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LOW ENERGY SUPERGRAVITY AND SUPERSTRING-INSPIRED MODELS

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ABSTRACT

We study the process of gauge symmetry breaking in superstring-inspired models with two, one and no extra Z^0 gauge bosons. We present analytic expressions for the solutions of the renormalization group equations for the masses and couplings of an arbitrary extra $U(1)$ model. A specific model with an extra Z^0 (the only one consistent with a heavy ν_R) is studied numerically and results for the sparticle spectrum, as well as for the constraints amongst parameters, are given. We also consider a minimal "stringy" model with no extra Z^0 's. Consistent gauge symmetry breaking in this model requires the t-quark mass $m_t \gtrsim 70$ GeV, the sneutrino (or a Higgsino) being very often the lightest SUSY particle. We argue that the models with an extra Z^0 , (although viable), are rather artificial and emphasize the importance of obtaining compactification schemes leading to rank = 4 low energy groups.

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1. - INTRODUCTION

In the last year, there has been a lot of effort in trying to make contact between superstrings^{1),2)} and low-energy phenomenology³⁾⁻²⁷⁾. It has been shown that under certain circumstances one may compactify the heterotic $E_8 \times E_8$ superstring²⁾ to obtain some extension of the standard model coupled to $N = 1$ supergravity³⁾. This fits nicely with the "low-energy supergravity" versions of the standard model worked out in the last few years²⁸⁾⁻³²⁾. In these models supersymmetry breaking takes place in a "hidden sector" of the theory²⁹⁾ and it is transmitted to the observable quark-lepton world only through gravitational interactions. One is left at low energies with a theory with softly broken supersymmetry with soft parameters m (universal scalar masses), M (universal gaugino masses) and a trilinear scalar coupling proportional to a dimensionless constant A -times m .

Interestingly enough, the $E_8 \times E_8$ string has already a "hidden sector" (the hidden E_8 + some singlets) built in, which can trigger $N = 1$ supersymmetry breaking dynamically if gaugino condensation^{8),10)} occurs in the extra E_8 . Let us however note, as we will point out in Section 2 in more detail, that the supersymmetry breaking mechanism is still not well understood^{23),26)} at the moment. The effective low-energy theory obtained is a sort of truncated version⁵⁾ of an E_6 model, i.e., the gauge interactions are a subgroup of E_6 and the chiral superfields are truncations of fundamental 27 's of E_6 . In order to obtain reasonable results for $\sin^2 \theta_W$ and the unification scale, one considers as candidate effective gauge theories

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times (U(1))^2 \quad (1.a)$$

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1) \quad (1.b)$$

$$SU(3)_c \times SU(2)_L \times U(1)_Y \quad (1.c)$$

i.e., models with two, one or no extra Z_0 's but no non-Abelian extensions of these^{7),12)}. We discuss the general form and quantum numbers of these rank = 6, 5 and 4 models in Chapter 3. In the present paper, we present a detailed study of these "string-inspired" models with special emphasis on the process of gauge symmetry breaking through radiative corrections. In Chapter 4, the general model with an extra $U(1)$ interaction is discussed and we give results for the solutions of the renormalization group equations for mass and coupling parameters. A specific extra Z^0 model, the only one consistent with a superheavy right-handed

neutrino, is studied numerically in Section 5. The minimal "stringy" model involving just the standard gauge interactions (l.c) is discussed in Chapter 6. We show our conclusions in Chapter 7.

2. - SUPERSYMMETRY BREAKING

Two possible sources of $N = 1$ supersymmetry breaking have been discussed in the literature in the context of superstring models. A vacuum expectation value (VEV) for the antisymmetric³³⁾ tensor $\langle H_{ijk} \rangle = c \xi_{ijk}$ (which can be seen as a constant superpotential in $d = 4$) gives rise to supersymmetry breaking⁸⁾⁻¹⁰⁾. This generates SUSY-breaking low-energy soft terms^{8),12)}:

$$M = m_{3/2} \quad ; \quad m_{3/2} W(C_x) \quad (2)$$

where M are gaugino masses and $W(C_x)$ is the low energy superpotential of the observable fields C_x . The gravitino mass^{*}) is $m_{3/2} = |c| / (4S^{1/2} T^{3/2})$ and one expects $|c|$ to be a small number compared to M_P so that the "gauge hierarchy" is preserved. Such a small value for $\langle H_{ijk} \rangle$ could come from a small miscancellation¹²⁾ of Yang-Mills and Lorentz-Chern-Simons symbols:

$$H_{ijk} \sim \omega_{ijk}^{(YM)} - \omega_{ijk}^{(L)} \sim c \xi_{ijk} M_P^3 \quad (3)$$

or else from just a VEV of the low-energy superpotential¹²⁾. The other possible source of SUSY breaking is gaugino condensation^{8),10)} in the hidden sector of the theory. This second mechanism is appealing since it can potentially generate a small scale dynamically. It also gives rise to soft SUSY-breaking terms at low energies. One problem of both SUSY-breaking mechanisms is that they generate a large cosmological constant. However, the situation changes if one considers simultaneously both mechanisms of supersymmetry breaking. It was pointed out¹⁰⁾ that H_{mnp} and the relevant gaugino χ condensate appear in the Lagrangian in the form of a perfect square:

$$\varphi^{-3/2} \left[H_{mnp} + \frac{i\sqrt{2}}{24} \varphi^{3/4} \text{Tr}(\bar{\chi} \Gamma_{mnp} \chi) \right]^2 \quad (4)$$

^{*}) S and T are the dilaton fields introduced in Ref. 4).

This allows for the possibility of cancelling both contributions to the cosmological constant against each other and still having broken supersymmetry. This cancellation could take place in two ways. One possibility which we find particularly interesting is one in which the gauge coupling constants and grand unification scales were determined upon compactification. Then, the gaugino condensate $\langle \bar{\chi}\chi \rangle$ scale is fixed and H_{ijk} adjusts itself to cancel it. This possibility, suggested in Ref. 12), has the advantage that the small scale is determined dynamically and is set by the size of the condensation scale. The other possibility is to assume an input VEV for H_{ijk} but an undetermined value for the gauge coupling constant and compactification scales (or $\langle S \rangle$ and $\langle T \rangle$). Then the condensate itself runs to cancel the VEV of H_{ijk} . This situation is easily realized¹⁰⁾ in the truncation of Ref. 4) but has the disadvantage that the SUSY-breaking scale is fixed by hand when one sets a value for $\langle H \rangle = c$.

