RESOLVING THE INDEFINITENESS PROBLEM IN FREE STRING FIELD THEORIES

I. Bengtsson and H. Hüffel

CERN -- Geneva

A B S T R A C T

The Euclidean action for free bosonic string fields is not positive definite, hence the standard Euclidean path integral is ill-defined. We propose a solution of this problem using stochastic quantization as a guiding principle. Starting from convergent Langevin equations we give a prescription which makes the Fokker-Planck equation well defined. In the resulting equilibrium distribution the path integral is taken along the imaginary axis in some directions in field space.

The so-called duality problem in string field theory - how to reconcile Feynman diagram combinatorics with string combinatorics - is solved in the light front formulation [1] through the use of "time" ordered perturbation theory, where the "time" parameter is given by $\tau = x^+$. A similar solution has been proposed also for the covariant formulation, where the interaction terms [2] contain an extra parameter, which again is crucial for the resolution of the duality problem, but whose origin is so far not clearly understood. A challenging suggestion [3] is that stochastic concepts [4] and specifically stochastic quantization [5] (for a forthcoming review see [6]) might offer a natural setting for string field theory, implying e.g. a new interpretation of this parameter.

On the other hand the Euclidean string field theory actions, in common with ordinary field theories containing fields with spin ≥ 2 , are indefinite, so that straightforward quantization in Euclidean space is not possible. For Einstein gravity modifications of the standard Euclidean path integral have been proposed [7], and several different attempts for quantization within the stochastic scheme have been put forward as well [8,9]

The indefiniteness problem appears already in the free theory; the purpose of this letter is to see how it can be dealt with for free bosonic string fields, using the stochastic scheme as a guiding principle. We will see that the associated Langevin equation exhibits a single projection operator in its drift term, so that it may easily be solved and that the large t limit of gauge invariant quantities is convergent as it stands. The indefiniteness problem manifests itself when one tries to give a probabilistic interpretation of the noise. We offer a rather natural prescription for how to deal with this and study the equilibrium limit of the corresponding Fokker-Planck distribution.

To make our assertions more transparent we will discuss the massless sector of the closed string in detail; it will be seen that our approach implies in fact Rumpf's stochastic treatment [9] of Einstein gravity.

The gauge invariant classical action for free bosonic strings as given in [10] reads (see also [11]; a particularly elegant form of the action can be found in [12], but for our purposes [10] is most suitable):

$$S = \int \langle \psi(x) | H - q T_3^{-1} q | \psi(x) \rangle d^2x \quad (1)$$

Here $|\Psi\rangle$ is a vector in the Fock space spanned by the harmonic oscillators a_n^+ , b_n^+ , $\bar{b}_n^+$ (in the open string case), of which the latter two are fermionic (ghosts and antighosts of the first quantized theory). It is subject to two constraints, one of which says that the shifted first quantized ghost number is zero, and a further symmetrization constraint, which ensures that the operator T_3 can be inverted. The operators in (1) are defined by expanding the first quantized BRS-charge of the string ^[13] in terms of ghost zero modes:

$$Q = b_0 H - \bar{b}_0 T_3 + q \quad (2)$$

The fact that $Q^2 = 0$ (in $D = 26$ dimensions) implies

$$[H, T_3] = [H, q] = [T_3, q] = 0 \quad q^2 = T_3 H \quad (3)$$

Explicit mode expansions are given in ^[10]. Here we just note that H is a positive operator except for the tachyonic sector, which we disregard. If the expectation values with respect to the ghost oscillators are performed, one sees that the action (1) is not positive definite (and one also finds the connection to ^[11]).

For closed strings the action is still given by (1), but the interpretation differs, since $|\Psi\rangle$ is now part of the Fock space spanned by a_n^{+R} , a_n^{+L} , etc., i.e. both the right and left movers of the first quantized string. Eq. (2) reads in the closed case

$$\begin{aligned} Q &= b_0 H + \tilde{b}_0 \Delta H - \bar{b}_0 T_3 - \tilde{\bar{b}}_0 \Delta T_3 + q = \\ &= b_0 (H^L + H^R) + \tilde{b}_0 (H^L - H^R) - \bar{b}_0 (T_3^L + T_3^R) - \\ &\quad - \tilde{\bar{b}}_0 (T_3^L - T_3^R) + q^L + q^R \end{aligned} \quad (4)$$

and the field obeys the further constraint $\Delta H |\Psi\rangle = 0$. Then (3) remain valid modulo terms which vanish when they act on $|\Psi\rangle$.

