CERN-TH.4402/86

COMMENT ON THE SCALAR GLUONIUM DECAY INTO TWO PHOTONS

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ABSTRACT

We discuss the coupling of scalar gluonium σ to two photons. Naive use of the anomalous triangle diagram would lead to a surprisingly large $\Gamma(\sigma \rightarrow \gamma\gamma)$ decay rate, but we argue that this type of argument in fact applies to the decay rate $\Gamma(S \rightarrow \gamma\gamma)$ of scalar quarkonium S . A better estimate of $\Gamma(\sigma \rightarrow \gamma\gamma)$ is obtained using the higher order Euler-Heisenberg box diagram for the two-gluon/two-photon coupling, which also gives a better estimate of the $\Gamma(J/\psi \rightarrow \sigma\gamma)$ decay rate. We discuss the effects of mixing between the scalar quarkonium and gluonium states, and give plausible upper and lower limits on $\Gamma(\sigma \rightarrow \gamma\gamma)$.

In a previous paper¹⁾ we have reanalyzed the couplings of scalar gluonium σ to $\pi\pi$ and to $\gamma\gamma$, using the low-energy theorems of broken chiral symmetry and scale invariance^{2),3)}. The anomalous trace of the hadronic energy-momentum tensor²⁾ and the ITEP⁴⁾ value of the gluon condensate were incorporated in a phenomenological Lagrangian⁵⁾, leading to the estimate $\Gamma(\sigma \rightarrow \pi\pi) \approx 0.6 \text{ GeV} \times (m_\sigma/1 \text{ GeV})^5$. This result suggested that a scalar gluonium state would be unobservably wide if its mass were above 1 GeV. We also estimated $\Gamma(\sigma \rightarrow \gamma\gamma)$ and $\Gamma(J/\psi \rightarrow \sigma\gamma)$ in our previous paper, using the canonical trace anomaly coupling the trace of the hadronic energy-momentum tensor to two photons³⁾. In this paper we correct⁶⁾ our previous estimates of $\Gamma(\sigma \rightarrow \gamma\gamma)$ and $\Gamma(J/\psi \rightarrow \sigma\gamma)$, and present new ones based on more sophisticated dynamical assumptions which we now prefer.

Naive use of the canonical anomaly triangle diagram of Fig. 1 for $\Gamma(\sigma \rightarrow \gamma\gamma)$ would give a large rate of the order of several keV, which is unreasonably large. Instead, we believe this estimate is applicable to the two-photon decay of scalar quarkonium $\Gamma(S \rightarrow \gamma\gamma)$. A more reasonable estimate of $\Gamma(\sigma \rightarrow \gamma\gamma)$ is obtained using the higher-order Euler-Heisenberg⁷⁾ quark box diagram of Fig. 2 for the two-gluon-two-photon coupling. This suppresses the contributions of heavy quarks and gives⁸⁾ a plausible rate for $\Gamma(J/\psi \rightarrow \sigma\gamma)$. It indicates a much lower rate for $\Gamma(\sigma \rightarrow \gamma\gamma)$ via light quark loops, but this cannot be computed reliably. Mixing between scalar gluonium σ and scalar quarkonium S is probably important, and we estimate its possible effects in a simple model⁹⁾. In this way we get an upper limit on $\Gamma(\sigma \rightarrow \gamma\gamma)$ which is rather smaller than our previous estimate, and a lower limit which is considerably smaller still.

The canonical trace anomaly³⁾ due to the triangle diagram of Fig. 1 gives the following two-photon coupling $g_{\epsilon\gamma\gamma}$ for a generic scalar meson ϵ :

$$\mathcal{L}_{\epsilon\gamma\gamma}^{\text{CA}} = g_{\epsilon\gamma\gamma}^{\text{CA}} \epsilon(x) F_{\mu\nu}(x) F^{\mu\nu}(x) \quad (1)$$

where

$$f_\epsilon g_{\epsilon\gamma\gamma}^{\text{CA}} = \frac{\alpha R}{6\pi} \quad (2)$$

with the decay constant f_ϵ defined by the matrix element

$$\langle 0 | (\theta_\mu^\mu)_{\text{hadronic}} | e \rangle = -m_e^2 f_e \quad (3)$$

