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ON THE MIKHEYEV-SMIRNOV-WOLFENSTEIN (MSW) MECHANISM
OF AMPLIFICATION OF NEUTRINO OSCILLATIONS IN MATTER

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A B S T R A C T

Mikheyev and Smirnov have recently proposed a novel and plausible solution of the solar neutrino problem, based on the resonant amplification of the neutrino oscillations in matter. We comment on several aspects of this mechanism. (a) For the values of neutrino masses and mixing angles predicted by the seesaw model of grand unified theories, the MSW effect may take place most naturally in the Sun leading to a considerable reduction of the flux of solar electron neutrinos, the dominant transition being $\nu_e \rightarrow \nu_\tau$ (rather than $\nu_e \rightarrow \nu_\mu$). (b) Oscillations between the ordinary neutrinos (ν_e, ν_μ, ν_τ) can affect primordial nucleosynthesis, but the effect is small (i.e., the abundance of ^4He is predicted to change by less than 1.3×10^{-3}). (c) A comparison of some of the general properties of neutrino oscillations in matter and in vacuum is given.

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INTRODUCTION

For some years the event rate in the $^{37}\text{Cl} \rightarrow ^{37}\text{Ar}$ solar neutrino experiment¹⁾ (2.0 ± 0.3 SNU) has been considerably below the prediction²⁾ 5.8 ± 2.2 SNU of the standard solar model. One explanation for the discrepancy is the existence of vacuum oscillations³⁾ of the ν_e into other neutrinos. These could be important for neutrino mass-squared differences $\Delta m^2 \equiv m_1^2 - m_2^2$ as small as $\Delta m^2 \sim (10^{-11} - 10^{-10}) \text{ eV}^2$, but only if the mixing angles are large. Other canonical explanations involve non-standard solar models. The ^{37}Cl detector is mainly sensitive to the relatively high energy (from 0.81 MeV up to 14 MeV) neutrinos from ^8B decay. The flux of these ^8B neutrinos depends very sensitively on the temperature of the solar core and could be changed significantly by modifications of the standard solar model.

A $^{71}\text{Ga} \rightarrow ^{71}\text{Ge}$ experiment could distinguish between these two possibilities. Most of the expected ^{71}Ga event rate is from the low energy pp neutrinos, the flux of which can be inferred from the over-all solar luminosity and is relatively insensitive to the temperature of the solar core. The predicted ^{71}Ga event rate of ≈ 107 SNU can be reduced at most to around 78 SNU in most non-standard solar models^{2), 4)}. The traditional view has been that a flux lower than this would imply large vacuum oscillations, which would reduce the ^{71}Ga rate by a factor comparable to the ^{37}Cl event rate reduction for most oscillation parameters (e.g., to around 40 SNU).

Recently, Mikheyev and Smirnov⁵⁾ have proposed an elegant new solution to the solar neutrino problem^{*}). The mechanism of reduction of the solar neutrino flux suggested by them is based on Wolfenstein's observation⁷⁾ [see also Ref. 8)] that neutrino oscillations are modified for neutrinos propagating through matter because the $\nu_e + e^- \rightarrow \nu_e + e^-$ forward scattering amplitude, which has both charged and neutral current contributions, is different from that for $\nu_\mu - e^-$ or $\nu_\tau - e^-$ scattering. If an appropriate relation between the vacuum oscillation length

^{*}) Let us note that if the ν_e is a Dirac particle and has a magnetic (or electric dipole) moment much larger than predicted in the standard theory with right-handed (RH) neutrinos a part of the ν_e flux may be transformed on its way out of the Sun by the solar magnetic field into a flux of (sterile or active) RH neutrinos not detectable in the ^{37}Cl experiment⁶⁾. Neutrino magnetic (electric dipole) moments which may cause such a transition are possible in the gauge theories with RH currents⁶⁾.

$$l_\nu = \frac{4\pi E_\nu}{|\Delta m^2|} \quad (1)$$

and the electron number density n_e is satisfied then a resonance occurs in which even a tiny vacuum mixing is amplified to a maximal (45°) mixing. If the electron density in the core of the Sun (where the ν_e are produced) is higher than the resonance density the ν_e must encounter a resonance region on their way out of the Sun, and, as Mikheyev and Smirnov showed, if the electron density decreases slowly enough there will be an essentially total conversion of the electron neutrinos into another flavour as they pass through the resonance layer.

To see this in detail, consider the case of two neutrino weak eigenstates, e.g., the ν_e and ν_μ for definiteness, which are mixtures

$$\begin{aligned} |\nu_e\rangle &= |\nu_1\rangle \cos\theta_\nu + |\nu_2\rangle \sin\theta_\nu \\ |\nu_\mu\rangle &= -|\nu_1\rangle \sin\theta_\nu + |\nu_2\rangle \cos\theta_\nu \end{aligned} \quad (2)$$

with mixing angle θ_ν , of the (vacuum) mass eigenstates ν_1 and ν_2 with masses m_1 and m_2 , respectively. It is easy to show that for $E_\nu \gg m_1, m_2$ the state

$$|\nu(t)\rangle = \nu_e(t) |\nu_e\rangle + \nu_\mu(t) |\nu_\mu\rangle \quad (3)$$

will propagate in vacuum according to the Schrödinger-like equation

$$i \frac{d}{dt} \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = M_0 \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} \quad (4)$$

where

$$M_0 = \begin{pmatrix} \frac{\Delta m^2}{2p} \cos 2\theta_\nu & -\frac{\Delta m^2}{4p} \sin 2\theta_\nu \\ -\frac{\Delta m^2}{4p} \sin 2\theta_\nu & 0 \end{pmatrix} = 2\epsilon \begin{pmatrix} \pm \frac{\cos 2\theta_\nu}{l_\nu} & \mp \frac{\sin 2\theta_\nu}{2l_\nu} \\ \mp \frac{\sin 2\theta_\nu}{2l_\nu} & 0 \end{pmatrix} \quad (5)$$

p is the neutrino momentum and the $\pm$ is the sign of Δm^2 . In (5) we have subtracted off a multiple $[p + (m_1^2 \sin^2 \theta_\nu + m_2^2 \cos^2 \theta_\nu)/2p]$ of the identity, which only contributes an over-all phase and does not affect the $\nu_e - \nu_\mu$ transition amplitude. From (4) one can easily derive the usual oscillation formula

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta_\nu \sin^2 \left(\frac{\Delta m^2 r}{4p} \right) \quad (6)$$

where r is the distance travelled by the neutrinos.

