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PROMPT MESON PRODUCTION AT LARGE p_T

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ABSTRACT

We study the higher twist production mechanism of all s-wave quarkonia as well as the three lowest-lying 2-gluon gluonia. After reviewing the basic notions which express the conditions under which perturbative QCD applies to prompt production, we work out all the differential cross-sections of hard processes in which, apart from the prompt meson, there are either only partons ($q, \bar{q}, g$) or partons and a single photon. After pointing out some general remarks about the properties of the higher twist mechanism which follow from these results, we give numerical evaluations of prompt production in itself and compare it with what is expected from the parton fragmentation picture alone. To this end, extensive use is made of the Lund high- p_T Monte Carlo as well as its modified versions which include the prompt production mechanism. Interesting channels are quoted, in hadron-hadron and photon-hadron scatterings, for which the higher twist signal is expected to be enhanced compared to the parton fragmentation background.

1. Introduction

Large p_T hadrons are normally expected to come, according to perturbative QCD, from the fragmentation process of partons (quarks or gluons) produced at large p_T . Computer simulations, such as the Lund Monte-Carlo ⁽¹⁾ for instance, widely use this basic scheme to reproduce with success a large number of features of the experimental data. However, an independent production mechanism can also occur where the hadron is directly produced at large p_T in the elementary reaction among partons (see fig(1)). Immediately after it has been produced, this hadron has a size considerably smaller than its standard one and the price to be paid for such an exceptional configuration is similar to that occurring in the electromagnetic form factor, e.g. an inverse power of the transverse momentum squared. This class of processes is then expected to scale as p_T^{-6} instead of p_T^{-4} as in the normal large p_T hadron production through the parton fragmentation mechanism. These kinds of process represent a correction to the standard picture of large hadron scattering which dies out as a power of the momentum transfer squared and are called higher twist processes.

Recently an experiment ⁽²⁾ has been especially designed to single out such an effect and its preliminary results ⁽³⁾ seem to answer positively the question for production of ρ^0 mesons at medium- to high- p_T . What first motivated the present work was to provide quantitative estimates to see whether the higher twist mechanism could explain this high- p_T excess. The theoretical results for inclusive pion production in hadron-hadron scattering ⁽⁴⁾, for the photoproduction of mesons ⁽⁵⁾ have already appeared in the literature. The purpose of this paper has been extended to provide the results for other types of mesons such as $\omega, \varphi, \eta, \eta', \rho, K^*$ and also for the lowest lying 2-gluons glueballs. The last case is, a priori, a promising one, given the relevance of the gluon-gluon scattering process already in the standard large p_T physics for not too high values of p_T . The existing preliminary results of the WA77 experiment ⁽³⁾ already provide interesting bounds on gluonium wave function for some glueball candidates.

The comparison of results provided by the standard Lund high- p_T Monte Carlo ⁽¹⁾ -which deals with the fragmentation mechanism only- with the corresponding ones coming from its higher twist partner ⁽⁶⁾ will give information

about the relative importance of the phenomenon we are interested in. This will allow one to single out those reactions where the higher twist mechanism is favoured.

Section 2 provides the general scheme for computing prompt meson production and examines the question of the meson wave functions. It mainly consists of a brief review on that subject centred around peculiarities needed later. In Section 3, we describe the elementary processes we shall be interested in, the set of notations used and the differential cross sections of prompt meson production. Let us notice here that we give the results for pure QCD subprocesses as well as some QCD-QED mixed ones.

In Section 5, we shall give some general properties common to all the prompt meson productions ; in particular we shall point out their deep dependence upon the helicity state.

In Section 6, using Monte Carlo estimates, we shall examine the relevance of each meson production in order to single out the "higher twist signal". This could suggest some experiments to be done to get a definitive answer to the question of the relevance of the higher twist production mechanism. Section 7 summarizes the conclusions.

2. The Higher twist mechanism

The general schema for calculating the rates of prompt resonance production has been widely discussed in the literature ^{(5) (6) (7) (8)}. Here, we shall only recall the basic steps and tools. As is often the case in perturbative QCD, the main point is the factorization of the short distance physics (e.g. production at large p_T of a parton pair) from the long distance one (e.g., the amplitude for the formation of a given meson from a parton pair). This is pictured in figure (1), where we denote by $\hat{\sigma}(k_1, k_2, k_3, p_1, p_2)$ (or in short $\hat{\sigma}(p_1, p_2)$) the short distance part of the production amplitude, and by $\Gamma_R(p_1, p_2, p)$ the long distance one. Before factorization, the full amplitude reads :

$$A \sim \int d^4 p_1 d^4 p_2 \delta^4(p_1 + p_2 - p) \left[\Gamma_{\alpha' \beta' \alpha' b'}(p_1, p_2, p) \left(\frac{1}{p_1} \right)_{\alpha' \alpha'} \left(\frac{1}{p_2} \right)_{\beta \beta'} \hat{\sigma}_{\alpha \beta \alpha' b'}(p_1, p_2) \right] \quad (2.1)$$

for the quarkonium mesons. Here summation over repeated spinor (greek) and color (latin) indices is understood. For a gluonium meson, the expression corresponding to (2.1) is :

$$A \sim \frac{1}{2!} \int d^4 p_1 d^4 p_2 \delta^4(p_1 + p_2 - p) \left[\Gamma_{\mu\nu, \alpha'\beta'}(p_1, p_2, p) \frac{R_{\mu\mu'}(p_1) \delta_{\alpha\alpha'}}{p_1^2} \frac{R_{\nu\nu'}(p_2) \delta_{\beta\beta'}}{p_2^2} \hat{\sigma}_{\mu\nu, \alpha\beta}(p_1, p_2) \right], \quad (2.2)$$

using the same summation convention as above, except that now greek letters denote Lorentz indices. The $1/2!$ which appears in rel (2.2) - and not in rel (2.1) - is here to avoid double counting when integrating over gluon 4-momenta into the whole phase space. Working in the axial gauge, the projector part of the gluon propagator is :

$$R_{\mu\nu}(p) = -g_{\mu\nu} + \frac{p_\mu \eta_\nu + p_\nu \eta_\mu}{p \cdot \eta}, \quad (\eta^2 = 0) \quad (2.3)$$

The factorization procedure involves some steps, the simplest one being the treatment of color. Indeed, given that the produced meson is colorless, one can single out the color singlet component of the color propagators $\delta_{\alpha\alpha'} \delta_{\beta\beta'}$ and drop out all other components (octet, decuplet...). This can be done by means of a Fierz-like expansion which provides :

$$\delta_{\alpha\alpha'} \delta_{\beta\beta'} \implies \left(\frac{1}{\sqrt{3}} \right)^2 \delta_{\alpha\beta} \delta_{\alpha'\beta'}, \quad \alpha, \beta, \alpha', \beta' = 1, 2, 3 \quad (2.4)$$

for the (normalized) color propagator of a quarkonium, while :

$$\delta_{\alpha\alpha'} \delta_{\beta\beta'} \implies \left(\frac{1}{\sqrt{8}} \right)^2 \delta_{\alpha\beta} \delta_{\alpha'\beta'}, \quad \alpha', \beta', \alpha, \beta = 1, \dots, 8, \quad (2.5)$$

for the propagator of a 2-gluon valence gluonium.

To abbreviate we shall redefine, from now on, the short distance amplitude as :

$$\hat{\sigma}_{\alpha\beta}(p_1, p_2) \equiv \hat{\sigma}_{\alpha\beta, \alpha\beta}(p_1, p_2) \frac{1}{\sqrt{3}} \delta_{\alpha\beta} \quad (2.6.1)$$

resp :

$$\hat{\sigma}_{\mu\nu}(p_1, p_2) \equiv \hat{\sigma}_{\mu\nu, \alpha\beta}(p_1, p_2) \frac{1}{\sqrt{8}} \delta_{\alpha\beta} \quad (2.6.2)$$

for quarkonium, resp. gluonium.

2.1. Factorization of the momentum integral

The leading behaviour of the amplitude is obtained by making the "collinear approximation" for the short distance part of the amplitude. This amounts to setting :

$$\hat{G}(p_1, p_2) = \hat{G}(x_1 p, x_2 p) \quad (2.7.1)$$

with

$$p_1 \simeq x_1 p, \quad p_2 \simeq x_2 p, \quad x_1 + x_2 = 1 \quad (2.7.2)$$

i.e. one neglects all transverse momentum and off-shell effects inside the produced parton pair. The meson is assumed massless ; this supposes $M_R^2/4p_T^2 \ll 1$ a condition admittedly not always realized in practice^(*).

2.1.1. The quarkonium meson case

This case has been already examined in the literature^{(4) (5) (7)}. However we recall here a few details which will be useful for comparison with the gluonium case.

The two propagators in eq (2.1) can be approximated by

$$\begin{aligned} \left(\frac{1}{\not{p}_1}\right)_{\alpha'\alpha} \left(\frac{1}{\not{p}_2}\right)_{\beta\beta'} &= \frac{\not{p}_1}_{p_1^2} \frac{\not{p}_2}_{p_2^2} \\ &\simeq - \sum_{s, s'} \frac{u_{\alpha'}(p_1, s) \bar{u}_{\alpha}(p_1, s) v_{\beta}(p_2, s') \bar{v}_{\beta'}(p_2, s')}{p_1^2 p_2^2}, \end{aligned} \quad (2.8)$$

the parton pair being assumed to be nearly on shell ; here, we have used the expression of energy-momentum projectors for massless fermions (see ref.9). The factorization procedure transforms eq (2.8) into :

$$\left(\frac{1}{\not{p}_1}\right)_{\alpha'\alpha} \left(\frac{1}{\not{p}_2}\right)_{\beta\beta'} \simeq - \frac{u_{\alpha'}(p_1, s) \bar{v}_{\beta'}(p_2, s')}{p_1^2 p_2^2} \delta_{s s_1} \delta_{s' s_2} \bar{u}_{\alpha}(x_1 p, s_1) v_{\beta}(x_2 p, s_2) \quad (2.9)$$

(*) For consistency, one assumes also the quark masses to be zero. For the u, d and s quarks, this approximation seems reasonable. This assumption has already been done to write down rel.(2.1).

where the summation over repeated helicity indices s, s', s_1, s_2 must be understood. As a last step, one has to rearrange further expression (2.9) in such a way that the spin-parity and helicity coupling of the $q\bar{q}$ pair become explicit. This is done by means of the Fierz expansion :

$$\delta_{ss_1} \delta_{s's_2} = \frac{1}{2} \sum_{p=0}^3 \sigma_{ss'}^p \sigma_{s_2s_1}^p, \quad (2.10)$$

where σ_0 is the 2x2 unit matrix and the σ^k ($k = 1, 2, 3$) coincide with the usual Pauli matrices. We can then rewrite rel (2.9) under the form :

$$\left(\frac{1}{p_1}\right)_{\alpha'\alpha} \left(\frac{1}{p_2}\right)_{\beta\beta'} = -\frac{1}{2} \sum_{p=0}^3 \left\{ \frac{u_{\alpha'}(p_1, s_1) \bar{u}_{\beta'}(p_2, s_2) \sigma_{s_2s_1}^p}{p_1^2 p_2^2} \right\} \left\{ \bar{u}_{\alpha}(x_1, p, s) v_{\beta}(x_2, p, s') \sigma_{ss'}^p \right\} \quad (2.11)$$

which exhibits the independent helicity couplings :

$$P_{\alpha\beta}^p = \frac{1}{\sqrt{2}} \bar{u}_{\alpha}(x_1, p, s) v_{\beta}(x_2, p, s') \sigma_{ss'}^p, \quad p = 0, 1, 2, 3, \quad (2.12)$$

among the quark and antiquark produced in the short distance process.

Using the usual spinors ⁽⁹⁾ for zero mass fermions, expressed in the helicity frame (the z axis taken along $\vec{p}$), these expressions give ^(*) :

$$\begin{cases} P_{\alpha\beta}(0_A) = \frac{1}{\sqrt{2}} \gamma^5 \not{p} \\ P_{\alpha\beta}(0_S) = \frac{1}{\sqrt{2}} \not{p} \\ P_{\alpha\beta}^i(1_S) = \frac{1}{\sqrt{2}} \not{p}^i \not{p}, \quad i = 1, 2 \end{cases} \quad (2.13.1)$$

(*) a factor $\sqrt{x_1 x_2}$ which should appear in expression (2.13) is dropped from there and will be included in the definition of the probability amplitude (see rel(2.19) below).

which is the usual set of helicity wave functions^{(5) (*)} of quarkonia. Here O_A , O_S stand for helicity zero antisymmetric and symmetric states, while 1_S stands for the helicity ± 1 state. These wave functions are usually identified with J^P non-zero mass quarkonia wave functions :

$$P(O_A) = \mathbb{P}(J^P = 0^-), P(O_S) = \mathbb{P}(J^P = 1^-, \text{hel} = 0), P(1_S) = \mathbb{P}(J^P = 1^-, \text{hel} = \pm 1). \quad (2.13.2)$$

It must be noticed that, when using expression (2.10), what naturally appears are the linear polarization vectors, rather than the circular ones. These linear polarization vectors satisfy :

$$S_i \cdot S_j = -\delta_{ij}, \quad S_i \cdot p = 0, \quad i, j = 1, 2 \quad (2.14)$$

and we can choose as closure relation :

$$\sum_{i=1}^2 S_i^\mu S_i^\nu = -g^{\mu\nu} + \frac{p^\mu \eta^\nu + p^\nu \eta^\mu}{p \cdot \eta} \quad (2.15)$$

where η is any arbitrary lightlike vector, not parallel to p . Because $p^2=0$, the $p^\mu \eta^\nu$ terms in (2.15) do not play any role and in practical computations only the $g^{\mu\nu}$ term will survive.

The term in rel(2.11) :

$$\frac{1}{\sqrt{2}} \frac{u_{\alpha'}(p_1, s_1)}{p_1^2} \frac{\bar{v}_{\beta'}(p_2, s_2)}{p_2^2} \sigma_{s_2 s_1}^p \quad (2.16)$$

when acting on :

$$\Gamma_{\alpha' \beta', a' b'} \sim \langle R | \bar{\Psi}_{\alpha', a'} \Psi_{\beta', b'} | 0 \rangle \quad (2.17)$$

will select the term which has the quantum numbers of the quarkonium meson R (pseudo scalar, or helicity zero vector, or transversely polarized vector meson).

