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Photon plus missing energy in polarized e^- and e^+ collisions

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Abstract

We studied quantitatively the reactions $e_R^- e^+ + \gamma \nu \bar{\nu}$, $\gamma \nu \nu^*$ and $e_{R,L}^- e^+ + \gamma \gamma \gamma$. For sneutrino counting the polarized e^- beam is very useful as the reaction depends only on sneutrino mass as the only unknown. If not found limits on the parameters of sneutrinos can be established without large ambiguity. The asymmetry in the cross sections between left and right-handed-electrons is best suited for photon searches. For neutrino counting at the Z^0 peak no advantage or disadvantage is found when a e_R^- beam is used.

In this paper we study the effects of using a polarized e^- beam in the reaction

$$e^+ e_{L,R}^- + \gamma + \text{missing energy} \quad (I)$$

We concentrate on three reactions where a left(right)-handed polarized $e_L(R)$ beam can be of help in exploring new physics beyond the $SU(2) \times U(1)$ electroweak gauge theory with three quark and lepton families. Hereafter this is referred to as the standard model. Since our interest lies in getting at new phenomena the effects of the standard model become important background processes. To suppress this source of events in $e^+ e^-$ collisions two techniques immediately spring to mind. The first one is to make use of some kind of asymmetry. One example is to study the scattering of e_R^- versus a left-handed e_L^- which is particularly useful in getting rid of electrodynamic backgrounds. Indeed this was successfully used at SLAC for measuring the parity violating effect of neutral currents in the electron sector.¹ Another tool which is less emphasized is the use of an incoming e_R^- beam. Obviously, with an incoming e_R^- state the conventional charged weak current will not participate in the reaction. In this paper we study the use of the above two techniques in the following three reactions:

$$e_R^- e^+ + \gamma \nu_l \bar{\nu}_l, \quad (IA)$$

$$e_R^- e^+ + \gamma \bar{\nu}_l \nu_l^*, \quad (IB)$$

and

$$e_{R(L)}^- e^+ + \gamma \gamma \gamma, \quad (IC)$$

where ν_l denotes the three standard neutrinos with $l=e,\mu,\tau$ plus possible more families; $\bar{\nu}_l$ are the corresponding scalar neutrinos and γ

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denotes the photino. The last two are important ingredients in any supersymmetric extension of the standard model. The neutrinos will escape the detectors as missing energy. If the sneutrinos (photino) are the lightest supersymmetric particle they will also escape detection as missing energy.

Reaction (IA) with unpolarized incoming beams has been studied extensively as a neutrino counting experiment² or as a means of probing the internal structure of the Z^0 -boson.³ With a e_R^- beam the W-boson contribution will be switched off and the cross section for (IA) will only be a simple function to the number of neutrinos, N_ν , kinematically accessible. In Sec. II we will discuss this in detail. It will be shown that additional families of light neutrino will be clearly distinguished from the standard model. An additional neutrino mass between 20 GeV/c² and 40 GeV/c² can be distinguished from a light one at SLC or LEP. On the other hand if the additional neutrinos are lighter than 20 GeV/c² they will be indistinguishable from being massless.

One currently popular theoretical endeavour is the supersymmetric extension of the standard model. This model predicts the existence of a supersymmetric partner for each particle of the standard model. Hence, one expects at least three scalar neutrinos, $\tilde{\nu}_L$ ($\tilde{\nu}=e, \mu, \tau$) associated with the standard model. Due to conservation of R parity⁴ these will be stable if they are the lightest mass eigenstates.⁵ In this case reaction (IB) using a e_R^- beam is particularly clean for the study of sneutrinos. This is due to the advantage that an e_R^- initial state not only switches off the W^- -boson contribution it also eliminates the effect charge-gaugino exchange which is left-handed. Thus, a major uncertainty of supersymmetric model is eliminated. The charged higgsino contribution

is negligible since its coupling to the $e\bar{\nu}$ system is proportional to the electron mass, m_e . Hence, reaction (IIB) is sensitive to only the mass of the sneutrino and e_R^- will be valuable for testing the hypothesis that sneutrinos are the lightest supersymmetric object.