Whichever of these two mechanisms (if any) is at work, the cancellation of the bracket in Eq. (4) means that one has no soft SUSY-breaking terms in four dimensions

$$A = M = m = 0 \quad (5)$$

where M and m refer to observable gaugino and squark-slepton masses. Of course, this is only true at the tree level and one expects that soft SUSY-breaking terms should appear radiatively. It turns out that the required soft terms are not so easy to generate, at least if one only uses the effective low-energy Lagrangian of Refs. 4) and 10). If one starts from $A = M = 0$ the effective theory has an R -symmetry which forbids non-vanishing A or M to be generated. In this connection, we disagree with the mechanism proposed in Ref. 20) for generating a gaugino mass. The observable sector radiative corrections (such as, e.g., those corresponding to the Georgi-Quinn-Weinberg result) do not induce any new dependence of the gauge function f_{ab} on any (not initially present) chiral field. This is because f_{ab} is an analytic function of chiral fields and the radiative corrections in the present case always involve non-analytic combinations of chiral fields. Masses for observable scalars are not generated²³⁾⁻²⁵⁾ at one loop (at least in the truncated model we are discussing), but do have contributions²⁴⁾ at two loops. However, this source for SUSY breaking would be in general problematic since it gives rise to squark-slepton masses

$$m_{\tilde{q}, \tilde{\ell}} \sim h_{q, \ell} \frac{m_{3/2}}{(4\pi)^2} \quad (6)$$

and hence the soft scalar masses would not be universal but proportional to each fermion partner's mass. This would be a disaster for the suppression of flavour-changing neutral currents (FCNC). In general, if the origin of soft SUSY-breaking terms comes only from couplings involving the superpotential, universal soft terms are not expected. Thus although one obtains SUSY-breaking soft terms in higher loops from the truncated Lagrangian of Ref. 4), the phenomenological prospects do not seem very promising.

The situation changes if one includes the effect of one-loop terms coming from the exchange of heavy string modes (see Fig. 1). Such types of one-loop terms are the "Wess-Zumino" couplings¹⁾ which one has to add to the ten-dimensional supergravity + Yang-Mills Lagrangian in order to obtain a gauge and Lorentz-covariant action. These terms introduced by Green and Schwarz¹⁾ require the existence of additional ten-dimensional terms so that the effective $d = 4$ theory is supersymmetric²⁶⁾. The truncated Lagrangian of Ref. 4) gets modifications coming from these one-loop couplings. Amongst those modifications one obtains different gauge kinetic terms for hidden and observable sectors^{12), 26)} so that even if the minimization conditions of the scalar potential require a vanishing hidden-gaugino mass M_8 , there is still an observable-gaugino mass²⁶⁾

$$M_6 \sim \frac{m_{3/2}}{(2\pi)^5} \quad (7)$$

Under certain conditions²⁶⁾, one still gets a vanishing cosmological constant and $m, A = 0$. This provides us with an explicit mechanism which transmits SUSY breaking from the hidden to the observable sector. It has the attractive feature of relating this transmission to the cancellation of anomalies in ten dimensions.

From the above discussion, it seems that plausible values for the soft SUSY-breaking terms at the compactification scale would be $M_6 \approx (2\pi)^{-5} m_{3/2}$, $m \approx 0$, $A \approx 0$. However, we think that until the complete subject of residual SUSY breaking is better understood, it is premature to stick to any specific ansatz for those parameters. Even if the mechanism for transmitting SUSY-breaking to the observable sector is the one discussed above, only for certain values of the coefficients in the $D = 10$ local terms, does one get $m = A = 0$ and a vanishing cosmological constant²⁶⁾. Furthermore, there are indications that the effective potential of the "T" field is unbounded below^{24), 25)} and hence it is quite possible that non-perturbative effects may be important in the determination of the soft SUSY-breaking parameters. In this situation we think that the only sensible attitude at the moment is to assume general SUSY-breaking parameters for the effective low-energy Lagrangian. This is what we will do in the rest of the paper although, of course, we will keep in mind the plausibility of the

parameters discussed above. On the other hand, we will see that consistent radiative gauge symmetry breaking will restrict the possible values of M , m and A allowed. Interestingly enough, a situation with $M \ll m$ is usually inconsistent with appropriate radiative breaking of residual gauge symmetries (unless the top quark mass is rather large).

3. - LOW ENERGY SUPERSTRING-INSPIRED MODELS

It is well known that the simple identification of $SU(3)$ Lorentz and gauge connections give rise to a breaking $E_8 \times E'_8 \rightarrow E_6 \times E'_8$ in the heterotic string³⁾. The group E_6 (and in fact also E'_8) can be further broken if the compact manifold is not simply connected and non-trivial configurations of the E_6 gauge fields occur^{3),5)}. This can break E_6 to subgroups such as, for example, $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)^2$ of rank 6. If the compact manifold has a certain non-Abelian discrete symmetry acting freely, one can break the symmetry down to the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'''$ where the quantum numbers of the $U(1)'''$ will be described below⁵⁾. These are not the only possible low-energy groups, since one can also obtain other rank 5 and 4 examples. This may be done by using the chiral fields in the $\underline{27} + \overline{\underline{27}}$ representations which models usually have^{7),11),12)}. A VEV for them can lower the rank from 6 down to 5 or 4. This symmetry breaking can occur at an intermediate scale (e.g., $\sim 10^{10}$ GeV) by radiative corrections. Recently, it has been observed³⁴⁾ that there exists a class of compact manifolds in which this symmetry breaking may occur at the compactification scale already. This allows for the breaking of E_6 down to rank 4 or 5 groups like $SU(5)$ or $SO(10)$, still maintaining an unbroken supersymmetry. Then the usual Wilson loop mechanism can give rise to, e.g., the standard model^{34),35)} $SU(3)_C \times SU(2)_L \times U(1)_Y$ directly.

