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SPIN EFFECTS IN ELASTIC ANTIPROTON NUCLEUS SCATTERING

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ABSTRACT

Elastic scattering of 180 MeV antiprotons by several closed-shell nuclei are analyzed using Glauber's multiple scattering theory. Spin effects are studied and polarization in elastic scattering is obtained within this framework. The sensitivity of the polarizations to the spin-flip amplitudes are discussed in terms of the $\bar{N}N$ potential models used for the calculations. Spin effects in angular distribution are found to be small.

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1. - INTRODUCTION

Glauber's multiple scattering theory¹⁾ is known to be a powerful tool to describe the high-energy hadron-nucleus scattering, both in the coherent and incoherent regions. It relates the scattering from a nucleus to that from individual nucleons, and has been quite successful in describing elastic scattering of hadrons even with rather simplified pictures of nuclear structure and of the elementary hadron-hadron scattering amplitudes. Recently, experimental data on $\bar{p}$ -nucleus scattering from LEAR have become available at low and intermediate energies. The elastic angular distribution data²⁾ exhibit a pronounced diffractive structure typical of a strongly-absorbed particle. Hence it would be interesting to study elastic scattering of antiprotons using Glauber's multiple scattering theory, which is a convenient, efficient model for exploring the spin effects.

In the present work, we use Glauber's model to analyze the elastic $\bar{p}$ -nucleus scattering data at 180 MeV and to study the spin effects in elastic $\bar{p}$ -nucleus scattering within this framework. Although Glauber's theory is valid at high energies, one of the basic assumptions of this model, i.e., the strong forward peak in the elementary $\bar{p}p$ amplitude, is fulfilled in $\bar{p}$ -A scattering at intermediate and low energies. The elementary $\bar{p}p$ scattering exhibits a strongly-pronounced forward peak which indicates that a number of partial waves with non-zero orbital angular momentum ($\ell > 0$) contribute even at low energies. Recent analysis of the $\bar{p}p$ scattering data³⁾ show that even at low energies ($p_{lab} \sim 180$ MeV/c) there is substantial contribution from partial waves $\ell > 0$. The appearance of a narrow forward cone at low energies is due to the fact that several partial waves with non-zero angular momentum contribute. The interference between the different partial waves is reflected in the appearance of a narrow forward peak in the $\bar{p}N$ scattering. Eikonal and non-adiabatic corrections become weak due to the strong forward peak in the elementary $\bar{p}N$ amplitude, which has been observed in a recent analysis⁴⁾ of $\bar{p}$ -¹²C data at 46 MeV. Furthermore, recent analyses of the $\bar{p}$ -nucleus data by several authors⁵⁾ based on impulse approximation and multiple scattering theory show that the elastic angular distribution is fairly well reproduced even at low energies, where the validity of impulse approximation is an open question. The successes of the impulse approximation in the relativistic⁶⁾ and non-relativistic⁷⁾ treatment of $\bar{p}$ -nucleus scattering analyses indicate that nuclear medium effects most probably do not play a significant role, and are weak for antiproton nucleus scattering. This is due to the fact that the antiproton annihilates with nucleons into meson channels. This interaction occurs at the nuclear surface and hence medium effects become negligible.

In the present work, we have used the simple Glauber model to analyze the data²⁾. Corrections to Glauber's model have been proposed, but it has been stressed by Saudinos and Wilkin⁸⁾ that many of them tend to cancel each other at low momentum transfers; hence we have not taken these into account in the present work. It is well known from the analysis of proton-nucleus elastic scattering⁹⁾ that polarization depends mainly on the central and spin-flip parts of the two-body amplitudes, and is very sensitive to the imaginary part of the spin-dependent amplitude. We hope therefore to obtain information on the spin-dependent components of the elementary $\bar{N}N$ amplitude which enter as input parameters in our calculation. In the next section the formalism is briefly outlined, and in Section 3 the results are given, together with discussions. The last section contains the conclusions.

