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LOW ENERGY GAUGE COUPLINGS AND THE MASS GAP
OF $N = 1$ SUPERSYMMETRY

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E R R A T U M

The one loop β -functions of the Yukawa couplings above the supersymmetry threshold we have used are incorrect. The terms arising from the gauge interactions in the right-hand side of Eq. (26) have to be all multiplied by $\frac{1}{2}$. The numerical values of the low-energy gauge couplings change very little. For example, at $\Lambda = 10^7$ and $M_{SS} = 2.0$ TeV, the corrected values of α_3 , α and $\sin^2\theta_W$ are 0.108, 0.00784, 0.238 respectively, to be compared with the values: 0.114, 0.00794 and 0.235, reported in Table 1. Our conclusions, in particular those concerning the supersymmetry mass gap, are unchanged. On the contrary, the one loop upper bounds to the heaviest quark and lepton masses are reduced. We find: $m_0 < 210$ GeV, $m_L < 170$ GeV instead of 290 and 210 as reported previously (the maximal value of 176 GeV for the v.e.v. of the relevant Higgs field is assumed).

We thank G. Kane and D.R.T. Jones for calling our attention to the paper by J.E. Björkman and D.R.T. Jones, Nucl. Phys. B259 (1985) 533, where the correct β -functions are reported.

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A B S T R A C T

The low energy values of the gauge coupling constants are very insensitive to those at a large energy scale if the corresponding theory is asymptotically divergent. This in general requires more than the standard three generations of quarks and leptons. We explore this scenario within the extension of the standard model provided by $N = 1$ supersymmetry with n quark and lepton generations. We introduce as a parameter the mass threshold M_{ss} of supersymmetric partners of ordinary particles. We find that, if all gauge theories enter the non-perturbative regime at an energy of the order of 10^{17} GeV, the low energy values of gauge coupling constants are well reproduced with five quark and lepton generations and $M_{ss} < 5$ TeV. Furthermore, all fermion masses should be below 250-300 GeV.

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Renormalizable field theories yield a finite relation between the coupling constants defined at two different energy scales, μ and Λ , of the form:

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(\Lambda)} + b \ln(\Lambda/\mu) \quad (1)$$

with b a calculable constant. We will take $\mu \ll \Lambda$ in the following. Asymptotically free theories are characterized by a negative value of b . In this case, the actual value of $g(\mu)$ results from a very delicate balance of two large quantities, i.e., $g(\Lambda)^{-2}$ and $b \ln(\Lambda/\mu)$. This is particularly annoying if we give a physical meaning to the large energy scale, Λ , for example associating it with a further unification into a more complete theory which possesses an intrinsic length scale, such as gravity. For $\Lambda \sim M_{\text{Planck}}$ and $b < 0$, low energy physics depends very critically on the properties of the fundamental theory, i.e., the value of $g(\Lambda)$. Another way of seeing the same problem is to consider, for $b < 0$, the value $\bar{\mu}$ at which $g(\bar{\mu})$ is of order unity (in QCD, $\bar{\mu} = \Lambda_{\text{QCD}}$):

$$\bar{\mu} = \Lambda_{\text{QCD}} = \text{const} \cdot \Lambda e^{\frac{1}{b g(\Lambda)^2}} \quad (2)$$

With Λ fixed by independent physical considerations, Eq. (2) shows that Λ_{QCD} depends exponentially upon the value of $g(\Lambda)$.

The situation looks better in an asymptotically divergent theory, $b > 0$. In the latter case¹⁾, the value of $g(\mu)$ is very insensitive to the value of $g(\Lambda)$. Indeed, to a first approximation, we may even drop the $g(\Lambda)^{-2}$ term in the right-hand side of Eq. (1), and see that $g(\mu)$ is essentially determined by the value of Λ only. The standard theory of particle interactions, with three-quark and lepton generations, does not enjoy the property of being completely divergent in the ultra-violet. With the latter particle content, both the $SU(3)_{\text{colour}}$ and the weak $SU(2)$ gauge interactions are asymptotically free. The simplest way to implement the ultra-violet divergent scenario is to increase to eight or nine the number of quark and lepton generations. In this way²⁾, a qualitatively correct prediction of the low energy gauge couplings has been obtained. The picture of Ref. 2), however, suffers from the theoretical drawback that it does not contain supersymmetry as a mechanism to stabilize the low energy versus the high energy scale, the well-known hierarchy problem³⁾. Without supersymmetry, the corrections to the curvature of the Higgs potential are of the form:

$$m^2 = m_0^2 + d C \Lambda^2 \quad (3)$$

with C a model-dependent constant, and extraordinary fine tuning is required to keep m as low as the Fermi scale:

$$m \simeq \langle H \rangle_0 \simeq G_F^{-1/2} \ll \Lambda \quad (4)$$

Supersymmetry turns Eq. (3) into:

$$m^2 = m_0^2 + \alpha C \Delta M^2 \quad (5)$$

where ΔM is the mass gap between normal particles and their supersymmetric partners. With:

$$\Delta M \lesssim (\alpha G_F)^{-1/2} \simeq 3 \text{ TeV} \quad (6)$$

the inequality in Eq. (4) is not spoiled by higher order corrections.

Low energy supersymmetry has been included in the scenario of Ref. 2) by Cabibbo and Farrar⁴⁾. Assuming that the theory is fully supersymmetric above $(G_F)^{-1/2}$, with five supersymmetric generations, two Higgs doublets and all gauginos required by $SU(3) \times SU(2) \times U(1)$, they find:

$$\alpha_{e.w.}(300 \text{ GeV}) = .00781 \quad (\text{input}) \quad (7)$$

$$\Lambda = 2.4 \cdot 10^{16} \text{ GeV}$$

$$\alpha_3(300 \text{ GeV}) = .09 \quad (8)$$

$$\sin^2 \theta_W(300 \text{ GeV}) = .247 \quad (9)$$

Prompted by the precise values of $\sin^2 \theta_W$ and α_3 which have become available recently, we have reanalyzed the supersymmetric scenario, allowing for a non-vanishing mass gap between the normal particles and their supersymmetric partners. The aim is to see whether one can put an upper bound to ΔM , to be compared with the theoretical estimate, Eq. (6). Indeed, we find this to be the case. With respect to Ref. 4), we have included in the evolution of the gauge couplings the effects of the Yukawa couplings. The latter have to be considered for consistency, since the scenario itself predicts additional generations with

masses between M_W and $G_F^{-1/2}$, and therefore rather large Yukawa couplings. We illustrate first the physical assumptions underlying our calculation.

We consider an $SU(3) \times SU(2) \times U(1)$, supersymmetric gauge theory with n supersymmetric generations and two Higgs chiral supermultiplets. We assume that supersymmetry is broken by soft⁵⁾ mass terms, which split squarks and sleptons from their fermionic counterparts and give masses to the gauginos. All these mass terms are independent of each other, but, for simplicity, we will assume that they are of the same order of magnitude. Thus, sparticles and gauginos start contributing to the β -functions above a single threshold, M_{ss} . Quarks and leptons take masses from the Yukawa couplings to the Higgs doublets. The latter couplings cannot diverge too early, which gives a general bound⁶⁾ of the order of 200-250 GeV to quark and lepton masses. To be definite, we assume that quarks and leptons of the yet unseen, $n-3$, generations have a mass above the W -mass and smaller than Λ_F :

$$\Lambda_F \equiv 250 \text{ GeV} \quad (10)$$

Similar bounds apply to the Higgs scalar masses. Excluding ad hoc cancellations, this implies the whole Higgs supermultiplet to be below Λ_F .

Present experiments determine the values of the gauge couplings at a mass scale of the order of M_W . For colour interactions, we take:

$$\alpha_3(M_W) = .12^{+.01}_{-.02} \quad (11)$$

corresponding to⁷⁾

$$\Lambda_{QCD} = 150^{+150}_{-100} \text{ MeV} \quad (12)$$

and

$$\alpha_{e.m.}(M_W) = .00772 \quad (13)$$

$$\sin^2 \theta_W(M_W) = .231 \pm .007 \quad (14)$$

The latter value results from the average of the CDHS⁸⁾ and CHARM⁹⁾ results:

$$\sin^2 \theta_W(M_W) \Big|_{CDHS} = .227 \pm .005 \pm .005 \quad (15)$$

$$\sin^2 \theta_w(M_W) \Big|_{\text{CHARM}} = .236 \pm .005 \quad (16)$$

Evolution of the above values from M_W to 250 GeV has been computed with three or five quark and lepton generations, and the mean value of the two results has been taken as a reference value, to be compared with the $\alpha_i(\Lambda_F)$, $i = 1, 2, 3$, given by our calculation. The resulting reference values are reported in the Table. The errors quoted result from the uncertainty in the extrapolation, combined with the original errors, Eqs. (11) to (14).