Since it seems that one may obtain low energy groups with rank 4, 5 and 6, it is useful to discuss what extra gauge interactions (if any) one may have at low energies. We are going to confine ourselves to models involving only (at most) extra $U(1)$'s since non-Abelian generalizations of these lead in general to problems^{7),12)} with the predictions for $\sin^2 \theta_W$ and M_X . Extra $U(1)$ interactions are conveniently analyzed in terms of the diagonal generators^{5),13)} of the $SU(3)_C \times SU(3)_L \times SU(3)_R$ subgroup of E_6 . Apart from the Cartan subalgebra of the $SU(3)_C \times SU(2)_L$ group, one has the extra diagonal generators

$$\begin{aligned}
 T_L &= \begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}_L \\
 T_R &= \frac{1}{2} \begin{pmatrix} -2 & & \\ & 1 & \\ & & 1 \end{pmatrix}_R \\
 T_N &= \frac{1}{2} \begin{pmatrix} 0 & & \\ & 1 & \\ & & -1 \end{pmatrix}_R
 \end{aligned} \tag{8}$$

The quantum numbers of the fields in a 27 of E_6 with respect to these generators are shown in Table 1. This table shows also our notation for the quark, lepton and Higgs superfields. One can write for the usual hypercharge generator

$$Y = \sqrt{\frac{3}{5}} \left(\frac{2}{3} T_R - \frac{1}{6} T_L \right). \tag{9}$$

Any combination of T_L , T_R and T_N orthogonal to Y is in principle a candidate for extra $U(1)$ and some candidates are shown in Table 1.

Before discussing the different possibilities, let us consider the Yukawa couplings expected in the low-energy theory. The superpotential may in general contain⁵⁾ any term present in the $(27)^3 E_6$ coupling, although the Yukawa couplings will not, in general, obey any E_6 relationship since the Wilson-loop breaking alters those. Under these conditions the most general superpotential originated in the $(27)^3$ coupling will be

$$\begin{aligned}
 W = \sum_{\text{families}} & h_L H L e_L^c + h_d H Q d_L^c + h_u \bar{H} Q u_L^c + \\
 & + h_\nu \bar{H} L \nu_L^c + h'_\nu D d_L^c \nu_L^c + \\
 & + \lambda_2 N H \bar{H} + \lambda_3 N D \bar{D} + \\
 & + \lambda_L L Q \bar{D} + \lambda'_L e_L^c u_L^c D + \\
 & + \lambda_B Q Q D + \lambda'_B u_L^c d_L^c \bar{D}
 \end{aligned} \tag{10}$$

It is important to remark at this point that the names we have given to the fields in Table 1 and Eq. (10) are only tentative. Which fields correspond to the observed quarks and leptons will depend on what couplings are present or absent in Eq. (10) and also on what are the physical Higgses which break

SU(2)×U(1). In particular, from the point of view of the standard model one can make the replacements

$$\begin{aligned} d_L^c &\longleftrightarrow \bar{D} \\ L &\longleftrightarrow H \\ \nu_L^c &\longleftrightarrow N \end{aligned} \quad (11)$$

and nothing changes. The identification with the physical d_L^c , L and ν_L^c is only done "a posteriori". Let us assume without loss of generality that the assignments of Table 1 are correct. It is well known that if all the couplings in (10) are present, several phenomenological disasters may occur:

- a) The $D, \bar{D}$ fields mediate fast proton decay unless $\lambda_B = \lambda_B' = 0$ and/or $\lambda_L = \lambda_L' = 0$ (or those fields are heavier than $\sim 10^{10}$ GeV).
- b) There are unacceptable Dirac neutrino masses unless $h_\nu = 0$ (or the ν_L^c 's have Majorana masses bigger than ~ 10 TeV).
- c) Unless only one of the three sets of Higgses $H+\bar{H}$ couples to quarks and leptons, tree-level FCNC will appear (or the extra two Higgses are heavy enough).

One also has to be careful that the $\tilde{\nu}_L^c$'s do not get VEVs, since (if $h_\nu' \neq 0$) the GIM mechanism would be ruined and FCNC would again appear. This latter condition is easy to fulfil in practice.

All these problems become less severe the smaller the rank of the low-energy group is. For a rank = 6 model, the only possibility is to assume that some symmetries exist⁵⁾ forbidding the dangerous couplings, since the gauge symmetries do not allow us to give mass to any of the dangerous particles. In rank = 5 models, one may get rid of some of the unwanted particles ($D, \bar{D}, H, \bar{H}$ or ν_L^c 's) as we will discuss below, so that one only needs to forbid some of the couplings. Finally, in rank 4 models the possibility exists that no dangerous particles at all remain in the low energy spectrum. Still one has to be sure that Weinberg-Salam (WS) doublets remain light. We now consider these three cases in turn^{7),12),13),16)}.

i) Rank = 6 models

As we mentioned above we consider only models with just extra $U(1)$'s and not any non-Abelian extension. The unique gauge group of this type is

$$G_6 = SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'' \times U(1)''_L \quad (12)$$

where the two additional $U(1)$'s can be any of the ones inside E_6 which commute with $SU(5)$. There is no physical distinction between any possible choice as long as the gauge coupling constants are equal at the compactification scale. The example chosen in (12) corresponds to the extra $U(1)$ inside $SO(10)$ commuting with $SU(5)$ [$U(1)''$ in Table 1] and the $U(1)$ inside E_6 commuting with $SO(10)$ [$U(1)''_L$ in Table 1]. In this model all of the 27 states in a fundamental of E_6 remain light since they all are chiral under the G_6 group. Thus one has to assume there is some symmetry forbidding dangerous couplings. Even assuming that these symmetries are present, the triple symmetry breaking

$$SU(2)_L \times U(1)_Y \times U(1)'' \times U(1)''_L \longrightarrow U(1)_{e.m.} \quad (13)$$

seems complicated to obtain. In principle, as discussed in Refs. 12) and 16), radiative corrections may induce non-vanishing VEVs for some $\langle H \rangle \neq 0$, $\langle \bar{H} \rangle \neq 0$, $\langle \tilde{\nu}_L^c \rangle \neq 0$ and $\langle N \rangle \neq 0$. However, several diseases will in general appear. A VEV for a $\tilde{\nu}_L^c$ will give rise to d_L^c -D and L-H mixing through the h_ν and h'_ν couplings. This would ruin the GIM mechanism. Also, a VEV for ν_L^c would break lepton number conservation. It also in general induces a VEV for the left-handed s-neutrino $\tilde{\nu}_L$ through the coupling in the potential $\lambda_2^* h_\nu (L \nu_L^c)(NH) + h.c.$, since N , H and ν_L^c have VEVs. All these problems, along with the fact that a good number of symmetries forbidding certain Yukawa couplings are needed, make this model quite contrived. We will not consider it here any longer.

ii) Rank = 5 models

Here we consider models whose low-energy group (i.e., gauge symmetry not very far above the weak scale) is of the form^{13),14),16),17),19)}

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1) \quad (14)$$

There are several ways in which the rank of the gauge group can be reduced to five, as we discuss below. The extra $U(1)$ may be any of the linear combinations of T_L , T_R and T_N which commute with the hypercharge. The more general form for it will be:

$$Y_{\text{extra}} = (\text{constant}) (T_L + T_R + d T_N) \quad (15)$$

as one can easily check. Thus, there is a one-parameter (d) family of possible extra $U(1)$'s. There are, however, three cases of special physical interest which correspond to the hypercharges Y' , Y'' , and Y''' in Table 1.