2. - FORMALISM

In Glauber's model the elastic scattering amplitude for an A-particle system is given by

$$F_{fi}(q) = \frac{ik}{2\pi} \int e^{i\vec{q} \cdot \vec{b}} d^2b \langle \psi_f | \left\{ 1 - \prod_{j=1}^A [1 - \Gamma_j(\vec{b} - \vec{s}_j)] \right\} | \psi_i \rangle \quad (1)$$

where k is the wave number of the incident particle, $\vec{q}$ is the momentum transfer, $\vec{b}$ is the impact parameter and $\psi_{i,f}$ are the initial- and final-state nuclear wave functions. The profile function $\Gamma_j(\vec{b} - \vec{s}_j)$ is the two-dimensional Fourier transform of the scattering amplitude f_j , $\vec{s}_j$ being the projection of the radius vectors $\vec{r}_j$ of the j th particle on the plane perpendicular to the direction of $\vec{k}$ chosen along the z -axis. The profile function is written as

$$\Gamma(\vec{b} - \vec{s}_j) = \frac{1}{2\pi i k} \int e^{-i\vec{q} \cdot (\vec{b} - \vec{s}_j)} f_j(q') d^2q' , \quad (2)$$

where $f_j(q)$ is the scattering amplitude for the single j th nucleon. In order to perform the integration over the nuclear wave function in Eq. (1), a separable ground state density for the nucleus is assumed:

$$|\psi(r_1, \dots, r_j, \dots, r_A)|^2 = \prod_{j=1}^A \rho_j(r_j) \quad (3)$$

where ρ_j 's are the single particle densities.

The elementary scattering amplitude is assumed to have the following form, with central and spin-flip components given by

$$f(q) = A(q) + C(q) \vec{\sigma} \cdot (\vec{q} \times \hat{k}) \quad (4)$$

where $\vec{\sigma}$ is the spin operator for the antiproton, $\hat{k}$ is the direction of the incident momentum and $\vec{q}$ is the momentum transfer. Although the most general representation of the two-body scattering matrix consists of five invariants, the scattering is dominated¹⁰⁾ by the two terms which are retained in the above form. Such an approximation is well justified for elastic scattering on spin-zero target nuclei which are considered in the present work. Finally, the optical theorem is used to relate the imaginary part of the scattering amplitude to the total cross-section.

The central and spin-flip amplitudes are assumed to be Gaussian and for small momentum transfers are parametrized as¹¹⁾

$$A(q) = \frac{k\sigma}{4\pi} (i + \alpha) \exp(-\beta q^2/2) \quad (5)$$

$$C(q) = \frac{k\sigma_s}{4\pi} (i + \alpha_s) \exp(-\beta_s q^2/2) \quad (6)$$

where σ is the total $\bar{N}$ -N cross-section and σ_s determines the strength of the spin-dependent amplitude, α and α_s are the ratio of the real to imaginary part of the forward scattering amplitudes, and β and β_s are the slope parameters for the central and spin-flip amplitudes respectively, k and q being the incident momentum and momentum transfer respectively. These are generally obtained from the fit to the experimental scattering data. For the spin-dependent amplitudes, the parameters σ , α and β are taken from the analysis of the $p\bar{p}$ scattering data at

590 MeV/c¹²⁾. For the $\bar{p}n$ scattering, no experimental data exist, however. The parameters for the spin-flip amplitudes were obtained from the fit to the $\bar{N}N$ amplitudes (at low momentum transfer) derived from the two popular $\bar{N}N$ potential models^{13),14)}, which differ basically in their treatment of the strong short-range annihilation part of the $\bar{N}N$ potential. In the Table, the average values of the parameters for the $\bar{p}p$ and $\bar{p}n$ scatterings are given, together with the experimental values¹²⁾ for central amplitudes for the $\bar{p}p$ scattering.

