusive* cross-sections at the parton level as an interpretation of the QCD matrix elements. Partons are classified as distinguishable jets if they have $p_T > 12$ GeV and inter-jet angles greater than 40° . These cut-offs resemble those used in the present analysis, e.g. if we translate our invariant mass cut-off to the inter-jet angle for a third jet with $x_3 = 0.5$, we get a cut-off angle of 47° .

The inclusive and the exclusive 3-jet cross-sections are not the same. However, we will assume here that the probability of the presence of an additional jet is independent of the number of jets present already (as supported by the observation of jets associated with W-bosons [23]). This implies that the ratio of the 3-jet and 2-jet cross-sections are the same with the inclusive and the exclusive definitions, so that the derived value of α_s will be the same. Therefore we have placed the UA2 result in Table 5 under the heading '*resolution of partons*'.

The UA1 Collaboration select events with two or three jets with a total jet mass in excess of 150 GeV in $p\bar{p}$ collisions at $\sqrt{s} = 546$ GeV, implying dominantly

qq and $q\bar{q}$ scattering. They define several samples of 2- and 3-jet events. Here we refer to their samples of events where the jets emerge at large angles to the beam, since these events are kinematically most similar to those used in the AFS and the UA2 analyses. (Since most of the energy is deposited in the central calorimeter, this choice also leads to a larger degree of cancellation of the systematic errors in the energy scales of the calorimeters.) For these events the average total jet E_T is approximately 140 GeV.

The UA1 Collaboration have not performed a Monte Carlo simulation of complete 2- and 3-jet events as the AFS and UA2 Collaboration have done. Instead they have studied single jets in their apparatus and calculated correction tables for jet energies, using the ISAJET Monte Carlo, i.e. a 'jet' is a final-state cascade of partons with subsequent independent fragmentation [24]. Thus the classification of parton final states is of the type 'final-state cascade'.

In the evaluation of the results from the AFS, UA2 and UA1 Collaborations in Table 5, we make the following remarks:

- i) The value of α_s measured at different energy scales

$$Q^2 = 1/3 (\sum E_T^{\text{jets}})^2 \quad (15)$$

should be related by the renormalization group equation:

$$\alpha_s(Q^2, n_f) = \frac{12\pi}{(33-2n_f) \log(Q^2/\Lambda_{\text{QCD}}^2)} \quad (16)$$

For a fixed number of flavours $n_f = 4$, and $\Lambda_{\text{QCD}} = 0.2$ GeV, the coupling runs from 0.17 to 0.13 when Q increases from 17 to 80 GeV. However, when the 'colour charge' of the partons is probed at smaller and smaller distances, the screening cloud of virtual beauty (and maybe top) quark-antiquark pairs is penetrated. Thus a stronger colour is revealed at shorter distances owing to this effect, which partly compensates the energy dependence. Indeed, the formula above shows that the addition of more flavours reduces the change of α_s with energy.

- ii) The three determinations of α_s all rely on the independent fragmentation scheme. If string fragmentation were used instead, the values of α_s could well come out larger in analogy with experience from e^+e^- annihilations. The differences in α_s due to different fragmentation schemes are expected to decrease for increasing jet energy.
- iii) The couplings given in Table 5 are $\alpha_s K_3/K_2$ and the unknown ratio of K factors could be different for $q\bar{q}$, $q\bar{q}$, qg , and gg scattering. Indeed the lowest-order matrix elements show that the process $q\bar{q}' \rightarrow q\bar{q}'g$ has 50% larger cross-section than $q\bar{q}' \rightarrow q\bar{q}'g$ (see Table 1), a colour dynamical

effect leading to a different 3-jet/2-jet ratio for pp and $p\bar{p}$ collisions. This is, of course, taken into account in these analyses, but there might be differences of the same order of magnitude in the higher-order corrections which are not known.

The agreement between the measurements by the AFS and UA2 Collaborations is excellent, whereas the UA1 value of α_s seems to be a factor 1.5-1.7 too high relative to the expectation from the AFS measurement, although the deviation is within the statistical and systematic errors. All three comments above are important for the comparison of the UA1 and AFS values, so we conclude that the agreement between the three measurements is satisfactory.