In matter M_0 is replaced by⁷⁾

$$M' = 2F \begin{pmatrix} \pm \frac{\cos 2\theta_\nu}{2l_\nu} + \frac{1}{l_0} & \mp \frac{\sin 2\theta_\nu}{2l_\nu} \\ \mp \frac{\sin 2\theta_\nu}{2l_\nu} & 0 \end{pmatrix} \quad (7)$$

where⁹⁾

$$\frac{2F}{l_0} = \sqrt{2} G_F n_e = \langle \nu_e e^- | H_{eff} | \nu_e e^- \rangle = - \frac{2FF(0)n_e}{p} \quad (8)$$

is the charged-current contribution to the $\nu_e e^-$ forward scattering amplitude in a medium with electron density n_e (the neutral current contribution is the same for ν_e and ν_μ and has again been subtracted off). In the language of optics, the weak amplitude contributes a relative contribution

$$n - 1 = \frac{2FF(0)n_e}{p^2} = - \frac{\sqrt{2} G_F n_e}{p} \quad (9)$$

to the ν_e index of refraction. In (8) and (9), $F(0)$ is the forward scattering amplitude normalized such that the total cross-section is $4\pi \text{Im}F(0)/p$. Equation (7) implies matter eigenstates $|\nu_1^m\rangle$ and $|\nu_2^m\rangle$, which are related to $|\nu_e\rangle$ and $|\nu_\mu\rangle$ by an equation analogous to (2), only with θ_ν replaced by the effective matter oscillation angle θ_m defined by

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta_\nu}{1 \pm 2 \frac{l_\nu}{l_0} \cos 2\theta_\nu + \frac{l_\nu^2}{l_0^2}} \quad (10)$$

The oscillation length in matter is

$$l_m = \frac{l_\nu}{\left(1 \pm 2 \frac{l_\nu}{l_0} \cos 2\theta_\nu + \frac{l_\nu^2}{l_0^2}\right)^{1/2}} \quad (11)$$

Resonance occurs when the diagonal element in M' vanishes, namely for

$$\frac{\ell_v}{\ell_0} = \cos 2\theta_v \quad (12)$$

with $\Delta m^2 < 0$ (one can take $0 < \theta_v < 45^\circ$ without loss of generality). At resonance one has a maximal mixing $\theta_m = 45^\circ$, and a matter oscillation length $\ell_m^{\text{res}} = \ell_v / \sin 2\theta_v$. From (10) and (11) one observes that the resonance half-width in n_e over which there is a significant enhancement is given by

$$\Delta \left(\frac{\ell_v}{\ell_0} \right) = \sin 2\theta_v. \quad (13)$$

For small vacuum mixing angle θ_v the oscillation length is very large and the resonance is very narrow.

For neutrinos, resonance only occurs if $\Delta m^2 < 0$. Assuming a small vacuum mixing angle this implies that resonance occurs if the dominant component of the ν_e is lighter than the dominant ν_μ component⁹⁾. This is expected intuitively and is given by most specific models, but it is by no means certain. The sign of the $\bar{\nu}_\ell - e^-$ scattering amplitude is opposite to that of the $\nu_\ell - e^-$ scattering amplitude [i.e., ℓ_0 is replaced by $(-\ell_0)$ in (7), (10) and (11)]. Hence, resonance can occur either for neutrinos or for antineutrinos, depending on whether Δm^2 is negative or positive, but not for both.

Equation (8) is valid for oscillations between ν_e and another doublet neutrino such as ν_μ or ν_τ (first-class oscillations). For (second-class) oscillations between ν_e and a sterile neutrino [a hypothetical $SU(2)_L \times U(1)$ singlet neutrino with no weak interactions except those generated by mixing with the ν_e] one must include the neutral current contribution to the $\nu_e - e^-$ scattering amplitude. The factor $\sqrt{2}G_F n_e$ in (8) is replaced by¹⁰⁾

$$\sqrt{2} G_F \sum_a g_v^a n_a \quad (14)$$

where the g_v^a are the vector couplings for $\nu_e - a$ elastic scattering. For a neutral target with density n_e of electrons and protons and density n_n of neutrons this implies

$$\frac{\ell_v}{\ell_0} = \sqrt{2} G_F \left(n_e - \frac{n_n}{2} \right) \quad (15)$$

Resonance still requires $\Delta m^2 < 0$ as long as $n_n < 2n_e$. Corrections to the index of refraction due to the neutrino mass (if it is not negligible compared to p) or for a polarized medium may be found in Ref. 10).

Mikheyev and Smirnov also considered the passage of a neutrino through a medium of varying density. If the density varies sufficiently slowly an adiabatic approximation applies. In particular, for $\Delta m^2 < 0$, ν_e coincides with ν_2^m for n_e much greater than the resonance density n_{Res} and with ν_1^m for $n_e \ll n_{Res}$. Hence, ν_e 's produced in the core of the Sun will be almost totally converted to ν_μ 's for small θ_ν provided that the density at the core exceeds n_{Res} and the adiabatic condition

$$\Delta z \equiv \frac{\Delta(\frac{l_\nu}{l_0})}{\frac{d(l_\nu/l_0)}{dz}} = \frac{\sin 2\theta_\nu}{\frac{1}{n_e} \frac{dn_e}{dz}} > l_m^{res} = \frac{l_\nu}{\sin 2\theta_\nu} \quad (16)$$

holds at the resonance^{*)}.

A number of authors^{5), 4), 11), 12)} have performed detailed analyses of the implications of the MSW effect for the ^{37}Cl experiment. The observed suppression can be accounted for by two bands of parameters: (a) for $-\Delta m^2 = 5 \times 10^{-5} \text{ eV}^2$, and any $\sin^2 2\theta_\nu > 4 \times 10^{-4}$, all neutrinos with energies $> 5 \text{ MeV}$ encounter a resonance layer on their way out of the Sun. The adiabatic condition is satisfied, so the high energy neutrinos are converted, accounting for the ^{37}Cl results. Lower energy neutrinos do not encounter a resonance layer and are not affected. Hence, the ^{71}Ga experiment should see an event rate almost as large as if there were no oscillations. (b) A second set of solutions¹²⁾ satisfies $-\Delta m^2 \times \sin^2 2\theta_\nu = 10^{-7.5} \text{ eV}^2$. This set of solutions intersects with the first set of solutions for small $\sin^2 2\theta_\nu$. For relatively large $\sin^2 2\theta_\nu$, essentially all solar neutrinos encounter a resonance layer, but the adiabatic condition is only marginally satisfied, leading to a suppression of the ^{37}Cl rate by a factor ≈ 3 . The suppression for ^{71}Ga depends on the value of $-\Delta m^2$. These two sets of solutions are similar in their implications for ^{71}Ga to non-standard solar models and large vacuum oscillations, respectively.

*) Numerical studies indicate that substantial conversions still occur if (16) is relaxed somewhat. For example, $\Delta r = 0.1 l_m^{res}$ typically yields a 50% conversion probability.