Varying the action (1) with respect to $\langle\Psi|$, we obtain the equation of motion

$$\begin{aligned} 0 &= (H - q T_3^{-1} q) |\Psi\rangle \equiv H (1 - \frac{1}{H} q T_3^{-1} q) |\Psi\rangle \equiv \\ &\equiv H P |\Psi\rangle \end{aligned} \quad (5)$$

Using (3) we see that P is a projector, i.e.

$$P^2 = P \quad (6)$$

Thus the equation of motion contains a single projector. Note that this is true for both open and closed strings; only the interpretation of (5) differs.

According to the standard procedure of stochastic quantization [5] we now supplement the (Euclidean) string field with an additional "fictitious time" co-ordinate t and associate to (5) the following Langevin equation:

$$\frac{\partial}{\partial t} |\psi(x, t)\rangle = -H P |\psi(x, t)\rangle + |\eta(x, t)\rangle \quad (7)$$

The same projection operator P as in the classical field equation appears now in the drift term of the stochastic differential equation, where also a noise string field $|\eta\rangle$ has been introduced. Denoting stochastic averages by round brackets, we demand

$$(|\eta\rangle) = (\langle\eta|) = 0 \quad (8)$$

$$(|\eta(x, t)\rangle \langle\eta(x', t')|) = 2 \delta(x - x') \delta(t - t') \mathbb{1}, \text{ etc.} \quad (9)$$

Eqs. (7) - (9) constitute the most natural extension of the stochastic quantization concept from ordinary field theory to string field theory; given the fact that $|\psi\rangle$ and $|\eta\rangle$ carry no indices, (7) - (9) appear to be rather unique.

The solution of (7) can easily be found in analogy to the Maxwell field [5] and is given in Fourier space by

$$|\psi(k, t)\rangle = \int_0^t d\tau [e^{-H(t-\tau)} P + (\mathbb{1} - P)] |\eta(k, \tau)\rangle. \quad (10)$$

As a first example we calculate the two point correlation

$$(|\psi(k, t)\rangle \langle\psi(k', t)|) = \left[\frac{1}{H} P (1 - e^{-2Ht}) + 2t (\mathbb{1} - P) \right] \delta(k + k') \quad (11)$$

which exhibits in its "longitudinal" part the well known linear t -factor, which is characteristic of the random walk behaviour in gauge parameter space [5]. Calculating a gauge invariant quantity as e.g. $(\langle\psi| HP |\psi\rangle)$ we obtain from (11)

$$(\langle\psi(k, t)| HP |\psi(k', t)\rangle) = \text{tr} [P (1 - e^{-2Ht})] \delta(k + k') \quad (12)$$

where the trace is defined with respect to a complete set of states in the first quantized Fock space. Note that the longitudinal modes have been projected away in (12), as is expected from general principles [5] for all gauge invariant quantities. Similarly we want to recall that the stochastic scheme is perfectly consistent without having added gauge fixing terms or ghost fields to the action (1).

Given that H is a positive operator, strict relaxation to equilibrium is obtained for $t \rightarrow \infty$ and the quantum expectation value of (12) reads

$$\lim_{t \rightarrow \infty} (\langle \psi(k, t) | H P | \psi(k', t) \rangle) = \text{tr } P \delta(k + k') \quad (13)$$

Although stochastic quantization based on the Parisi - Wu form [5] of the Langevin equation is consistent without any gauge fixing, a generalization of it to the stochastic formalism is possible [14]. In the present case, it means that the Langevin equation can be rewritten as

$$\frac{\partial}{\partial t} |\psi\rangle = -H P |\psi\rangle + g |\Lambda\rangle + |\eta\rangle \quad (14)$$

where $|\Lambda\rangle$ is - apart from the constraints mentioned in the beginning - an arbitrary string field. Choosing e.g. $|\Lambda\rangle = T_3^{-1} q |\psi\rangle$ we obtain the simple Langevin equation

$$\frac{\partial}{\partial t} |\psi\rangle = -H |\psi\rangle + |\eta\rangle \quad (15)$$

So far we formulated stochastic quantization in terms of Langevin equations. The connection to conventional Euclidean quantization is done by studying the corresponding Fokker - Planck equation. Formal application of this procedure to the present case leads however to an equilibrium distribution which is ill defined, since it is given as an exponential of the indefinite Euclidean action. The reason is that, due to the fermionic oscillator structure, the noise correlations (9) are non - Gaussian and do not allow a conventional probability interpretation.