The evolution of the gauge couplings is described by two-loop, coupled renormalization group equations:

$$\frac{d\alpha_i}{d\ln E} = \frac{(\alpha_i)^2}{2\pi} A_i + \sum_k \frac{(\alpha_i)^2 \alpha_k}{8\pi^2} B_{ik} - \frac{\alpha_i}{8\pi^2} \left[\frac{Y_4^i(F)}{4\pi} \right] \quad (17)$$

where the last term¹⁰⁾ arises from the Yukawa couplings. For E less than the supersymmetry threshold, M_{ss} , our assumptions about the particle spectrum yield the following values^{*)} for the constants A and B ¹¹⁾:

$$\begin{aligned} A_1 &= \frac{1}{2} n_H + \frac{20}{9} n \\ A_2 &= \frac{1}{2} n_H + \frac{4}{3} n - \frac{22}{3} \\ A_3 &= \frac{4}{3} n - 11 \end{aligned} \quad (18)$$

$$B_{ik} = \begin{bmatrix} \frac{3}{4} n_H + \frac{95}{27} n & \frac{9}{4} n_H + n & \frac{44}{9} n \\ \frac{3}{4} n_H + \frac{1}{3} n & \frac{25}{4} n_H + \frac{49}{3} n - \frac{136}{3} & 4 n \\ \frac{11}{18} n & \frac{3}{2} n & \frac{76}{3} n - 102 \end{bmatrix} \quad (19)$$

For each generation, we write the Yukawa couplings according to:

$$\begin{aligned} \mathcal{L}_Y &= g_u \bar{q}_L u_R H + g_d \bar{q}_L d_R H^c + g_e \bar{l}_L e_R H^c \\ H &= \begin{pmatrix} H^0 \\ H^- \end{pmatrix} ; \quad H^c = \begin{pmatrix} H_c^+ \\ H_c^0 \end{pmatrix} \end{aligned} \quad (20)$$

*) In obtaining Eqs. (18) and (19) from Ref. 11), one must remember that the Higgs supermultiplet is fully active below M_{ss} . In the following, $n_H = 2$ will be used.

Below M_{ss} , the quantities $Y_4^i(F)$ take the form¹⁰⁾:

$$\begin{aligned} Y^1 &= T_2 \left(\frac{17}{6} g_u^2 + \frac{5}{6} g_d^2 + \frac{5}{2} g_e^2 \right) \\ Y^2 &= T_2 \left(\frac{3}{2} g_u^2 + \frac{3}{2} g_d^2 + \frac{1}{2} g_e^2 \right) \\ Y^3 &= T_2 \left(2 g_u^2 + 2 g_d^2 \right) \end{aligned} \quad (21)$$

where trace goes over generations.

For $E > M_{ss}$, the corresponding quantities, distinguished with a prime in the following, are:

$$\begin{aligned} A'_1 &= \frac{1}{2} n_H + \frac{10}{3} n \\ A'_2 &= \frac{1}{2} n_H + 2n - 6 \\ A'_3 &= 2n - 9 \end{aligned} \quad (22)$$

$$B'_{ik} = \begin{bmatrix} \frac{1}{2} n_H + \frac{190}{27} n & \frac{3}{2} n_H + 2n & \frac{88}{9} n \\ \frac{1}{2} n_H + \frac{2}{3} n & \frac{7}{2} n_H + 14n - 24 & 8n \\ \frac{11}{9} n & 3n & \frac{68}{3} n - 54 \end{bmatrix} \quad (23)$$

$$\begin{aligned} (Y^1)' &= T_2 \left(\frac{26}{3} g_u^2 + \frac{14}{3} g_d^2 + 6 g_e^2 \right) \\ (Y^2)' &= T_2 \left(6 g_u^2 + 6 g_d^2 + 2 g_e^2 \right) \\ (Y^3)' &= T_2 \left(4 g_u^2 + 4 g_d^2 \right) \end{aligned} \quad (24)$$