We note that the UA2 and the AFS measurements are performed with similar detector coverage and similar Monte Carlo simulations of complete events. The comparison is especially interesting because the colliding partons under study are dominantly gluons and quarks respectively, which are expected to have a different colour structure, leading to a different gluon bremsstrahlung probability. Figure 8a shows that the 3-jet/2-jet ratio is approximately 1.8 times larger for gg than for qq scattering at fixed parton energies. This factor was of course taken into account in the analyses, and the fact that the values of α_s deduced by the two experiments are consistent is a confirmation of QCD.

7. SUMMARY AND CONCLUSIONS

We have observed the production of 2-jet and 3-jet events and related it to the partons emerging from the $2 \rightarrow 2$ and $2 \rightarrow 3$ QCD processes. The jets were reconstructed by a window jet algorithm in a large-aperture calorimeter. We used the observed cross-sections for a determination of the strong coupling α_s to lowest order in QCD. We find

$$\alpha_s = 0.18 \pm 0.03 \pm 0.04 \quad (17)$$

at $Q^2 = 300 \text{ GeV}^2$.

We found that the corrections to the window definition of a jet are essential, so we embarked on a study of the fragmentation of quarks, gluons, and beam remnants, within the framework of independent fragmentation based on the ISAJET model, and a good description of the event structures using a 'KNO-scaled' minimum-bias event to simulate the beam jets. The scattered partons are allowed to branch into several partons with a cut-off for the invariant mass of a parton pair of 6 GeV, and a 'non-perturbative' fragmentation of quarks and gluons with a 33% larger transverse width for gluons.

The energy flow for star 3-jet events (Fig. 12) is in perfect agreement with our model.

The parton dynamics in our model is based on ISAJET, which has incomplete matrix elements for wide-angle 3-jet production, mainly because of the neglect of the initial state as a source of bremsstrahlung, leading to a factor of 1.45 too few star 3-jet events. Therefore the model is used for acceptance and fragmentation corrections only.

Finally, we have compared our results with measurements done at the SPS Collider. Owing to various theoretical uncertainties we conclude that the consistency of the measurements is satisfactory. Of particular interest is the comparison of the 3-jet production in quark-quark and gluon-gluon collisions studied by the AFS and the UA2 Collaborations, respectively. The agreement on the value of α_s deduced by these two experiments is an indication of the fundamental property of QCD, that the gluon carries a larger 'colour charge' than the quarks.

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APPENDIX A

DESCRIPTION OF THE RAPIDITY CORRECTION PROCEDURE

To compare the measured cross-section with simple theoretical expressions, a correction is applied to transform the data to the exact transverse configuration. These cross-sections are related to the observed cross-sections by the rapidity correction factors f_2 and f_3 :

$$\left. \frac{d\sigma^{2\text{jets}}}{d\eta_1 d\eta_2} \right|_{\eta_i=0} = f_2(\eta_0) \frac{1}{(2\eta_0)^2} \sigma^{2\text{jets}}(|\eta_i| \leq \eta_0) \quad (\text{A1})$$

$$\left. \frac{d\sigma^{3\text{jets}}}{d\eta_1 d\eta_2 d\eta_3} \right|_{\eta_i=0} = f_3(\eta_0) \frac{1}{(2\eta_0)^3} \sigma^{3\text{jets}}(|\eta_i| \leq \eta_0) .$$

The correction factors are determined from the data in the following way: the 2- and the star 3-jet cross-sections are parametrized by a quadratic function of the pseudorapidities:

$$\begin{aligned} \frac{d\sigma^{2\text{jets}}}{d\eta_1 d\eta_2} &= A(1 - 3\alpha \eta_1^2 - 3\alpha \eta_2^2 + K\eta_1\eta_2) \\ \frac{d\sigma^{3\text{jets}}}{d\eta_1 d\eta_2 d\eta_3} &= B \left(1 - 3\beta_1 \eta_1^2 - 3\beta_2 \eta_2^2 - 3\beta_3 \eta_3^2 + \sum_{i,j=1}^3 K_{ij}\eta_i\eta_j \right) . \end{aligned} \quad (\text{A2})$$

This form takes rapidity correlations into account.