The elastic scattering amplitude $F_{ii}(q)$ of a spin-zero nucleus takes the form¹⁵⁾

$$F_{ii}(q) = F(q) + G(q) \vec{\sigma} \cdot (\hat{q} \times \hat{k}) \quad (7)$$

and the differential cross-section and polarization are given by

$$d\sigma/d\Omega = |F(q)|^2 + |G(q)|^2 \quad (8)$$

$$P = 2 \operatorname{Re} |F(q)G(q)^*| / (d\sigma/d\Omega) \quad (9)$$

In the absence of Coulomb interaction, the nuclear amplitudes $F(q)$ and $G(q)$ are given by

$$F(q) = ik \int_0^{\infty} b db J_0(qb) \left\{ 1 - \exp(-\lambda_c T(b, \beta)) \right. \\ \left. * \cos\left(\lambda_s \frac{d}{db} T(b, \beta_s)\right) \right\} \quad (10)$$

$$G(q) = -ik \int_0^{\infty} b db J_1(qb) \left\{ \exp(-\lambda_c T(b, \beta)) \right. \\ \left. * \sin\left(\lambda_s \frac{d}{db} T(b, \beta_s)\right) \right\} \quad (11)$$

with

$$\lambda_c = \frac{1}{2} \sigma (1 - i\alpha) , \quad \lambda_s = \frac{1}{2} \sigma_s (i + \alpha_s) \quad (12)$$

and $T(b, \beta)$ is the target thickness function defined as

$$T(b, \beta) = \int d^3r \rho(r) \frac{1}{2\pi\beta} e^{-\frac{(\vec{b}-\vec{s})^2}{2\beta}} \quad (13)$$

$\rho(r)$ being the nuclear matter distribution.

For the lighter nuclei ^{12}C and ^{16}O , the nuclear density was taken to be of harmonic oscillator form and the target thickness function was evaluated exactly. For the heavier nucleus ^{40}Ca , the nuclear matter density was taken to be of two-parameter Woods-Saxon form

$$\rho(r) = \rho_0 \left[1 + \exp\left(\frac{r-c}{a}\right) \right]^{-1} \quad (14)$$

and the target thickness function was evaluated in the thick nucleus approximation¹⁶⁾. The nuclear parameters were chosen from fits to elastic electron scattering data. For ^{12}C and ^{16}O , the oscillator parameters were chosen to be 1.64 fm and 1.70 fm respectively. For ^{40}Ca the values of c and a are taken to be 3.66 fm and 0.54 fm respectively.

3. - RESULTS AND DISCUSSIONS

The differential cross-section and polarization were calculated using the values of the parameters of the spin-independent amplitude taken from the analysis of the recent $\bar{p}p$ scattering data¹²⁾. For the spin-flip amplitude these were taken either from the values given in the Table, as derived from the Dover-Richard (DR) model¹³⁾ or from the model of Coté et al. (PARIS)¹⁴⁾. The calculations will be referred to as the DR or PARIS model, according to whether the spin-flip amplitudes are chosen from the DR or the PARIS model.

The results of elastic angular distribution for $\bar{p}-^{12}\text{C}$ scattering at 180 MeV are displayed in Fig. 1 together with the experimental data²⁾. The contributions due to spin-independent and spin-flip amplitudes are shown separately for comparison. It can be seen that the spin-flip amplitude has negligible contribution compared to the spin-independent one except at large angles. The elastic angular distribution is primarily determined by the spin-independent term of the two-body

amplitude [Eq. (4)]. The spin-flip contributions are seen to be much smaller compared to the non-spin-flip contributions and are in phase with them, and hence the minima of the angular distribution remain unaffected due to spin-flip contributions. This was seen to be similar to the analysis of proton-nucleus scattering at 1 GeV¹¹⁾. The DR and PARIS models give different amounts of contributions due to the spin-flip components, but the differential cross-section remains practically unaffected except at very large angles. The overall agreement with experimental data is fairly well reproduced, except at large momentum transfers.