We are primarily interested in the values of the gauge couplings. To this aim, it is consistent to determine the evolution of the Yukawa couplings to one-loop level only. For $E < M_{ss}$, the relevant equations have been given in Ref. 6a) for the case of one Higgs doublet. By trivial modifications^{6b)}, we have in the present case ($H_C \neq H^+$):

$$\begin{aligned} 16\pi^2 \frac{dg_u}{d \ln E} &= \left[\frac{3}{2} g_u^2 + \frac{1}{2} g_d^2 + 3 T_2 g_u^2 - 4n \left(8d_3 + \frac{9}{4} d_2 + \frac{17}{12} d_1 \right) \right] g_u \\ 16\pi^2 \frac{dg_d}{d \ln E} &= \left[\frac{1}{2} g_u^2 + \frac{3}{2} g_d^2 + T_2 (3g_d^2 + g_e^2) - \right. \\ &\quad \left. - 4n \left(8d_3 + \frac{9}{4} d_2 + \frac{5}{12} d_1 \right) \right] g_d \end{aligned} \quad (25)$$

$$16\pi^2 \frac{dg_e}{d\ln E} = \left[\frac{3}{2} g_e^2 + \text{Tr}(3g_d^2 + g_e^2) - 4\pi \left(\frac{9}{4} d_2 + \frac{15}{4} d_1 \right) \right] g_e \quad (25)$$

Contd.

for $E < M_{ss}$, and:

$$16\pi^2 \frac{dg_u}{d\ln E} = \left[3g_u^2 + g_d^2 + 3\text{Tr} g_u^2 - 4\pi \left(\frac{32}{3} d_3 + 6d_2 + \frac{26}{9} d_1 \right) \right] g_u$$

$$16\pi^2 \frac{dg_d}{d\ln E} = \left[g_u^2 + 3g_d^2 + \text{Tr}(3g_d^2 + g_e^2) - 4\pi \left(\frac{32}{3} d_3 + 6d_2 + \frac{14}{9} d_1 \right) \right] g_d$$

$$16\pi^2 \frac{dg_e}{d\ln E} = \left[3g_e^2 + \text{Tr}(3g_d^2 + g_e^2) - 4\pi (6d_2 + 6d_1) \right] g_e$$

(26)

for the supersymmetric case, $E > M_{ss}$.

The integration of Eqs. (17), (25) and (26) has been performed numerically, starting from Λ and going downwards. We expect the final values, $\alpha_i(\Lambda_F)$, to be insensitive to the values of the gauge and of the Yukawa couplings at Λ . We have explicitly checked this point by varying independently each of the gauge couplings between 1 and 10. The Yukawa couplings at Λ have also been varied independently, allowing for vanishing values, still with marginal effects on $\alpha_i(\Lambda_F)$. In the following, we report detailed results for the initial values:

$$\begin{aligned} d_{1,2,3}(\Lambda) &= 1 \\ \frac{g_{u,d,e}^2}{4\pi}(\Lambda) &= 1 \end{aligned} \quad (27)$$

and assuming only one generation to dominate the Yukawa contribution to Eq. (17). The relevant parameters are n , Λ and M_{ss} . n is essentially fixed to the first two integers for which colour interactions above M_{ss} are not asymptotically free to one loop, $n = 5, 6$. Larger values would make colour interactions to diverge too early, for $M_{ss} \ll \Lambda$. For $n = 5$ or 6 , we have varied independently Λ and M_{ss} , in the range:

$$10^{14} \text{ GeV} < \Lambda < 10^{19} \text{ GeV}$$

$$\Lambda_F < M_{ss} < 10^7 \text{ GeV} \quad (28)$$

searching for a "triple coincidence" of $\alpha_i(\Lambda_F)$ with the reference values reported in the Table. Our results for $n = 5$ generations are illustrated in Figs. 1 to 3 and the Table. A good, simultaneous fit to the gauge couplings is obtained for:

$$\Lambda = 1 \cdot 10^{17} \text{ GeV}$$

$$M_{ss} = 2 \text{ TeV} \quad (29)$$

The no-threshold situation⁴⁾, $M_{ss} \approx \Lambda_F$, still leads to an acceptable solution. Increasing Λ , the curve giving $\sin^2 \theta_W$ as a function of M_{ss} remains quite stable, while α_3 and $\alpha_{e.m.}$ shift both downwards. As their steepness is quite different, the triple coincidence is lost at some value of Λ . The opposite effect on α_3 and $\alpha_{e.m.}$ is produced when lowering Λ .