The inclusive rapidity distributions for 2- and star 3-jet events with all jets inside $\pm\eta_0$ are then given by

$$\begin{aligned} \frac{dn^{2\text{jets}}}{d\eta_j} &= A'(1 - 3a \eta_j^2) \\ \frac{dn^{3\text{jets}}}{d\eta_j} &= B'(1 - 3b \eta_j^2) , \end{aligned} \quad (\text{A3})$$

where

$$a = \frac{\alpha}{1 - \alpha \eta_0^2}$$

$$b_1 = \frac{\beta_1}{1 - (\beta_1 + \beta_2)\eta_0^2}, \quad b_2 = \frac{\beta_2}{1 - (\beta_1 + \beta_3)\eta_0^2}, \quad b_3 = \frac{\beta_3}{1 - (\beta_1 + \beta_2)\eta_0^2}. \quad (A4)$$

Figure 7 gives the inclusive η distribution for 2-jet and star 3-jet events. The distributions for all three jets are given in one plot, since the three distributions are consistent with being the same, i.e. $b_1 = b_2 = b_3 = b = \beta/(1-2\beta\eta_0^2)$. Also shown in Fig. 7 are the fits to the expressions above, from which the parameters a and b are determined:

$$\begin{aligned} a &= 1.18 \pm 0.06 \\ b &= 0.83 \pm 0.10. \end{aligned} \quad (A5)$$

The rapidity correction factors are given by α and β , which in turn are determined from a and b :

$$\begin{aligned} f_2(\eta_0) &= \frac{1}{1 - 2\alpha \eta_0^2} \\ f_3(\eta_0) &= \frac{1}{1 - 3\beta \eta_0^2}. \end{aligned} \quad (A6)$$

Inserting the values for a and b determined from the data gives

$$f_2(\eta_0 = 0.3) = 1.24 \pm 0.01, \quad f_3(\eta_0 = 0.4) = 1.46 \pm 0.07. \quad (A7)$$

With these values of the rapidity correction factors the data in Fig. 6 have been extrapolated to the transverse configuration.

The rapidity distributions of the Monte Carlo are not in perfect agreement with the data. However, the procedure for extrapolating the data to the transverse configuration is used throughout, eliminating these deviations, i.e. the correction factors in Table 2 refer to events in the transverse configuration.

APPENDIX B

ALTERNATIVE PARTON CLASSIFICATION

In Section 5.7 we described our preferred classification of parton final states into 2- and 3-jet states, the so-called 'final-state cascade' definition of a jet. By analogy with e^+e^- annihilations we expect the corresponding K factor ratio K_3/K_2 (defined in Section 6.1) to be small.

Here the importance of the specification of the classification scheme is illustrated by presenting an alternative scheme which gives a different value of the lowest-order α_s corresponding to different K factors.

It is based on the idea that two partons produced in a hard process are resolved as two jets, if and only if the invariant mass of the two partons is greater than some cut-off value. The cut-off is chosen to be 6 GeV as used in the parton cascade, so that the final-state partons correspond to the jets of the event. The alternative scheme for classification of the events in 2- or 3-jet events is now defined by the two or three final partons in the event giving the highest total E_T . Additional partons are not included in the definition of the E_T of the 2-jet (or 3-jet) event. This classification is illustrated in Fig. 11c, and leads to a smaller 2-jet cross-section in the model, because an event with bremsstrahlung gets a lower 2-jet E_T and thus contributes at lower E_T , where it has smaller relative importance. The star 3-jet cross-section is also reduced but not as much, since here the phase space for emission of a resolvable gluon is smaller. At $\sqrt{s} = 30$ GeV we find that compared to the 'final-state cascade' classification the 2-jet cross-section is reduced by 31% and the star 3-jet cross-section by 6%, so that the nominal 3-jet/2-jet ratio is increased by a factor of 1.36 in the model. The value of α_s determined with this definition is consequently 1.36 times larger than the one determined with the final-state cascade scheme, i.e.

$$\alpha_s^{\text{res}} = 1.36\alpha_s = 0.25 \pm 0.04 \pm 0.05 .$$

We label this classification scheme 'resolution of partons'. It has the advantage that the definitions of 2- and 3-jet events are closer to what is actually observed as 2-jet or 3-jet events in our study. For events with two centrally scattered partons with $\sqrt{s} = 30$ GeV, 17% of the quarks and 30% of the gluons emit a quantum ($q \rightarrow qg$ and $g \rightarrow gg$ or $g \rightarrow q\bar{q}$). Less than 0.2% emit two quanta. In 66% of the events neither parton emits anything. These numbers are for 'untriggered' events, i.e. events selected on the parton level. For the reconstructed jet events 91%, i.e. almost all of the reconstructed 2-jet events,

are non-radiating. For the star 3-jets, only 1% are non-radiating ('fake' 3-jets), 94% are singly radiating, and 5% are doubly radiating (see Table 4).