Gathering all these results in rel(2.1), we get the full amplitude factorized :

$$A_{J^P, \text{hel}} \sim \int_0^1 dx_1 dx_2 \delta(1-x_1-x_2) \phi_{J^P, \text{hel}}(x_1, x_2) \hat{\sigma}(x_1 p, x_2 p)_{\alpha\beta} \mathbb{P}(J^P, \text{hel})_{\beta\alpha} \quad (2.18)$$

(*) There is a misprint in the last formula (11) of ref(5) : their $\mathcal{E}_\pm$ must be understood "slashed".

where :

$$\phi_{J^P, \text{ReP}}(x_1, x_2) = \int \frac{d^2 p_1 d^2 p_2}{2 x_1} \frac{\sqrt{x_1 x_2}}{P^2 (P-p_1)^2} u_{\alpha'}(p_1, s_1) \bar{v}_{\beta'}(p_2, s_2) \frac{\sigma^P}{\sqrt{2}} s_2 s_1 \quad (2.19.1)$$

$$* \Gamma_{\alpha' \beta', \alpha' b'}(p_1, P-p_1, P) \frac{1}{\sqrt{3}} \delta_{\alpha' b'}$$

is the long distance part of the amplitude, generally named in the literature probability amplitude⁽⁷⁾ of recombination of the quark pair into a meson. To get rel(2.19.1), we have used :

$$d^4 p_1 = \frac{1}{2} d^2 p_1 d^2 p_2 \frac{dx_1}{x_1} . \quad (2.19.2)$$

2.1.2. The gluonium case

We shall be interested in the case of gluonium made of 2 valence gluons only (not three). The factorization procedure closely follows the case of quarkonium, with some differences however. After the step represented by the approximation (2.7.1.), the next one is to factorize both gluon propagators in rel(2.2). To this purpose, we recall that, the gluon pair being assumed nearly on shell, we have :

$$\sum_{\lambda=1}^2 \varepsilon_i^\lambda(p_j) \varepsilon_i^{\lambda'}(p_j) \simeq R^{\lambda\lambda'}(p_j) \quad , \quad j = 1, 2 \quad , \quad (2.20)$$

where R here is given by rel(2.3). As already mentioned, we prefer to work with (real) linear polarization vectors rather than circular ones. The factorization hypothesis consists of making the replacement :

$$\frac{R^{\mu\mu}(p_1)}{p_1^2} \frac{R^{\nu\nu}(p_2)}{p_2^2} \implies \varepsilon_i^\mu(p) \varepsilon_j^\nu(p) \frac{\varepsilon_i^{\mu'}(p_1)}{p_1^2} \frac{\varepsilon_j^{\nu'}(p_2)}{p_2^2} \quad (2.21)$$

(implicit summation over repeated polarization indices $i, j=1, 2$ is always understood). To write down rel(2.21), we have used the property that $\varepsilon_i^\mu(\lambda p) = \varepsilon_i^\mu(p)$ for $\lambda > 0$ and we have chosen the same polarization frame for both collinear gluons. To complete the factorization procedure on rel(2.21), we must further expand this expression in helicity couplings. This can again be done by means of rel(2.10), to get :

$$\frac{R^{\mu\mu'}(p_1)}{p_1^2} \frac{R^{\nu\nu'}(p_2)}{p_2^2} \Rightarrow \frac{1}{2} \sum_{\rho=0}^3 \left[\varepsilon_i^\mu(p) \varepsilon_j^\nu(p) \sigma_{ij}^\rho \right] \left[\frac{\varepsilon_{i'}^{\mu'}(p_1)}{p_1^2} \frac{\varepsilon_{j'}^{\nu'}(p_2)}{p_2^2} \sigma_{j'\nu'}^\rho \right] \quad (2.22)$$

Each term in this expansion clearly corresponds to different helicity couplings, namely :

- i) $\sigma_{ij}^0 \equiv \delta_{ij}$ corresponds to a scalar symmetric coupling
- ii) σ_{ij}^2 corresponds to a scalar antisymmetric coupling
- iii) $\sigma_{ij}^1, \sigma_{ij}^3$ corresponds to two independent symmetric and traceless couplings.

The connection can be made between these couplings and the already known couplings of two gluons to gluonia^{(10) (11)}

$$\begin{cases} T_{\mu\nu,ab} \sim \langle R | G_{\mu\rho}^a G_{\nu\rho}^b | 0 \rangle \\ T_{\mu\nu,ab} \sim \langle R | G_{\mu\rho}^a \tilde{G}_{\nu\rho}^b | 0 \rangle \\ T_{\mu\nu,ab} \sim \langle R | -G_{\mu\rho}^a G_{\nu\rho}^b + \frac{1}{4} G_{\mu\rho}^a G_{\nu\rho}^b g_{\mu\nu} | 0 \rangle \end{cases} \quad (2.23)$$

which are respectively the symmetric (O_S) and the antisymmetric (O_A) helicity zero couplings and helicity ± 2 of the symmetric helicity 2 coupling (2_S). As is well known⁽¹⁰⁾, there is no helicity ± 1 coupling of two gluons to gluonium, at least in the collinear approximation. Therefore each coupling (2.23) selects one term in (2.22) : the first and second coupling in (2.23) will make only the σ^0 and σ^2 couplings respectively survive, whereas the last coupling (2.23) will give rise to helicity ± 2 coupling. In fact, σ^1 and σ^3 couplings correspond to linear polarization states rather than helicity states^(*).

By introducing all these results in rel(2.2), we obtain :

$$A_\rho \sim \frac{1}{2!} \int_0^1 dx_1 dx_2 \delta(1-x_1-x_2) \phi^\rho(x_1, x_2) \left[\hat{\sigma}_{\mu\nu}(x_1 p, x_2 p) \sum_{i,j=1}^2 \varepsilon_i^\mu(p) \varepsilon_j^\nu(p) \sigma_{ij}^\rho \frac{1}{\sqrt{2}} \right] \quad (2.24)$$

(*) There is here a difference with respect to the quarkonium case : there is an exchange in the role of σ^2 and σ^3 with respect to helicity coupling.

with

$$\phi^p(x_1, x_2) = \int \frac{d^2 p_1 d^2 p_2}{2 x_1} \frac{\varepsilon_{i'}^{\mu'}(p_1)}{p_1^2} \frac{\varepsilon_{j'}^{\nu'}(p-p_1)}{(p-p_1)^2} \frac{\sigma_{ij'}^p}{\sqrt{2}} \Gamma_{\mu'\nu'; a'b'}(p_1, p-p_1, p) \frac{1}{\sqrt{2}} \delta_{a'b'} \quad (2.25)$$

for the probability amplitude to recombine both gluons into a gluonium.

2.1.3. Helicity wave function of the gluonia

The expression

$$\mathbb{P}^p = \frac{1}{\sqrt{2}} \varepsilon_i^\mu(p) \varepsilon_j^\nu(p) \sigma_{ij}^p, \quad p=0, \dots, 3 \quad (2.26)$$

for the gluonium wave functions is not easy to handle in covariant calculations of amplitudes. When computing the "short distance cross sections" :

$$X^p = \left| \hat{\sigma}_{\mu\nu}(x_1 p, x_2 p) \varepsilon_i^\mu(p) \varepsilon_j^\nu(p) \frac{1}{\sqrt{2}} \sigma_{ij}^p \right|^2 \quad (2.27.1)$$

we have :

$$X^p = \frac{1}{2} \hat{\sigma}_{\mu\nu}(x_1 p, x_2 p) \hat{\sigma}_{\lambda\sigma}^*(x_1 p, x_2 p) \varepsilon_i^\mu(p) \varepsilon_j^\nu(p) \varepsilon_{i'}^{\lambda'}(p) \varepsilon_{j'}^{\sigma'}(p) \sigma_{ij}^p \sigma_{i'j'}^{p'}, \quad (2.27.2)$$

which can be written :

$$X^p = \hat{\sigma}_{\mu\nu}(x_1 p, x_2 p) \hat{\sigma}_{\lambda\sigma}^*(x_1 p, x_2 p) K_p^{\mu\nu\lambda\sigma}. \quad (2.28)$$

With the help of rel(2.3), one can prove that :

$$\left\{ \begin{array}{l} K_0^{\mu\nu\lambda\sigma} \equiv K_{0s}^{\mu\nu\lambda\sigma} = \frac{1}{2} R^{\mu\nu}(p) R^{\lambda\sigma}(p) \\ K_2^{\mu\nu\lambda\sigma} \equiv K_{0a}^{\mu\nu\lambda\sigma} = \frac{1}{2} [R^{\mu\lambda}(p) R^{\nu\sigma}(p) - R^{\mu\sigma}(p) R^{\nu\lambda}(p)] \\ K_1^{\mu\nu\lambda\sigma} + K_3^{\mu\nu\lambda\sigma} \equiv K_{2s}^{\mu\nu\lambda\sigma} = \frac{1}{2} [R^{\mu\lambda}(p) R^{\nu\sigma}(p) + R^{\mu\sigma}(p) R^{\nu\lambda}(p) - R^{\mu\nu}(p) R^{\lambda\sigma}(p)] \end{array} \right. \quad (2.29)$$

which are easy to use in any calculation.

We can find an alternative set of wave functions (compared to (2.26)) which provides the same operators $K^{\mu\nu\lambda\sigma}$ as those in rels (2.29). To get it, one must expand the tensor product $\varepsilon_i^\mu(p)\varepsilon_j^\nu(p) \equiv \varepsilon_i^\mu \varepsilon_j^\nu$ into definite quantum numbers couplings. One has the following identity :

$$\begin{aligned} \varepsilon_i^\mu \varepsilon_j^\nu = & -\frac{R^{\mu\nu}(p)}{2} \varepsilon_i \cdot \varepsilon_j + \frac{1}{2} [\varepsilon_i^\mu \varepsilon_j^\nu - \varepsilon_i^\nu \varepsilon_j^\mu] \\ & + \frac{1}{2} [\varepsilon_i^\mu \varepsilon_j^\nu + \varepsilon_i^\nu \varepsilon_j^\mu + R^{\mu\nu}(p) \varepsilon_i \cdot \varepsilon_j] \end{aligned} \quad (2.30)$$

which is an extension to polarization vectors of the well-known expansion of tensor product of two 3-vectors into irreducible tensorial sets of definite rank⁽¹²⁾. The first term is helicity zero symmetric gluonium state (0_S) corresponding to the helicity 0 state of both the 0^{++} and 2^{++} gluonium) ; the second term corresponds to the single antisymmetric helicity state of the 0^{-+} gluonium (state 0_A), whereas the third gathers the helicity states +2 and -2 of the 2^{++} gluonium. This corresponds to what is expected⁽¹³⁾⁽¹⁴⁾.

This expansion thus defines the following set of gluonium wave functions :

$$\begin{cases} Q_{ij}^{\mu\nu}(0_S) = -\frac{1}{2} R^{\mu\nu}(p) \varepsilon_i \cdot \varepsilon_j \\ Q_{ij}^{\mu\nu}(0_A) = \frac{1}{2} [\varepsilon_i^\mu \varepsilon_j^\nu - \varepsilon_i^\nu \varepsilon_j^\mu] \\ Q_{ij}^{\mu\nu}(2_S) = \frac{1}{2} [\varepsilon_i^\mu \varepsilon_j^\nu + \varepsilon_i^\nu \varepsilon_j^\mu + R^{\mu\nu}(p) \varepsilon_i \cdot \varepsilon_j] \end{cases} \quad (2.31)$$

which can be compared with the corresponding set in ref(13).

As a matter of fact, there is a simple connection between formulae (2.31) and (2.26) ; one can indeed check that :

$$\frac{1}{\sqrt{2}} Q_{ij}^{\mu\nu}(0_S) \sigma_{ij}^0 = \mathbb{P}^0, \quad \frac{1}{\sqrt{2}} Q_{ij}^{\mu\nu}(0_A) \sigma_{ij}^2 = \mathbb{P}^2, \quad (2.32)$$

$$\frac{1}{\sqrt{2}} Q_{ij}^{\mu\nu}(2_S) \sigma_{ij}^k = \mathbb{P}^k, \quad k=1,2, \quad ,$$

while all the other possible contractions give zero.

When computing cross sections, we have :

$$X(hel) = \hat{\sigma}_{\mu\nu}(x_1 p, x_2 p) \hat{\sigma}_{\lambda\sigma}^*(x_1 p, x_2 p) Q_{ij}^{\mu\nu}(hel) Q_{ij}^{\lambda\sigma}(hel) . \quad (2.33)$$

By using rel(2.3), one can further check that the contracted products $Q_{ij}^{\mu\nu}(hel.) Q_{ij}^{\lambda\sigma}(hel.)$ are exactly the $K^{\mu\nu\lambda\sigma}$ operators as already defined by eqs(2.29). This finally proves the complete equivalence of the helicity wave function sets of eqs (2.26) and (2.31).

2.1.4 Summary

The full amplitude for each helicity coupling has been written, in both the quarkonium and the gluonium cases, as the product of the production amplitude of a parton pair with definite quantum numbers (short distance amplitude) by a recombination amplitude of both partons into a meson having the same set of quantum numbers (long distance amplitude).

The choice of the quantum numbers of the parton pair in the short distance amplitude amounts to replacing, in the Feynman amplitudes, the $\bar{u}_\alpha(x_1 p, s) v_\beta(x_2 p, s')$ by one of the operators (2.13), or the $\varepsilon_i^\mu(p) \varepsilon_j^\nu(p)$ by one of the operators (2.31), depending on which meson we are dealing with. With this at hand, the use of the Feynman rules^{(9) (15)} allows the computation of any short distance amplitude at any order in perturbative QCD.

2.2. The probability amplitudes $\phi(x_1, x_2)$

The function $\phi(x_1, x_2)$ contains all the information on the long distance part of the amplitude (see rels(2.19) and (2.25)). This function $\phi(x_1, x_2; Q^2)$ is the probability amplitude for finding a valence parton pair (q, $\bar{q}$, g) carrying fractions (x_1, x_2) of the meson longitudinal 4-momentum at distances of order $1/Q^2$ with :

$$p_1 \simeq x_1 p \quad , \quad p_2 \simeq x_2 p \quad , \quad x_1 + x_2 = 1 ; \quad (2.34)$$

sea parton contributions are neglected.