Here $(\theta_\mu^\mu)_{\text{hadronic}}$ is the trace of the hadronic energy-momentum tensor including both the quark mass terms and the QCD anomaly²⁾:

$$(\theta_\mu^\mu)_{\text{hadronic}} = \sum_q m_q \bar{q}q + (\theta_\mu^\mu)_{\text{anom}} \quad (4)$$

and

$$R \equiv \frac{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu^+\mu^-)} \quad (5)$$

The coupling $g_{e\gamma\gamma}$ (1) gives³⁾ a decay width

$$\Gamma(e \rightarrow \gamma\gamma) = g_{e\gamma\gamma}^2 \frac{m_e^3}{4\pi} = \left(\frac{\alpha R}{3\pi f_e} \right)^2 \frac{m_e^3}{16\pi} \quad (6)$$

if we use the canonical trace anomaly estimate (2)*).

What happens if we apply this estimate to the scalar gluonium state σ ? The parameter f_σ has been estimated¹⁾ in terms of the mass m_σ of the gluonium state and of the gluon condensate $G_0 \equiv \langle 0 | (\alpha_s/\pi) G_{\mu\nu}^a G_a^{\mu\nu} | 0 \rangle$:

$$f_\sigma = \frac{1}{m_\sigma} \sqrt{\frac{b G_0}{2}} \quad (7)$$

where $b = (11/3)N_C - (2/3)N_F$, with N_C and N_F being the numbers of colours and flavours respectively. Taking the ITEP value⁴⁾ of $G_0 = 0.012 \text{ GeV}^4$, $N_C = 3$, $N_F = 6$ and $R = 5$ for all the six "known" quark flavours, Eqs. (6) and (7) yield

*) The corresponding expressions (29) to (31) of our previous paper¹⁾ were incorrect. See also Ref. 6).

$$\Gamma(\sigma \rightarrow \gamma\gamma) = 7 \left(\frac{m_\sigma}{1 \text{ GeV}} \right)^5 \text{ keV} \quad (?) \quad (8)$$

which is a surprisingly large value. It is comparable with the $\gamma\gamma$ decay widths of conventional $\bar{q}q$ resonances such as the $f[\Gamma(f \rightarrow \gamma\gamma) \sim 3 \text{ keV}^{10}]$, whereas one would have expected a smaller width for the gluonium state σ , since it does not contain charged constituents before mixing. Therefore, we believe that the estimate (2) and the result (8) cannot be applied to scalar gluonium.

Instead, we recall (4) that (θ_μ^μ) contains both the gluonic anomaly piece²⁾ $(\theta_\mu^\mu)_{\text{anom}} \equiv (-b\alpha_s/8\pi) G_{\mu\nu}^a G^{\mu\nu}_a$ and the quark mass terms $m_q \bar{q}q$, and believe that the canonical trace anomaly estimate (6) applies to the scalar quarkonium mesons which dominate matrix elements of $m_q \bar{q}q$, not to the scalar gluonium dominating $(\theta_\mu^\mu)_{\text{anom}}$ in the absence of mixing. Indeed, the result (6) was originally derived³⁾ at a time when the QCD trace anomaly²⁾ was unknown and the strong interactions were treated as if they were superrenormalizable, and then applied to scalar quarkonium mesons at a time when scalar gluonia had not yet been invented by theorists.

In support of this belief, we now show that the estimate (2) of the scalar $\gamma\gamma$ coupling cannot be used to describe effectively the coupling between scalar gluonium, photons and heavy $(\bar{c}c, \bar{b}b, \bar{t}t)$ quarkonia. Instead, the appropriate description of these couplings is given by the Euler-Heisenberg type⁷⁾ of effective Lagrangian given by Fig. 2 for the gluon-photon interactions^{7),8)}:

$$\mathcal{L}_{\gamma\gamma gg}^{\text{EH}} = \frac{3}{2} \frac{\alpha\alpha_s}{180m_q^4} Q_q^2 (F_{\rho\sigma} F^{\rho\sigma}) (G_{\mu\nu}^a G^{\mu\nu}_a) \quad (9)$$

This can be transformed into a scalar gluonium $\sigma\gamma\gamma$ coupling:

$$\mathcal{L}_{\sigma\gamma\gamma}^{\text{EH}} = \frac{3}{2} \frac{\alpha\alpha_s}{180} A \langle 0 | G_{\mu\nu}^a G^{\mu\nu}_a | 0 \rangle \sigma(x) F_{\rho\sigma} F^{\rho\sigma} \quad (10)$$

$$: A = \sum_q \frac{Q_q^2}{m_q^4}$$

where

$$\langle 0 | \alpha_s G_{\mu\nu}^a G_a^{\mu\nu} | \sigma \rangle = \frac{8\pi}{b} m_\sigma^2 f_\sigma \quad (11)$$