A more commonly used hypothesis in supersymmetric models is to assume that the photino is the lightest mass eigenstate. In this case reaction (IC) will be the new physics one would encounter in a missing energy plus one photon experiment. Unlike the previous two cases where the Z^0 is an efficient source of neutrinos and/or sneutrinos, photinos do not couple directly to the Z^0 . Reaction (IC) proceeds via selection exchanges in the t-channel. In this case we suggest that the best variable to measure will be the asymmetry in the cross section σ_R of a right-handed electron (e_R^-) versus that of a left-handed electron (e_L^-). We define

$$\Delta\sigma \equiv \sigma_R - \sigma_L. \quad (1.1)$$

We calculated in Sec. II that $\Delta\sigma$ vanishes at $\sqrt{s}=46$ and 128 GeV for the standard model. At these points the background from standard physics will be negligible and thus will be a good place to look for photinos.

In Sec. III we discuss our results and compare them with the case of unpolarized collisions. We present cross sections with the phase-space of the two missing particles integrated over. One can perform the final integration over the detected photon as is appropriate for the particular experimental setup. For illustration we have chosen the cutoff of the photon energy E_γ at 200 MeV⁶ and angular cuts of the photon at 3° from the incoming axis. We emphasize that a close to 4π coverage is crucial to this type of experiment. One must detect as many charged particles as

possible. With large angular cuts one is also sacrificing a large portion of the cross section which one can ill afford to do when looking for new physics.

II. CALCULATIONS OF $e\bar{e}_R(L) \rightarrow \gamma + \text{MISSING ENERGY}$

We present here details of the calculations of reaction I(A-C). The total cross section as a function of $\sqrt{s}$, the centre of mass energy, as well the photon spectrum for each case will be given.

$$A. \bar{e}\bar{e}_R + \gamma\nu$$

Here we are interested in the simplest possible extension of the standard model; i.e. the addition of one or more families to the standard model. The added neutrinos can be either massless or massive. Only two Feynman diagrams are involved in the calculation and are given in Fig. (I-1). In using an $\bar{e}_R$ beam we have switched off the t-channel W-boson exchange process which contributes to the $\gamma\nu_e\nu_e$ reaction for unpolarized beams collisions. The effect of the W-boson is not large near the Z^0 -peak. For us only the two s-channel diagrams contribute. This makes the calculation simpler and the cross section for the reaction should be directly proportional to the number of families of neutrinos away from the Z^0 -peak. Near $\sqrt{s}=90$ GeV, this linear relationship is spoiled by the fact that the width (Γ) of the Z^0 increases as the number of families increases. One can show that for each additional family, the width increases by $\Delta\Gamma = .18[1-4m_W^2/M_Z^2]^{3/2}$ GeV where m_W is the mass of the neutrino considered and M_Z the mass of the Z-boson. In this paper, we assumed a width of 3.0 GeV from the minimal standard model and increased it by $\Delta\Gamma$ for each family. We also assume that additional charged fermions are heavy enough as not to contribute to the Z^0 -width significantly.

For each neutrino of mass m_ν , the photon spectrum is easily calculated to be

$$\frac{d\sigma}{dx} = \frac{\alpha^3}{6 \cos^4 \theta_W s} \frac{x(1-x)}{\left[(1-x) - \frac{M_Z^2}{s} \right]^2 + \frac{(1+\Delta\Gamma)^2 M_Z^2}{s^2}} \sqrt{1 - \frac{4m_\nu^2}{s(1-x)}} \left[\left(\frac{1+(1-x)^2}{x^2} \right) \ln \left(\frac{1+\cos\theta_C}{1-\cos\theta_C} \right) - 1 \right], \quad (2.1)$$

where $x \equiv \frac{2\omega}{\sqrt{s}}$ (2.2).

with ω being the photon energy. The angle θ_C is the smallest angle away from the beam axis that an emitted photon can be detected. For our numerical work we choose it to be 3° .