- Y' hypercharge¹⁹⁾. It is defined as the one under which the ν_L^c field is inert, $Y'(\nu_L^c) = 0$. It is given by

$$Y' = \frac{1}{2\sqrt{10}} (T_L + T_R - 5 T_N) \quad (16)$$

where the normalization is the same as the usual hypercharge. This charge is physically interesting since it allows for a solution of the neutrino mass problem: a large Majorana mass for the ν_L^c 's is allowed by the gauge symmetries. Some of the mechanisms discussed in Ref. 12) may be the source of these ν_L^c masses or they may be present to start with if the model is obtained as in Ref. 34). An unbroken gauge symmetry $U(1)'$ is obtained¹⁹⁾ starting from a rank = 6 group if a field N_R with the quantum numbers of a ν_L^c gets a VEV, breaking the orthogonal symmetry $U(1)'$:

$$Y'_\perp = \frac{1}{2\sqrt{6}} (T_L + T_R + 3 T_N) \quad (17)$$

In this rank = 5 model 26 states per family remain light. In fact, it can be obtained after Wilson-loop breaking of an $SO(10)$ model with fermion content $3 \times (\underline{16} + \underline{10})$. However, the embedding of a standard family is unusual in that $\underline{16} = (Q_L, U_L^c, e_L^c; H, \bar{D}, N)$, $\underline{10} = (L, d_L^c; \bar{H}, D)$. Thus N does not behave like a right-handed neutrino (it is harmless) and L, d_L^c are inside a $\underline{10}$ and not inside a $\underline{16}$. These assignments are as natural as the standard $SO(10)$ ones, but have the advantage that there is no neutrino mass problem.

Another interesting property of this $U(1)'$ is that all known quark and lepton superfields have $U(1)'$ assignments with the same sign as the field N .

As a result, when N eventually gets a VEV to break $U(1)'$, all squarks and sleptons get a large contribution to their masses from the D^2 term. This helps in understanding why, for example, H gets a VEV but L does not.

- Y'' hypercharge. This is the one under which the field N is inert, $Y''(N) = 0$. It is given by

$$Y'' = \frac{1}{2\sqrt{10}} (T_L + T_R + 5 T_N) \quad (18)$$

In fact, this is obtained from Y' by changing $T_N \rightarrow -T_N$ and just corresponds to $U(1)'$ after you redefine your fields $(d_{L,L}^c, v_L^c) \leftrightarrow (\bar{D}, H, N)$. $U(1)''$ is the $U(1)$ inside $SO(10)$ which commutes with $SU(5)$ and the fermion content is also $3 \times (\underline{16} + \underline{10})$. However, in this case we have the standard embedding $\underline{16} = (Q_L, u_L^c, e_L^c, d_{L,L}^c, v_L^c)$, $\underline{10} = (H, \bar{D}; \bar{H}, D)$. This model can be obtained from a rank = 6 model if a field with the quantum numbers of N gets a VEV breaking the orthogonal combination

$$Y''_{\perp} = \frac{1}{2\sqrt{6}} (T_L + T_R - 3 T_N) \quad (19)$$

This may happen radiatively as discussed in Refs. 11), 12) and 15). Alternatively, it may be obtained from an $SO(10)$ model after Wilson-loop breaking. Models with $SO(10)$ low energy group may be obtained under certain conditions^{34), 35)}, as we mentioned above.

A model with a Y'' hypercharge is only physically distinguishable from a Y' model if you destroy the symmetry $T_N \rightarrow -T_N$ in the low-energy spectrum. This is what is done in the models with an intermediate symmetry breaking scale ($\langle N \rangle \sim 10^{10-14} \text{ GeV}$) of Refs. 7), 11), 12) and 15). In those models a Higgs field N breaks $Y''_{\perp}$ and at the same time gives masses to the unwanted D , $\bar{D}$, H and $\bar{H}$ fields so that one is left at low energies with three standard $SO(10)$ families, $(Q_L, u_L^c, e_L^c, d_{L,L}^c, v_L^c)$. In this way we may get rid of the fast proton decay problem since the D , $\bar{D}$ are now superheavy. The low energy superpotential is then just

$$W'' = \sum_{\text{families}} h_L H L e_L^c + h_d H Q d_L^c + h_u \bar{H} Q u_L^c + \\ + h_\nu \bar{H} L \nu_L^c + \lambda_2 N H \bar{H} \quad (20)$$

where we obviously assume that a couple of WS doublets H , $\bar{H}$ remain light.

Unfortunately, in order to forbid Dirac neutrino masses, we have to set $h_v = 0$ and the v_L^c fields are left with no Yukawa couplings at all. Then it is hard to see how $\tilde{v}_L^c$ may get a negative (mass)² and break U(1)". Furthermore, even if one could maintain a $(\bar{H}v_L^c)$ coupling (maybe for the third or fourth generation), the soft term $m A(\bar{H}\tilde{v}_L^c)$ will induce a VEV for the left-handed sneutrino $\tilde{v}_L$. Then, lepton number would be broken leading to possible phenomenological problems. The chances of getting an unwanted VEV for $\langle \tilde{v}_L \rangle$ are increased by the structure of the D^2 term in the scalar potential

$$\left(\frac{D''}{2}\right)^2 = \frac{g''^2}{2} (3|L|^2 - 2|H|^2 + 2|\bar{H}|^2 - 5|\tilde{v}_L^c|^2)^2 \quad (21)$$

A VEV for v_L^c gives a larger negative mass to L than to $\bar{H}$. Although one might attempt to live with lepton number broken, the whole scheme seems a bit contrived.