We will discuss this issue in detail for the massless sector of the closed string (at the same time making clear the connection to [9]); the extension to the full string field will be obvious. Expanding the closed string field as

$$|\psi\rangle = \left[\dots + \frac{1}{2} (h_{\mu\nu} + i A_{\mu\nu}) \alpha_{\mu}^L \alpha_{\nu}^{+R} + \frac{i}{\sqrt{2}} \phi b_i^{+R} \bar{b}_i^{+L} + \dots \right] |0\rangle \quad (16)$$

where $h_{\mu\nu}$ and $A_{\mu\nu}$ are symmetric and antisymmetric tensor fields, respectively, the action for the massless sector becomes

$$S = - \int \chi^T \Gamma \Pi P \chi d^{26}x \quad (17)$$

where

$$\chi = \begin{pmatrix} A_{\mu\nu} \\ h_{\mu\nu} \\ \varphi \end{pmatrix} \quad \Gamma = \begin{pmatrix} 1^a & & \\ & 1^s & \\ & & -1 \end{pmatrix} \quad (18)$$

$$P = \begin{pmatrix} P_b^i & 0 & 0 \\ 0 & P^2 - P_w^0 + P_s^0 & -\sqrt{2}L \\ 0 & \sqrt{2}L & 2 \end{pmatrix} \quad (19)$$

Here δ means the Euclidean metric, $L = \partial\partial/\alpha$. P^2 , P_w^0 , P_s^0 and P_b^1 are members of complete orthogonal sets of projection operators [15] on the space of symmetric and antisymmetric tensor fields, respectively; 1^s and 1^a denote the corresponding unit operators.

The action can be diagonalized using

$$\chi = M \hat{\chi} \quad M = \begin{pmatrix} 1^a & 0 & 0 \\ 0 & 1^s & -\frac{1}{\sqrt{D-2}}\delta \\ 0 & -\frac{1}{\sqrt{2}}\delta & \sqrt{\frac{2}{D-2}} \end{pmatrix} \quad (20)$$

so that

$$S = - \int \hat{\chi}^T \Pi \hat{\chi} d^{26}x \quad (21)$$

where

$$\Pi = M^T \Gamma P M = \begin{pmatrix} P_b^i & 0 & 0 \\ 0 & P^2 - (D-2)P_s^0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (22)$$

which is the usual indefinite action. Note that Π is no projector (although P is).

Expanding the noise string field in the same way as (16), we extract from (7) the component field Langevin equation

$$\frac{\partial}{\partial t} \chi = \square P \chi + \xi \quad (23)$$

where

$$\xi = \begin{pmatrix} \gamma^A \\ \gamma^h \\ \gamma^\varphi \end{pmatrix} \quad (24)$$

The two point correlation of ξ can be read off from (9)

$$(\xi, \xi^\tau) = \begin{pmatrix} 1^A & & \\ & 1^S & \\ & & -1 \end{pmatrix} 2 \delta(x-x') \delta(t-t') \quad (25)$$

As anticipated we note as a general feature that due to the ghost oscillator structure in (16) the noise component fields are Gaussian only up to signs, i.e. they are correlated as plus minus the appropriate unit operators. This is how the indefiniteness problem appears within the stochastic context.

The diagonalized version of the Langevin equation (23) is

$$\frac{\partial}{\partial t} \hat{\chi} = \square \hat{P} \hat{\chi} + \hat{\xi} \quad (26)$$

with

$$\hat{\xi} = M^{-1} \xi \quad \hat{P} = M^{-1} P M = \begin{pmatrix} P'_b & \\ & P_R \\ & & 1 \end{pmatrix} \quad (27)$$

where

$$P_R = P^2 - (D-2) P^0_S + \delta(\delta-L) ; \quad P^2_R = P_R. \quad (28)$$

For the physical noise correlations we find straightforwardly

$$(\hat{\xi}, \hat{\xi}^\tau) = \begin{pmatrix} 1^a & & \\ & 1^S - \frac{1}{D-2} \delta\delta & \\ & & 1 \end{pmatrix} 2 \delta(x-x') \delta(t-t') . \quad (29)$$

We remark that the symmetric tensor part in (26) - (29), corresponding to linearized Einstein gravity, is identical to Rumpf's stochastic treatment of Einstein gravity [9]. He considered, guided by

the principle of general covariance with respect to field redefinitions, a one-parameter family of Langevin equations for the metric tensor field, pointing out a preferred parameter value. It seems noteworthy that the string approach gives directly Rumpf's Langevin equation with the preferred parameter value.

For stochastic quantization of antisymmetric tensors, see [16].