THE SEESAW MODEL FOR THE NEUTRINO MASSES AND THE SOLAR NEUTRINO FLUX

It is interesting to compare the parameter ranges required to explain the solar neutrino problem with theoretical expectations. There are many classes of models of neutrino mass¹³⁾, but the ones with the most predictive power are the seesaw models¹⁴⁾ of Gell-Mann, Ramond and Slansky and of Yanagida. For one family the neutrino mass term is

$$\mathcal{L}_M = - \frac{1}{2} (\bar{\nu}_L \quad \bar{N}_L^c) \begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_R^c \\ N_R \end{pmatrix} + h.c. \quad (17)$$

In (17), ν_L is the field of the ordinary $[SU(2)_L \text{ doublet}]$ left-handed (LH) neutrino and RH antineutrino, $\nu_R^c = C \bar{\nu}_L^T$ (C is the charge conjugation matrix) is a RH field^{*}), and N_R and $N_L^c = C \bar{N}_R^T$ are $SU(2)_L \times U(1)$ singlet neutrino fields describing a RH neutrino N_R and LH antineutrino $\bar{N}_L$. The mass m_D is an ordinary Dirac mass, generated by the vacuum expectation value of a Higgs doublet, while M is an $SU(2)_L \times U(1)$ singlet Majorana mass^{**)} for the RH neutrino N_R . M can be a bare mass or can be generated by a Higgs singlet. No tree level Majorana mass term for the ν_L is allowed unless Higgs triplets are introduced into the theory.

Diagonalization of (17) yields two mass eigenstate Majorana neutrinos ν_1 and ν_2 , with masses m_1 and m_2 ($m_1, m_2 > 0$) respectively, and with left-handed components given by

$$\begin{aligned} \nu_L &= \nu_{1L} \cos \theta + \nu_{2L} \sin \theta \\ N_L^c &= -\nu_{1L} \sin \theta + \nu_{2L} \cos \theta \end{aligned} \quad (18)$$

where $\tan^2 \theta = m_1/m_2$. (A γ_5 transformation has been performed to make $m_1 > 0$.) The case $M \gg m_D$ is particularly interesting. Then,

$$\begin{aligned} m_1 &\approx \frac{m_D^2}{M} \ll m_D, \\ m_2 &\approx M \gg m_1. \end{aligned}$$

^{*}) The RH neutrino field ν_R^c carries the quantum numbers of the field $\bar{\nu}_L$ (i.e., of the Dirac conjugate of ν_L) and transforms as ν_L under the proper Lorentz transformations.

^{**)} Let us note that a neutrino mass Lagrangian containing both Dirac and Majorana terms was considered first by Bilenky and Pontecorvo¹⁵⁾.

One therefore has a natural explanation for very small neutrino masses. In particular, in typical grand unified theories [e.g., the simplest $SO(10)$ model]¹³⁾ one has $M = M_{GUT} = 10^{14}$ GeV and $m_D(M_{GUT}) = m_u(M_{GUT})$. Renormalizing down to low momenta this implies^{16), 17)} $m_D = m_u/K = 5$ MeV/K, where $K = 4.7$, yielding $m_1 = 10^{-11}$ eV. [Left-right symmetric $SU(2)_L \times SU(2)_R \times U(1)$ models¹⁸⁾, on the other hand, typically have $m_D = m_e$ and $M = 1$ TeV, yielding $m_1 = 0.25$ eV. Therefore for the range of neutrino masses they predict, the resonant amplification of the neutrino transitions cannot take place in the Sun and we shall not consider them further.]

The seesaw model can easily be generalized to n families. Equation (17) is replaced by

$$\mathcal{L}_M = -\frac{1}{2} (\bar{\nu}_L \quad \bar{N}_L^c) \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_R^c \\ -N_R \end{pmatrix} + R.c. \quad (19)$$

where ν_L and N_L are now n component vectors of doublet and singlet neutrinos, respectively, M_D is an $n \times n$ Dirac matrix, and M_R is a symmetric $n \times n$ Majorana mass matrix. Equation (18) implies the existence of n light and n heavy Majorana mass eigenstates. The light eigenstates are mainly mixtures of the ν_L [except for small $O(M_D/M_R)$ admixtures of the N_L] and have the mass matrix

$$M_\nu = M_D M_R^{-1} M_D^T \quad (20)$$

while the heavy neutrinos consist mainly of the N_L and have a mass matrix M_R .

Let us now consider simple models such as the minimal version of $SO(10)$ in which M_D , M_e , M_u and M_d , which are the $n \times n$ Dirac mass matrices for neutrinos, electrons, u quarks, and d quarks, respectively, are symmetric and are generated by $SO(10)$ 10's while M_R is generated by the $SU(3)_C \times SU(2)_L \times U(1)$ singlet component of a 126. Models of this type have well-known difficulties¹³⁾ in their predictions for m_d/m_s and for m_s , and of course for proton decay, but for the rough guidance we are seeking they should suffice. We will make the further simplifying assumptions that M_R is a multiple MI of the identity and neglect CP violating phases in M_D . Because of the ad hoc nature of these assumptions the results must be regarded as illustrations of typical possibilities and not as concrete predictions.

In such models, $M_D(M_{\text{GUT}}) = M_U(M_{\text{GUT}})$ and $M_e(M_{\text{GUT}}) = M_d(M_{\text{GUT}})$. Hence the neutrino and electron mass matrices at low momenta are

$$M_\nu = M_u M_R^{-1} M_u^T / K^2, \quad M_e = M_d / K \quad (21)$$

where $K \approx 4.7$ if M_u and M_d are evaluated at 1 GeV^* . For $M_R = M_I$ and M_u real this implies that M_ν has the eigenvalues

$$m_i = \frac{m_{u_i}^2}{MK^2} \quad (22a)$$

where m_{u_i} are the eigenvalues of M_u . For three families with¹⁷⁾ $m_u(1 \text{ GeV}) \approx 5 \text{ MeV}$ and $m_c(1 \text{ GeV}) \approx 1.35 \text{ GeV}$, and arbitrarily taking $m_t(1 \text{ GeV}) \approx 50 \text{ GeV}$, (22) implies

$$\begin{aligned} m_1 &\approx 10^{-11} \text{ eV}, \\ m_2 &\approx 8 \times 10^{-7} \text{ eV}, \\ m_3 &\approx 10^{-3} \text{ eV}. \end{aligned} \quad (22b)$$

Of course, for small mixing angles the ν_1 , ν_2 and ν_3 correspond approximately to the ν_e , ν_μ , and ν_τ , respectively.