- Y''' hypercharge⁵⁾. It is just the U(1) generator orthogonal to Y and T_N

$$Y''' = \frac{1}{\sqrt{15}} (T_L + T_R) \quad (22)$$

The U(1)''' quantum numbers of the different particles are shown in Table 1. All the particles are chiral under this U(1)''' so that the complete 27 states must remain light^{14),27)}. This model can be obtained from E_6 through Wilson-loop breaking. If there is a non-Abelian discrete symmetry in the compact manifold which is mapped into the $SU(2)_N$ subgroup inside $SU(3)_R$, a non-trivial gauge configuration may directly give rise to the gauge group⁵⁾

$$SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)''' \quad (23)$$

Unlike Y' and Y'' , Y''' is typical of an E_6 model and cannot be obtained from an initial $SO(10)$ symmetry. Another typical property is that the fields (d_L^c, L, v_L^c) and $(\bar{D}, H, N)$ have respectively the same gauge quantum numbers. This is in fact the only extra U(1) for which this is the case (because it is the only one orthogonal to T_N). This extra U(1)''' is an interesting U(1), the only problem being that one has to rely on the existence of appropriate symmetries forbidding all of the disasters a)-c) discussed above. Particularly, one has to set the couplings $h_v = 0$ to avoid Dirac neutrino masses. The right-handed neutrinos are then

massless and this may have some cosmological problems, since six massless neutrinos are probably too much for consistent nucleosynthesis.

Apart from these three physically interesting $U(1)$'s, there is a family of other¹³⁾ possible $U(1)$'s as one varies the parameter d in Eq. (15). They may be obtained, e.g., by giving VEVs both to N_R and N in a rank = 6 model. Then, depending on the size of the VEVs, one gets an unbroken $U(1)$:

$$Y_{extra} = \cos \alpha Y' + \sin \alpha Y'' , \quad \tan \alpha = N/N_R \quad (24)$$

The $U(1)'$, $U(1)''$ and $U(1)'''$ above correspond to the values $\alpha = 0, \pi/2, \pi/4$. The new possibilities in (24) can only come from an E_6 [not $SO(10)$] initial symmetry and have no special advantage over the Y''' case so that we will not consider them any longer here. As we have discussed above, the models based on $U(1)'$ and $U(1)'''$ seem to be the least contrived. The first one especially, $U(1)'$, has the advantage of being the only string model which is consistent with the Gell-Mann-Ramond-Slansky-Yanagida³⁶⁾ solution of the neutrino mass problem. In the $U(1)'''$ case, one has to forbid the h_ν neutrino couplings and one really loses an understanding of why the observed neutrino is massless and any other quark or lepton is not. That is why we will explore in more detail numerically the model $U(1)'$ in Chapter 5. However, we will consider the general extra $U(1)$ case in the next chapter so that the equations can be easily employed for any extra $U(1)$.

iii) Rank = 4 models. Witten³⁴⁾ has recently shown the possibility of having compact manifolds of $SU(3)$ holonomy leading to unbroken supersymmetry and $SO(10)$ or $SU(5)$ gauge groups. The possibility of getting $SU(5)$ is specially interesting, since in such a case one might have models with none of the problems (p-decay, ν -masses, FCNC) we discussed above without invoking opportune symmetries. The symmetry can be further broken to $SU(3)_C \times SU(2)_L \times U(1)_Y$ through the Wilson loop mechanism^{34),35)}. One still has to make sure that we have a set of light WS Higgses $H+\bar{H}$ to break $SU(2) \times U(1)$, but that no dangerous $SU(5)$ partners $\bar{D}+D$ triplets remain light. This looks problematic at first sight, since any non-trivial gauge configuration will be proportional to the hypercharge operator Y and $H+\bar{H}$ have non-vanishing hypercharge. However³⁵⁾, if $H+\bar{H}$ come from some $\underline{5}+\bar{\underline{5}}$ not related to the existence of the Kähler form ($b_{1,1} > 2$), they may have additional transformation properties in such a way that [in the notation of Ref. 5)] $H+\bar{H}$ are invariant under $G \oplus \bar{G}$ and $D+\bar{D}$ are not. This leads to the desired doublet-triplet splitting without any fine-tuning³⁵⁾.

Let us comment at this point that for $b_{1,1} > 2$, one may also break a rank = 6 model like $SU(3) \times SU(2) \times U(1)^3$ down to the standard model. To do this one has to give VEVs both to Higgses transforming like N and v_L^c . One cannot expect to have both Higgses N and N_L^c light in a model with $b_{1,1} = 1$ since this would imply a bigger unbroken gauge symmetry¹²⁾. However, if there is more than one $\overline{27}$, one can obtain models with both fields light and the possibility exists that they may get a VEV (e.g., radiatively) at a large scale.

A rank = 4 model will be very similar to the "minimal low-energy supergravity" models²⁸⁾⁻³²⁾ discussed in the literature, but will have some differences which make it worthwhile to explore this case again in some detail. This we do in Chapter 6. Let us discuss first the case of a general model with an extra $U(1)$.

4. - MODELS WITH AN EXTRA $U(1)$: RADIATIVE SYMMETRY BREAKING

Here we will study how radiative corrections could induce the gauge symmetry breaking

$$SU(2)_L \times U(1)_Y \times U(1) \longrightarrow U(1)_{e.m.} \quad (25)$$

in models with an extra $U(1)$. We will do so for a general $U(1)$ but will keep in mind the models based on the $U(1)'$ and $U(1)'''$ of the previous section. An important role in symmetry breaking is played by the superpotential which, for a realistic model, would be a reduced version of the one in Eq. (10). In particular, either $\lambda_B = \lambda'_B = 0$ or $\lambda_L = \lambda'_L = 0$ (but not both) in order to avoid fast proton decay. They cannot all vanish because then some of the $D, \bar{D}$ fields would be absolutely stable, leading to cosmological problems²⁷⁾. In the $U(1)'''$ model $h_\nu = 0$ to avoid Dirac neutrino masses²⁷⁾. In the one based on $U(1)'$, both couplings h_ν and h'_ν are not present since the v_L^c fields decouple at low energies. Ignoring possible (safe) baryon- or lepton-violating couplings which we assume not too strong, one has a relevant superpotential:

$$W = \sum_{\text{families}} h_L H L e_L^c + h_d H Q d_L^c + h_u \bar{H} Q u_L^c + \\ + \lambda_2 N H \bar{H} + \lambda_3 N D \bar{D} \quad (26)$$

To avoid FCNC one has further to assume¹²⁾ that only one of the three sets of Higgses $H+\bar{H}$ couples significantly to quarks and leptons. We also then assume that only one of the three N-fields couples strongly enough to $H, \bar{H}$ and one set $D, \bar{D}$. This is not really required but simplifies the computations. We will also neglect in the computations the Yukawa couplings of all quarks and leptons except for the one of the top quark. Thus, our relevant superpotential for the renormalization group corrections will be

$$W = h_t \bar{H} Q u_L^c + \lambda_2 N H \bar{H} + \lambda_3 N D \bar{D} \quad (27)$$

and we use conventions for the resulting Lagrangian similar to those in Ref. 32).