The total cross section for the standard model with 3 light neutrinos is given by integrating over the appropriate limits of x and multiplying by 3. The results of this integration are shown in Fig. 2 where we also display the effects of a massive neutrino for $m_\nu=0$, 20 and 40 GeV/ c^2 .

It appears that a clear distinction between three and four light neutrinos is relatively easy to observe. Purely due to kinematics a neutrino heavier than 40 GeV/ c would be difficult unless one has good control over the systematics.

Besides the total cross section, the photon spectrum also contains useful information. For this part of the analysis, we used an incoming beam energy of 50 GeV. Again, we considered the examples of the fourth neutrino as having $m_\nu=0$, and 40 GeV/ c^2 .

For massive neutrinos the photon spectrum in general reveals a kink at the kinematic limit of producing them. For $\sqrt{s}=100$ and $m_\nu=40$ GeV/ c^2

the kink appears at $\omega=18$ GeV. Each massive neutrino will give rise to one kink in the photon spectrum. The lighter the neutrino the further out the kink will occur and the smaller the differential cross section will be. As shown in Fig. 3 even for a 40 GeV/c² neutrino the kink is barely discernible.

$$B. \bar{e}e_R \rightarrow \tilde{\gamma}\tilde{\nu}_e^*$$

The diagrams contributing to the reaction $\bar{e}e + \tilde{\gamma}\tilde{\nu}^*$ are shown in Fig. 4 II-1. If one polarizes the incoming e^- beam to a right-handed helicity, only the s-channel diagrams of Fig. 4 contribute. There are two reasons for their simplification: firstly, the charged gaugino, $\tilde{w}$, contributes only to the left-handed electron just as the W-boson of the standard model does, and secondly, the higgsino, $\tilde{h}$, coupling to $e^-\tilde{\nu}$ is proportional to the mass of the electron and thus is negligible.

Hence, with a e_R^- initial state we are then left with the s-channel annihilation. The incoming e_R^- beam frees us from the uncertainties in the mass of the $\tilde{w}$ and the mixing strength with the $\tilde{h}$.

The differential cross section for the production of one sneutrino of mass $m_{\tilde{\nu}}$ is given by

$$\frac{d\sigma}{dx} = \frac{\alpha^3}{12 \cos^4 \theta_W s} \frac{x(1-x)}{\left[(1-x) - \frac{M_Z^2}{s} \right] + \frac{[\Gamma + \Delta\Gamma]^2 M_Z^2}{s^2}} \quad (2.3)$$

$$\left[1 - \frac{4m_{\tilde{\nu}}^2}{s(1-x)} \right]^{3/2} \left\{ \log \left[\frac{1+\cos\theta_C}{1-\cos\theta_C} \right] \left[\frac{1+(1-x)^2}{x^2} \right] - 1 \right\}.$$

Equation (2.3) shows the characteristic p-wave coupling of scalars to the Z⁰-boson. In this case also one must take into account the fact

that the width, Γ , of the Z₀ will increase with the number of sneutrinos. Here, we must add

$$\Delta\Gamma_{\tilde{\nu}} = .09 \left[1 - \frac{4m_{\tilde{\nu}}^2}{M_Z^2} \right]^{3/2} \text{ GeV} \quad (2.4)$$

per sneutrino to the standard model width of 3 GeV. In the minimal supersymmetric extension of the standard model one must have at least 3 scalar neutrinos. It is interesting to note that if all three sneutrinos are light they contribute 270 MeV to the width of the Z⁰. This corresponds to one and one-half families of massless neutrinos. Hence, we treat three cases as follows: we add to the minimum standard model (i) three light, i.e. less than 20 GeV/c², scalar neutrinos (ii) four light sneutrino and (iii) one 25 GeV/c² sneutrino only. As before, the integration of Eq. (2.3) to obtain the total cross section was performed numerically. The total cross sections for these cases are shown in Fig. 5. It is clear from the graph that the difference between three and four light sneutrinos will be difficult to distinguish unless one has very good statistics. However only one heavy sneutrino is easy to discern. It is important to note that the standard model contribution has been added in.

