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TOPOLOGICALLY MASSIVE GRAVITY IN THE DREIBEIN LIGHT-FRONT GAUGE

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A B S T R A C T

The unconstrained light-front action of third-order topologically massive gravity is given. The generator of its dynamical evolution along the light-front time is found and it is shown to be non-negative. The analysis is performed in the dreibein light-front gauge we introduce here. This natural gauge choice allows us to stress the dreibein gauge structure of the theory.

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The quantum consistent coupling of fermions to bosons in the context of realistic low energy 10D pointlike field theories derived from superstrings requires the presence of a local Lorentz Chern-Simons (LCS) type of coupling between the antisymmetric second-rank tensor and the vielbein gravity field¹⁾. This local LCS coupling generates a sixth-order modification to the basic ingredient of the action, the standard Einstein second-order gravity action. One can interpret the presence of this topological contribution as the needed slight quantum increase of the classical D-dimensions of the bosonic space-time one starts with, required by the presence of fermions which are intrinsically quantum entities.

In the 3D case, the LCS term²⁾ gives rise to a conformally invariant contribution to the full action which now has room enough to permit the propagation of a massive spin-two excitation. It is known that Einstein's 3D action leads to a locally trivial dynamics.

Since the 3D LCS term originates an action (instead of a local coupling as it happens in $D = 10$ and a fortiori in $D = 4$), topological massive gravity is a third-order theory (instead of being a sixth-order one), and it is very likely that the qualitative modifications it will bring to the dynamics of the spin-two field will be milder than those which occur in $D = 4$ and in $D = 10$ when one has a local LCS coupling.

Presently there are three locally supersymmetric theories containing LCS terms: two in $D = 4$ ^{3), 4)} and one in $D = 3$ ⁵⁾.

In the following we shall study the dynamical properties of $D = 3$ topologically massive gravity, which is the simplest case where LCS terms appear.

We start from the dreibein form of the action⁵⁾

$$I_{\mu} = I_E + I_{CS} = \frac{1}{k^2} \langle e_{pc} \epsilon^{pmn} R_{mn}^{*c}(\omega) \rangle + \\ + \frac{1}{2k^2\mu} \langle \omega_{ma} \epsilon^{mnp} R_{np}^{*a}(\omega) - \frac{1}{3} \epsilon^{mnp} \epsilon^{abc} \omega_{ma} \omega_{nb} \omega_{pc} \rangle, \quad (1a)$$

where $R_{mn}^{*c} \equiv \partial_m \omega_n^c - \partial_n \omega_m^c + \epsilon^{cab} \omega_{ma} \omega_{nb}$, $\epsilon^{012} = +1$, $-\eta^{00} = \eta^{11} = \eta^{22} = +1$, first letters indicate dreibein indices, middle of the alphabet the world ones. It is a third-order theory since in this action the ω_{ma} are defined to be the torsionless

solutions corresponding to considering I_E as a first-order action, i.e.:

$$T_{mn}^a \equiv \partial_m e_n^a - \partial_n e_m^a + \epsilon^{abc} \omega_{mb} e_{nc} = 0. \quad (1b)$$

In this case, I_E can be written in the form

$$I_E = -\frac{1}{k^2} \langle \epsilon^{mnp} \epsilon^{abc} e_{ma} \omega_{nb} \omega_{pc} \rangle. \quad (1c)$$

This action is invariant under two independent sets of local transformations: the diffeomorphisms and the Lorentz ones. Both have the typical gauge structure:

$$\delta_\xi e_r^a = D_r^\omega \xi^a \equiv \partial_r \xi^a + \epsilon^{abc} \omega_{rb} \xi_c, \quad (2)$$

$$\delta_l e_r^a = \epsilon^a_{bc} e_r^b l^c, \quad \delta_l \omega_r^a = D_r^\omega l^a. \quad (3)$$

Notice that the gauge field associated with the local Lorentz transformations is the affinity ω_r^a .

We shall work in a local Lorentz null frame defined by:

$$e_r^\pm = \frac{1}{\sqrt{2}} (e_r^0 \mp e_r^2), \quad e_r^1, \quad (4)$$

and will systematically denote the co-ordinates by (u, v, l) , already thinking in terms of making u become a light-front one, i.e., such that the hypersurfaces $u = \text{const.}$ will be characteristics of the hyperbolic propagating system. The flat metric has the light-front form $-\eta^{+-} = \eta^{11} = +1$, and $\epsilon^{-+1} = \epsilon^{\text{vul}} = +1$.