The scalar potential along the neutral directions will be

$$\begin{aligned} V = & \frac{g_1'^2}{2} (X_{\bar{H}} |\bar{H}|^2 + X_H |H|^2 + X_N |N|^2)^2 + \frac{(g_1'^2 + g_2'^2)}{8} (|H|^2 - |\bar{H}|^2)^2 + \\ & + \lambda_2^2 (|N|^2 |H|^2 + |N|^2 |\bar{H}|^2 + |H|^2 |\bar{H}|^2) + m_N^2 |N|^2 + m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 + \\ & + m A_2 \lambda_2 (N H \bar{H}) + h.c. \end{aligned} \quad (28)$$

where $X_{\bar{H}}, X_H, X_N$ are the extra $U(1)$ charges of the corresponding fields. For the $U(1)'$ and $U(1)''$ models they are

$$U(1)': \quad X_{\bar{H}} = 2 \quad ; \quad X_H = 3 \quad ; \quad X_N = -5 \quad (29.a)$$

$$U(1)'': \quad X_{\bar{H}} = 2 \quad ; \quad X_H = 1/2 \quad ; \quad X_N = -5/2 \quad (29.b)$$

We have included in the potential the usual soft SUSY-breaking terms, i.e., scalar mass terms and trilinear scalar couplings. Choosing by convention positive VEVs for the neutral components of the scalars:

$$\langle H_0 \rangle = v \quad ; \quad \langle \bar{H}_0 \rangle = \bar{v} \quad ; \quad \langle N \rangle = N \quad (30)$$

one gets the following extremal conditions (for all VEVs non-vanishing)

$$X_H g_1'^2 (X_{\bar{H}} \bar{v}^2 + X_H v^2 + X_N N^2) + \frac{(g_1'^2 + g_2'^2)}{4} (v^2 - \bar{v}^2) + \lambda_2^2 (\bar{v}^2 N^2) + m_H^2 + m \lambda_2 A_2 \frac{N \bar{v}}{v} = 0 \quad (31.a)$$

$$X_{\bar{H}} g_1'^2 (X_{\bar{H}} \bar{v}^2 + X_H v^2 + X_N N^2) - \frac{(g_1'^2 + g_2'^2)}{4} (v^2 - \bar{v}^2) + \lambda_2^2 (v^2 N^2) + m_{\bar{H}}^2 + m \lambda_2 A_2 \frac{N v}{\bar{v}} = 0 \quad (31.b)$$

$$X_N g'^2 (X_{\bar{H}} \bar{v}^2 + X_H v^2 + X_N N^2) + \lambda_2^2 (v^2 + \bar{v}^2) + m_N^2 + m \lambda_2 A_2 \frac{v \bar{v}}{N} = 0 \quad (31.c)$$

Obviously the minimization conditions are much more complicated than in the "minimal low-energy supergravity" model in which the possible minima can be parametrized in terms of a single parameter $\tan \theta \equiv \bar{v}/v$. In Ref. 16), it was pointed out that radiative symmetry breaking with an extra $U(1)$ is problematic. However, these authors assume that one has $A_2 = 0$, which is in our opinion a not very compelling assumption. In fact, it is not surprising that one finds no solution with all VEVs non-vanishing and $A_2 = 0$. This already happens in the "minimal low-energy supergravity (MLES) model in which there is no solution^{30),31)} for $v = 0$ and $\mu_2^2 = 0$ (in standard notation) unless one fine-tunes $m_H^2 = -\frac{m^2}{H}$.

All the parameters in the scalar potential are supposed to be taken at the biggest of the two symmetry-breaking scales, i.e., at $Q = M'_{Z0}$. One also expects M'_{Z0} not to be very much larger than M_{Z0} (e.g., $M'_{Z0} \lesssim 500-600$ GeV), otherwise we would need to do some unnatural fine-tuning^{17),19)} to avoid radiative corrections giving an unwanted contribution to M_{Z0} . We have then to compute how the parameters m_H^2 , $m_{\bar{H}}^2$, m_N^2 , λ_2 and A_2 are renormalized in going from the compactification scale down to low energies. This, in turn, requires the computation of the evolution of other parameters: m_Q^2 , m_U^2 , m_D^2 , $m_{\bar{D}}^2$, λ_3 , h_t , A_3 and A_t . The renormalization group equations for all these parameters are shown for completeness in Appendix A. Notice that an important difference with the "MLES" model is that there is no bilinear Higgs mass term μ in the superpotential; its role is played here by the VEV $\langle \lambda_2 N \rangle$.

All the renormalization group equations (RGE) in Appendix A are coupled differential equations without an analytical solution. It is interesting to look for a reasonable approximation so that one can compute analytically as many things as possible without relying completely on numerical analysis. This allows for a more transparent analysis of the symmetry-breaking process. An interesting approximation is to neglect the effect of the coupling λ_2 in the renormalization group analysis. This is reasonable on several grounds. First, the coupling λ_2 always appears in the r.g.e. with smaller numerical coefficients than the other Yukawas, λ_3 and h_t . Secondly, λ_2 involves non-coloured particles only so that it gets much less renormalization than the other two couplings. Also, in Eqs. (A.1), Y_2 multiplies only $(\text{mass})^2$ terms of colourless particles which are usually much smaller than those of coloured particles. For all these reasons, even if $Y_2 \sim Y_3$ at the compactification scale, Y_2 gets smaller compared to Y_3 and Y_t at low energies and the effects of a non-vanishing Y_2 in the renormalization group equations become more and more negligible. Of course, we only neglect the

influence of λ_2 on the computation of the renormalized value of m_H^2 , $m_{\bar{H}}^2$, m_N^2 and A_2 , but we do not set $\lambda_2 = 0$ in the potential Eq. (28) since the coupling $\lambda_2 A_2 m_H \bar{H} N$ plays in general an important role in the minimization process.