Comparing Figs. 2 and 5 shows that there is a spectacular change in the total cross section from the standard model result if all three sneutrinos are light. The effect is not as great as that of three extra light neutrinos but larger than only one extra light neutrino. However, if only one sneutrino is light, say, $\tilde{\nu}_e$ then its effect on the total cross section is small and indistinguishable from the contribution of a heavy neutrino ($m_{\nu} > 40$ GeV/c²). To distinguish between these two latter cases one has to go to the photon spectrum and observe a kink for

a heavy neutrino. A light sneutrino will not give a visible kink in the photon spectrum.

$$C. \bar{e} e_{L,R} + \gamma \gamma$$

We now turn to the case where the missing energy is carried off by photinos. For unpolarized beams the reaction has been studied by many authors.⁷ The Feynman diagrams to be evaluated are shown in Fig. 6.

Notice here that we have two orthogonal sets of diagrams: one that is right-handed and goes through the right-handed selection, $\tilde{e}_R$, and one that is left-handed and goes through the left-handed selection, $\tilde{e}_L$.

In general the mass of these two particles are different; and since they have the same quantum numbers, there will be mixing between them. If we denote the mass eigenstates by $\tilde{e}_1$ and $\tilde{e}_2$ they are related to $\tilde{e}_L$ and $\tilde{e}_R$ via:

$$\tilde{e}_R = \tilde{e}_1 \cos \tilde{\theta} + \tilde{e}_2 \sin \tilde{\theta} \quad (2.5)$$

$$\tilde{e}_L = -\tilde{e}_1 \sin \tilde{\theta} + \tilde{e}_2 \cos \tilde{\theta} \quad (2.6)$$

where $\tilde{\theta}$ is the mixing angle.

The mixing angle $\tilde{\theta}$ is in general proportional to the off diagonal term of the mass matrix. In the minimal supersymmetric standard^{9,10,11} model this is proportional to the ratio of the mass of the electron to that of supersymmetry breaking scale which is expected to be at least that of the Fermi scale. Thus, $\tilde{\theta}$ is expected to be small.

If the mixing $\tilde{\theta}$ is indeed small then the cross sections σ_R and σ_L will depend only on $\tilde{m}_1$ and $\tilde{m}_2$ respectively where they are the corresponding masses of $\tilde{e}_1$ and $\tilde{e}_2$. The explicit expressions for σ_R and σ_L are lengthy. They are given in Appendix I. We note that the cross section for photino production is the order of 1 pb with the cuts we imposed. This is about two orders of magnitude lower than that of the standard

model. Representative curves of σ_R as a function of $\sqrt{s}$ is given in Figs. 7 and 8 for $m_{\tilde{\nu}}=0$ and $m_{\tilde{\nu}}=30$ GeV/c² with various values of $\tilde{m}_1$.

Our results agree with that of Ref. 9 wherever they overlap. These results hold for σ_L with the replacement of $\tilde{m}_1$ by $\tilde{m}_2$. Unless $\tilde{m}_1$ and $\tilde{m}_2$ are degenerate or very close in mass $\sigma_R > \sigma_L$ if $\tilde{m}_1 < \tilde{m}_2$, and conversely $\sigma_L > \sigma_R$ if $\tilde{m}_1 > \tilde{m}_2$.