In conclusion, for $n = 5$, we can reproduce satisfactorily the values of the gauge couplings^{*)}, for:

$$10^{16} \text{ GeV} \leq \Lambda \leq 5 \cdot 10^{17} \text{ GeV} \quad (30)$$

and for

$$M_{ss} \leq 5 \text{ TeV} \quad (31)$$

No lower bound to M_{ss} results.

We find it quite satisfactory that a good fit to the low energy couplings requires a value of the mass-gap of $N = 1$ supersymmetry of the order given in Eq. (31), and well consistent with the value of M_{ss} needed to stabilize the low energy scale, Eq. (6).

*) A by-product of the present analysis is an evaluation of the bounds to quark and lepton masses, in the supersymmetric framework. We find $m < 290 \text{ GeV}$ for quarks of up or down type, and $m < 210 \text{ GeV}$ for an electron-like lepton. These bounds are accurate to one loop only and quite similar to those found in Refs. 6).

The case $n = 6$ also admits a solution but only for considerably larger values of M_{ss} :

$$M_{ss} \simeq 10^3 \text{ TeV} \quad (32)$$

The reason is that colour interactions diverge more rapidly above M_{ss} . This must be compensated by delaying the mass-scale where all particles contribute, i.e., by increasing M_{ss} . In contrast with the previous case, the value Eq. (32) seems too large to be reconciled with the bound in Eq. (6). Models with $N = 2$ supersymmetry and three generations yield three $N = 1$ left-handed generations and their mirror. The latter being indistinguishable from the former in what concerns the evolution of gauge couplings, our $n = 6$ case corresponds closely to the $N = 2$ supersymmetric extension considered in Ref. 12). A large value for the threshold of the first supersymmetry is also indicated by their results.

In conclusion, we have further explored the idea that: (i) gauge interactions are not asymptotically free at large energy, but rather diverge at a common energy scale Λ ; (ii) $N = 1$ supersymmetry holds below Λ . A good fit to the low energy gauge couplings is obtained for five supersymmetric generations. At the same time, a very significant upper bound to the mass-gap of $N = 1$ supersymmetry obtains, M_{ss} less than a few TeV. This is remarkably consistent with what is needed for the hierarchy problem, and very promising for the next generation of accelerators. The range of values which we find for Λ is close enough to the Planck mass, not to exclude that the strong regime of the gauge interactions is related to a further unification which includes gravity.

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$\Lambda(\text{GeV})$	$M_{ss}(\text{TeV})$	$\alpha_3(\Lambda_F)$	$\alpha_{e.m.}(\Lambda_F)$	$\sin^2\theta_W(\Lambda_F)$
$3 \cdot 10^{16}$	0.25	0.096(0.088)	0.00801(0.00775)	0.241(0.248)
	0.50	0.102	0.00808	0.240
	1.0	0.109	0.00816	0.237
$1 \cdot 10^{17}$	0.25	0.094	0.00773	0.241
	2.0	0.114	0.00794	0.235
	4.0	0.122	0.00801	0.233
$3 \cdot 10^{17}$	3.0	0.115	0.00773	0.234
	5.0	0.121	0.00778	0.232
	10.0	0.131	0.00785	0.230
Reference values		0.099 (12)	0.00791 (7)	0.237 (8)

TABLE

Values of the gauge couplings at $\Lambda_F = 250$ GeV as functions of the ultra-violet cut-off, Λ , and of the mass threshold of $N = 1$ supersymmetry, M_{ss} . Five generations are assumed. The numbers in parentheses (first line) are computed for vanishing Yukawa couplings, and reproduce the results of Cabibbo and Farrar, Ref. 4). The last line gives the values obtained from present experiments (see text) with errors on the last digits quoted in parentheses.

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FIGURE CAPTIONS

Fig. 1 The strong coupling constant, α_3 , at $\Lambda_F = 250$ GeV as a function of the supersymmetry threshold M_{ss} for five generations of quarks and leptons. Values of the ultra-violet cut-off, Λ , are indicated. The shaded region represents the range allowed by present experimental data; see the Table.

Fig. 2 Same as Fig. 1, for the electromagnetic coupling, $\alpha_{e.m.}$.

Fig. 3 Same as Fig. 1, for the weak mixing parameter, $\sin^2\theta_W$.

- Figure 1 -

- Figure 2 -

- Figure 3 -