The approximations discussed above allow us to compute the renormalized values of all the relevant parameters in terms of certain known functions and a few quadratures. One has for example for the Yukawa couplings:

$$Y_t(t) = Y_t(0) E_t(t) / (1 + 6 Y_t(0) F_t(t)) \quad (32.a)$$

$$Y_3(t) = Y_3(0) E_3(t) / (1 + 5 Y_3(0) F_3(t)) \quad (32.b)$$

$$Y_2(t) = Y_2(0) E_2(t) / \left[(1 + 6 Y_t(0) F_t(t))^{1/2} (1 + 5 Y_3(0) F_3(t))^{3/5} \right] \quad (32.c)$$

where one defines

$$E_t(t) = (1 + \beta_3 t)^{16/3 b_3} (1 + \beta_2 t)^{3/b_2} (1 + \beta_1 t)^{13/9 b_1} (1 + \beta' t)^{4\Omega_t/b'} \quad (33.a)$$

$$E_3(t) = (1 + \beta_3 t)^{16/3 b_3} (1 + \beta_1 t)^{4/9 b_1} (1 + \beta' t)^{4\Omega_3/b'} \quad (33.b)$$

$$E_2(t) = (1 + \beta_2 t)^{3/b_2} (1 + \beta_1 t)^{1/b_1} (1 + \beta' t)^{4\Omega_2/b'} \quad (33.c)$$

and one has $\beta_i = \alpha_i(0) b_i / (4\pi)$, b_i being the beta-function coefficients, as given in Appendix B. The F-functions are

$$F_i(t) = \int_0^t E_i(t') dt' \quad (34)$$

It is equally easy to obtain the renormalized values of the trilinear scalar couplings

$$A_t(t) = \frac{A}{(1 + 6 Y_t^0 F_t)} + \frac{M}{m} \left(H_2^t - \frac{6 Y_t^0 H_3^t}{(1 + 6 Y_t^0 F_t)} \right) \quad (35.a)$$

$$A_3(t) = \frac{A}{(1 + 5 Y_3^0 F_3)} + \frac{M}{m} \left(H_2^3 - \frac{5 Y_3^0 H_3^3}{(1 + 5 Y_3^0 F_3)} \right) \quad (35.b)$$

$$A_2(t) = A \left(1 - \frac{3Y_t^0 F_t}{(1+6Y_t^0 F_t)} - \frac{3Y_3^0 F_3}{(1+5Y_3^0 F_3)} \right) + \frac{M}{m} \left(H_2^2 - \frac{3Y_t^0 H_3^t}{(1+6Y_t^0 F_t)} - \frac{3Y_3^0 H_3^t}{(1+5Y_3^0 F_3)} \right) \quad (35.c)$$

where $Y_i^0 \equiv Y_i(t=0)$ and H_2^i, H_3^i are known functions of the gauge coupling constants only and are shown also in Appendix B. We have taken the usual boundary conditions at the unification scale

$$A_3 = A_2 = A_t = A \quad ; \quad M_3 = M_2 = M_1 = M' = M$$

$$m_{\tilde{g}} = m_{\tilde{t}} = m_H = m_{\bar{H}} = m \quad (36)$$

A little more effort gives us the result for the renormalized values of the masses in the scalar potential, $m_N^2, m_{\bar{H}}^2, m_H^2$. Using a notation very similar to that in Refs. 31) and 32) (although not identical), one can write down expressions for those masses in which the explicit dependence on the soft parameters m, M and A is shown. Also one can see the dependence on the Yukawa couplings Y_t and Y_3 . Thus for the Higgs $\bar{H}$ one obtains:

$$m_{\bar{H}}^2(t) = m^2(h(t) - k(t)A^2) + e(t)M^2 + f(t)mMA \quad (37)$$

The coefficients h, k, e and f are as follows:

$$h(t) = (3/D(t) - 1)/2 \quad ; \quad K(t) = 3Y_t^0 F_t / D^2(t)$$

$$D(t) = (1 + 6Y_t^0 F_t) \quad ; \quad f(t) = -6Y_t^0 H_3^t / D^2(t) \quad (38)$$

$$e(t) = (G_1 + Y_t^0 G_2)/D(t) + \frac{1}{2} (H_2^t + 6Y_t^0 H_4^t)^2 / D^2(t)$$

where the functions G_1, G_2, H_2^t, H_3^t and H_4^t depend only on the gauge coupling constants and are given in Appendix B. A similar computation can be done to obtain the renormalized value of m_N^2 . One finds in this case an analogous result (notice, however, the difference in some of the coefficients)

$$m_N^2(t) = m^2(\hat{h}(t) - \hat{k}(t)A^2) + \hat{e}(t)M^2 + \hat{f}(t)mMA \quad (39)$$

One has for the functions, $\hat{h}, \hat{k}, \hat{e}, \hat{f}$:

$$\begin{aligned}
 \hat{h}(t) &= (1 - 4 Y_3^0 F_3) / \hat{D}(t) & ; & \quad \hat{K}(t) = 3 Y_3^0 F_3 / \hat{D}(t)^2 \\
 \hat{D}(t) &= (1 + 5 Y_3^0 F_3) & ; & \quad \hat{f}(t) = -6 Y_3^0 H_3^3 / \hat{D}(t)^2 \\
 \hat{e}(t) &= (\hat{G}_1 + Y_3^0 \hat{G}_2) / \hat{D}(t) + \frac{3}{5} (H_2^3 + 5 Y_3^0 H_4^3)^2 / \hat{D}(t)^2
 \end{aligned} \tag{40}$$

where again the functions $\hat{G}_1$, $\hat{G}_2$, H_2^3 , H_3^3 , H_4^3 depend only on gauge coupling constants and are given in Appendix B. The other relevant mass parameter, that of the Higgs H , is easy to compute since H does not couple strongly with any of the "large" Yukawas λ_3 or h_t . One gets

$$m_H^2(t) = m^2 + g(t) M^2 \tag{41}$$

where $g(t)$ is a function of gauge coupling constants and its expression is given in Appendix B.

Formulae (32.c), (35.c), (37), (39) and (41) give us the value of all the renormalized quantities appearing in the scalar potential (28). We have now to impose the minimization conditions in (31) in order to see what constraints we get between the different parameters. Notice that, to start with, the free parameters are

$$N, \nu, \bar{\nu} ; A, m, M ; \lambda_2, \lambda_3, h_t \tag{42}$$

There are however a number of constraints: the three minimization conditions plus the condition

$$M_w^2 = \frac{g_2^2}{2} (\nu^2 + \bar{\nu}^2) \tag{43}$$

There is a fifth condition $h_t(m_t)\bar{\nu} = m_t$ if a given top-quark mass is assumed. Since the dependence of the scalar masses on the Yukawa couplings is complicated, the most convenient way to impose the minimization conditions, when doing explicit numerical calculations, is to assume input values for N , $\bar{\nu}$, λ_2 , λ_3 and h_t . Then Eqs. (31.a-c) form a set of three coupled quadratic equations for A , m and M which can be solved numerically. One has also to verify that we are not dealing with a maximum or a saddle point. The easiest numerical way to check it

is to verify that the 3×3 matrix of double derivatives of the scalar potential has no negative eigenvalue.