For three or more neutrino flavours and small vacuum mixing angles, only two flavours will be important for a given density and neutrino energy (unless there are degeneracies). To see this, let us write the analogue of (7) for n neutrinos:

$$M' = U^V \begin{pmatrix} \frac{m_1^2}{2p} & 0 & \dots & 0 \\ 0 & \frac{m_2^2}{2p} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \frac{m_n^2}{2p} \end{pmatrix} U^{V\dagger} + \frac{2\pi}{\ell_0} \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \quad (23)$$

where U^V is the unitary matrix which relates the LH components of the weak and (vacuum) mass eigenstate neutrino fields ν_α and ν_i :

$$\nu_{\alpha L} = \sum_{i=1}^n U_{\alpha i}^V \nu_{iL} \quad (24)$$

For small mixing angles (i.e., U^V is close to the identity) resonance occurs when $\Delta m_{j1}^2 / 2p = (m_j^2 - m_1^2) / 2p$ approximately equals $2\pi / \ell_0 = \sqrt{2} G_F n_e$. Because of the degeneracy between M' and M'_{jj} , the eigenstates in matter will be approximately

*) In the calculation of the running quark mass matrices we have made the simplifying assumption that the heavy quark threshold effects are negligible.

mixtures of ν_1 and ν_j with maximal mixing angle ($\sim 45^\circ$) even for small off-diagonal elements in M' . Unless there are degeneracies in the m_i^2 the other neutrinos will play little role at the resonance (except for small effects involving the vacuum mixing angles), and one has an effective two-state problem. For the case $m_j^2 \gg m_1^2$, which occurs in the seesaw model, the effective mixing angle is $|\theta| = |U_{ej}^v|$, as can easily be verified by comparing (5) and (23).

First, consider $\nu_e - \nu_\mu$ oscillations, which correspond to the mass difference $\Delta m_{21}^2 = m_2^2 - m_1^2$. The present model predicts $\Delta m_{21}^2 = 6 \times 10^{-13} \text{ eV}^2$, so that the adiabatic condition (16) is badly violated and essentially no $\nu_e - \nu_\mu$ conversions can occur^{5),4),11),12)}. In contrast, $\nu_e - \nu_\tau$ conversions are associated with $\Delta m_{31}^2 = m_3^2 - m_1^2 = 10^{-6} \text{ eV}^2$. This value is well within the range needed to account for the result of the ^{37}Cl experiment (especially given that reasonable variations on the assumptions could easily change Δm_{31}^2 by a factor of 10^2 in either direction).

The relevant mixing angles can also be estimated in this simplified seesaw model. From (21) it is apparent that M_ν and M_u are both diagonalized by the same unitary matrix U_u , while M_e and M_d are both diagonalized by U_d . Hence, the unitary matrix U^v which relates the weak and mass eigenstate neutrinos in (24) is just V^+ , where $V = U_u^\dagger V_d$ is the Kobayashi-Maskawa mixing matrix. In particular, the effective mixing angle for $\nu_e - \nu_\tau$ oscillations is $|\theta| = |U_{e3}^v| = |V_{td}|$. The phenomenological estimates¹⁹⁾ $|V_{td}| = 0.005 - 0.02$ are well within the range needed to explain the result of the ^{37}Cl experiment. It should be noted that while the numerical values of the neutrino masses (22a) depend sensitively on the simplifying assumption $M_R = MI$, the order of magnitude of the predicted value of θ holds for a wide class of matrices M_R .

There are a number of uncertainties and ad hoc assumptions in these estimates. However, they suggest that the range of parameters needed to account for the solar neutrino results is perfectly consistent with typical predictions of the seesaw model. In particular, the seesaw model favours the small $\sin^2 2\theta_\nu$ region in which the two solutions described in the Introduction coincide and for which there should be little suppression of the flux of ν_e detectable in the $^{71}\text{Ga} \rightarrow ^{71}\text{Ge}$ experiments. Moreover the relevant process may well be $\nu_e \rightarrow \nu_\tau$ rather than $\nu_e \rightarrow \nu_\mu$.

NUCLEOSYNTHESIS AND NEUTRINO OSCILLATIONS IN MATTER

Primordial nucleosynthesis²⁰⁾ provides a probe of cosmological neutrino oscillations and conversions. Indeed, Kristev, Mikheyev and Smirnov²¹⁾ have noted that since either ν_e or $\bar{\nu}_e$ transitions but not both may be affected by resonant amplifications in matter, oscillations may create a ν_e - $\bar{\nu}_e$ asymmetry prior to nucleosynthesis. If such an asymmetry can be created while the reactions

$$\nu_e n \rightleftharpoons e^- p, \quad \bar{\nu}_e p \rightleftharpoons e^+ n \quad (25)$$

are still occurring, the neutron-to-proton ratio and, therefore, the predicted primordial abundance of ${}^4\text{He}$ and the limits to the number of neutrino flavours²²⁾ can be affected^{*)}.

Kristev, Mikheyev and Smirnov²¹⁾ considered the conversions of ν_e into sterile $[\text{SU}(2)_L \times \text{U}(1)]$ singlet] neutrinos (ν) and concluded the effects could be significant for $\sin^2 2\theta_\nu > 0.05$ and $(-\Delta m^2) \approx (10^{-6} - 10^{-9}) \text{ eV}^2$. Although oscillations of ordinary neutrinos into sterile neutrinos are a logical possibility, they may take place only under rather specific circumstances. In particular, the necessary masses and mixings could only occur if there are both Dirac mass terms (connecting the ν_e neutrino and the sterile neutrino) and Majorana masses (for ν_e , the ν , or both) in the range $(10^{-6} - 10^{-4}) \text{ eV}^{**})$. Such small Dirac and Majorana masses are not achieved simultaneously in any of the simpler models without fine tunings. Furthermore, if the small masses are generated by new Higgs fields with very small vacuum expectation values (vevs), then most likely there would be a phase transition in the early Universe, above which the vevs would be zero. Hence, for a vev $\ll 1 \text{ MeV}$, the relevant temperature-dependent mass would actually be zero²⁴⁾ at a time relevant for nucleosynthesis and no oscillations could take place. Finally, we will argue

*) The possibility that the oscillations of ν_e and $\bar{\nu}_e$ into sterile neutrinos may generate an asymmetry between the densities of ν_e and $\bar{\nu}_e$ prior to nucleosynthesis and therefore may affect the predictions for the ${}^4\text{He}$ abundance and, consequently, the limits on the number of neutrino flavours, was discussed without taking matter effects into account in Ref. 23).

***) The indicated range corresponds, of course, to the case of non-degenerate neutrino mass spectrum.

argue below that such oscillations would have to be into a different type of neutrino (if it is sterile) than those which are produced by the solar neutrino oscillations.

We have therefore considered instead the effects of ordinary conversions of the ν_e into the ν_μ or ν_τ . It will turn out that resonance can be achieved prior to nucleosynthesis for $(-\Delta m^2)$ of order 10^{-7} eV^2 , which is consistent with the expectations of the seesaw model. Furthermore, in the seesaw model, all mass terms are associated with Higgs fields with vevs very much greater than 1 MeV, so the masses and mixing angles can be regarded as constants during the relevant period. However, it will be argued that it is difficult to satisfy the adiabatic condition with mixing angles as small as those typically predicted by the seesaw model.