Now let us return to the problem of the non - Gaussian noise, eqs. (24,25). The simplest prescription we can think of is to set $\eta^\varphi = i \eta_p^\varphi$, where η_p^φ is real. Then η_p^φ is Gaussian, and a probabilistic interpretation is possible. The Langevin equation then implies that the fields are complex in general (this does not affect Green functions, since noises are always contracted pairwise in stochastic perturbation theory). If we distinguish between the real and imaginary parts of the fields, and consider the stochastically gauge fixed process (15), we find that the Fokker - Planck equation corresponding to the massless sector of (15) becomes

$$\frac{\partial}{\partial t} P_{FP} = \int d^4x \left\{ \frac{\delta}{\delta h_{Re}} (-\square h_{Re} + \frac{\delta}{\delta h_{Re}}) + \frac{\delta}{\delta h_{Im}} (-\square h_{Im}) + \frac{\delta}{\delta \varphi_{Re}} (-\square \varphi_{Re}) + \frac{\delta}{\delta \varphi_{Im}} (-\square \varphi_{Im} + \frac{\delta}{\delta \varphi_{Im}}) \right\} P_{FP}. \quad (30)$$

The equilibrium distribution is easily found as

$$P_{FP}^{eq} = \delta(\varphi_{Re}) \delta(h_{Im}) e^{-\int (\partial h_{Re} \partial h_{Re} + \partial \varphi_{Im} \partial \varphi_{Im}) d^4x} \quad (31)$$

Thus our prescription is equivalent to performing the path integral along the imaginary axis for the troublesome directions in field configuration space.

Our calculation can straightforwardly be repeated for the diagonalized fields; we find an equilibrium distribution which is not quite the same as that corresponding to the approach of [7], however it implies the same propagators.

We finally note that a generalization of stochastic quantization working directly in Minkowski space [17,18] can be applied also to the above analysis: The noise correlations (8) - (9) then remain unchanged, whereas the Langevin equation reads

$$\frac{\partial}{\partial t} |\psi\rangle = i H P |\psi\rangle + |\eta\rangle \quad (32)$$

necessitating as in [17] a reinterpretation of the equilibrium limit of the (complex) stochastic process.

In conclusion, if we insist on a probabilistic interpretation of stochastic quantization, we are led to a well defined prescription for the Euclidean path integral over free string fields, despite the fact that the action is indefinite. It seems an exciting challenge to apply the stochastic methods also for the interacting case.

Acknowledgements:

We thank Poul Damgaard and André Neveu for helpful discussions and Helmut Rumpf for crucial help concerning the noise reinterpretation.

REFERENCES

- [1] M.Kaku and K.Kikkawa, Phys.Rev. D10 (1974) 1110, 1823
- [2] H.Hata, K.Itoh, T.Kugo, H.Kunimoto and K.Ogawa, Kyoto preprints KUNS-814 HE(TH) 85/10 (1985), KUNS-817 HE(TH) 85/13 (1985), KUNS-829 HE(TH) 86/3 (1986)
A.Neveu and P.West, CERN preprints TH 4315/85 (1985), TH 4358/86 (1986)
- [3] W.Siegel, Phys. Lett. 124B (1984) 276
- [4] G. Parisi and N. Surlas, Phys. Rev. Lett. 43 (1979) 744
- [5] G. Parisi and Wu Yongshi, Scientia Sinica 24 (1981) 483
- [6] P.H. Damgaard and H. Hüffel, "Stochastic Quantization", Phys. Reports C, in preparation
- [7] G.W. Gibbons, S.W. Hawking and M.J. Perry, Nucl. Phys. B138 (1978) 141
- [8] J. Greensite and M.B. Halpern, Nucl. Phys. B242 (1984) 167,
H. Hüffel and H. Rumpf, Z. Phys. C29 (1985) 319,
T. Fukai and O. Okano, Prog. Theor. Phys. 73 (1985) 790
- [9] H. Rumpf, Phys. Rev. D33 (1986) 942
- [10] W. Siegel and B. Zwiebach, Berkeley preprint UCB-PTH-85/30 (1985),
K. Itoh, T. Kugo, H. Kunitomo and H.Ooguri, Kyoto preprint KUNS-800 HE(TH) 85/04 (1985)
- [11] A. Neveu and P. West, Nucl. Phys. B268 (1986) 125,
T. Banks and M.Peskin, Nucl. Phys. B264 (1986) 513
- [12] A. Neveu, H. Nicolai and P. West, Phys. Lett 167B (1986) 307,
E. Witten, Nucl. Phys. B268 (1986) 253
- [13] M. Kato and K. Ogawa, Nucl. Phys. B212 (1983) 443,
S. Hwang, Phys. Rev. D28 (1983) 2614
- [14] D. Zwanziger, Nucl. Phys. B192 (1981) 259
- [15] P. van Nieuwenhuizen, Nucl. Phys. B60 (1973) 478
- [16] P.A. Amundsen, P.H. Damgaard and B.S. Skagerstam, Univ. of Göteborg preprint 86-15 (1986)
- [17] H. Hüffel and H. Rumpf, Phys. Lett. 148B (1984) 104
- [18] E. Gozzi, Phys. Lett. 150B (1985) 119