Such Lagrangians have already been used⁸⁾ to estimate the decays $J/\psi \rightarrow \gamma + X$, where X = pseudoscalars η, η' or scalar gluonium σ . Taking just the c -quark contribution to (9) and (10), and assuming J/ψ vector meson dominance for one of the photons, one has⁸⁾

$$\begin{aligned} \Gamma(J/\psi \rightarrow \sigma\gamma) &= \frac{\alpha^3}{\pi \times 5^2 \times 3^8 \times 2^3} \frac{m_{J/\psi}^4}{m_c^8} \frac{|\langle 0 | \alpha_s G_{\mu\nu}^a G_a^{\mu\nu} | \sigma \rangle|^2}{\Gamma(J/\psi \rightarrow e^+e^-)} \\ &\times \left(1 - \frac{m_\sigma^2}{m_{J/\psi}^2}\right)^3 \end{aligned} \quad (12)$$

Using $m_c = 1.25$ GeV, formula (12) yields

$$B(J/\psi \rightarrow \sigma\gamma) \equiv \frac{\Gamma(J/\psi \rightarrow \sigma\gamma)}{\Gamma(J/\psi \rightarrow \text{all})} \simeq 1.8 \times 10^{-3} \left(\frac{m_\sigma}{1 \text{ GeV}}\right)^2 \left(1 - \frac{m_\sigma^2}{m_{J/\psi}^2}\right)^3 \quad (13)$$

which seems a reasonable estimate. In contrast, the canonical anomaly estimate (1), (2) applied unquestioningly to estimate $g_{\sigma\gamma\gamma}^{\text{CA}}$ would yield, by comparison with $g_{\sigma\gamma\gamma}^{\text{EH}}$ estimated from (10) and (11):

$$\frac{g_{\sigma\sigma\sigma}^{\text{CA}}}{g_{\sigma\sigma\sigma}^{\text{EH}}} = \frac{15}{\pi^2} \left(\frac{m_q^4}{G_0}\right) \quad (14)$$

for any heavy flavour q . In the case of the c -quark, using $m_c = 1.25$ GeV and $G_0 = 0.012$ GeV⁴ as before, we find $g_{\sigma\gamma\gamma}^{\text{CA}}/g_{\sigma\gamma\gamma}^{\text{EH}} \simeq 310$, which would give $B(J/\psi \rightarrow \gamma\sigma) \simeq O(100)!$ Even worse results would be obtained for heavier flavours such as b and t . The plausible conclusion is that the heavy flavour loops in the canonical trace anomaly triangle diagrams can only be used to estimate the photonic couplings of heavy scalar quarkonium states. Here "heavy" quarks are those weighing more than the critical value

$$m_0 = \left[\left(\frac{\pi^2}{15} \right) G_0 \right]^{1/4} \approx 300 \text{ MeV} \quad (15)$$

for which $g_{\sigma\gamma\gamma}^{\text{CA}} = g_{\sigma\gamma\gamma}^{\text{EH}}$, including certainly the c, b and t quarks. This means that one should use $R \leq 2$ corresponding to the u, d and s quarks when using the canonical trace anomaly result (2) for light scalar mesons.

Presumably, mixing between scalar gluonium and the light scalar quarkonia cannot be neglected, though it is difficult to estimate reliably. We explore the possible magnitude of mixing effects using a simple model⁹⁾ containing a scalar gluonium field $\sigma(x)$, pion fields $\pi_i(x)$ ($i = 1, 2, 3$) and an isosinglet scalar (u,d) quarkonium field $S(x)$. The quark matter fields S and π_i are assumed to form the venerable linear sigma model for chiral $SU(2) \times SU(2)$ symmetry, while the flavourless gluonium field σ is invariant under the chiral $SU(2) \times SU(2)$ transformations. The Lagrangian is of the form⁹⁾

$$\mathcal{L} = \frac{1}{2} [(\partial_\mu S)^2 + (\partial_\mu \pi_i)^2] + \frac{1}{2} (\partial_\mu \sigma)^2 - V(\sigma, S, \pi_i) \quad (16)$$

where the potential V is chiral invariant and obeys the trace anomaly equation:

$$\left(\Theta^\mu_\mu \right)_{\text{anom}} = 4V - \sigma \frac{\partial V}{\partial \sigma} - S \frac{\partial V}{\partial S} - \pi_i \frac{\partial V}{\partial \pi_i} \quad (17)$$