For simplicity, we will consider conversions involving two flavours only, and will usually assume that the second flavour is the same as the one that is relevant to the solar neutrino problem. We will assume that this is the ν_τ for definiteness, but similar considerations would apply to the ν_μ . At resonance ν_e 's are converted into ν_τ 's and vice versa, so a net change can only occur if there is a difference between the number (or momentum distributions) of ν_e and ν_τ to begin with^{*}). A small initial $\nu_e - \nu_\tau$ asymmetry is in fact expected²⁵⁾ because the ν_τ (and ν_μ) decouple somewhat earlier than the ν_e (at $T_\tau^* = 5 \text{ MeV}$ as compared with $T_e^* = 3 \text{ MeV}$). e^+e^- annihilations between T_τ^* and T_e^* therefore increase the ν_e temperature and density slightly compared to that of ν_τ . If a $\nu_e - \nu_\tau$ resonance conversion occurs the net effect will be to decrease the number of ν_e compared with $\bar{\nu}_e$ (which are not affected). The resulting $\bar{\nu}_e - \nu_e$ asymmetry will therefore increase the abundance of neutrons relative to protons, leading to an increased ^4He abundance.

^{*}) In the case of sterile neutrinos considered in Refs. 23) and 21), one expects a large initial excess of ν_e over the sterile ν since the latter, which are not produced by ordinary weak processes, would have decoupled early in the evolution of the Universe.

If the assumption that the conversion is into the same flavour in the early Universe as in the Sun is relaxed, then there is another possibility^{*)}. Namely, the dominant component of the ν_e could be the intermediate mass state in a three-neutrino hierarchy: i.e., $m_1 < m_2 < m_3$, where the ν_1 , ν_2 , and ν_3 are the dominant components of the ν_a , ν_e , and ν_b , respectively. In that case, the $\nu_e - \nu_b$ conversion could occur in the Sun and the $\bar{\nu}_e - \bar{\nu}_a$ conversion in the early Universe. Then the neutron and the ^4He fractions would decrease. The decrease of the ^4He abundance could be compensated by additional neutrinos or other new light particles. We will not consider this possibility further.

When, during the early evolution of the Universe, the bulk of the $e^\pm$ pairs annihilates, the neutrinos are - to a good approximation - decoupled. The photons, therefore, are heated with respect to the neutrinos; after $e^\pm$ annihilation is complete, $T_\gamma/T_\nu = (11/4)^{1/3}$. However, the weak rates, Eq. (25), which determine the n/p ratio and, therefore, influence the ^4He abundance, are extremely sensitive to the neutrino temperature. It is, thus, valuable to consider the effects of the deviations from the approximation of perfect decoupling²⁵⁾. Because electron neutrinos ν_e have charged current as well as neutral current weak interactions, they remain coupled longer than do the ν_μ or ν_τ . Dicus et al.²⁵⁾ find for the fractional change in the neutrino temperatures (with respect to the standard-decoupled-result), $\delta = \Delta T/T$,

$$\delta(T) = 5.9 \times 10^{-3} \left(\frac{a}{3}\right)^{2/3} \exp\left(\frac{aT^3}{3}\right) \int_{\frac{aT^3}{3}}^{\infty} \frac{e^{-x}}{x^{2/3}} dx \quad (26)$$

where for ν_e $a = 0.8$ and, for ν_μ and ν_τ $a = 0.2$. For $T = T_{n/p} = 0.7$ MeV, $\delta_e = 4 \times 10^{-3}$ whereas $\delta_\mu = \delta_\tau = 2 \times 10^{-3}$. For T in excess of $T_{n/p}$, the δ 's decrease and the difference between δ_e and δ_μ , δ_τ becomes smaller. Since $\nu_e - \nu_\tau$ conversions rely on the δ_e , δ_τ difference, the cosmological effect of such conversions becomes negligible for $T \gtrsim 2$ MeV; we, therefore, limit our considerations to $T \lesssim 2$ MeV.

Neglecting conversions, but accounting for the slight temperature differences between ν_e and ν_μ , ν_τ , the predicted ^4He mass fraction changes by²⁵⁾

*) It is necessary to relax this assumption in the case of conversions into sterile neutrinos [Ref. 21)] because of the much larger initial excess of electron neutrinos over the sterile neutrinos.

$$\Delta Y \approx -0.10 \delta_e + 0.04 (\delta_\mu + \delta_\tau) \quad (27)$$

If, now, MSW oscillations succeed in exchanging ν_e with ν_τ this change in Y will be modified to

$$\Delta Y^{(1)} \approx -0.06 \left(\frac{\delta_e + \delta_\tau}{2} \right) + 0.04 \delta_\mu, \quad (28)$$

since the ν_e and ν_τ are swapped but the antineutrinos are untouched.

There is, in the case of $\nu_e - \bar{\nu}_e$ conversions an additional effect. After MSW oscillations, the "new" ν_e are less abundant than the $\bar{\nu}_e$; there is a $\nu_e - \bar{\nu}_e$ asymmetry which affects the rates (25):

$$\frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_{\nu_e} + n_{\bar{\nu}_e}} \approx -3 (\delta_e - \delta_\tau). \quad (29)$$

This $\nu_e - \bar{\nu}_e$ asymmetry may be expressed⁽²²⁾ in terms of the "degeneracy parameter" $\xi = \mu/T$,

$$L_{\nu_e} = \frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_\gamma} = \frac{x^2}{123(3)} \left(\frac{T_{\nu_e}}{T_\gamma} \right)^3 \xi \left[1 + \left(\frac{\xi}{x} \right)^2 \right] \quad (30)$$

Since, to sufficient accuracy, $\xi \ll 1$ and $(n_{\nu_e} + n_{\bar{\nu}_e})/n_\gamma = 3/4 (T_{\nu_e}/T_\gamma)^3$,

$$\xi \approx -\frac{273(3)}{x^2} (\delta_e - \delta_\tau) \approx -3.3 (\delta_e - \delta_\tau) \quad (31)$$

The neutron-to-proton ratio, for $T > T_{n/p}$, is modified to

$$\frac{n}{p} \approx \exp \left(-\frac{m_n - m_p}{T} - \xi \right) \quad (32)$$

so that the fractional change in n/p is

$$\frac{\Delta(n/p)}{(n/p)} \approx -\xi \approx 3.3 (\delta_e - \delta_\tau) \quad (33)$$

The predicted ${}^4\text{He}$ abundance will change, due to this effect of asymmetry, by an amount which may be estimated as

$$\frac{\Delta Y}{Y}^{(2)} \approx \left(1 - \frac{Y}{2}\right) \left[\frac{\Delta(n/p)}{(n/p)}\right] \approx 3.3 \left(1 - \frac{Y}{2}\right) (\delta_e - \delta_\tau) \quad (34)$$

For $Y \approx 0.24$, the total estimated change in the predicted ${}^4\text{He}$ mass fraction is

$$\Delta Y = \Delta Y^{(1)} + \Delta Y^{(2)} \approx 0.66 \delta_e - 0.68 \delta_\tau \quad (35)$$