Up to now, all the methods which provide $\phi(x_1, x_2; Q^2)$ follow from the expansion of this function in terms of Gegenbauer polynomials as first proved by Brodsky and Lepage⁽⁷⁾ for the quarkonium case and extended later to various kinds of 2-gluon gluonia⁽¹¹⁾⁽¹³⁾⁽¹⁶⁾⁽¹⁷⁾.

In practice, one has to truncate the series to a few terms (generally the first one) and the question arises how to take properly into account the effects of the neglected part of the series.

Indeed, for the quarkonium mesons ($J^P = 0^-, 1^-$), the Brodsky-Lepage results predicts a behaviour^(*) :

$$\phi(x_1, x_2) \sim (x_1, x_2) \delta(1-x_1-x_2) , \quad (2.35)$$

up to a smooth $\log Q^2/\Lambda^2$ dependence for helicity states ± 1 . This however leads to pessimistic values of α_s ($\simeq 1$) when evaluating the pion electromagnetic form factor. The way suggested by Brodsky and Lepage to circumvent this disease consists in replacing eq(2.35) by :

$$\phi(x_1, x_2) \sim (x_1, x_2)^{1/4} \delta(1-x_1-x_2) , \quad (2.36)$$

which gives the expected electromagnetic form factor with more reasonable values of α_s ($\simeq 0.3$). Such a modification can be considered as an empirical way to account for the neglected part of the Gegenbauer series of $\phi(x_1, x_2; Q^2)$.

Another approach to the same question has been initiated by Chernyak et al⁽¹⁸⁾⁽¹⁹⁾. It is based on the technique of QCD sum rules and gives reliable evaluations of the first few moments :

$$\langle (x_1 - x_2)^n \rangle \equiv \int_{-1}^{+1} d(x_1 - x_2) \phi_M(x_1, x_2; Q^2) (x_1 - x_2)^n , \quad (2.37)$$

* From now on, we shall include the $\delta(1-x_1-x_2)$ in rels (2.20) and (2.26) into the definition of $\phi(x_1, x_2; Q^2)$.

($n = 0, \dots, 4$) ; for any meson M, the function $\phi_M(x_1, x_2; q^2)$ is assumed to be of the form

$$\phi_M(x_1, x_2) \sim x_1 x_2 [4A(x_1 - 1/2)(x_2 - 1/2) - B] \delta(1 - x_1 - x_2), \quad (2.38)$$

and the parameters A and B are suitably optimized for each meson. The resulting functions seem to agree with the available experimental data⁽¹⁸⁾⁽¹⁹⁾ ; for instance, the pion electromagnetic form factor is well reproduced with α_S ranging in (0.2, 0.3) as expected.

2.3. Symmetry properties of $\phi(x_1, x_2)$

Fortunately, in order to work out completely all higher twist production cross sections of s-wave mesons (quarkonia or gluonia), one does not need the detailed structure of the probability amplitudes $\phi(x_1, x_2; q^2)$. It is sufficient to assume symmetry properties :

$$\phi(x_1, x_2; q^2) = \phi(x_2, x_1; q^2) \quad (2.39)$$

or

$$\phi(x_1, x_2; q^2) = -\phi(x_2, x_1; q^2) \quad (2.40)$$

From the Brodsky-Lepage analysis, property (2.39) is found to hold for quarkonium s-wave mesons. For 2-gluon gluonium mesons, property (2.39) again holds for the states denoted 0_S and 2_S which cover the case of 0^{++} and 2^{++} low lying states of the gluonia⁽²⁰⁾, whereas the probability amplitude of the 0^{-+} gluonium⁽¹¹⁾⁽¹³⁾⁽¹⁶⁾⁽¹⁷⁾ (state already named 0_A) has the symmetry (2.40). Let us notice that these properties for gluonia follow from the projectors (2.31), if one wants to preserve Bose statistics in the interchange of both gluons.

For practical purposes, we shall factor out the normalization of the probability amplitude⁽¹⁸⁾⁽¹⁹⁾ :

$$\phi(x_1, x_2; q^2) = \alpha_0 \tilde{\phi}(x_1, x_2; q^2) \quad (2.41)$$

with :

$$a_0 = \sqrt{3} f_{ps} \quad \text{or} \quad a_0 = \sqrt{6} f_v \quad (2.42.1)$$

for, respectively, pseudoscalar and vector quarkonium mesons ,
($f_{ps} \sim 0.093$ GeV, $f_v \sim 0.107$ GeV). We shall follow the same convention for the gluonia :

$$a_0 = f_g , \quad (2.42.2)$$

without extracting here any additional numerical factor. By doing this, the normalization of the reduced probability amplitudes $\tilde{\phi}(x_1, x_2)$ becomes :

$$3 \int_{-1}^{+1} d(x_1 - x_2) C_0^{3/2}(x_1 - x_2) \tilde{\phi}(x_1, x_2; Q^2) = 1, \quad (2.43.1)$$

for s-wave quarkonia and for the $J^{PC} = 0^{++}, 2^{++}$ gluonia, and

$$\frac{21}{5} \int_{-1}^{+1} d(x_1 - x_2) C_1^{5/2}(x_1 - x_2) \tilde{\phi}(x_1, x_2; Q^2) = 1, \quad (2.43.2)$$

for the 0^{-+} gluonium. Let us here recall that the value of f_g is largely arbitrary and will remain such until one will identify gluonium states among the known mesons.

The reduced probability amplitudes, normalized consistently with rel(2.43) are :

$$\tilde{\phi}(x_1, x_2) = \frac{1}{6 B(\beta+1, \beta+1)} (x_1 x_2)^\beta \delta(1-x_1-x_2) \quad (2.44)$$

when dealing with functions "à la Brodsky-Lepage" ; B is the usual Euler Beta function and expression (2.44) leads to convergent results for any $\beta > 0$. The alternative class of functions proceeding from modifications "à la Chernyak et al", normalized consistently with rel(2.43) are :

$$\tilde{\phi}(x_1, x_2) = \frac{-5}{A+5B} (x_1 x_2) \left[4A(x_1 - 1/2)(x_2 - 1/2) - B \right] \times \delta(1-x_1-x_2) . \quad (2.45)$$

These expressions are in principle valid for either s-wave quarkonia or the 0^{++} and 2^{++} gluonia. For the 0^{-+} gluonium, one assumes a behaviour like⁽¹⁶⁾:

$$\tilde{\phi}(x_1, x_2) \sim (x_1 x_2)^b (x_1 - x_2) \delta(1-x_1-x_2), \quad (2.46)$$

which leads to finite results only for finite $b > 0$. From a practical point of view, neglecting all masses, the expressions like (2.45) are equivalent for some fixed β to an expression (2.44). The reason is that all higher twist production amplitudes only depend on $\tilde{\phi}$ through moments like :

$$c(\gamma_1, \gamma_2) = \int_{-1}^{+1} dx_1 dx_2 x_1^{\gamma_1} x_2^{\gamma_2} \tilde{\phi}(x_1, x_2), \quad (2.47)$$

with a limited range of values of (γ_1, γ_2) :

$$\begin{aligned} & (0,0), (1,0), (0,1), (-1,-1), \\ & (-1,0), (0,-1), (-1,1), (1,-1). \end{aligned} \quad (2.48)$$

It has already been pointed out⁽¹⁵⁾ that all these moments are algebraically related among themselves. Namely, if one chooses as fundamental parameters :

$$c_0 \equiv c(0,0), \quad c_1 \equiv c(-1,0), \quad c_2 \equiv c(1,0), \quad (2.49)$$

for quarkonium and the 0^{++} and 2^{++} gluonia, we have :

$$\begin{aligned} c(\gamma_1, \gamma_2) &= c(\gamma_2, \gamma_1), \\ c(-1,-1) &= 2c_1, \quad c(1,0) = c_0/2, \quad c(1,-1) = c_1 - c_0, \end{aligned} \quad (2.50)$$

whereas for the gluonium $0^{-+} \equiv 0_A$, we have :

$$\begin{aligned} c(\gamma_1, \gamma_2) &= -c(\gamma_2, \gamma_1), \\ c(1,-1) &= -c_1. \end{aligned} \quad (2.51)$$

The constant c_2 , which may in principle appear only in the production of the 0^{-+} gluonium, is not present in practice in the Feynman amplitudes.

We shall choose to express all our results in terms of c_0 and c_1 ^(*).

3. Higher-Twist production of $q\bar{q}$ or gg pairs with definite quantum numbers

We begin by evaluating the higher twist production cross sections of a $q\bar{q}$ or gg pair with definite quantum numbers. For most cases, namely quarkonia

(*) The authors of ref (5) prefer to use $c(-1, -1)$ instead of $c(-1,0)$ as fundamental parameter.

with $I_3 \neq 0$, these cross sections are exactly those of the "physical" s-wave mesons ; however, for some of them ($\rho^0, \pi^0, \eta, \eta', \omega, \psi$), we must make a coherent superposition of $q\bar{q}$ waves of different flavours. This requires some subtleties and this is why we postpone to the next paragraph the discussion of the direct production cross sections of all mesons.

3.1. General features and hypotheses

As already mentioned, every calculation will be done by performing the factorization procedure described in the previous paragraph. Let us further notice that, by this method, one can only reach the s-wave couplings of the $q\bar{q}$ mesons and only the lowest three multipole couplings of the gg mesons, namely the $0^{++}, 0^{-+}, 2^{++}$ gluonia.

Our predictions are only valid for directly produced mesons of transverse momentum large compared with any meson mass (typically $p_T \gtrsim 1.5$ to 2.0 GeV/c).

We work out the 2 partons $\rightarrow$ 3 partons amplitude following the usual Feynman rules ⁽⁹⁾⁽¹⁵⁾ ; the parton pair which will recombine into the (prompt) meson is first put onto its "color shell" by contracting the Feynman amplitude with the required color tensor (see rel(2.6)). The spin-parity and helicity state is afterwards chosen by replacing in the Feynman amplitude the $\bar{u}(x_1, p) v(x_2, p)$ by one of the wave functions (2.13), or the $\epsilon_i^\mu(x_1, p) \epsilon_j^\nu(x_2, p)$ by one of the wave functions (2.33). At this point, we are left with an effective 4-body amplitude, which is a function of the Mandelstam variables (written s, t, u rather than $\hat{s}, \hat{t}, \hat{u}$), which satisfy :

$$s+t+u = 0 \quad (3.1)$$

and of the fraction pair (x_1, x_2) . The result is the short distance amplitude which is multiplied by the convenient probability amplitude $\phi(x_1, x_2)$ and finally integrated over x_1 , and x_2 to get the - leading - full production amplitude. Let us here recall that only a couple of moments of $\phi(x_1, x_2)$ are needed, at most, and that any change in the function $\phi(x_1, x_2)$ will result in a change of global scale of all the cross sections.

3.2. Pure QCD basic processes

3.2.1. In this subsection, we shall only be concerned with pure QCD processes (at third order in the coupling constant) :

$$q_1 g_2 \longrightarrow q'_3 M , \quad (3.2.1)$$

$$q_1 \bar{q}'_2 \longrightarrow g_3 M , \quad (3.2.2)$$

$$g_1 g_2 \longrightarrow g_3 M , \quad (3.2.3)$$

where M stands for a $(q\bar{q})$ or (gg) white pair in a definite spin-parity and/or helicity state. Here and throughout, labels 1,2,3 refer to the 4-momenta (k_1, k_2, k_3) of the partons, while the M (meson) 4-momentum is denoted by p . The Mandelstam variables are defined by :

$$\begin{cases} s = (k_1 + k_2)^2 = (k_3 + p)^2 \\ t = (k_1 - k_3)^2 = (k_2 - p)^2 \\ u = (k_2 - k_3)^2 = (k_1 - p)^2 \end{cases} . \quad (3.3)$$

Crossing symmetry among the invariant amplitudes associated with the first two processes (3.2) will be directly always checked by performing the change :

$$s \longleftrightarrow -t , \quad u \longleftrightarrow u , \quad (3.4)$$

in the invariant amplitude squared (summed over spin and color indices). The same property for the process (3.2.3) will results in a s, t, u symmetry of its invariant amplitude squared.

3.2.2 When dealing with quarkonia, the full set of non-trivially zero (for color reasons) Feynman diagrams for the process (3.2.2) are those depicted in figure (2). We shall here distinguish the amplitude associated with the first five diagrams from that associated with the last two ; the former will be denoted by A , the later by B . The amplitudes A and B clearly have different status because, assuming exact $SU(3)$ flavour symmetry, amplitude B can only contribute to isospin zero meson production amplitude, whereas amplitude A contributes for any isospin. Remember that, from usual Feynman rules ⁽⁹⁾, there is an extra minus sign in going from amplitude A to amplitude B .

We do not provide the set of diagrams corresponding to the process (3.2.1) ; it can be deduced from figure (2) by exchanging in each of its graphs the outgoing gluon and the incoming antiquark lines (which become a quark line). Here again, we split up the set into A and B subsets with the same definition and properties as just mentioned.

The full set of non-trivially zero diagrams associated with the process (3.2.3) is given in figure (3). Assuming exact SU(3) flavour symmetry, the corresponding amplitude will only contribute to direct production of isospin zero mesons. As far as gluonia are concerned, the sets of non-trivially zero diagrams associated with the first process (3.2.1) are deductible from figure (4) by crossing two legs, as already noted.

A check of gauge invariance of each set of diagrams is performed in getting zero by replacing in the 5 body amplitude any polarization vector $\epsilon^\mu(q_i)$ by q_i^μ .

All calculations of invariant amplitudes squared have been done using REDUCE⁽²¹⁾, apart from the three processes involving gluonia (O_S , O_A , 2_S) :

$$g + g \longrightarrow g + \text{gluonium}$$

which have been processed using SCHOONSHIP⁽²²⁾ to overcome the difficulties encountered by REDUCE in dealing with too big intermediate results.

In these computations, we have always worked in the Feynman gauge. The question of the unwanted longitudinal degrees of freedom which would thus come with the gluons has been overcome using the trick recalled by Cutler and Sivers⁽¹⁵⁾ : The closure relation used for the polarization vector set of each external gluon is that of rel(2.3), however η is chosen to be the momentum of an accompanying parton of the same s-channel (see rel (3.5), (3.6), in ref (15)).