We neglect the quark mass term and assume that $SU(2) \times SU(2)$ symmetry is spontaneously broken. We reparametrize σ and S by $\tilde{\sigma}$ and $\tilde{S}$:

$$\sigma = f_\sigma \exp \left(\frac{\tilde{\sigma}}{f_\sigma} \right) \quad ; \quad S = f_s + \tilde{S} \quad (18)$$

where $f_\sigma = \langle 0 | \sigma | 0 \rangle$, $f_s = \langle 0 | S | 0 \rangle$ and $\langle 0 | \pi_i | 0 \rangle = 0$. The potential V does not contain terms linear in the $\tilde{\sigma}$ and $\tilde{S}$ fields and can only be a function of the two $SU(2) \times SU(2)$ invariants σ and $\sqrt{S^2 + \pi^2}$. One obtains two sum rules

$$f_\sigma^2 m_{\sigma\sigma}^2 - f_s^2 m_{SS}^2 = 4H_0 \quad (19)$$

where $H_0 \equiv -\langle 0 | (\theta_\mu^\mu)_{\text{anom}} | 0 \rangle = (b/8) \langle 0 | (\alpha_s/\pi) G_{\mu\nu}^a G_a^{\mu\nu} | 0 \rangle = (b/8) G_0$, and

$$f_\sigma m_{\sigma\sigma}^2 = -2 f_s m_{ss}^2 \quad (20)$$

where the m_{ij}^2 are entries in the squared mass matrix for the $\tilde{\sigma}$ and $\tilde{S}$ fields. The physical mass eigenstates are S_1 and S_2 :

$$\left. \begin{aligned} \tilde{\sigma} &= S_1 \cos \phi + S_2 \sin \phi \\ \tilde{S} &= -S_1 \sin \phi + S_2 \cos \phi \end{aligned} \right\} \quad (21)$$

where

$$\tan \phi = \frac{m_{\sigma s}^2}{m_{ss}^2 - m_{\sigma\sigma}^2} \quad (22)$$

Unfortunately, we do not know the elements in the squared mass matrix with much reliability.

In this model, the old canonical trace anomaly result (2) is generalized to

$$f_\sigma g_{\sigma\sigma\sigma} + f_s g_{s\sigma\sigma} = \frac{\alpha R}{6\pi} \quad (23)$$

where $R = 5/3$ for the u and d quarks, and the mixing (21), (22) can be used in (23) to relate the $\gamma\gamma$ couplings of the physical mass eigenstates S_1 and S_2 . Since the gluonium field σ is an $SU(2) \times SU(2)$ singlet, it does not change the form of the $SU(2)$ axial current $A_\mu^i = \sigma \partial_\mu \pi_i - \pi_i \partial_\mu \sigma$, so we can use $\langle 0 | A_\mu^i | \pi_j \rangle = \delta_{ij} P_\mu f_\pi$ to deduce that

$$f_s = -f_\pi \quad (24)$$

where $f_\pi = 93$ MeV is the pion decay constant. The sum rule (23) can be used to estimate the scalar $\gamma\gamma$ couplings even if $g_{\sigma\gamma\gamma} = 0$. Indeed, in this case, using (23) and (24) with $R = 5/3$, we obtain

$$\Gamma(S \rightarrow \gamma\gamma) \approx 4 \times \left(\frac{m_S}{1 \text{ GeV}} \right)^3 \text{ keV} \quad (25)$$

for the scalar quarkonium in the limit of small mixing $\phi \rightarrow 0$. This result seems reasonable for $m_S \sim m_\delta \sim 980 \text{ MeV}$, when compared with $\Gamma(f \rightarrow \gamma\gamma) \approx 3 \text{ keV}^{10}$. We conclude that the canonical trace anomaly result (2), even with $R = 2$ for the light quarks alone, can only be regarded as an upper bound for the decay of scalar gluonium into $\gamma\gamma$. Presumably, after mixing, the eigenstate S_1 in (21) which is mainly gluonic has a decay width smaller than the unmixed scalar quarkonium value (25).