Thus, MSW oscillations between ν_e and ν_τ lead to an increase in the predicted ${}^4\text{He}$ mass fraction. To be at all effective, the oscillations cannot occur too early ($T \lesssim 2 \text{ MeV}$) nor too late ($T \gtrsim T_{n/p} \approx 0.7 \text{ MeV}$)²³⁾. The maximal effect is for complete conversion at $T \approx T_{n/p}$, for which

$$\Delta Y \approx 0.32 \delta_e \lesssim 1.3 \times 10^{-3} \quad (36)$$

Any increase in the predicted abundance of ${}^4\text{He}$ implies a smaller upper bound on the permitted number of extra flavours of light neutrinos²⁰⁾:

$$-\Delta N_\nu \approx \frac{\Delta Y}{0.014} \lesssim 0.09 \quad (37)$$

Now let us consider the conditions for conversion to occur between $T = 2 \text{ MeV}$ and $T = 0.7 \text{ MeV}$. As the Universe expands and cools, the density decreases. To estimate the resonance temperature (T_R) at which $l_\nu = l_0 \cos 2\theta$, E_ν is replaced^{*)} in (1) by the average energy, $\langle E_\nu \rangle = 3.15 T$, at temperature $T^{**})$. Since the $\nu_e e^-$ and $\nu_e e^+$ scattering amplitudes have opposite signs, (8) is replaced by

$$\frac{2\pi}{l_0} = \sqrt{2} G_F \sum_{\vec{p}, \vec{s}} \left[\langle \nu_e e^- | H_{eff} | \nu_e e^- \rangle - \langle \nu_e e^+ | H_{eff} | \nu_e e^+ \rangle \right] \quad (38)$$

where the matrix elements are summed over the electron and positron momentum distributions and spins at temperature T . As long as the distributions are isotropic and unpolarized, (38) reduces to

*) The energy dispersion $\Delta E = 0.55 \langle E \rangle$ changes the resonance temperature for individual neutrinos by less than 15% which is not significant.

**) Although the neutrino temperatures differ from the photon temperature, this difference has a negligibly small effect on the neutrino oscillations. Therefore here we take $T_{\nu_e} = T_{\nu_\tau} = T_\gamma = T$.

$$\frac{\delta F}{\ell_0} = \sqrt{2} G_F \Delta n_e \quad (39)$$

where Δn_e is the excess density of electrons over positrons. Since charge conservation ensures that Δn_e is equal to the proton density, Δn_e may be related to the photon density (n_γ) by

$$\frac{\Delta n_e}{n_\gamma} = X_p \left(\frac{B}{N_{\gamma 0}} \right) \left(\frac{N_{\gamma 0}}{N_\gamma} \right) = \frac{11}{4} \eta X_p(T) \quad (40)$$

where $\eta = B/N_{\gamma 0}$ is the present ratio of baryons to photons, $N_{\gamma 0}/N_\gamma = 11/4$ is the ratio of photons (per comoving volume) at present to those at $T \gtrsim T_{n/p}$, and $X_p = n_p/(n_p + n_n)^{-1}$ is the proton mass fraction. Although $\Delta n_e/n_\gamma$ changes because of neutron-to-proton (plus electron) conversions, the variation in X_p for $2 \text{ MeV} \gtrsim T \gtrsim T_{n/p}$ is not large. The condition for resonance in the early Universe follows from (1), (39) and (40)

$$-\Delta m^2 \cos 2\theta_\nu = 3.9 \times 10^{-8} \left(\frac{\eta}{5 \times 10^{-10}} \right) X_p(T_R) T_R^4 \quad (41)$$

In (41), Δm^2 is in eV^2 , all temperatures are in MeV, and we have assumed $E_\nu = \langle E_\nu \rangle$ and $T_\nu = T$; nucleosynthesis in the standard model suggests that²⁰⁾ $\eta = (5 \pm 2) \times 10^{-10}$.

For total $\nu_e - \nu_\tau$ conversion to occur, at resonance, the adiabatic condition, $(\Delta t/\ell_m)_R > 1$, should be satisfied, where

$$\left(\frac{\Delta t}{\ell_m} \right)_R = \frac{\Delta t}{\ell_m^{\text{res}}} = \frac{1}{\ell_m^{\text{res}}} \Delta \left(\frac{\ell_\nu}{\ell_0} \right) \left[\frac{d}{dt} \left(\frac{\ell_\nu}{\ell_0} \right) \right]_R^{-1} = \frac{\sin^2 2\theta_\nu}{\ell_\nu} \left[\frac{d}{dt} \left(\frac{\ell_\nu}{\ell_0} \right) \right]_R^{-1} \quad (42)$$

At resonance, we have:

$$\ell_\nu = \left(\frac{\sqrt{2} \pi \cos 2\theta_\nu}{G_F \Delta n_e} \right)_R \quad (43)$$

and

$$\left| \frac{d}{dt} \left(\frac{\ell_\nu}{\ell_0} \right) \right|_R = \frac{2 \cos 2\theta_\nu}{t(T_R)} \quad (44)$$

In (44) the logarithmic derivative (with respect to temperature) of X_p has been neglected. For $T \approx$ a few MeV, when there are $43/4$ relativistic degrees of freedom, $t^{-1} = 10.85 T^2/M_p$, and we obtain

$$\left(\frac{\Delta t}{\ell_m}\right)_R = 0.54 (\tan^2 2\theta_v) \left(\frac{\eta}{5 \times 10^{-10}}\right) X_p(T_R) T_R \quad (45)$$

Numerical studies show that a significant conversion is possible even if the adiabatic condition ($\Delta t > \ell_m^{\text{res}}$) is relaxed significantly. For example, for $\Delta t \approx 0.08 \ell_m^{\text{res}}$, 50% conversion occurs.

Equation (45) may be solved for the value of $\sin^2 2\theta_v$ which, for fixed η and T_R , corresponds to a specific value of $(\Delta t/\ell_m)_R$:

$$\sin^2 2\theta_v = \left(\frac{\Delta t}{\ell_m}\right)_R \left[\left(\frac{\Delta t}{\ell_m}\right)_R + 0.54 \left(\frac{\eta}{5 \times 10^{-10}}\right) T_R X_p(T_R) \right]^{-1} \quad (46)$$

For each value of η and T_R , there is a minimum value of θ_v determined by setting $(\Delta t/\ell_m)_R = (\Delta t/\ell_m)_{R\text{min}} = 0.08$. Then, corresponding to this choice of $\theta_v = \theta_{v\text{min}}$, the minimum value of Δm^2 is found by substituting in (41):

$$-\Delta m_{\text{min}}^2 = 7.1 \times 10^{-8} T_R^3 \left[0.54 \left(\frac{\eta}{5 \times 10^{-10}}\right) T_R X_p(T_R) \left(0.08 + 0.54 \left(\frac{\eta}{5 \times 10^{-10}}\right) T_R X_p(T_R)\right)^{-1/2} \right]^2 \quad (47)$$

It must be checked that the conversion of ν_e to ν_τ occurs sufficiently rapidly to affect the neutron-proton ratio. If Δt is compared with $t_R \equiv t(T_R)$, it follows that

$$\left(\frac{\Delta t}{t}\right)_R = \frac{1}{2} \tan^2 2\theta_v \quad (48)$$

If T_f is defined by $t(T_f) \equiv t_R + \Delta t$, then $T_f/T_R = [1 + (\Delta t/t)_R]^{-1/2}$. For $T_f \gtrsim T_{n/p} \approx 0.7 \text{ MeV}$, we are restricted to $T_R \gtrsim 0.8 \text{ MeV}$.