The fundamental invariant amplitudes squared which appear for processes (3.2) are :

$$\begin{aligned}
F_1(s,t,u) &= \frac{512}{27} s \frac{t^2+u^2}{t^2 u^2} \\
F_2(s,t,u) &= \frac{32}{3} \frac{t^2+u^2}{s^3} \\
F_{12}(s,t,u) &= -\frac{512}{9} \frac{t^2+u^2}{s t u} \\
F_3(s,t,u) &= \frac{4096}{27} \frac{s^3}{t^2 u^2} \\
F_4(s,t,u) &= 8 \frac{[s^2+t^2+u^2]^4}{s^3 t^3 u^3} \\
D_1(s,t,u) &= \frac{1}{9} \frac{t^2+u^2}{s^3 t^2 u^2} [9(t^2+u^2)-s^2]^2 \\
D_2(s,t,u) &= 27 \frac{[s^2+t^2+u^2]^4}{s^3 t^3 u^3}
\end{aligned} \tag{3.5}$$

The factor containing the effect of the wave functions and a $g^6 = (4\pi\alpha_s)^3$ has been grouped together with the phase space factor and the color and spin mean term in a so called "extra factor" to connect to $d\sigma/dt$. All the results for this class of processes (pure QCD) are listed in table I.

3.2.3 Remarks on table I

The results presented in table I contain factors c_1, c_0 as defined in §2 ; one may notice that they could differ from one meson to another.

The quantities f_{ps}, f_v are the pseudoscalar and vector meson decay constant. Their values are :

$$f_{ps} = 93 \text{ MeV}, \quad f_v = 107 \text{ MeV}, \tag{3.6}$$

the accompanying factors $\sqrt{3}$ or $\sqrt{6}$ (see ref (7)) are taken into account in the extra factor. The gluonium decay constant f_{0a}, f_{0s}, f_{2s} are taken without extra numerical factors (see §2 above). The factor $(1/2!)^2$ which avoids double counting when dealing with gluonia is taken into account in the expressions for D_1 and D_2 .

Finally, the interference term F_{12} between the sets A and B, is complete ($= 2 \text{ Re } (AB^*)$).

3.3. QCD-QED mixed processes

In this section, we shall focus on QCD processes involving an external photon and which are of order $g^2 e (\alpha_s^2 \alpha_{em})$ for the cross sections). They are

$$q_1 \gamma_2 \rightarrow q'_3 M \quad (3.7.1)$$

$$q_1 \bar{q}'_2 \rightarrow \gamma_3 M \quad (3.7.2)$$

$$g_1 \gamma_2 \rightarrow g_3 M \quad (3.7.3)$$

$$g_1 g_2 \rightarrow \gamma_3 M \quad (3.7.4)$$

All the conventions listed in §(3.1) and §(3.2) are still valid ; here γ denotes a photon. The invariant amplitudes squared associated with the first two processes are crossing symmetric (eq(3.4)) and likewise for the last two.

If we consider first the case of M being a quarkonium, the set of diagrams associated with the process (3.7.2) is depicted in figure (6). Because the produced quarkonia are white, there is no contribution from what was previously called amplitude B (see § 3.2). The set of diagrams associated with the process (3.7.2) can be deduced from figure (6) by crossing.

Taking now M to be a gluonium, the set of diagrams depicting the process (3.7.2) is given in figure (8) ; the set associated with process (3.7.1) can be got by crossing.

Finally, in this same case, there is no third order process $g\gamma \rightarrow gM$ or $gg \rightarrow \gamma M$ because of the lack of coupling $g-\gamma$.

The calculation of all the invariant amplitudes squared has been performed using "REDUCE" and they are all listed in Table II, together with the required extra factors to connect them with the differential cross section $d\sigma/dt$.

The fundamental invariant amplitudes squared are :

$$\begin{aligned} F'_1(s, t, u) &= \frac{128}{9} \frac{t^2+u^2}{s} \frac{[u\lambda_q - t\lambda_{q'}]^2}{t^2u^2} \\ F'_3(s, t, u) &= \frac{1024}{9} \frac{s}{t^2u^2} [u\lambda_q - t\lambda_{q'}]^2 \\ D'_1(s, t, u) &= \frac{64}{3} s \frac{t^2+u^2}{t^2u^2} \lambda_q^2 \end{aligned} \quad (3.8)$$

Here again the factor coming from the meson wave function, together with $g^4 e^2 = (4\pi\alpha_s)^2 (4\pi\alpha_{em})$ has been grouped with the spin and color mean factor and the phase space term into the so called extra factor which connects to $d\sigma/dt$. As previously, the factor $(1/2!)^2$ is included in the expression for D'_1 .

Let us point out that λ_q and $\lambda_{q'}$, in all processes, are the quark charges and not that of antiquarks.

The gauge chosen to work out these amplitudes is here again the Feynman gauge and the problem of ghosts (longitudinal components of the gluons) has been overcome using the trick of ref (15) op. cit.

4. From the invariant amplitudes to the cross section estimates

With the results for processes (3.2) (see table I) and (3.7) (see table II), we can write down the differential cross sections of prompt production of s-wave quarkonium mesons as well as that of the lowest lying glueballs of helicity 0_A , 0_S , 2_S . As already pointed out, what is called here 0_A may be identified with the usual gluonium 0^{++} , while the other helicity states concern both the 0^{++} and 2^{++} gluonia. Let us notice that no other coupling of a parton pair can be reached using collinear partons.

4.1 Quarkonium mesons

For $I = 1/2$ as well as $I=1$ charged 0^- and 1^- mesons, the cross sections $d\sigma/dt$ of prompt meson production simply follow by multiplying the invariant amplitude squared by the corresponding "extra factor". One has to take into account a flavour constraint (see table III-VIII) which forbids or allows a process depending on the quark content of the produced meson and the interacting q and $\bar{q}$.

These constraints are expressed by a trivial factor $U_M(q, q')$:

$$U_M(q, q') = \delta_{qu}\delta_{q'd}, \delta_{qd}\delta_{q'u}, \frac{1}{2} \delta_{qq'}(\delta_{qu} + \delta_{qd}), \quad (4.1)$$

for the $\pi^+(\rho^+), \pi^-(\rho^-), \pi^0(\rho^0)$ meson production respectively, or

$$U_M(q, q') = \delta_{qd}\delta_{q's}, \delta_{qu}\delta_{q's}, \delta_{qs}\delta_{q'u}, \delta_{qs}\delta_{q'd}, \quad (4.2)$$

for the $K^0(K^{*0}), K^\pm(K^{*\pm}), \bar{K}^0(\bar{K}^{*0})$ meson production respectively. In all the other cases, $U_M(q, q')$ is directly replaced by its explicit form :

$$U_M(q, q') = \delta_{qq'} \quad (4.3)$$

We recall that the remarks and definitions of §3.2.3 remain still valid for all tables.

We now proceed to examine some questions related to ($I = 1, I_3 = 0$) and ($I=0$) mesons.

4.2. π^0 and ρ^0 production

At any order in the coupling constants (g, e), any complete Feynman amplitude can be split into two well-defined parts :

$$T = T^a + T^{na}$$

T^a will be named annihilation amplitude, T^{na} non-annihilation amplitude. In T^a we gather all the Feynman diagrams where the q and the $\bar{q}$ of the produced meson are from the same quark line. In T^{na} we gather all the Feynman diagrams where the q and/or the $\bar{q}$ of the prompt meson are (is) connected with an entering quark and/or antiquark line. Any diagram of any of the processes (3.2) and (3.7), clearly belong either to class T^a or T^{na} (exclusive of each other).

Let us now consider the flavour wave function of the π^0 (or ρ^0) meson :

$$|\pi^0\rangle = \frac{1}{\sqrt{2}} (|u\bar{u}\rangle - |d\bar{d}\rangle) \quad (4.5)$$

Focusing on processes (3.2), let us define :

$$\left\{ \begin{array}{l} A(q, q') = \langle qM | T^{na} | q\bar{q}' \rangle, \quad B(q, q') = \langle qM | T^a | q\bar{q}' \rangle \\ D'(q, q) = \langle qM | T^{na} | qq \rangle \equiv 0, \quad D(q, q) = \langle qM | T^a | qq \rangle \\ \bar{A}(q, q') = \langle q'M | T^{na} | qq \rangle, \quad \bar{B}(q, q') = \langle q'M | T^a | qq \rangle. \end{array} \right. \quad (4.6)$$

The meaning of each amplitude is obvious : the ket describes the initial state while the bra describes the final states ; M is the $(q\bar{q}')$ pair with definite quantum numbers. Amplitudes A and B here correspond to the diagram sets A and B of figure (2)^(*), amplitude D corresponds to the set depicted in figure (3).

(*) recall that A and B have a destructive interference.

Let us now consider the prompt π^0 production amplitude through one of the processes (3.2) :

$$T(\dots \rightarrow \pi^0) = \langle \pi^0 | T | \dots \rangle ; \quad (4.7)$$

this can be rewritten using (4.5) :

$$T(\dots \rightarrow \pi^0) = \frac{1}{\sqrt{2}} \{ \langle u\bar{u} | T | \dots \rangle - \langle d\bar{d} | T | \dots \rangle \} . \quad (4.8)$$

Spitting out T through the relation (4.4) and using (4.6), we get :

$$\begin{cases} T(q\bar{q}' \rightarrow \pi^0 q) = \frac{1}{\sqrt{2}} A(q, q') \delta_{qq'} (\delta_{qu} + \delta_{qd}) \sqrt{3} f_\pi \\ T(qq' \rightarrow \pi^0 q') = \frac{1}{\sqrt{2}} \bar{A}(q, q') \delta_{qq'} (\delta_{qu} + \delta_{qd}) \sqrt{3} f_\pi \\ T(qq' \rightarrow \pi^0 q) = 0 \end{cases} \quad (4.9)$$

These relations are formally the same when going to the ρ^0 case ($\sqrt{3} f_\pi \rightarrow \sqrt{6} f_\rho$). By looking at formulae (4.9), one can remark that amplitudes B, $\bar{B}$, D do not contribute to π^0 and ρ^0 production. This is because, assuming exact SU(3) flavour symmetry, the T^a part of the amplitude drops out in the difference (4.8). Corresponding results can easily be derived for processes (3.7). By squaring expression (4.9), we obtain the results for π^0 (and ρ^0) as they appear in tables III and IV.

4.3 Isospin zero quarkonium meson production

We shall only speak about the η and η' mesons here ; the corresponding results for ω and φ can be derived by making the changes :

$$\{ \eta \rightarrow \varphi , \quad \eta' \rightarrow \omega , \quad \theta_s \rightarrow \theta_v \} . \quad (4.10)$$

The singlet and octet flavour wave functions are :

$$\begin{cases} |\eta_1\rangle = \frac{1}{\sqrt{3}} [|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle] \\ |\eta_8\rangle = \frac{1}{\sqrt{6}} [|u\bar{u}\rangle + |d\bar{d}\rangle - 2|s\bar{s}\rangle] \end{cases} , \quad (4.11)$$

while, assuming no (gg) component in η , η' mesons :

$$\begin{cases} |\eta\rangle = \cos \theta_S |\eta_8\rangle - \sin \theta_S |\eta_1\rangle \\ |\eta'\rangle = \sin \theta_S |\eta_8\rangle + \cos \theta_S |\eta_1\rangle \end{cases}, \quad (4.12)$$

where $\theta_S = -11^\circ$ ($\theta_V = +35^\circ$).

In this case, we also use decomposition (4.4) and expressions (4.6) to get (recall that exact SU(3) flavour symmetry holds) :

$$\begin{cases} T(q\bar{q}' \rightarrow \eta_1 g) = \frac{1}{\sqrt{3}} [A(q, q') + 3B(q, q')] \delta_{qq'} \sqrt{3} f_{\eta_1} \\ T(qg \rightarrow \eta_1 q') = \frac{1}{\sqrt{3}} [\bar{A}(q, q') + 3\bar{B}(q, q')] \delta_{qq'} \sqrt{3} f_{\eta_1} \\ T(qg \rightarrow \eta_1 g) = \sqrt{3} D(q, q') \delta_{qq'} \sqrt{3} f_{\eta_1} \end{cases}, \quad (4.13)$$

and

$$\begin{cases} T(q\bar{q}' \rightarrow \eta_8 g) = \frac{1}{\sqrt{6}} A(q, q') \delta_{qq'} [1 - 3\delta_{q5}] \sqrt{3} f_{\eta_8} \\ T(qg \rightarrow \eta_8 q') = \frac{1}{\sqrt{6}} \bar{A}(q, q') \delta_{qq'} [1 - 3\delta_{q5}] \sqrt{3} f_{\eta_8} \\ T(qg \rightarrow \eta_8 g) = 0 \end{cases}, \quad (4.14)$$

these formulae contain the singlet and octet decay constants which could well be different ⁽²³⁾, however recent results on photon-photon collisions ⁽²⁴⁾ strongly support the "nonet symmetry" for the pseudoscalar mesons :

$$f_{\eta_1} \simeq f_{\eta_8} \simeq f_{\pi} \quad (4.15)$$

as was already admitted for vector mesons.

From formulae (4.13) and (4.14) we see that there is no B, $\bar{B}$ or D contribution to the η_8 component (as for the π^0/ρ^0), while they are present when producing η_1 . The final reason is algebraic : for the η_8 (ω_8) as well as the π^0 (ρ^0), the sum of the Clebsch-Gordan coefficients is zero. By defining :

$$\begin{cases} \lambda_1 = \left[\frac{\cos \theta_s}{\sqrt{6}} (1-3 \delta_{qs}) f_{\eta_8} - \frac{\sin \theta_s}{\sqrt{3}} f_{\eta_1} \right] \\ \lambda_2 = -\sqrt{3} \sin \theta_s f_{\eta_1} \end{cases} \quad (4.16)$$

we get from rels.(4.12) to (4.14)^(*) :

$$\begin{cases} T(q\bar{q}' \rightarrow \eta q) = \sqrt{3} \{ \lambda_1 A(q, q') - \lambda_2 B(q, q') \} \delta_{qq'} \\ T(qq \rightarrow \eta q') = \sqrt{3} \{ \lambda_1 \bar{A}(q, q') - \lambda_2 \bar{B}(q, q') \} \delta_{qq'} \\ T(qq \rightarrow \eta q) = \sqrt{3} \lambda_2 D(q, q') \delta_{qq'} \end{cases} \quad (4.17)$$

The same expressions hold for the η' by changing in rels (4.17) :

$$\lambda_1 \rightarrow \mu_1, \quad \lambda_2 \rightarrow \mu_2, \quad (4.18)$$

with :

$$\begin{cases} \mu_1 = \left[\frac{\sin \theta_s}{\sqrt{6}} (1-3 \delta_{qs}) f_{\eta_8} + \frac{\cos \theta_s}{\sqrt{3}} f_{\eta_1} \right] \\ \mu_2 = \sqrt{3} \cos \theta_s f_{\eta_1} \end{cases} \quad (4.19)$$

The results when one gluon is exchanged with a photon follows the same lines, as well as those for ω and φ .