The minima of the elastic angular distribution are found to be very sensitive to the value of α , the ratio of the real to the imaginary part of the forward scattering amplitude. In Fig. 2 the results of angular distribution for $\bar{p}$ -¹²C elastic scattering at 180 MeV are shown as obtained with the spin-independent amplitude [Eq. (5)] only. The contributions due to the real and imaginary parts of the scattering amplitude are shown separately. From Fig. 2 it is seen that the minima of the imaginary part correspond to the maxima of the real part of the scattering amplitude, which is determined by the value of α . The elastic cross-section is determined by the imaginary part, i.e., $|\text{Im } F(q)|^2$ except at the minima which get the contribution from the real part, i.e., $|\text{Re } F(q)|^2$, and is proportional to $|\alpha|^2$. The depth of the diffraction minima is very sensitive to the value of α and is filled up from the contribution of the real part of the scattering amplitude. Hence, from the analysis of elastic scattering cross-section minima, one may determine the absolute value of the parameter α , provided the scattering is truly elastic¹⁷⁾. However, it may be borne in mind that, in particular for heavy nuclei, the Coulomb effects are considerable and change the diffractive minima depths to quite a large extent¹⁸⁾. For lighter nuclei, Coulomb effects are not very considerable except at very forward angles.

In Fig. 3a the results of elastic angular distribution by 180 MeV antiproton scattering by ¹²C, ¹⁶O and ⁴⁰Ca are shown together with the experimental data. The value of $\alpha = 0.25$ was used for these calculations. Fairly good agreement with the experimental data is obtained for all the three nuclei studied.

The polarizations in elastic $\bar{p}$ -A scattering were calculated using the DR and PARIS models. The PARIS model predicts a larger amount of polarization than the DR case. This feature was also observed in a recent analysis¹⁹⁾ of $\bar{p}$ -A elastic scattering data. In Fig. 3b, we display the polarization results obtained with the DR model only. The polarization is found to be almost stable from nucleus to nucleus, apart from places where $\text{Im } F(q) = 0$ and which are determined by the nuclear size. This indicates a correlation between the polarization in $\bar{p}$ -nucleus

and $\bar{p}$ -N scattering⁸⁾, and one may obtain some information on the elementary (averaged) amplitudes from the polarization data. The presence of correlation between $d\sigma/d\Omega$ and polarization P is also observed (Figs. 3a and 3b); the extrema of P correspond to inflection points at $d\sigma/d\Omega$ and vice-versa. Similar behaviour is also obtained in the analysis of proton-nucleus scattering²⁰⁾.

In Fig. 4, the polarization results for elastic $\bar{p}$ - ^{12}C are shown, together with the few data points²¹⁾ at forward angles. It is seen that although the elastic angular distribution is insensitive to the spin-flip components, the polarization results as obtained by the two models predict different amounts of polarization, the PARIS model predicting more than the DR model. Similar results were also observed in a recent analysis of elastic $\bar{p}$ - ^{12}C scattering based on relativistic impulse approximation¹⁹⁾. The few data points favour the DR model; however, no firm conclusions can be drawn from this observation. The polarization at forward angles is determined primarily by the values of the imaginary part of the scattering amplitudes, and remains practically unaffected by the change in the real part of the amplitudes.

The polarization was also calculated with the values of α (the ratio of the real to the imaginary amplitude) as determined from the model amplitudes given in the Table. In Fig. 5, we show the results of polarization for $\bar{p}$ - ^{12}C elastic scattering due to variation of the parameter α . The overall polarization remains unaffected, except that the detailed shape of the polarization curve changes with the appearance of structures around the diffraction minima. The details of polarization depend not only on α , but also on the value of the slope parameters of the two-body amplitudes, in particular on the difference between the slopes of the central and spin-flip amplitudes [Eqs. (5) and (6)]. In Fig. 6, the results of polarization for different values of $(\beta - \beta_s)$ are shown for comparison. Hence we observe that although the polarization is determined primarily by the imaginary parts of the central amplitude and the spin-flip amplitudes, the detailed shape is determined mainly by the value of α and also to a lesser extent by the difference of the slope parameters for the scattering amplitudes. However, the lack of experimental information on spin polarization observables for $\bar{N}N$ and $\bar{N}$ -nucleus scattering does not allow us to draw a definite conclusion about the spin components of the two-body amplitudes.

4. - CONCLUSIONS

From our present study we show that Glauber's multiple scattering theory provides satisfactory fits to the angular distribution for elastic antiproton scattering by ^{12}C , ^{16}O and ^{40}Ca at intermediate energies and small momentum transfers. The spin effects are found to be negligible for the elastic angular distribution.