Finally, we note that there is an upper bound to the vacuum mixing angle of interest here. Large vacuum mixing angles reduce the effects on nucleosynthesis described above because they reduce the $\nu_e - \nu_\tau$ asymmetry and, also, because they lead to $\bar{\nu}_e - \bar{\nu}_\tau$ oscillations. However, the presence of matter reduces the mixing angles for neutrinos before resonance and for antineutrinos at all T [see Eq. (10)]. As a result, we limit our considerations to $\sin^2 2\theta_v \lesssim \frac{1}{2}$, for which vacuum oscillations are small though not entirely negligible.

Our results are summarized graphically in the Figure where the region in the $\Delta m^2 - \sin^2 2\theta_v$ plane is shown for $\eta = 5 \times 10^{-10}$ and $0.8 \text{ MeV} < T_R < 2 \text{ MeV}$. For each value of T_R (and η), $\sin^2 2\theta_v$ lies between $\sin^2 2\theta_{v\min}$ [Eq. (46) with $(\Delta t/\lambda_m)_R = 0.08$] and $\sin^2 2\theta_{v\max} = \frac{1}{2}$. Also shown in the Figure is the "solution" to the solar neutrino problem: $(-\Delta m^2) \sin^2 2\theta_v = 10^{-7.5} \text{ eV}^2$; there is overlap with early Universe oscillations for $1.1 \text{ MeV} \lesssim T_R \lesssim 1.8 \text{ MeV}$.

GENERAL PROPERTIES

We would like to conclude with a comparison between the general properties of the neutrino oscillations in vacuum and in matter. If the charged and neutral current weak interactions are described by the standard model, then $(\bar{\nu}_\mu^-)$ and $(\bar{\nu}_\tau^-)$ interact with matter in the same way while the interaction of $(\bar{\nu}_e^-)$ is different from that of $(\bar{\nu}_\mu^-)$ and $(\bar{\nu}_\tau^-)$. If CPT invariance holds, the probabilities of the transitions of interest $\nu_e^- \rightarrow \nu_\ell^-$ and $(\bar{\nu}_\ell^-) \rightarrow (\bar{\nu}_e^-)$ ($\ell = e, \mu, \tau$) in vacuum $P^v(\nu_e^- \rightarrow \nu_\ell^-)$ and $P^v(\bar{\nu}_\ell^- \rightarrow \bar{\nu}_e^-)$ satisfy the following conditions:

$$\begin{aligned} P^v(\nu_e^- \rightarrow \nu_\ell^-) &= P^v(\bar{\nu}_\ell^- \rightarrow \bar{\nu}_e^-), \\ P^v(\nu_\ell^- \rightarrow \nu_e^-) &= P^v(\bar{\nu}_e^- \rightarrow \bar{\nu}_\ell^-), \\ \ell &= e, \mu, \tau. \end{aligned} \quad (49)$$

In particular, it follows from CPT invariance that

$$P^v(\nu_e^- \rightarrow \nu_e^-) = P^v(\bar{\nu}_e^- \rightarrow \bar{\nu}_e^-) \quad (50)$$

Further, if CP invariance holds in the leptonic sector, one has²⁶⁾:

$$\begin{aligned} P^v(\nu_e^- \rightarrow \nu_\ell^-) &= P^v(\bar{\nu}_e^- \rightarrow \bar{\nu}_\ell^-), \\ P^v(\nu_\ell^- \rightarrow \nu_e^-) &= P^v(\bar{\nu}_\ell^- \rightarrow \bar{\nu}_e^-), \\ \ell &= e, \mu, \tau. \end{aligned} \quad (51)$$

Equation (50) implies, however, that the study of the transitions $\nu_e \rightarrow \nu_e$ and $\bar{\nu}_e \rightarrow \bar{\nu}_e$ in vacuum cannot give information about possible CP violation in the weak interaction of leptons.

In contrast, none of the relations (49) to (51) are valid for the probabilities of transitions of neutrinos propagating through matter. This is true even if the interactions of neutrinos are both CPT and CP conserving. In particular, as we have seen, in the case of two flavours,

$$P^m(\nu_e \rightarrow \nu_e) \neq P^m(\bar{\nu}_e \rightarrow \bar{\nu}_e)$$

because ν_e and $\bar{\nu}_e$ scatter differently off electrons. This drastic difference between the properties of the probabilities of neutrino transitions in vacuum and of neutrino transitions in matter is a consequence of the fact that, due to the absence of positrons and antinucleons, ordinary matter is neither CPT nor CP symmetric, while the vacuum is (at least to a good approximation).

Let us suppose further that CP invariance does not hold in the leptonic sector and therefore the lepton mixing matrix contains non-trivial CP violating phases. The possible number of these depends on the type of massive neutrinos. In particular, if neutrinos do not take part in weak interaction with right-handed currents, which shall be assumed further, for n lepton flavours there may be $((n-1)(n-2)/2)$ CP violating phases if the neutrinos with definite mass are Dirac particles and $n(n-1)/2$ CP violating phases in the case of Majorana mass eigenstate neutrinos^{27),28)}. We shall show next that the probabilities of the neutrino transitions in matter, like the transition probabilities in vacuum^{27),28)}, do not depend on the additional $(n-1)$ CP violating phases which are characteristic for the existence of massive Majorana neutrinos. For brevity we shall call these $(n-1)$ phases Majorana phases, while the term Dirac phases shall be used for the other $\frac{1}{2}(n-1)(n-2)$ CP violating phases since they may be present in the lepton mixing matrix also in the case of massive Dirac neutrinos.

For the state of the left-handed flavour neutrino ν_ℓ produced at the moment $t = 0$ in some weak process we have in the case of interest

$$|\nu_\ell\rangle = \sum_{k=1}^n U_{\ell k} |\chi_k(p; L)\rangle, \quad \ell = e, \mu, \tau, \dots \quad (52)$$

Here U is an $n \times n$ unitary matrix - the lepton mixing matrix in vacuum [it is the complex conjugate of the matrix U^V defined in (24)], $|\chi_k(p; L)\rangle$ is the state of

the Majorana neutrino χ_k with mass m_k and definite four-momentum p_k . In particular,

$$H_0 |\chi_k(p_k; L)\rangle = E_k |\chi_k(p_k; L)\rangle \quad (53)$$

where H_0 is the free neutrino Hamiltonian and $E_k = \sqrt{p^2 + m_k^2}$. In the limit $m_k = 0$ the state $|\chi_k(p_k; L)\rangle$ describes a left-handed neutrino.