All the results for quarkonia^(**) are listed in tables III-IV. Apart from the already defined λ_1 , λ_2 (see rel (4.16)) and μ_1 , μ_2 (see rel. 4.19), they use the following parameters :

$$\begin{cases} \lambda'_1 = \left[\frac{\cos \theta_v}{\sqrt{6}} (1-3 \delta_{qs}) f_{\omega_8} - \frac{\sin \theta_v}{\sqrt{3}} f_{\omega_1} \right] \\ \mu'_1 = \left[\frac{\sin \theta_v}{\sqrt{6}} (1-3 \delta_{qs}) f_{\omega_8} + \frac{\cos \theta_v}{\sqrt{3}} f_{\omega_1} \right] \end{cases} \quad (4.20)$$

(*) The minus sign in front of λ_2 accounts for the destructive interference between A and B amplitudes (resp. $\bar{A}$ and $\bar{B}$).

(**) In the vector meson production case, we have summed up all the helicities states. It is however simple to split them : the F_1 or F'_1 terms always refer to helicity zero while the F_3 or F'_3 terms always refer to ± 1 helicity states.

Let us point out that we recover the results of ref(4) for π^+ prompt production in hadron-hadron scattering and that of ref(5) for photoproduction of ρ^+ mesons (by setting their I_M^2 to $24 f_V^2 c_1^2$, that is $6f_V^2$ for $\beta = 1$). The formulae for the invariant amplitude squared⁽²⁵⁾ of $\pi q \rightarrow \gamma q$ agrees with ours, as well as that of ref(5) ; however, we disagree with the results for $q\bar{q}' \rightarrow \pi \gamma$ quoted in the appendix of ref (25), which seems to be inconsistent with crossing symmetry.

4.4. Gluonium production

The cross sections for prompt production of gluonia are simply deduced from the results quoted in tables I and II, by multiplying the invariant amplitudes squared by the corresponding extra factors. The resulting formulae are gathered in tables VII and VIII.

5. General properties of the higher twist productions of mesons

There are two main peculiarities common to all prompt production cross sections. Both have been previously noticed in the literature^{(4) (5)} for the few cases already treated :

- i) the dependence upon the meson wave function results in a global scale factor in the cross sections,
- ii) there is a sharp dependence upon the helicity state of the directly produced meson.

This last property is the main difference between the higher twist mechanism and what is expected from the quark or gluon fragmentation mechanism.

To be more precise, let us focus on $\pi^+ p$ collisions, for instance. In this case, it appears that the dominant process to produce a π^0 or a ρ^0 is the $q\bar{q}'$ hard scattering, because all the required quarks or antiquarks are available as valence partons. From our results, we get^(*) :

(*) Assuming the value of c_1 to be equal for π^0 and any helicity state of the ρ^0 .

$$\frac{\frac{d\sigma}{dt}(q\bar{q}' \rightarrow \pi^0 g)}{\frac{d\sigma}{dt}(q\bar{q}' \rightarrow \rho^0(\text{hel}=0)g)} = \frac{f_\pi^2}{2f_\rho^2} \simeq 0.38 \quad (5.1)$$

and

$$\frac{1}{16} \leq \frac{\frac{d\sigma}{dt}(q\bar{q}' \rightarrow \rho^0(\text{hel}=0)g)}{\frac{d\sigma}{dt}(q\bar{q}' \rightarrow \rho^0(\text{hel}=\pm 1)g)} \leq \frac{1}{8} \quad (5.2)$$

which seem to provide some rules :

iii) Pseudoscalar s-wave quarkonium mesons are much less produced than their vector partners,

iv) the zero helicity state of any vector meson is much less produced than its ± 1 helicity state.

The higher twist version of the Lund Monte Carlo⁽⁶⁾ confirms this trend and we get, by cumulating all parton-parton hard processes in π^-p collisions :

$$\frac{\sigma_{HT}(\pi^0)}{\sigma_{HT}(\rho^0, \text{hel}=0)} = 0.37 \quad (5.3)$$

and

$$\frac{\sigma_{HT}(\rho^0, \text{hel}=0)}{\sigma_{HT}(\rho^0, \text{hel}=\pm 1)} = 0.11 \quad , \quad (5.4)$$

(the subscript HT means Higher Twist mechanism). Therefore, we can argue that one of the best ways to single out the higher twist production mechanism is to measure, at medium to high- p_T , vector meson production cross sections as a whole and for each helicity state. Any significant excess of ± 1 helicity compared with 2/3 would be a clear signature of the effect we are looking for.

For gluonium production, the helicity dependence of the cross section is at the same time simpler and nicer. For the 2^{++} glueball, we have :

$$\frac{\sigma_{HT}(G(2^{++}, \text{hel}=0))}{\sigma_{HT}(G(2^{++}, \text{hel}=\pm 2))} = \frac{(c_1 - c_0)^2}{2 c_1^2} \quad (5.5)$$

Assuming c_1 does not crucially depend on the helicity state, this ratio is of the order 10-20% for $\beta \approx 1$ (see rel (2.44)), while

$$\sigma_{HT}(G(2^{++}, \text{hel}=\pm 1)) = 0,$$

at least when neglecting transverse momentum within the gluonium. We can thus summarize the major features of higher twist production of quarkonium mesons by the points i) to iv) already noticed.

For the 2-gluon gluonia, all which can be noticed is summarized in one relation :

$$\frac{1}{c_1^2 \rho_{0A}^2} \frac{d\sigma}{dt}[G(0_A)] = \frac{1}{(c_1 - c_0)^2 \rho_{0S}^2} \frac{d\sigma}{dt}[G(0_S)] = \frac{1}{2 c_1^2 \rho_{2S}^2} \frac{d\sigma}{dt}[G(2_S)] \quad (5.6)$$

which expresses that the differential cross sections of all 2-gluon gluonia are proportional, and no helicity $\neq 1$ is expected for the lowest lying gluonia. Relations (5.6) expresses a nice property, never encountered for any quarkonium meson ; it implies that ratios of glueball production cross sections are independent of the initial parton densities.

The higher twist production cross section for any s-wave quarkonium meson is proportional to c_1^2 :

$$c_1 = \int_0^1 \int_0^1 dx_1 dx_2 \frac{1}{x_1} \tilde{\phi}(x_1, x_2) \quad ; \quad (5.8)$$

therefore, if one has at one's disposal a reliable simulation of the minimum twist production mechanism of any meson (through parton fragmentation), then

a comparison with experiment in the expected range of validity of our results ($p_T \gtrsim 2 \text{ GeV}/c$) could provide a numerical evaluation of $|c_1|$. This is in principle possible for any meson for which the prompt production occurs significantly above the fragmentation background. The sum rule (5.8) provides, in principle, a criterium to distinguish among different models for $\Phi_M(x_1, x_2)$. For instance, the comparison of our predictions with preliminary results of the WA77 experiment ^{(2) (3)} seems to favour a function $\tilde{\Phi}_M$ with $\beta = 1/4$ (see rel (2.44)) ; if it is confirmed, this should rule out the form proposed for the ρ meson by Chernyak et al ^{(18) (19)}, which is equivalent to a $\tilde{\Phi}_M$ with $\beta \simeq 0.7$. Let us notice that the preliminary results of the WA77 experiment ⁽³⁾ do not show any of the expected glueballs in $\pi\pi$ or $K\bar{K}$; from these results one can draw rough bounds for the wave function of two glueball candidates, namely the S^* (0^{++} , 975) and Θ (2^{++} , 1690). Assuming they are glueballs, assuming that all events above background in the mass plot can be attributed to them as soon as they lie in the right mass region, neglecting also -for the 2^{++} gluonium - the helicity zero contribution compared to helicity ± 2 , we can obtain :

$$|f_{S^*} c_1| \lesssim 50 \text{ MeV} \quad (5.9.1)$$

and

$$|f_{\Theta} c_1| \lesssim 120 \text{ MeV} \quad (5.9.2)$$

We have used here the branching ratio $\Theta \rightarrow K\bar{K} \simeq 2/3$ - thus $1/3$ in $K^+ K^-$ - as it can roughly be deduced by comparing the data reported in ref (26).

Before leaving this section let us point out a remark on photoproduction processes. As can be seen from tables XI and VIII, the subprocesses $\gamma g \rightarrow M g$ (and $g g \rightarrow M \gamma$) do not contribute to the cross section at the (lowest) order $\alpha_s^2 \alpha_{em}$. This peculiarity of processes involving an incoming or a prompt photon makes photoproduction a priori different from hadron-hadron scattering.

6. Monte Carlo Studies

It seems desirable to have an idea about the order of magnitude of the effects expected from the higher twist mechanism, as well as a comparison between prompt production and fragmentation production of mesons. All this information could lead to a definition of the best ways to look for strong evidence of the higher twist mechanism, if any. Results have been obtained, for the "minimum twist" (fragmentation), by the standard Lund Monte Carlo for high- p_T physics^{(1)(*)}; for the higher twist mechanism we have used the Monte Carlo especially designed for this kind of processes⁽⁶⁾. All the results we shall hereafter give have been obtained for fixed target proton and 350 GeV/c beams of $\pi^\pm$, p $\bar{p}$. The parton transverse momentum has been chosen to be larger than or equal to 2 GeV/c.

For the higher twist mechanism some further parameters have been chosen :

- i) The decay constants of all pseudoscalar mesons :

$$f_{PS} = f_\pi (\simeq f_\kappa \simeq f_{\bar{\kappa}} \simeq f_{\eta_8} \simeq f_{\eta_1}) = 93 \text{ MeV}$$

- ii) The decay constant of all vector mesons :

$$f_V = f_\rho (\simeq f_{\kappa^*} \simeq f_{\bar{\kappa}^*} \simeq f_{\omega_8} \simeq f_{\omega_1}) = 107 \text{ MeV}$$

- iii) The decay constant of the gluonia, has been conservatively chosen as :

$$f_G = 200 \text{ MeV}$$

(*)Default values for the Lund Monte Carlo (JETSET 5.2) parameters have always been assumed. This covers the choice of the structure functions of $\pi^\pm$ mesons and nucleons (coming from refs(27) and (28)), that of the running coupling constant $\alpha_s(Q^2) = 12 \pi / ((33-2n_F) \log Q^2/\Lambda^2)$ with $n_F = 3$, $\Lambda = 0.3 \text{ GeV}$, $Q^2 = 2 \text{ stu}/(s^2 + t^2 + u^2)$. The η/π ratio used is 0.9. Other parameters⁽²⁹⁾ of importance for us are relative probabilities to create mesons when fragmenting the string, and mainly the ratios $\pi^0 : \eta : \eta'$ which are chosen to be 2 : 1 : 1. This last choice mainly comes from SU(6), the ratio $\eta : \eta'$ as 1 : 1 might have to be measured. Notice that the relative probabilities just mentioned imply that $\pi^0 : \eta$ is 2 : 1 which agrees with the result quoted by the NA14 experiment⁽³⁰⁾, (see § 6.3 below).

Let us recall that a $\sqrt{3}$ (or $\sqrt{6}$) coming with the f_{ps} (or f_v) is already taken into account in the so-called "extra factors" in tables I and II and that no such factor has been taken to define f_G (see rels (2.42)).

What remains is to deal with the moments c_1 and c_0 which appear in our cross sections. Referring to the known results for the electromagnetic form factor of the pion and to the preliminary results of WA77 experiment for the prompt production of φ^0 mesons, it seems that a value $\beta \simeq 1/4$ (see rel(2.44)) can be assumed for all quarkonium mesons ; this implies :

$$c_1 = \frac{2\beta+1}{6\beta} \simeq 1 . \quad (6.1)$$

For gluonia, however, such a striking change from the nominal value $\beta = 1$ is not presently expected, so we have kept this value for the O^{++} and 2^{++} gluonia, which implies :

$$c_1 = \frac{1}{2} , \quad c_0 = \frac{1}{6} , \quad (6.2)$$

for the helicity states $2_S, O_S$.

For simplicity reasons, we have also chosen :

$$c_1 = \frac{1}{2}$$

for the O^{-+} gluonium as well (helicity state O_A). This can be improved and the results in the tables would only have to be rescaled.

6.1. Higher twist cross section of various processes

With the hypotheses just listed, we have computed the prompt production cross sections of any O^- and 1^- quarkonium mesons and of the $O_A, O_S, 2_S$ gluonia, in $\pi^- p, \pi^+ p, \bar{p} p, pp$ at beam $(\pi^+, \pi^-, p, \bar{p})$ momentum 350 GeV/c. All our results are gathered in tables IX to XII. These tables are arranged as follows : the first three columns display the total cross sections for pure QCD processes (see rels (3.2)) subdivided among the 3 kinds of basic processes which can occur in purely hadronic initial states ($q\bar{q}, qg$ and $\bar{q}g, gg$) ; the fourth

column displays the results for associated production of a prompt meson and a prompt photon with all initial states ($q\bar{q}$, gg , $\bar{q}g$, $g\bar{q}$) cumulated. The last column gives the cumulated results for any prompt meson. All numbers are given in nanobarns. The accuracy of these results can be evaluated to about 5%-15%^(*).

The main remark about these results is the surprisingly low rate of production of pseudoscalar quarkonia with the noticeable exception of the η and η' . One can immediately conclude from this that prompt π or K are surely the least favoured channels to amount the higher twist effects. Conversely, at beam momentum of 350 GeV/c, the best channels seem to be η , η' , ϕ , ω and perhaps K^* or $\bar{K}^*$ (depending on which collision we are speaking of).