Looking at the gauge transformations (2) and (3), it seems natural to assume that a natural gauge choice would be close to assuming $e_v^a = 0$, $\omega_v^a = 0$. However, since e_r^a has to be non-degenerate, and because the absence of torsion (1b) relates the dreibein with the affinity, one has to be more precise. The consistent choice we make is:

$$\begin{aligned} e_v^a &\equiv \delta_-^a; & e_r^+ &\equiv \delta_r^+; & e_r^I &\equiv e \delta_r^I, \\ e_u^- &\equiv n; & e_l^- &\equiv N; \end{aligned} \quad (5a)$$

for the dreibein field and

$$\omega_{va} \equiv \omega_{v,+} \delta_a^+, \quad (5b)$$

for the Lorentz gauge field. The inverse dreibein is given by $e_a^u = \delta_a^+$, $e_a^1 = e^{-1} \delta_a^1$, $e_+^v = -n$; $e_-^v = 1$, $e_{\bar{1}}^v = -e^{-1} N$. (We are using barred subindices to indicate they are Lorentz vectors.) The metric tensor $g_{rs} = e_r^a \eta_{ab} e_s^b$ takes the form

$$ds^2 = -2du(n du + dv + N dx) + e^2 dx^2 \quad (6)$$

while $g^{uu} = 0$ showing that our choice corresponds to having imposed to the surfaces $u = \text{const.}$ to be characteristic surfaces. Equations (5) define the dreibein light-front gauge. Introducing these values (5) into the torsion equation (1b) one obtains the values of the Lorentz affinities

$$-e\omega_{v,+} = \frac{1}{2} N' = e\omega_{u,-} = \omega_{1,\bar{1}}, \quad (7a)$$

$$\omega_{1,-} = -e' ; \quad \omega_{u,\bar{1}} = n' , \quad (7b)$$

$$\omega_{1,+} = \dot{e} - ne' - (2e)^{-1} NN' , \quad (7c)$$

$$\omega_{u,+} = -\dot{N} + n_{,1} + \frac{1}{2} n N' - n' N. \quad (7d)$$

where $\dot{f} \equiv \partial f / \partial u$ $f' \equiv \partial f / \partial v$.

To obtain the value of I_μ one introduces these expressions into (1a), where I_E has the form (1c) and I_{CS} is written, for convenience, in the form

$$\mu \kappa^2 I_{CS} = \langle \omega_{pa} \epsilon^{pmn} \partial_m \omega_{na} \rangle + \frac{1}{3} \langle \epsilon^{mnp} \epsilon^{abc} \omega_{ma} \omega_{nb} \omega_{pc} \rangle_{(1d)}$$

It is amusing to note that, if one goes to the weak limit, $e_{ma} \rightarrow \eta_{ma}$, the last term of (1d) has to be neglected and one recovers the recently found self-dual massive spin-two action⁶⁾ if ω_{na} is considered as an independent variable.

The full action becomes

$$\mu\kappa^2 I_\mu = 2 \left\langle \frac{e''}{e} \dot{N} \right\rangle + 2 \left\langle n \cdot \left[\frac{1}{2} N''' - 2 \left(\frac{e''}{e} \right) N' - \left(\frac{e''}{e} \right)' N + \right. \right. \right. \quad (8a)$$

$$\left. \left. - 2\mu \left\langle \frac{1}{4e} N'^2 \right\rangle + \left(\frac{e''}{e} \right)_{,1} + \mu e'' \right] \right\rangle$$

where the contribution stemming from the Einstein action is

$$\mu\kappa^2 I_E = 2\mu \left\langle n e'' - \frac{1}{4e} N'^2 \right\rangle. \quad (8b)$$

Written in the form (8a), it is immediate to understand how the system works. n is the Lagrange multiplier associated with the ordinary (third-order) differential constraint

$$C \equiv \frac{1}{2} N''' - 2 \left(\frac{e''}{e} \right) N' - \left(\frac{e''}{e} \right)' N + \left(\frac{e''}{e} \right)_{,1} + \mu e'' = 0, \quad (9)$$

whose origin can be traced back to the holding of the Bianchi identity linking the field equations due to the off-shell Poincaré gauge invariance (2).

This ordinary differential constraint is very similar to what happens in the 4D case⁷⁾, the qualitative difference with this case being the presence of a third-order derivative (and of course of the mass term). It has to be thought as providing the value of $N = N(e)$ in order to obtain, after insertion of this value into the action, the unconstrained action in terms of the unique, massive spin 2, dynamical variable which must appear. If one intends that N is a short notation for $N(e)$, the unconstrained action turns out to be:

$$\frac{1}{2} \mu\kappa^2 I_\mu = \left\langle \frac{e''}{e} \dot{N}(e) \right\rangle - \mu \left\langle \frac{1}{4e} N'^2 \right\rangle, \quad (10)$$

where the last term is the light-front generator of the evolution of the system along u . Notice that in a similar way as happens for the 4D massless case, it is an explicitly non-negative quantity [due to the light front character of the co-ordinate u , $\langle (4e)^{-1} N'^2 dv dx \rangle$ is a non-conserved quantity whose intrinsic meaning, in the 4D case, has been related to one of the Newman-Penrose variables⁸⁾].