The smallness of the photino production cross section calls for using the asymmetry $\Delta\sigma \equiv \sigma_R - \sigma_L$ to suppress the effects from the standard model. In Fig. 9 we show $\Delta\sigma/(\sigma_R + \sigma_L)$ as a function of $\sqrt{s}$. It is seen that the left and right-handed cross sections become equal at $\sqrt{s}=46$ and 128 GeV. If $\tilde{m}_1 \neq \tilde{m}_2$ these two energies will be most suitable for photino searches. To give an indication of the size of the signals we display the cross sections at these energies for different values of $\tilde{m}_1$ in Table I. All of these cross sections are small.

III. CONCLUSIONS

We have considered in detail the effects of having a right-handed polarized e_L^- beam in the reactions IA-C. We note in passing that having a further polarized e_L^+ beam will increase the cross sections we presented by a factor of two due to no spin averaging. A summary of our results is given in Table II. We briefly describe our results as follows:

- 1) For neutrino counting no significant advantage is achieved by using e_R^- . On the other hand no significant loss in signal will occur by using it. One or more light neutrinos beyond that of the standard model can be easily measured. Mass effects for neutrino in the mass range 20 to 40 GeV/c² can show up in cross section measurements. Kinks in the photon spectrum can be used to detect heavy neutrinos.

2) A e_R^- beam is particularly clean for sneutrino searches. It is free of the unknown gaugino mass and its mixing. If all the three sneutrinos in the minimal broken supersymmetric standard model are light the cross section near the Z^0 -peak is large compared to the standard model. If only one of them is light the effect in the cross section will be similar to having an extra heavy neutrino. One has to use the photon spectrum to separate the two cases.

3) Due to the small cross section, photino searches are very difficult. However, if the selectron masses $\tilde{m}_1$ and $\tilde{m}_2$ are not degenerate the best energies for such searches are at $\sqrt{s} \sim 46$ and 128 GeV where $\Delta\sigma \sim 0$ for the standard model.

We have presented our studies for reactions I(A-C) in cases where new physics occur. Even if no new physics are found at SLC and LEP one can still use our results for case B and C to set lower limits on $m_{\tilde{\chi}_1^0}$, $m_{\tilde{\chi}_2^0}$ and $\tilde{m}_1/\tilde{m}_2$. Having polarized beams¹² will be valuable in such studies.

Finally, we note that our analysis indicates that if new generations of neutrinos (sneutrinos) exist and have masses between 0.5 and 20 GeV/c² then neutrino counting at the Z^0 -peak will detect them as "light" neutrinos (sneutrinos). On the other hand, neutrino counting with the experiment $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ will not be sensitive to these new neutrinos (sneutrinos). Hence, it will be of interest to see if the two experiments give the same number for neutrino counting.

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APPENDIX I

In this appendix we give the complete scattering matrix element obtained from calculating the diagrams given in Fig. 6 where the kinematics are defined. Full mixing of the scalar electrons are given.

As mentioned in the text, the s lepton eigenstates do not necessarily have the same mass. We express the mass eigenvalues by m_{e1} and the mixing angle by $\tilde{\theta}$ [see Eq. 2.5-2.6]. We adopt the conventions of Ref. 11 and the Feynman rules are given there. With the kinematics defined in Fig. 6, after a tedious but straightforward calculation, we obtain the following:

$$T1 = \left\{ \frac{16e^6(k \cdot l_1)(\bar{p} \cdot l_2)}{(p \cdot k)[M_1^2 - 2p \cdot l_2][M_3^2 - 2p \cdot l_2]} + \left(\frac{p \leftrightarrow \bar{p}}{l_1 \leftrightarrow l_2} \right) \right\} + (l_1 \leftrightarrow l_2), \quad (A.1)$$

$$T2 = \frac{64e^6(p \cdot l_1)(\bar{p} \cdot l_2)(2(p \cdot l_1) - m_e^2)}{[M_1^2 - 2(p \cdot l_1)]^2[M_3^2 - 2p \cdot l_2]^2} + (l_1 \leftrightarrow l_2), \quad (A.2)$$