The surprising high rate of η' (in absolute magnitude and compared with the η) must be noticed. It corresponds to a SU(3) mixing angle of -11° ; going from -11° to -23° (value also quoted in the literature) only lessens this cross section by 20% (while it increases the η production by a factor 3). Thus, in any hadronic interaction :

$$6\% \leq \frac{\sigma_{HT}(\eta)}{\sigma_{HT}(\eta')} \leq 18\% \quad . \quad (6.3)$$

Among the vector quarkonia, only the ψ meson is promptly produced at a negligible rate.

Let us now go to gluonium production. In the four examined cases, the rates of such mesons (for which a comparison with standard fragmentation is lacking) are surprisingly high, even with our choices for f_G , c_1 , c_0 . The fact that no glueball candidate appears in the WA77 experiment⁽³⁾ allows one to put some bound on $|f_G \cdot c_1|$ which must be refined when the final results will become available (see rels (5.9)).

(*) This accuracy is essentially due to statistical fluctuations. This value is expected for all our Monte Carlo results ; of course, we have not estimated the uncertainties inherent to the Lund Model.

6.2 Minimum twists versus higher twist

To compare prompt production of mesons with the usual fragmentation mechanism, we have computed the cross sections of :

$$(\pi^\pm, \rho, \bar{\rho}) \quad p \rightarrow M(p_T \geq 2.5 \text{ GeV}/c) + X$$

where M is any of the $J^P = 0^-$ and $J^P = 1^-$ quarkonia. To simplify our analysis, we have also required that in X , there is no other candidate to the specific trigger meson ; for instance, if we trigger on a meson M , say a ρ^0 with $p_T \geq 2.5$ GeV/c, then the event is taken into account if, and only if, there is no other ρ^0 within the same range in p_T included in X . For such 4π acceptance experiments, the results are gathered in tables XIII and XIV. Let us point out that, there, the trigger meson with $p_T \geq 2.5$ GeV/c considered is not necessarily a prompt meson, but can follow from the decay of other prompt mesons as well as from fragmentation in any of the higher twist mechanisms taken into account. This explains why some higher twist cross sections of mesons with $p_T \geq 2.5$ GeV/c are much larger than the corresponding prompt production cross sections.

As a general conclusion, valid for any available beam hadron at 350 GeV/c, we can remark that^(*) :

$$\frac{\sigma_{MT}(\eta)}{\sigma_{MT}(\eta')} \simeq 1, \quad \frac{\sigma_{HT}(\eta)}{\sigma_{HT}(\eta')} \simeq \frac{1}{10}, \quad \frac{\sigma_{MT}(\eta')}{\sigma_{HT}(\eta')} \lesssim 1 \quad (6.4)$$

From there, it follows that ($\beta = 1/4$ being assumed), the measurement of :

$$K = \frac{\sigma(\eta, p_T \geq 2.5 \text{ GeV}/c)}{\sigma(\eta', p_T \geq 2.5 \text{ GeV}/c)} \quad (6.5)$$

provides the best way to probe the higher twist mechanism in hadronic interactions. If this mechanism is absent, one expects^(**) :

$$K \simeq 1, \quad (6.6.1)$$

(*) The subscripts MT and HT refer to minimum twist and higher twist mechanisms respectively. This notation will be used from now on .

(**) see note on page 30.

otherwise this parameter would satisfy :

$$0.30 \lesssim K \lesssim 0.65 \quad , \quad (6.6.2)$$

if one assumes that the power β lies within 1 and 1/4 for both the η and η' .

The absolute rates of some other mesons are also good candidates to single out the higher twist signal : In $\pi^- p$ collisions for instance the ρ^0 , ρ^- or ω high- p_T production are reasonable channels ; in $\pi^+ p$ the same channels, but for the change of ρ^- to ρ^+ , are good too. In pp experiments, the situation is less good, while in $\bar{p}p$, many channels are available at significant rates (η' , ρ^+ , ρ^0 , ω).

One may wonder why the η' has such a singular behaviour compared with all other quarkonia. This strong departure from the common rule can easily be traced back to the large coupling of the η and η' to the gluon-gluon hard process (see tables IX to XII) ; this lets both of these mesons have common features with gluonia. The particular choice of the hadron structure functions (refs (27), (28)), may also play a role, however it might not be crucial.

6.3. Photoproduction Monte Carlo results

The results on photoproduction have been obtained using the Lund Monte Carlo adapted to monochromatic photon beam by Weigend⁽³¹⁾ in the minimum twist version ; we have extended it so as to include the meson production through the various higher twist mechanisms given in tables VI and VIII. Running this program in the same conditions as before - see paragraph (6.2) above - except for the beam energy ($E_\gamma = 150$ GeV) we have got the results given in tables XI and XVI.

The results quoted in table XV only concern the higher twist mechanism. They clearly show that all pseudoscalar meson productions are at a level which seems to be below any experimental check ; among the vector mesons, only the charged K^* and ρ mesons exhibit reasonable cross sections and possibly observable charge asymmetries. Let us notice that no further cut on the p_T of the prompt meson has been used. We also have run the Lund Monte Carlo including higher twists for the reactions :

$$\gamma (p, n) \rightarrow M(p_T \geq 2.5 \text{ GeV}/c) + X$$

with the same conditions on X as those already described in §(6.2), with $E_\gamma = 150$ GeV/c and the $q_{T\min}$ parameter still equal to 2.0 GeV/c. The results obtained are gathered in table XVI. By weighting suitably the γp and γn cross sections, one can deduce the results for, in principle, any target.

In discussing the results in table XVI, let us focus on γp scattering. As a first remark, one can point out, concerning the magnitude of the cross sections, that the higher twist effect is low or very low for any pseudoscalar meson, while the situation appears different for some vector mesons, namely the ρ^+ , ρ^- and K^{*+} . Then, if one is able to extract reliably from experimental data the VDM (vector dominance) background, there exists some way to look at the higher twist signal.

Among other predictions involving the perturbative part of γp , some other results can be compared. For instance the ratio $\sigma(\eta)/\sigma(\pi^0)$ is :

$$\frac{\sigma(\eta)}{\sigma(\pi^0)} \Big|_{MT} = (49 \pm 5.) \% \quad \text{or} \quad \frac{\sigma(\eta)}{\sigma(\pi^0)} \Big|_{MT+HT} = (41. \pm 5.) \%$$

These values stay at the edge of the statistical errors. If we introduce the charge asymmetry ratio as :

$$A(M) = \frac{\sigma(M^+) - \sigma(M^-)}{\sigma(M^+) + \sigma(M^-)}$$

in the case of γp scattering we have^(*) :

$$\begin{aligned} A(\pi) \Big|_{MT} &= (4.4 \pm 6.1)\% \quad , \quad A(\pi) \Big|_{MT+HT} = (6.6 \pm 5.1)\% \\ A(K) \Big|_{MT} &= (10.3 \pm 10.4)\% \quad , \quad A(K) \Big|_{MT+HT} = (19.3 \pm 9.1)\% \end{aligned}$$

(*) Scattering on neutrons exhibits properties which are slightly different ; for details see table XVI.

$$A(\eta) \Big|_{nT} = (16.9 \pm 7.1)\% , \quad A(\eta) \Big|_{nT+nT} = (9.9 \pm 4.2)\%$$

$$A(K^*) \Big|_{nT} = (18.5 \pm 10.8)\% , \quad A(K^*) \Big|_{nT+nT} = (42.8 \pm 7.9)\% .$$

Taking into account the accuracy of our Lund results, we can confidently say that the higher twist mechanism does not contribute to the pion and kaon charge asymmetries at a measurable level (the kaon cross sections seem to be too low). However, the charge asymmetries of K^* and η meson production may provide a valuable test of the relevance of the higher twist mechanism.

To compare roughly with the results of the NA14 experiment at CERN, we have run our Monte Carlo at $E_\gamma = 90$ GeV - which is nearly the mean value of their photon beam spectrum - with a minimum q_T between the photon and any parton in the nucleons of 2.0 GeV/c. Taking only into account inclusive cross sections of mesons at $p_T \geq 2.5$ GeV/c, just as before, we have found^(*)

$$A(\pi + K) \Big|_{nT} = (0.4 \pm 6.1)\% , \quad , \quad A(\pi + K) \Big|_{nT+nT} = (0.0 \pm 5.4)\% ,$$

that is, pure QCD effects do not predict charge asymmetry, and the higher twist mechanism, even at this energy, is not visible. This seems to be in agreement with the NA14 results⁽³²⁾, taking into account the VDM background effects. As further results, the same simulation provides :

$$\frac{\sigma(\eta)}{\sigma(\pi^0)} \Big|_{nT} = (48 \pm 6) \% \quad \text{or} \quad \frac{\sigma(\eta)}{\sigma(\pi^0)} \Big|_{nT+nT} = (38 \pm 4) \% ,$$

with both agree with the NA14 result⁽³⁰⁾ :

$$\frac{\sigma(\eta)}{\sigma(\pi^0)} \Big|_{exp} = (52 \pm 15) \% .$$

(*) We have summed up π and K cross sections for the same charges, because in a range of p_L of 20 to 40 GeV/c, they are not distinguished. It appears that this cut in p_L does not affect these numerical results.

This result is not expected to be modified when subtracting the VDM background.

As tests of the higher twist mechanism (without any cut in p_L) looking at collisions on isoscalar targets, we can quote firstly charge asymmetries which stay, in principle, at a detectable level :

$$\begin{aligned} A(\rho)|_{MT} &= (5 \pm 6)\% \quad \text{or} \quad A(\rho)|_{MT+HT} = (-16 \pm 3)\% \\ A(K^*)|_{MT} &= (17 \pm 10)\% \quad \text{or} \quad A(K^*)|_{MT+HT} = (52 \pm 7)\% \end{aligned}$$

and secondly absolute inclusive cross sections (nb)^(*) :

$$\sigma(\pi^0)|_{MT} = 1.73 \pm 0.12 \quad \sigma(\pi^0)|_{MT+HT} = 2.38 \pm 0.13,$$

and

$$\sigma(\eta)|_{MT} = 0.83 \pm 0.08, \quad \sigma(\eta)|_{MT+HT} = 0.90 \pm 0.08,$$

for π^0 or η at $p_T \geq 2.5$ GeV/c. For the ρ and K^* mesons at $p_T \geq 2.5$ GeV/c, we also have :

$$\begin{aligned} \sigma(\rho^+)|_{MT} &= 1.63 \pm 0.12 \quad \text{or} \quad \sigma(\rho^+)|_{MT+HT} = 3.38 \pm 0.19, \\ \sigma(\rho^-)|_{MT} &= 1.49 \pm 0.11 \quad \text{or} \quad \sigma(\rho^-)|_{MT+HT} = 4.62 \pm 0.15, \\ \sigma(K^{*+})|_{MT} &= 0.68 \pm 0.08 \quad \text{or} \quad \sigma(K^{*+})|_{MT+HT} = 2.11 \pm 0.11, \\ \sigma(K^{*-})|_{MT} &= 0.48 \pm 0.06 \quad \text{or} \quad \sigma(K^{*-})|_{MT+HT} = 0.66 \pm 0.06, \end{aligned}$$

from which one gets :

$$\frac{\sigma(K^{*+}, p_T \geq 2.5 \text{ GeV/c})}{\sigma(K^{*-}, p_T \geq 2.5 \text{ GeV/c})} = 1.4 \quad \text{or} \quad 3.2,$$

for respectively the MT and MT + HT cases.

(*) the cross sections are normalized to the number of nucleons and not to that of Li^6 nuclei.

6.4. General remarks on the numerical results

As already explained at the beginning of this paragraph, we have chosen the decay constants at the known values or at values which are likely expected. The same remark can be applied to the moments c_1 and c_0 of the reduced wave function .

Getting the numerical results for other values can be immediately done by rescaling the numbers in tables IX to XVI, in the way deductible from table III-VII. The minimum twist results (fragmentation) are of course not affected by these changes.

7. Conclusions

After having introduced the general framework for calculating the higher twist processes responsible for prompt meson production, we have worked out all the relevant cross sections. We have thus covered the range of hadron-hadron and photon-hadron scatterings with prompt production of all s-wave quarkonia and the lowest lying gluonia. From Monte-Carlo estimates of the possible effects, it seems reasonable to hope for suitable tests of the relevance of prompt meson production with respect to parton fragmentation production.

As a matter of fact, many features are different when going from hadron-hadron to photon-hadron collisions. In the first case, many channels are available at significant levels ; among the best ones, one finds the ρ and K^* meson productions and, better, the comparative production of η and η' mesons. The surprisingly high production rates of these mesons proceed from their large coupling to gluons, which lets them look more like gluonia than like other quarkonia. It is however of importance to ascertain experimentally the relative production rate of the η and η' mesons in the fragmentation process. If this ratio is found to be nearly one, as expected from SU(6) and the η/π^0 ratio of the NA14 experiment, then the relative high- p_T production of η and η' mesons is one of the best channels available to probe the relevance of the higher twist mechanism.

Comparing the expected rates of gluonium production to still preliminary experimental results, one can put bounds on some glueball candidate wave functions at the origin ; these bounds will have to be refined when the final results will become available.

With the present experimental situation, one can tell that a prompt production signal seems to have been observed by the WA77 experiment at the expected rate, favouring a $(x_1 \cdot x_2)$ dependence of the wave function at the power $1/4$, just as for the pion.

In photon-hadron physics the situation is in fact less favourable in that only a limited class of measurements could, in principle, provide a clear signal of the higher twist mechanism. These are mainly the charge asymmetries and cross sections of ρ and K^* mesons. All other known results are compatible with minimum twist only as well as minimum plus higher twist.

In any case, the large number of clear signatures of prompt meson production leaves us with the hope of soon obtaining a definitive answer about the relevance of this mechanism.