It is worth analyzing what is the role played by the 3D Einstein action (8b). This action does not contain any dynamics. There is no time derivative in it that can drive the evolution of the variables it has. The constraint $e'' = 0$ leads to solutions which have a vanishing Ricci tensor; then, in 3D, the Riemann tensor vanishes too, i.e., they are flat spaces. On the other hand, the

Einstein action contains the full kinetic energy term of the whole system, as it can be read from (8b).

The role of the Chern-Simons action is not that of providing a mass to the system but rather of allowing the system to evolve dynamically along the light-front time u . This is the role played by the higher derivative part of the standard bosonic relativistic theories.

On the other hand, the Einstein's action provides the whole kinetic energy of the system. It is this term which really gives the energy to the system. This is in full agreement with the observation made in Ref. 2) that, when the conserved energy P^0 is evaluated as a 2D ($t=\text{const.}$) integral, the unique non-vanishing contribution arises from G_L^{00} , the linear part of the Einstein tensor. The contribution of C_L^{00} due to the CS action, which in principle is present, actually vanishes.

One can also realize by looking at Eq. (10) that the relative signs of I_E and I_{CS} have been taken in the proper way. Would the sign in front of I_{CS} in (1a) have been the opposite, the system would have had the ghost structure corresponding to the presence of a negative energy. This is in contrast with the vector case, where different signs in front of I_{CS} correspond both to physical meaningful theories, carrying an opposite pseudospin.

Having clarified the dynamical interplay of the different field variables which define the system, one might be interested in looking for non-trivial exact solutions. The search can be eased by requiring additional symmetries and by establishing some dynamical principle. Recalling the 4D results⁷⁾ concerning the existence of a wide family of exact solutions which have a vanishing light-front energy (which includes the Robinson-Trautman family) we propose that in this case too, one might try to obtain exact solutions by imposing the vanishing of the light-front energy, i.e., $N' = 0$.

In addition, we restrict our search to solutions not depending upon the spacelike co-ordinate $x \sim N, {}_1 = 0 = n, {}_1 = e, {}_1$. It can be seen that indeed,

$$e = \frac{2N_0}{c(u) - \mu v} ; \quad \eta = \frac{\dot{c}}{\mu} ; \quad N = N_0 \neq 0, \quad (11)$$

constitute an exact solution of the field equations having vanishing light-front energy. The associated metric can easily be seen which has the form:

$$ds^2 = -2du dv' + \frac{4}{[2c(u) - \mu v' + \mu x]^2} dx^2, \quad (12)$$

where $\mu v' \equiv c(u) + \mu v + \mu x$.

It is non-trivial, in the sense that its Riemann tensor does not vanish. Its physical interpretation is not clear at the moment. It seems to be of a wave type, since, if one relies in 4D similarities, the Bondi mass is given by integrating out a quantity proportional to n'' and in this case $n'' = 0$, there would not be a Bondi mass. Since in the process of obtaining this solution we have set to zero two integration constants, and for the sake of simplicity we have restricted the search to solutions not depending upon the spacelike co-ordinate x , one perhaps should have to relax some of these restrictions and look for the corresponding solutions in such more technically involved situations.

In conclusion, we have seen how the choice of a natural dreibein-associated-light-front gauge leads to a very simple form for the unconstrained action of topological massive gravity. The constraint, as usually happens in the light-front dynamics, is an ordinary - in this case third order - differential equations which, once solved, provide the value of N in terms of e , the unique spin-two propagating variable. The third-order (but first in the time derivatives) dynamical germ $\sim e^{-1}e''N$ stems in the Chern-Simons action while the light-front non-negative Hamiltonian arises from the Einstein action which actually becomes the energy [mass] generating part of the full action.

REFERENCES

- 1) M.B. Green and J.H. Schwarz - Phys.Lett. 149B (1984) 117.
- 2) S. Deser, R. Jackiw and S. Templeton - Ann.Phys. 140 (1982) 372.
- 3) S. Cecotti, S. Ferrara, L. Girardello and M. Porratti - Phys.Lett. 164B (1985) 46;
S. Cecotti, S. Ferrara, L. Girardello, M. Porratti and G. Pasquinucci - Phys.Rev. D33 (1986) 2509.
- 4) L. Baulieu and M. Bellon - Preprint LPTENS 85/32 (1985).
- 5) S. Deser and J. Kay - Phys.Lett. 120B (1983) 97.
- 6) C. Aragone and A. Khoudeir - Phys.Lett. B, to appear.
- 7) C. Aragone and A. Restuccia - Phys.Rev. D13 (1976) 207.
- 8) C. Aragone, R. Gambini and A. Restuccia - Phys.Rev. D13 (1976) 218.