Thus the jet algorithm selects events which are rather clean samples of events with the same classification on the parton level. The numbers quoted above correspond to the 45° window. For the 60° window 88% of the 2-jet events, are non-radiating. This is understood from the observation that a parton branching with invariant mass at the cut-off value 6 GeV has an opening angle of approximately 47°. When the window size is increased from 45° to 60°, more parton pairs are merged into single reconstructed jets.

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Table 1

Matrix elements of $2 \rightarrow 2$ and $2 \rightarrow 3$ QCD processes in the transverse configuration [7]. The processes are $ab \rightarrow 12$ and $ab \rightarrow 12g$. K and F^{sym} are dimensionless numbers expressing the matrix elements of the 2-parton and symmetric star 3-parton final states.

$ab \rightarrow 12g$	K	F^{sym}	F^{sym}/K
$qq \rightarrow qqg$	1.63	80.0	49
$qq' \rightarrow qq'g$	2.22	106.7	48
$qg \rightarrow qgg$	6.11	331.8	54
$gg \rightarrow ggg$	15.19	1215.0	80
$q\bar{q} \rightarrow q\bar{q}g$	2.59	172.4	67
$q\bar{q}' \rightarrow q\bar{q}'g$	2.22	160.0	72
$gg \rightarrow q\bar{q}g$	$0.73^{\text{a)}}$	$148.8^{\text{a)}}$	204
$e^+e^- \rightarrow q\bar{q}g$	$15.00^{\text{a)}}$	$320.0^{\text{a)}}$	21.3

a) Five flavours assumed

Table 2

The determination of α_s for the three different choices of window size. The first line is α_s uncorrected for effects of fragmentation and detector resolution. The second line is the correction factor calculated with the Monte Carlo model. The third line is the fully corrected α_s . Statistical errors only are given.

	Window size: $\Delta\phi \times \Delta\eta$		
	$35^\circ \times 0.80$	$45^\circ \times 0.90$	$60^\circ \times 1.05$
Data α_s (uncorr.)	0.16 ± 0.01	0.25 ± 0.01	0.29 ± 0.015
MC correction	1.05 ± 0.15	0.75 ± 0.10	0.63 ± 0.08
α_s (corr.)	0.17 ± 0.03	0.18 ± 0.03	0.18 ± 0.03

Table 3

Fragmentation model parameters. Those used in the present analysis are compared with the ones in the ISAJET Monte Carlo. The r.m.s. transverse momentum in jets is denoted by σ ; a and p are the parameters in the meson probability function $f(x) = 1 - a + (p+1) a(1-x)^p$ describing the fraction x of the parent quark $p_\perp = E + p_L$ taken by the meson [14]. These parameters are attached to our particular choice of the value of the cut-off (6 GeV) in the parton cascade.

	Our model		ISAJET
	q	g	q = g
σ	442 MeV	560 MeV	350 MeV
a	0.95	1.00	0.77
p	2.5	3.0	2.0

Table 4

Fraction of various types of two primary scattered partons in reconstructed jet events, according to the model.

(a) initial-state partons

	2-jet events (%)	Star 3-jet events (%)
qq	84	74
qg	16	25
gg	-	1

(b) final-state partons

	2-jet events (%)	Star 3-jet events (%)
qq	77	-
qg	14	1
gg	-	-
qq $\bar{q}$	1	3
qqg	7	71
qgg	1	20
ggg	-	-
qqgg	-	4
qggg	-	1

Table 5

Comparison of the determinations of α_s at the ISR and at the SPS Collider. The two values of the strong coupling, α_s and α_s^{res} , correspond to the two different ways of defining 2- and 3-jet events at the parton level (see Section 5.7 and Appendix B). Statistical and systematic errors are given.