The parameter α (ratio of real to imaginary part) plays an important role in describing the diffraction minima of the angular distribution. Hence from such an analysis one may determine the absolute value of α , i.e., this could be used as an independent method for determining the parameter α from analysis of $\bar{p}A$ elastic scattering data.

The polarization results as obtained in this work are model-dependent. This is due to lack of experimental information on spin-observables for $\bar{N}N$ scattering which forced us to have recourse to the $\bar{N}N$ potential models^{13),14)} in order to determine the parameters of the spin-flip amplitudes which are not yet fully constrained by experiment. The polarization depends on the strength of the two-body spin-flip amplitude, and the details of the polarization curve depend mainly on the parameter α and on the difference of the slope parameters, i.e., $(\beta - \beta_c)$. However, the forward polarization is determined by the imaginary parts of the scattering amplitudes and remains unaffected due to variation of α and $(\beta - \beta_c)$. This indicates that the form of the elementary amplitudes is very important for describing the detailed structure of the polarization. Comparison of the results of $\bar{p}-^{12}\text{C}$ elastic polarization with the few available data points favours the DR model. However, one cannot yet draw a conclusion from this observation. More experimental data are needed for a definite conclusion.

Refinements or corrections to these models are important for determining the spin-dependent components of the two-body amplitudes. However, we see that without improved and sufficient data on $\bar{N}-N$ and $\bar{N}$ -nucleus scattering on spin observables, it is neither informative nor justified to refine the model further.

Experimental data on spin observables at large angles as well as forward angles in $\bar{N}-N$ and $\bar{N}$ -nucleus scattering are needed in order to make a meaningful study, as well as to test the different $\bar{N}N$ potential models and to constrain the spin-dependent components of the two-body $\bar{N}N$ interaction.

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Parameters	PARIS	DR	Experimental value (*) for $\bar{p}p$ scattering at 590 MeV/c
σ fm^2	14.5	15.5	14.00 ± 0.26
α	+0.015	-0.04	0.203 ± 0.119
β fm^2	0.8586	0.9515	0.8877 ± 0.036
σ_s fm^3	1.8751	1.1689	
α_s	-0.017	+0.0355	
β_s fm^2	0.7347	0.8820	

(*) Brückner et al., Phys. Lett. 158B (1985) 180.

Table

Parameters for the averaged two-body amplitudes at 180 MeV

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FIGURE CAPTIONS

- Fig. 1 : The elastic differential cross-section for $\bar{p}$ - ^{12}C at 180 MeV are shown together with the experimental data²⁾. The contributions due to the spin-dependent (spin-independent) terms are shown by dashed (dotted-dashed) lines. The solid line gives the sum of the contributions due to the two terms.
- Fig. 2 : Elastic angular distribution for $\bar{p}$ - ^{12}C elastic scattering at 180 MeV due to the spin-independent part only is shown, together with the experimental data²⁾. The contribution due to the real and imaginary parts are shown by dashed and dotted-dashed lines. The solid line gives the full contribution.
- Fig. 3a : Elastic angular distribution for $\bar{p}$ scattering by ^{12}C , ^{16}O and ^{40}Ca at 180 MeV is shown together with the experimental data.
- Fig. 3b : Polarization in elastic scattering of antiprotons by ^{12}C , ^{16}O and ^{40}Ca at 180 MeV. The results are given for the DR model.
- Fig. 4 : Polarization in elastic $\bar{p}$ - ^{12}C scattering at 180 MeV is shown together with the experimental data. The solid (dashed) lines are the predictions of the PARIS and DR models.
- Fig. 5 : Polarization in elastic $\bar{p}$ - ^{12}C scattering at 180 MeV due to variation of the parameter α (the ratio of the real to the imaginary part of the forward scattering amplitude).
- Fig. 6 : Polarization in elastic $\bar{p}$ - ^{12}C scattering at 180 MeV due to variation of the difference of the slope parameters ($\beta - \beta_c$) of the spin-independent and spin-flip amplitudes.

Fig. 1

Fig. 2

Fig. 3a

Fig. 3b

Fig. 4

Fig. 5

Fig. 6