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Table I - Pure QCD processes
Invariant amplitudes squared and extra factors

Process	Diagram set A	Diagram set B	Interference A-B	Extra factors
$q\bar{q}' \rightarrow M(0^-) g$	$F_1(s, t, u)$	$F_2(s, t, u)$	$F_{12}(s, t, u)$	$\frac{\pi^2 \alpha_s^3}{3 s^2} c_1^2 p_{ps}^2$
$q\bar{q}' \rightarrow M(1^-, hel=0) g$	$F_1(s, t, u)$	0	0	$2 \frac{\pi^2 \alpha_s^3}{3 s^2} c_1^2 p_v^2$
$q\bar{q}' \rightarrow M(1^-, hel=\pm 1) g$	$F_3(s, t, u)$	0	0	$2 \frac{\pi^2 \alpha_s^3}{3 s^2} c_1^2 p_v^2$
$qg \rightarrow M(0^-) q'$	$F_1(-t, -s, u)$	$F_2(-t, -s, u)$	$F_{12}(-t, -s, u)$	$\frac{\pi^2 \alpha_s^3}{8 s^2} c_1^2 p_{ps}^2$
$qg \rightarrow M(1^-, hel=0) q'$	$F_1(-t, -s, u)$	0	0	$\frac{\pi^2 \alpha_s^3}{4 s^2} c_1^2 p_v^2$
$qg \rightarrow M(1^-, hel=\pm 1) q'$	$F_3(-t, -s, u)$	0	0	$\frac{\pi^2 \alpha_s^3}{4 s^2} c_1^2 p_v^2$
$gg \rightarrow M(0^-) g$	$F_4(s, t, u)$			$\frac{3\pi^2 \alpha_s^3}{64 s^2} c_1^2 p_{ps}^2$
$gg \rightarrow M(1^-, hel=0) g$	0			
$gg \rightarrow M(1^-, hel=\pm 1) g$	0			
$q\bar{q}' \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} g$	$D_1(s, t, u)$			$\frac{\pi^2 \alpha_s^3}{9 s^2} \begin{cases} c_1^2 p_{0A}^2 \\ (c_1 - c_0)^2 p_{0S}^2 \\ 2 c_1^2 p_{2S}^2 \end{cases}$
$qg \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} q'$	$D_1(-t, -s, u)$			$\frac{\pi^2 \alpha_s^3}{24 s^2} \begin{cases} c_1^2 p_{0A}^2 \\ (c_1 - c_0)^2 p_{0S}^2 \\ 2 c_1^2 p_{2S}^2 \end{cases}$
$gg \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} g$	$D_2(s, t, u)$			$\frac{\pi^2 \alpha_s^3}{64 s^2} \begin{cases} c_1^2 p_{0A}^2 \\ (c_1 - c_0)^2 p_{0S}^2 \\ 2 c_1^2 p_{2S}^2 \end{cases}$

Table II - QCD - QED mixed processes

Process	Invariant amplitude squared	extra factor
$q\bar{q}' \rightarrow M(0^-) \gamma$	$F'_1(s, t, u)$	$\frac{\pi^2 \alpha_s^2 \alpha_{em}}{3 s^2} c_1^2 f_{ps}^2$
$q\bar{q}' \rightarrow M(1^-, hel=0) \gamma$	$F'_1(s, t, u)$	$\frac{2\pi^2 \alpha_s^2 \alpha_{em}}{3 s^2} c_1^2 f_v^2$
$q\bar{q}' \rightarrow M(1^-, hel=\pm 1) \gamma$	$F'_3(s, t, u)$	$\frac{2\pi^2 \alpha_s^2 \alpha_{em}}{3 s^2} c_1^2 f_v^2$
$q\gamma \rightarrow M(0^-) q'$	$F'_1(-t, -s, u)$	$\frac{\pi^2 \alpha_s^2 \alpha_{em}}{s^2} c_1^2 f_{ps}^2$
$q\gamma \rightarrow M(1^-, hel=0) q'$	$F'_1(-t, -s, u)$	$\frac{2\pi^2 \alpha_s^2 \alpha_{em} c_1^2}{s^2} f_v^2$
$q\gamma \rightarrow M(1^-, hel=\pm 1) q'$	$F'_3(-t, -s, u)$	$\frac{2\pi^2 \alpha_s^2 \alpha_{em} c_1^2}{s^2} f_v^2$
$gg \rightarrow M(0^-) \gamma$	0
$gg \rightarrow M(1^-) \gamma$	0
$q\bar{q}' \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} \gamma$	$D'_1(s, t, u)$	$\frac{\pi^2 \alpha_s^2 \alpha_{em}}{9 s^2} \begin{cases} c_1^2 f_{0A}^2 \\ (c_1 - c_0)^2 f_{0S}^2 \\ 2 c_1^2 f_{2S}^2 \end{cases}$
$q\gamma \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} q'$	$D'_1(-t, -s, u)$	$\frac{\pi^2 \alpha_s^2 \alpha_{em}}{3 s^2} \begin{cases} c_1^2 f_{0A}^2 \\ (c_1 - c_0)^2 f_{0S}^2 \\ 2 c_1^2 f_{2S}^2 \end{cases}$
$gg \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} \gamma$	0
$gg \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} g$	0

Table III - Prompt production of $J^P = 0^-$ quarkonium mesons
Pure QCD processes

Process	$\frac{d\sigma}{dt}(s, t, u)$
$q\bar{q}' \rightarrow M(I \neq 0) g$	$\frac{\pi^2 \alpha_s^3}{3 s^2} c_1^2 p_{2s}^2 F_1(s, t, u) U_M(q, q')$
$qg \rightarrow M(I \neq 0) q'$	$\frac{\pi^2 \alpha_s^3}{8 s^2} c_1^2 p_{2s}^2 F_1(-t, -s, u) U_M(q, q')$
$gg \rightarrow M(I \neq 0) g$	$\bigcirc$
$q\bar{q}' \rightarrow \eta g$	$\frac{\pi^2 \alpha_s^3}{3 s^2} c_1^2 \left\{ \lambda_1^2 F_1(s, t, u) - \lambda_1 \lambda_2 F_{12}(s, t, u) + \lambda_2^2 F_2(s, t, u) \right\} \delta_{qq'}$
$qg \rightarrow \eta q'$	$\frac{\pi^2 \alpha_s^3}{8 s^2} c_1^2 \left\{ \lambda_1^2 F_1(-t, -s, u) - \lambda_1 \lambda_2 F_{12}(-t, -s, u) + \lambda_2^2 F_2(-t, -s, u) \right\} \delta_{qq'}$
$gg \rightarrow \eta g$	$\frac{3}{64} \frac{\pi^2 \alpha_s^3}{s^2} c_1^2 \lambda_2^2 F_4(s, t, u)$
$q\bar{q}' \rightarrow \eta' g$	$\frac{\pi^2 \alpha_s^3}{3 s^2} c_1^2 \left\{ \mu_1^2 F_1(s, t, u) - \mu_1 \mu_2 F_{12}(s, t, u) + \mu_2^2 F_2(s, t, u) \right\} \delta_{qq'}$
$qg \rightarrow \eta' q'$	$\frac{\pi^2 \alpha_s^3}{8 s^2} c_1^2 \left\{ \mu_1^2 F_1(-t, -s, u) - \mu_1 \mu_2 F_{12}(-t, -s, u) + \mu_2^2 F_2(-t, -s, u) \right\} \delta_{qq'}$
$gg \rightarrow \eta' g$	$\frac{3}{64} \frac{\pi^2 \alpha_s^3}{s^2} c_1^2 \mu_2^2 F_4(s, t, u)$

Table IV - Prompt production of $J^P = 0^-$ quarkonium mesons
QCD - QED mixed processes

Process	$\frac{d\sigma}{dt}(s, t, u)$
$q\bar{q}' \rightarrow M(I \neq 0) \gamma$	$\frac{\pi^2 \alpha_s^2 \alpha_{em}}{3 s^2} c_1^2 f_{PS}^2 F_1'(s, t, u) U_M(q, q')$
$q\bar{q} \rightarrow M(I \neq 0) q'$	$\frac{\pi^2 \alpha_s^2 \alpha_{em}}{s^2} c_1^2 f_{PS}^2 F_1'(-t, -s, u) U_M(q, q')$
$g\gamma \rightarrow M(I \neq 0) g$	0
$gg \rightarrow M(I \neq 0) \gamma$	0
$q\bar{q}' \rightarrow \eta \gamma$	$\frac{\pi^2 \alpha_s^2 \alpha_{em}}{3 s^2} c_1^2 \lambda_1^2 F_1'(s, t, u) \delta_{qq'}$
$q\bar{q} \rightarrow \eta q'$	$\frac{\pi^2 \alpha_s^2 \alpha_{em}}{s^2} c_1^2 \lambda_1^2 F_1'(-t, -s, u) \delta_{qq'}$
$g\gamma \rightarrow \eta g$	0
$gg \rightarrow \eta \gamma$	0
$q\bar{q}' \rightarrow \eta' \gamma$	$\frac{\pi^2 \alpha_s^2 \alpha_{em}}{3 s^2} c_1^2 \mu_1^2 F_1'(s, t, u) \delta_{qq'}$
$q\bar{q} \rightarrow \eta' q'$	$\frac{\pi^2 \alpha_s^2 \alpha_{em}}{s^2} c_1^2 \mu_1^2 F_1'(s, t, u) \delta_{qq'}$
$g\gamma \rightarrow \eta' g$	0
$gg \rightarrow \eta' \gamma$	0

Table V - Prompt production of $J^P = 1^-$ quarkonium mesons
Pure QCD Processes

Process	$\frac{d\sigma}{dt}(s,t,u)$
$q\bar{q}' \rightarrow M(I \neq 0) g$	$\frac{2\pi^2 \alpha_s^3}{3 s^2} c_1^2 p_V^2 [F_1(s,t,u) + F_3(s,t,u)] U_M(q,q')$
$qg \rightarrow M(I \neq 0) q'$	$\frac{\pi^2 \alpha_s^3}{4 s^2} c_1^2 p_V^2 [F_1(-t,-s,u) + F_3(-t,-s,u)] U_M(q,q')$
$gg \rightarrow M(I \neq 0) g$	0
$q\bar{q}' \rightarrow \varphi g$	$\frac{2\pi^2 \alpha_s^3}{3 s^2} c_1^2 \lambda_1'^2 [F_1(s,t,u) + F_3(s,t,u)] \delta_{qq'}$
$qg \rightarrow \varphi q'$	$\frac{\pi^2 \alpha_s^3}{4 s^2} c_1^2 \lambda_1'^2 [F_1(-t,-s,u) + F_3(-t,-s,u)] \delta_{qq'}$
$gg \rightarrow \varphi g$	0
$q\bar{q}' \rightarrow \omega g$	$\frac{2\pi^2 \alpha_s^3}{3 s^2} c_1^2 \mu_1'^2 [F_1(s,t,u) + F_3(s,t,u)] \delta_{qq'}$
$qg \rightarrow \omega q'$	$\frac{\pi^2 \alpha_s^3}{4 s^2} c_1^2 \mu_1'^2 [F_1(-t,-s,u) + F_3(-t,-s,u)] \delta_{qq'}$
$gg \rightarrow \omega g$	0

Table VI - Prompt production of $J^P = 1^-$ quarkonium mesons
QCD - QED mixed processes

Process	$\frac{d\sigma}{dt}(s, t, u)$
$q\bar{q}' \rightarrow M(I \neq 0)\gamma$	$\frac{2\pi^2\alpha_s^2\alpha_{em}}{3s^2} \cdot c_1^2 p_V^2 [F_1'(s, t, u) + F_3'(s, t, u)] U_M(q, q')$
$q\gamma \rightarrow M(I \neq 0)q'$	$\frac{2\pi^2\alpha_s^2\alpha_{em}}{s^2} \cdot c_1^2 p_V^2 [F_1'(-t, -s, u) + F_3'(-t, -s, u)] U_M(q, q')$
$g\gamma \rightarrow M(I \neq 0)g$	0
$gg \rightarrow M(I \neq 0)\gamma$	0
$q\bar{q}' \rightarrow \varphi\gamma$	$\frac{2\pi^2\alpha_s^2\alpha_{em}}{3s^2} \cdot c_1^2 \lambda_1'^2 [F_1'(s, t, u) + F_3'(s, t, u)] \delta_{qq'}$
$q\gamma \rightarrow \varphi q'$	$\frac{2\pi^2\alpha_s^2\alpha_{em}}{s^2} \cdot c_1^2 \lambda_1'^2 [F_1'(-t, -s, u) + F_3'(-t, -s, u)] \delta_{qq'}$
$gg \rightarrow \varphi\gamma$	0
$g\gamma \rightarrow \varphi g$	0
$q\bar{q}' \rightarrow \omega g$	$\frac{2\pi^2\alpha_s^2\alpha_{em}}{3s^2} \cdot c_1^2 \mu_1'^2 [F_1'(s, t, u) + F_3'(s, t, u)] \delta_{qq'}$
$q\gamma \rightarrow \omega q'$	$\frac{2\pi^2\alpha_s^2\alpha_{em}}{s^2} \cdot c_1^2 \mu_1'^2 [F_1'(-t, -s, u) + F_3'(-t, -s, u)] \delta_{qq'}$
$gg \rightarrow \omega\gamma$	0
$g\gamma \rightarrow \omega g$	0

Table VII - Prompt production of gluonium mesons
Pure QCD processes

Process	$\frac{d\sigma}{dt}(s,t,u)$
$q\bar{q}' \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} g$	$\frac{\pi^2 \alpha_s^3}{9 s^2} D_1(s,t,u) * \left\{ \begin{array}{l} c_1^2 f_{0A}^2 \\ (c_1 - c_0)^2 f_{0S}^2 \\ 2 c_1^2 f_{2S}^2 \end{array} \right\} * \delta_{qq'}$
$qg \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} q'$	$\frac{\pi^2 \alpha_s^3}{24 s^2} D_1(-t, -s, u) * \left\{ \begin{array}{l} c_1^2 f_{0A}^2 \\ (c_1 - c_0)^2 f_{0S}^2 \\ 2 c_1^2 f_{2S}^2 \end{array} \right\} * \delta_{qq'}$
$gg \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} g$	$\frac{\pi^2 \alpha_s^3}{64 s^2} D_2(s,t,u) * \left\{ \begin{array}{l} c_1^2 f_{0A}^2 \\ (c_1 - c_0)^2 f_{0S}^2 \\ 2 c_1^2 f_{2S}^2 \end{array} \right\} * \delta_{qq'}$