To estimate the $\gamma\gamma$ coupling of scalar gluonium in the limit of small mixing $\phi \rightarrow 0$, we return to the Euler-Heisenberg Lagrangian (10). We include all the quark box diagrams:

$$A = \sum_q \frac{Q_q^2}{m_q^4} \approx \frac{Q_u^2 + Q_d^2}{m_{u,d}^4} = \frac{5}{9 m_{u,d}^4} \quad (26)$$

and use the constituent masses $m_{u,d} = 336 \text{ MeV}^{11}$. Substituting (26) into (10), and using (11) with f_σ fixed by the sum rule (19) with $m_{SS}^2 = m_\delta^2$ and $f_S = 93 \text{ MeV}$ (24), we find

$$\Gamma(\sigma \rightarrow \gamma\gamma) = 0.2 \times \left(\frac{m_\sigma}{1 \text{ GeV}} \right)^5 \text{ keV} \quad (27)$$

for unmixed scalar gluonium. This value is reasonably small, as one would expect for a state without charged constituents. It is also interesting to substitute this result into the sum rule (23), because it has an important impact on our previous estimate of $\Gamma(S \rightarrow \gamma\gamma)$ (25). We find

$$f_S g_{S\gamma\gamma} = \frac{5\alpha}{18\pi} \left(1 - \frac{\pi^2}{15} \frac{G_0}{m_{u,d}^4} \right) \approx 0.38 \left(\frac{5\alpha}{18\pi} \right) \quad (28)$$

where we put $R = 5/3$ for consistency. This yields

$$\Gamma(S \rightarrow \gamma\gamma) = 0.56 \times \left(\frac{m_S}{1\text{GeV}}\right)^3 \text{ keV} \quad (29)$$

which is a factor 7 smaller than the naive estimate (25) with $g_{\sigma\gamma\gamma} = 0$ in the sum rule (23). This may give some appreciation of the uncertainties in estimating quarkonium couplings to photons.

It is amusing to translate the result (29) into a prediction for $\Gamma(\delta \rightarrow \gamma\gamma)$, assuming the quark model prediction $\Gamma(\delta \rightarrow \gamma\gamma)/\Gamma(S \rightarrow \gamma\gamma) = 9/25$. Taking $m_\delta = m_S = 980 \text{ MeV}$, Eq. (29) would imply

$$\Gamma(\delta \rightarrow \gamma\gamma) = 0.19 \text{ keV} \quad (30)$$

This can be compared with the recent experimental result¹²⁾ of the Crystal Ball group at DORIS:

$$\Gamma(S \rightarrow \gamma\gamma) B(S \rightarrow \pi\eta) = (0.19 \pm 0.07^{+0.10}_{-0.07}) \text{ keV} \quad (31)$$

which agrees surprisingly well with the prediction (30) if $B(\delta \rightarrow \pi\eta) \approx 1$.

It should be stressed that while one might expect that in the (σ, S) squared mass matrix $m_{SS}^2 = m_\delta^2 = (980 \text{ MeV})^2$, the physical scalar mesons S_1 and S_2 (21) generally have unknown masses depending on the pure gluonium squared mass $m_{\sigma\sigma}^2$ and on the off-diagonal mixing term $m_{\sigma S}^2$. Moreover, one expects the scalar mesons to be broad states¹⁾, and we have neglected the mixings with strangeonium states, $\bar{q}q\bar{q}q$ mesons and radial excitations. Modulo all these reservations, the light scalar meson couplings to $\gamma\gamma$ probably lie between the unmixed scalar gluonium estimate (27) and the range (29) to (25) estimated for unmixed scalar (u,d) quarkonium. All these values are considerably less than the naive value (8) which would have been extracted uncritically from the canonical trace anomaly (2).

ACKNOWLEDGEMENTS

One of us (J.E.) thanks L.B. Okun and his theoretical colleagues for their hospitality at the Institute for Theoretical and Experimental Physics during work on this paper. The other (J.L.) thanks V.M. Mesheryakov for his support.

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FIGURE CAPTIONS

Fig. 1 : Anomalous triangle diagram for the θ_μ^μ - $\gamma\gamma$ coupling.

Fig. 2 : Euler-Heisenberg box diagram for the gg - $\gamma\gamma$ coupling.

Fig. 1

Fig. 2