$$T3 = \frac{-16e^6}{(p \cdot k)(\bar{p} \cdot k)[M_1^2 - 2p \cdot l_2][M_3^2 - 2p \cdot l_1]} \left\{ \begin{aligned} & (p \cdot l_1)(\bar{p} \cdot l_2)(p \cdot k + \bar{p} \cdot k - 2p \cdot p) \\ & + (p \cdot p)((k \cdot l_2)(\bar{p} \cdot l_1) + (\bar{p} \cdot l_2)(k \cdot l_1)) \\ & - (p \cdot l_2)(p \cdot l_1)(\bar{p} \cdot k) - (\bar{p} \cdot l_1)(\bar{p} \cdot l_2)(p \cdot k) \end{aligned} \right\} + (l_1 \leftrightarrow l_2) \quad (A.3)$$

$$T4 = \left\{ \frac{32e^6(\bar{p} \cdot l_2)[2(p \cdot l_1)(k \cdot l_1) - 2(p \cdot l_1)^2 - (p \cdot k)m_e^2]}{(p \cdot k)[M_1^2 - 2p \cdot l_2]^2[M_3^2 - 2p \cdot l_1]} + \left(\frac{p \leftrightarrow \bar{p}}{l_1 \leftrightarrow l_2} \right) \right\} + (l_1 \leftrightarrow l_2), \quad (A.4)$$

$$T5 = \frac{16e^6 m_e^2(p \cdot p)(p \cdot k - p \cdot \bar{p} + p \cdot k)}{(p \cdot k)(\bar{p} \cdot k)[M_1^2 - 2p \cdot l_2][M_3^2 - 2p \cdot l_2]} + (l_1 \leftrightarrow l_2), \quad (A.5)$$

$$T6 = \left\{ \frac{-16e^6 m_e^2[-2(p \cdot p)(\bar{p} \cdot l_2) + (p \cdot p)(k \cdot l_2) + (\bar{p} \cdot k)(p \cdot l_2) - (p \cdot k)(\bar{p} \cdot l_2)]}{(p \cdot k)[M_1^2 - 2p \cdot l_2][M_3^2 - 2p \cdot l_2]} + \left(\frac{p \leftrightarrow \bar{p}}{l_1 \leftrightarrow l_2} \right) \right\} + (l_1 \leftrightarrow l_2) \quad (A.6)$$

$$T7 = \frac{-16e^6 m_Y^2 (p \cdot k)}{(\bar{p} \cdot k) [M_1^2 - 2p \cdot \ell_1] [M_2^2 - 2p \cdot \ell_2]} + (p \leftrightarrow \bar{p}), \quad (A.7)$$

$$T8 = \frac{64e^6 m_Y^2 (p \cdot \bar{p}) (\ell_1 \cdot \ell_2 - p \cdot \ell_1 - \bar{p} \cdot \ell_2)}{[M_1^2 - 2p \cdot \ell_1] [M_2^2 - 2p \cdot \ell_2] [M_3^2 - 2\bar{p} \cdot \ell_2] [M_3^2 - 2p \cdot \ell_1]}, \quad (A.8)$$

$$T9 = \frac{+32e^6}{[M_1^2 - 2p \cdot \ell_1]} \left[\frac{(p \cdot \bar{p}) m_Y^2}{[M_2^2 - 2p \cdot \ell_2] [M_3^2 - 2\bar{p} \cdot \ell_1]} - \frac{2(p \cdot \ell_1)(p \cdot \ell_2)}{[M_1^2 - 2p \cdot \ell_1] [M_3^2 - 2\bar{p} \cdot \ell_2]} \right] + (\ell_1 \leftrightarrow \ell_2), \quad (A.9)$$

$$\text{where} \quad M_1^2 \equiv m_Y^2 - m_1^2. \quad (A.10)$$

$$\text{and} \quad T(M_1, M_j) = \sum_{\ell=1}^9 T_\ell(M_1, M_j). \quad (A.11)$$