The matrix U can be represented in the form²⁹⁾⁻³¹⁾:

$$U = U^D P \quad (54)$$

Here U^D is an $n \times n$ unitary matrix which contains the $\frac{1}{2}(n-1)(n-2)$ Dirac CP violating phases, while P is an $n \times n$ diagonal unitary matrix

$$P_{jk} = \delta_{jk} e^{i\alpha_k}, \quad \alpha_n = 0 \quad (j, k = 1, 2, \dots, n) \quad (55)$$

where α_k , $k = 1, 2, \dots, n-1$ are $(n-1)$ CP violating phases associated with the Majorana character of the massive neutrinos.

After a time t one has:

$$|\nu_e\rangle_t = \sum_{\ell' = e, \mu, \tau, \dots} X_{\ell\ell'}(t) |\nu_{\ell'}\rangle \quad (56)$$

where $X_{\ell\ell'}(t)$ is the probability amplitude of the transition $\nu_{\ell'} \rightarrow \nu_{\ell}$, at time t . The time evolution of the state $|\nu_{\ell}\rangle_t$ is determined by the Schrödinger equation:

$$i \frac{d}{dt} |\nu_{\ell}\rangle_t = H |\nu_{\ell}\rangle_t \quad (57)$$

Here $H = H_0 + H_{\text{int}}$, where H_{int} is the neutrino interaction Hamiltonian. In the case of neutrinos propagating through matter, one has:

$$\langle \nu_{\ell'} | H_{\text{int}} | \nu_{\ell} \rangle = \delta_{\ell\ell'} H_{\ell}, \quad H_e \neq H_{\mu} = H_{\tau}. \quad (58)$$

(the validity of the standard model for the weak interaction of neutrinos has been assumed). From (57), using (52)-(54), (56) and (58), we obtain the following equation for the probability amplitude $X_{\ell\ell'}(t)$:

$$\begin{aligned}
 i \frac{d}{dt} X_{ee'}(t) &= \sum_{e''} X_{ee''}(t) \langle j_{e'} | H | j_{e''} \rangle = \\
 &= X_{ee'}(t) H_{e'} + \sum_{e''} X_{ee''}(t) \sum_j (U^D P)_{e''j} \cdot E \cdot (P^\dagger U^{D\dagger})_{je'} \quad (59)
 \end{aligned}$$

Obviously, it follows from (55) that

$$\sum_j (U^D P)_{e''j} \cdot E \cdot (P^\dagger U^{D\dagger})_{je'} = \sum_j U_{e''j}^D \cdot E \cdot U_{je'}^{D\dagger} \quad (60)$$

Consequently, the amplitude $X_{ee'}(t)$ of the transition $\nu_e \rightarrow \nu_{e'}$ in matter does not depend on the $(n-1)$ CP violating Majorana phases. It is not difficult to show that the same result is valid for the amplitude $\bar{X}_{ee'}(t)$ of the antineutrino transition $\bar{\nu}_e \rightarrow \bar{\nu}_{e'}$ in matter. In such a way, we arrive at the conclusion that it is impossible to determine the type of massive neutrinos by studying the oscillations of neutrinos propagating through matter. An analogous result has been previously obtained for the case of oscillations of neutrinos in vacuum^{27),28)}.

CONCLUSIONS

Mikheyev and Smirnov have proposed a novel and plausible solution to the solar neutrino problem, based on the resonant amplification of neutrino oscillations in matter. In particular, the results of the ^{37}Cl experiment can be accounted for if $\nu_e = \nu_1$ is the lightest neutrino with $(-\Delta m^2) = m_2^2 - m_1^2 = 5 \times 10^{-5} \text{ eV}^2$ even for very tiny vacuum mixing angle, or for $(-\Delta m^2) \sin^2 2\theta_v = 10^{-7.5} \text{ eV}^2$.

We have considered several aspects of the MSW effect. In particular, the masses and the mixing angles suggested by the ^{37}Cl results are consistent with the predictions of the seesaw model, with the large mass scale given by the typical GUT scale $M_{\text{GUT}} \sim 10^{14} \text{ GeV}$ and the neutrino and charged-lepton Dirac masses related to the quark masses according to the simple $\text{SO}(10)$ GUT prediction (21). Typical predictions for the neutrino masses are $m_1 = 10^{-11} \text{ eV}$, $m_2 = 8 \times 10^{-7} \text{ eV}$, $m_3 = 10^{-3} \text{ eV}$, where ν_1 , ν_2 and ν_3 massive neutrinos are the dominant components of the ν_e , ν_μ and ν_τ neutrinos, respectively. These values would be changed somewhat by reasonable modifications in the corresponding assumptions and should be considered as illustrative only. Nevertheless, they suggest that the relevant transition process for the solar neutrinos may be $\nu_e \rightarrow \nu_\tau$ rather than

$\nu_e \rightarrow \nu_\mu$ (for which the adiabatic condition is badly violated). For a wide class of heavy Majorana neutrino mass matrices, the order of magnitude of the effective mixing angle is predicted to be $|\theta| \sim |V_{bt}| \sim 0.005 - 0.02$, where V is the Cabibbo-Kobayashi-Maskawa matrix. This is within the range that can account for the ^{37}Cl results, and for which the expected suppression of the ^{71}Ga counting rate is small.

We have also considered cosmological implications of the MSW effect, concentrating on oscillations between the same two species that are relevant to the solar neutrino problem. There is a small range of parameters, shown in the Figure, for which resonant conversions between ν_e and ν_τ (or ν_e and ν_μ) can occur between the time of neutrino decoupling and freeze-out of the neutron-to-proton ratio. This range overlaps with the region of values of the parameters, which lead to a solution of the solar neutrino problem, but requires somewhat larger vacuum mixing angles than those predicted by most versions of the seesaw model. The ν_e - ν_τ conversions can affect nucleosynthesis because they induce a ν_e - $\bar{\nu}_e$ asymmetry (since resonance only affects neutrinos) and also because the ν_e - ν_τ interchange modifies the freeze-out temperature. However, the effect turns out to be very small: the ^4He abundance is predicted to increase by less than 1.3×10^{-3} , which is equivalent in its effect to the existence of less than 0.09 new neutrino species.

Finally, we have discussed some general properties of neutrino oscillations in matter. We have shown, in particular, that (just as in the case of vacuum oscillations) the probabilities of neutrino transitions in matter do not depend on the type (Dirac or Majorana) of the neutrinos with definite mass.

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FIGURE CAPTION

The region of the $\Delta m^2 - \sin^2 2\theta_v$ plane where significant $\nu_e - \nu_\tau$ conversions occur in the early Universe for $0.8 \text{ MeV} \lesssim T_R \lesssim 2 \text{ MeV}$ (and $\eta = 5 \times 10^{-10}$). Also shown is the "solution" to the solar neutrino problem: $(-\Delta m^2) \sin^2 2\theta_v = 10^{-7.5} \text{ eV}^2$. The region includes larger vacuum mixing angles than are predicted by most versions of the seesaw model.