Table VIII - Prompt production of gluonium mesons
QCD - QED mixed processes

Process	$\frac{d\sigma}{dt}(s, t, u)$
$q\bar{q}' \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} \gamma$	$\frac{\pi^2 \alpha_s^2 \alpha_{em}}{9 s^2} * \left\{ \begin{array}{c} c_1^2 f_{0A}^2 \\ (c_1 - c_0)^2 f_{0S}^2 \\ 2 c_1^2 f_{2S}^2 \end{array} \right\} * D'_1(s, t, u) \delta_{qq'}$
$q\gamma \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} q'$	$\frac{\pi^2 \alpha_s^2 \alpha_{em}}{s^2} * \left\{ \begin{array}{c} c_1^2 f_{0A}^2 \\ (c_1 - c_0)^2 f_{0S}^2 \\ 2 c_1^2 f_{2S}^2 \end{array} \right\} * D'_1(-t, -s, u) \delta_{qq'}$
$g\gamma \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} \gamma$	0
$g\gamma \rightarrow G \begin{pmatrix} 0_A \\ 0_S \\ 2_S \end{pmatrix} g$	0

Table IX : $\pi\bar{p}$ Scattering at 350 GeV/c - Higher twist cross sections

Prompt meson produced	Purely hadronic final state (nb) Initial state			Prompt photon and meson (nb) all initial states	Total cross section (nb)
	$q\bar{q}$	qg and $\bar{q}g$	gg		
π^+	3.9	12.9	0.	$.15 \cdot 10^{-1}$	16.9
π^-	21.1	20.0	0.	$.84 \cdot 10^{-1}$	41.2
K^+	3.4	11.7	0.	$.15 \cdot 10^{-1}$	15.2
K^-	1.5	9.2	0.	$.58 \cdot 10^{-2}$	10.7
K^0	2.7	13.7	0.	$.94 \cdot 10^{-3}$	16.4
$\bar{K}^0$	.3	2.0	0.	$.79 \cdot 10^{-4}$	2.3
π^0	20.5	17.1	0.	$.36 \cdot 10^{-1}$	37.6
η	14.9	45.9	51.9	$.19 \cdot 10^{-1}$	112.7
η'	30.4	654.0	1320.0	$.17 \cdot 10^{-1}$	2004.4
ρ^+	151.0	153.0	0.	.60	304.0
ρ^-	791.0	271.0	0.	3.15	1065.2
K^{*+}	144.0	145.0	0.	.58	289.6
K^{*-}	55.7	120.0	0.	.23	175.9
K^{*0}	104.0	161.0	0.	$.32 \cdot 10^{-1}$	265.0
$\bar{K}^{*0}$	10.7	18.4	0.	$.26 \cdot 10^{-2}$	29.1
ρ^0	758.0	214.0	0.	1.13	973.0
ω	759.0	208.0	0.	1.13	968.0
φ	5.2	13.5	0.	$.13 \cdot 10^{-2}$	18.7
gluonium 2_S	4.4	547.0	1190.0	29.5	1770.9
gluonium O_A	2.2	277.0	588.0	15.0	882.2
gluonium O_S	.8	132.0	261.0	6.7	400.5

Table X : $\pi\pi^+$ Scattering at 350 GeV/c - Higher twist cross sections

Prompt meson produced	Purely hadronic final state (nb) Initial state			Prompt photon and meson (nb)	Total cross section (nb)
	$q\bar{q}$	qg and $\bar{q}g$	gg		
π^+	39.9	27.0	0.0	.16	67.2
π^-	1.6	6.5	0.0	.64 10^{-2}	8.1
K^+	4.3	17.7	0.0	.20 10^{-1}	22.0
K^-	.3	1.3	0.0	.11 10^{-2}	1.6
K^0	1.2	5.3	0.0	.5 10^{-3}	6.5
$\bar{K}^0$	1.3	9.6	0.0	.5 10^{-3}	10.9
π^0	12.7	15.5	0.0	.9 10^{-2}	28.2
η	9.0	47.2	47.8	.5 10^{-2}	104.0
η'	19.0	666.0	1330.0	.5 10^{-2}	2015.0
ρ^+	1490.0	354.0	0.0	6.0	1844.0
ρ^-	62.4	72.4	0.0	.25	135.0
K^{*+}	189.0	239.0	0.0	.76	428.9
K^{*-}	9.9	19.5	0.0	.42 10^{-1}	29.4
K^{*0}	57.9	67.8	0.0	.17 10^{-1}	125.7
$\bar{K}^{*0}$	55.0	122.0	0.0	.18 10^{-1}	177.0
ϕ	456.0	205.0	0.0	.31	661.3
ω	456.0	206.0	0.0	.31	662.3
φ	4.7	15.3	0.0	.13 10^{-2}	20.0
gluonium 2_S	2.8	548.0	1190.0	9.68	1750.5
gluonium O_A	1.2	282.0	595.0	4.77	883.0
gluonium O_S	.6	130.0	266.0	2.08	398.7

Table XI : pp Scattering at 350 GeV/c - Higher twist cross sections

Prompt meson produced	Purely hadronic final state (nb) Initial state			Prompt photon and meson (nb)	Total cross section (nb)
	q $\bar{q}$	qg and $\bar{q}g$	gg	all initial states	
π^+	11.8	25.1	0.	.46 10^{-1}	36.9
π^-	3.6	12.1	0.	.17 10^{-1}	15.7
K^+	3.6	23.5	0.	.14 10^{-1}	27.1
K^-	.2	2.5	0.	.70 10^{-3}	2.7
K^0	1.4	9.8	0.	.43 10^{-3}	11.2
$\bar{K}^0$	.2	2.5	0.	.46 10^{-4}	2.7
π^0	8.0	16.9	0.	.83 10^{-2}	24.9
η	5.8	52.0	59.2	.43 10^{-2}	117.0
η'	13.0	724.0	1610	.40 10^{-2}	2347.0
ρ^+	440.0	314.0	0.	1.74	755.7
ρ^-	174.0	138.0	0.	.68	312.7
K^{*+}	141.0	307.0	0.	.56	448.6
K^{*-}	6.8	21.6	0.	.28 10^{-1}	28.4
K^{*0}	53.5	122.0	0.	.15 10^{-1}	175.5
$\bar{K}^{*0}$	7.0	23.5	0.	.17 10^{-2}	30.5
ρ^0	295.0	221.0	0.	.29	516.3
ω	296.0	215.0	0.	.29	511.3
ϕ	1.8	8.4	0.	.53 10^{-3}	10.2
gluonium 2_S	1.4	603.0	1460.	10.1	2074.5
gluonium O_A	.7	302.0	718.	4.96	1025.7
gluonium O_S	.3	139.0	317.	2.15	458.5

Table XII : $\bar{p}p$ Scattering at 350 GeV/c - Higher twist cross sections

Prompt meson produced	Purely hadronic final state (nb) Initial state			Prompt photon and meson (nb) all initial states	Total cross section (nb)
	$q\bar{q}$	qg and $\bar{q}g$	gg		
π^+	27.8	20.4	0.	.11	48.3
π^-	27.7	17.8	0.	.11	45.6
K^+	1.8	14.5	0.	$.76 \cdot 10^{-2}$	16.3
K^-	1.9	15.7	0.	$.75 \cdot 10^{-2}$	17.6
K^0	.8	6.0	0.	$.25 \cdot 10^{-3}$	6.8
$\bar{K}^0$	.8	6.9	0.	$.25 \cdot 10^{-3}$	7.7
π^0	35.8	19.8	0.	$.58 \cdot 10^{-1}$	55.6
η	25.5	54.3	59.0	$.30 \cdot 10^{-1}$	138.8
η'	54.8	722.0	1560.0	$.28 \cdot 10^{-1}$	2336.8
ρ^+	1050.0	242.0	0.	4.20	1296.2
ρ^-	1050.0	237.0	0.	4.20	1291.2
K^{*+}	72.2	171.0	0.	.30	243.5
K^{*-}	73.0	174.0	0.	.30	247.3
K^{*0}	30.7	81.6	0.	$.84 \cdot 10^{-2}$	112.3
$\bar{K}^{*0}$	30.5	73.4	0.	$.83 \cdot 10^{-2}$	103.9
ρ^0	1350.0	232.0	0.	1.83	1583.8
ω	1350.0	234.0	0.	1.83	1585.8
φ	2.4	9.3	0.	$.57 \cdot 10^{-3}$	11.7
gluonium 2_S	7.6	602.0	1400.0	50.6	2060.2
gluonium O_A	3.8	304.0	696.0	25.1	1028.9
gluonium O_S	1.7	142.0	307.0	11.5	462.2

Table XIII - π^+p Collision. Fragmentation vs Higher twist

High- p_T meson ($p_T \gg 2.5 \text{ GeV}/c$)	π^-p initial state Production through		π^+p initial state Production through	
	fragmentation	Higher twist	Fragmentation	Higher twist
π^+	622.8	50.3	875.8	117.1
π^-	808.8	88.4	411.9	32.6
K^+	245.4	21.5	233.5	18.2
K^-	224.3	7.3	122.3	11.0
K^0	253.3	16.1	156.3	14.9
$\bar{K}^0$	153.1	10.7	167.3	12.0
π^0	708.5	63.0	622.3	98.4
η	357.6	36.6	315.7	42.2
η'	368.1	468.6	330.7	446.4
ρ^+	564.7	77.1	895.9	551.0
ρ^-	803.5	318.3	339.7	39.4
K^{*+}	230.9	85.9	264.5	112.8
K^{*-}	207.1	47.4	98.2	10.6
K^{*0}	241.5	65.4	140.3	30.2
$\bar{K}^{*0}$	134.6	4.9	180.4	55.2
ρ^0	692.7	335.4	641.3	210.2
ω	723.0	295.3	631.3	201.6
φ	50.1	5.9	41.1	4.8

Table XIV - pp, $\bar{p}p$ collisions. Fragmentation vs higher twist

High meson ($p_T \geq 2.5 \text{ GeV}/c$)	pp initial state Production through		$\bar{p}p$ initial state Production through	
	fragmentation	Higher twist	Fragmentation	Higher twist
π^+	829.5	50.0	829.5	128.6
π^-	872.2	37.0	869.1	123.4
K^+	283.6	20.4	278.5	16.9
K^-	279.0	7.0	287.2	14.9
K^0	198.2	15.2	202.2	13.0
$\bar{K}^0$	215.0	4.4	206.6	15.6
π^0	911.8	33.9	920.4	112.4
η	375.1	35.2	367.9	50.0
η'	422.4	536.0	423.5	525.5
ρ^+	765.4	194.5	785.5	359.2
ρ^-	741.0	80.1	737.2	375.4
K^{*+}	282.1	130.1	285.8	68.2
K^{*-}	240.9	12.6	246.2	61.7
K^{*0}	172.3	48.3	174.4	26.6
$\bar{K}^{*0}$	176.9	7.8	171.5	26.6
ϕ	824.9	186.7	816.3	517.0
ω	799.0	128.8	791.4	463.8
φ	38.1	0.4	38.1	7.1

Table XV - γp and γn scattering at $E_\gamma = 150$ GeV
Higher twist cross sections

Prompt meson	γp scattering cross sections (nb)	γn scattering cross section (nb)
π^+	1.53	.97
π^-	1.28	2.22
K^+	1.12	.60
K^-	.23	.24
K^0	.09	.14
$\bar{K}^0$	.02	.02
π^0	.41	.28
η	.20	.14
η'	.09	.09
ρ^+	10.30	6.63
ρ^-	10.00	18.20
K^{*+}	7.36	4.24
K^{*-}	1.69	1.62
K^{*0}	1.07	2.09
$\bar{K}^{*0}$	.31	.31
ρ^0	5.49	3.85
ω	5.49	3.76
φ	.13	.11
Gluonium 2_S	.53	.41
Gluonium 0_A	.32	.27
Gluonium 0_S	.19	.11

Table XVI γp and γn cross sections at $E_\gamma = 150 \text{ GeV/c}$
Minimum twist versus higher twist

High- p_T meson ($p_T > 2.5 \text{ GeV/c}$)	γp Scattering cross sections (nb)		γn Scattering cross sections (nb)	
	fragmentation	Higher twist	Fragmentation	Higher twist
π^+	4.19	1.25	3.57	.69
π^-	3.67	1.10	3.59	1.76
K^+	1.66	.66	1.49	.45
K^-	1.35	.12	1.23	.13
K^0	1.51	.41	1.23	.29
$\bar{K}^0$	1.04	.11	1.18	.09
π^0	3.84	1.10	3.26	1.11
η	1.89	.12	1.61	.11
η'	2.29	.12	1.78	.15
ρ^+	4.30	3.06	3.00	1.96
ρ^-	3.06	2.97	3.44	5.77
K^{*+}	1.95	2.52	1.26	1.33
K^{*-}	1.34	.45	1.17	.57
K^{*0}	.97	.27	1.17	.66
$\bar{K}^{*0}$	.85	.16	.77	.11
ρ^0	3.90	1.60	3.17	1.07
ω	4.04	1.41	3.48	1.01
φ	0.40	.07	.36	.05

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FIGURE CAPTIONS

- Figure 1 :** Schematic figuration of the factorization of the prompt meson production full amplitude. The white blob figures $\hat{\sigma}(k_1, k_2, k_3, p_1, p_2)$, the dashed one $\Gamma_R(p_1, p_2, p)$.
- Figure 2 :** Full set of the Feynman diagrams of the $q\bar{q}' \rightarrow (q\bar{q}')g$ process. The first five diagram defines the set A, the last two the set B.
- Figure 3 :** Full set of the Feynman diagrams of the $gg \rightarrow (q\bar{q})g$ process.
- Figure 4 :** Full set of the Feynman diagrams of the $q\bar{q} \rightarrow (gg)g$ process. The gluonium is built up from the gluons numbered 1 and 2.
- Figure 5 :** Full set of the Feynman diagrams of the $gg \rightarrow (gg)g$ process. The gluonium is built up from the gluons numbered 1 and 2.
- Figure 6 :** Full set of the Feynman diagrams of the $q\bar{q}' \rightarrow (q\bar{q}')\gamma$ process.
- Figure 7 :** Full set of the Feynman diagrams of the $gg \rightarrow (q\bar{q})\gamma$ process.
- Figure 8 :** Full set of the Feynman diagrams of the $q\bar{q} \rightarrow (gg)\gamma$ process.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 5
(cont.)

Fig. 6

Fig. 7

Fig. 8