The complete matrix element squared for the scattering of $e_L(e_R)$ is given by

$$S_{(R)}^L = T(M_1, M_1) \begin{pmatrix} \cos^2 \tilde{\theta} \\ \sin^2 \tilde{\theta} \end{pmatrix} + T(M_2, M_2) \begin{pmatrix} \sin^2 \tilde{\theta} \\ \cos^2 \tilde{\theta} \end{pmatrix} \pm (T(M_1, M_2) + T(M_2, M_1)) \sin \tilde{\theta} \cos \tilde{\theta}. \quad (A.12)$$

If the mixing angle is small $\tilde{\theta}$ then only $T(M_1, M_1)$ gives the sizable contribution. With $\tilde{\theta}=0$ we recover the result of Ref. 8. Also if $\tilde{M}_2 > \tilde{M}_1$ then again $T(M_1, M_1)$ gives the major contribution. This is the case we examined numerically.

Table I. $m_Y = 1 \text{ GeV}$.

BEAM at 23 GeV		BEAM at 62 GeV	
$m_{eR}^2 (\text{GeV})$	$\sigma (\text{pb})$	$m_{eR}^2 (\text{GeV})$	$\sigma (\text{pb})$
10	9.5	1	3.0
20	5.2	10	2.2
30	2.8	20	1.8
40	1.6	30	1.5
50	0.9	50	0.9
80	0.2	80	0.5
100	0.1	100	0.3

Table II. Cross-section (pb) for different models

Beam Energy 1 GeV	Standard model (SM)	SM + 1 mass- less ν	SM + 1 ₍₄₀₎ ν GeV	SM + 3 ν mass- less	SM + 1 ₍₄₀₎ ν GeV
30	0.9	1.2	1.1	1.3	1.0
40	6.6	8.7	8.3	9.8	7.2
45	57	76	76	85	64
			63		58
50	190	239	231	262	209
			206		194
60	30	38	37	42	33
			33		31
70	13	17	16	18	15
			15		14
80	7.8	9.8	9.5	11	8.6
			8.6		8.0

Figure Captions

1. Feynman diagram to be considered for the reaction $e^+e^- \rightarrow \nu\bar{\nu}$.
2. Cross section for the reaction $e^+e^- \rightarrow \nu\bar{\nu}$. The standard model is given by the (dotted) line. Four massless neutrinos are given by the solid line; 3 massless and one 40 GeV/c² is given by dash-dot line and 3 massless plus a 20 GeV/c² neutrino is represented by dash line.
3. Photon spectrum when the 4th neutrino is massless (long dash) or has mass 40 GeV/c² (short-dash); as compared to the minimal standard model (solid line) for $\sqrt{s}=100$ GeV.
4. Feynman diagrams for the reaction $e^+e^- \rightarrow \nu\bar{\nu}$; (a) given the annihilation through Z^0 and (b) represents gaugino and higgsino exchange mechanism.
5. Total cross section for the reaction $e^+e^- \rightarrow \nu\bar{\nu}$. The standard model and 4 massless $\tilde{\nu}$ are given by the solid line; the standard model and 3 massless $\tilde{\nu}$ are given by the dash line and the standard model plus 1 $\tilde{\nu}$ of 25 GeV/c² is given by the dotted line.
6. Feynman diagram for the reaction $e^+e^- \rightarrow \nu\bar{\nu} + \nu\bar{\nu}$.
7. Cross section for the reaction $e^+e^- \rightarrow \nu\bar{\nu}$ as a function of the beam energy, for a massless photino and a selectron of 100 GeV/c², 50 GeV/c² and 30 GeV/c².
8. Cross section for the reaction $e^+e^- \rightarrow \nu\bar{\nu}$ as a function of the beam energy, for a photino of 30 GeV and a selectron of 100 GeV/c², 50 GeV/c² and 30 GeV/c².
9. Asymmetry for the reaction $e^+e^- \rightarrow \nu\bar{\nu}$ as a function of beam energy. The points where the asymmetry is 0 are of particular interest.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9