	$\sqrt{s}$ (GeV)	ΣE_T (GeV)	Dominant partons	Final-state cascade α_s	Resolution of partons α_s^{res}
AFS	63	30	qq	$0.18 \pm 0.03 \pm 0.04$	$0.25 \pm 0.04 \pm 0.05$
UA2	630	80	gg	-	$0.23 \pm 0.01 \pm 0.04$
UA1	540	140	$q\bar{q}$	$0.22 \pm 0.03 \pm 0.04$	-

Figure Captions

- Fig. 1 The AFS detector. The shaded areas are groups of calorimeter towers defining two of the 24 jet sums.
- Fig. 2 'Dalitz plot'; x_2 versus x_1 for 3-jet events in the transverse plane. a) AFS experimental data. The circles indicate the two events shown in Fig. 4. b) The geometry of 3-jet events in the 'Dalitz plot'. The inset gives the definition of the inter-jet angle ω . Lines of constant x_3 or ω are shown. The small diagrams are examples of the momentum vectors of the jets in 3-jet events.
- Fig. 3 'Dalitz plot' for Monte Carlo events. a) The event generator produces both 2-jet and multi-jet events, which are passed through the detector simulation. b) The event generator produces 2-jet events only. The region of star 3-jet events is indicated. The Monte Carlo data have been normalized to the same number of 2-jet events as the experimental data in Fig. 2a.
- Fig. 4 Two examples of star 3-jet events. The transverse energies of the clusters (particles) are represented by towers on a pseudorapidity-azimuth grid with cells $\Delta\phi \times \Delta\eta = 5^\circ \times 0.10$. The windows defining the jets are indicated. The insets give the jet configuration in the transverse plane.
- Fig. 5 The distribution of the uncorrected total 2-jet E_T and total 3-jet E_T for 2-jet and star 3-jet events. The inset gives the ratio of the two spectra. These spectra are uncorrected for instrumental effects and imperfections of the jet algorithm, so the variation of the ratio with E_T cannot be interpreted directly as the variation of the strong coupling α_s .
- Fig. 6 a) Distribution of the angle ω in the transverse plane between the third and the second jet in 3-jet events with $x_3 > 0.40$. b) Distribution of the fractional momentum x_3 of the third jet in 3-jet events with $\omega > 80^\circ$. The data are corrected for trigger and filter inefficiencies and for the finite size of the rapidity interval for jets. The solid lines 'QCD parton level' represent the lowest-order QCD matrix elements. The shaded bands represent the Monte Carlo model, i.e. including fragmentation effects and detector resolution. The dashed lines represent the Monte Carlo model with multi-jet events excluded from the event generator. The star 3-jet region is indicated in (a).
- Fig. 7 The inclusive rapidity distributions for jets in 2-jet events and star 3-jet events. The parabolic fits are used to extrapolate the data to the transverse configuration (see Appendix A).

- Fig. 8 The matrix elements in the transverse configuration for the $2 \rightarrow 3$ processes $ab \rightarrow 12g$, relative to the $2 \rightarrow 2$ processes $ab \rightarrow 12$. a) Distribution of inter-jet angle ω for $x_3 \geq 0.40$. b) The fractional momentum x_3 distribution for $\omega \geq 80^\circ$.
- Fig. 9 The transverse energy flow $dE_T/d\beta$ as a function of the angle β from the jet axis, E_T is the cluster E_T . The three classes of jets are as follows: a) Jets in 2-jet events. b) The two largest jets in star 3-jet events. c) The smallest jet in star 3-jet events. For these plots the rapidity cut for the jets is $|\eta_{jet}| \leq 0.40$. Detector resolution and edge effects have not been unfolded.
- Fig. 10 The distribution of spectator E_T in 2-jet events with no extra jet of $E_T^{jet} \geq 3$ GeV. The spectator region has azimuth 45° - 135° from the largest jet and $|\eta| \leq 0.70$.
- Fig. 11 Illustration of the classification of parton final states. a) Examples of final-state parton cascades. The momentum vectors of the partons originate from the end of the parent parton momenta. b) The classification 'final-state cascade jet' described in Section 5.7 is indicated for 2-jet events and 3-jet events. This scheme gives $\alpha_s = 0.18$. c) The classification 'resolution of partons' described in Appendix B. This scheme gives $\alpha_s = 0.25$.
- Fig. 12 The energy flow in star 3-jet events, $dE_T/d\phi$; E_T is the cluster E_T , and ϕ is the azimuth distance from the largest jet, oriented towards the second jet (see inset). Only clusters with $|\eta| \leq 0.70$ are used. AFS data and independent fragmentation model (modified ISAJET). As in Figs. 9 and 10, the data and the Monte Carlo are subjected to the same detector biases.

Fig. 1

Fig. 2

Fig. 3

The Axial Field Spectrometer

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9

Fig. 10

Fig. 11

Fig. 12