Some intuition into how the double symmetry breaking in this type of model occurs can be obtained from a discussion of the form of the scalar potential (28). Since we obviously want to obtain that $M'_{Z0} \gg M_{Z0}$ (because of neutral current constraints), the symmetry breaking should appear as follows. The parameter m_N^2 becomes negative not much above the weak scale and a VEV for N is induced. Upon renormalization one gets $m_{\bar{H}}^2 \ll m_H^2$ (although $m_{\bar{H}}^2$ need not be negative) and the extra- $U(1)$ D^2 terms give some (usually negative) contribution to the H and $\bar{H}$ effective masses. This, along with the trilinear term, induces VEVs for H and $\bar{H}$. The VEV for $\bar{H}$ is usually bigger than that for H because, as we said above, renormalization effects yield $m_{\bar{H}}^2 \ll m_H^2$ (the contribution of the D'^2 -term is not so important in this respect). Notice that, if we choose N , v and $\bar{v}$ positive, A_2 has to be negative if we want the minimum to be stable. Concerning the charged-current sector, the symmetry breaking proceeds as usual with a $W^\pm$ mass

$$M_W = \frac{g_2}{\sqrt{2}} \sqrt{v^2 + \bar{v}^2} \quad (44)$$

The same happens with the charged gaugino-Higgsino sector for which one obtains a mass matrix

$$\begin{array}{cc} & \begin{array}{cc} \tilde{W}^- & \tilde{H}^- \end{array} \\ \begin{array}{c} \tilde{W}^+ \\ \tilde{H}^+ \end{array} & \begin{pmatrix} M_2 & g_2 v \\ g_2 \bar{v} & -\lambda_2 N \end{pmatrix} \end{array} \quad (45)$$

with the well-known eigenvalues

$$M_{e,h}^2 = \frac{1}{2} \left\{ M_2^2 + \lambda_2^2 N^2 + 2M_W^2 \pm \sqrt{(M_2^2 - \lambda_2^2 N^2)^2 + 4M_W^4 \cos^2 2\theta + 4M_W^2 (M_2^2 + \lambda_2^2 N^2 + 2M_2 \lambda_2 N \sin 2\theta)} \right\} \quad (46)$$

However, the situation concerning the neutral currents is more complicated. The Higgs doublets H and $\bar{H}$ are in general charged under the two existing $U(1)$'s

so that when they get a VEV we get a non-diagonal mass matrix^{13),14),17),19),27)} for the Z^0 's:

$$M_{ab}^2 = M_Z^2 \begin{pmatrix} 1 & r0 \\ r0 & r^2 P \end{pmatrix} \quad (47)$$

where

$$0 \equiv (Q_Z(\bar{H}_0)X_{\bar{H}}\bar{\nu}^2 + Q_Z(H_0)X_H\nu^2) / (\nu^2 + \bar{\nu}^2) \quad (48.a)$$

$$P \equiv (X_{\bar{H}}^2\bar{\nu}^2 + X_H^2\nu^2 + X_N^2N^2) / (\nu^2 + \bar{\nu}^2) \quad (48.b)$$

$$r \equiv 2g' / \sqrt{g_1^2 + g_2^2} \quad (48.c)$$

$$M_Z^2 \equiv (g_1^2 + g_2^2)(\nu^2 + \bar{\nu}^2) / 2 \quad (48.d)$$

and Q_Z is the standard generator of the usual Z^0 boson. Since in general $0 \neq 0$, there is mixing and one obtains for the physical Z^0 and $Z^{0'}$ masses:

$$M_{Z^{0,2}}^2 = \frac{1}{2} M_Z^2 \left[1 + r^2 P \pm \sqrt{(1 - r^2 P)^2 + 4r^2 0^2} \right] \quad (49)$$

Notice that, since the value of the parameter M_Z^2 is fixed to be $M_Z^2 = ((g_1^2 + g_2^2)/g_2^2)M_W^2$, and M_W^2 is fixed experimentally, the lightest eigenvalue in (49) (which corresponds to the physical Z^0) may be too light if 0 is large. This may be a strong constraint in specific models.

The neutral gaugino-Higgsino sector is quite complicated in string-inspired models. One gets a mass matrix:

$$\begin{array}{c}
\tilde{W}_3 \\
\tilde{B} \\
\tilde{B}' \\
\tilde{H}_0 \\
\tilde{H}_0' \\
\tilde{N}
\end{array}
\begin{array}{c}
\begin{array}{c} - 21 - \\ \tilde{H}_0 \end{array} \\
\tilde{H}_0' \\
\tilde{N}
\end{array}
\begin{pmatrix}
M_2 & 0 & 0 & \frac{g_2 \nu}{\sqrt{2}} & -\frac{g_2 \bar{\nu}}{\sqrt{2}} & 0 \\
0 & M_1 & 0 & -\frac{g_1 \nu}{\sqrt{2}} & \frac{g_1 \bar{\nu}}{\sqrt{2}} & 0 \\
0 & 0 & M' & \sqrt{2} X_H g' \nu & \sqrt{2} X_H g' \bar{\nu} & \sqrt{2} X_N g' N \\
\frac{g_2 \nu}{\sqrt{2}} & -\frac{g_1 \nu}{\sqrt{2}} & \sqrt{2} X_H g' \nu & 0 & \lambda_2 N & \lambda_2 \bar{\nu} \\
-\frac{g_2 \bar{\nu}}{\sqrt{2}} & \frac{g_1 \bar{\nu}}{\sqrt{2}} & \sqrt{2} X_H g' \bar{\nu} & \lambda_2 N & 0 & \lambda_2 \nu \\
0 & 0 & \sqrt{2} X_N g' N & \lambda_2 \bar{\nu} & \lambda_2 \nu & 0
\end{pmatrix}
\quad (50)$$

in a self-explanatory notation. One has to compute the eigenvalues corresponding to physical particles numerically. It turns out that in specific models, the photino is not very often the lightest supersymmetric particle (LSP) but some combination of a Higgsino and a "singlino" $\tilde{N}$. In fact, in this type of superstring model, there are several other neutral Higgsinos (4) and singlinos (2) corresponding to the first two families. They are also natural candidates for being the LPS's, as we comment below.

Superstring-inspired models usually include, as we discussed in the previous paragraph, some new coloured particles, the "D-quarks" and D-squarks. The latter have a good chance of being relatively light, at least the ones of the third family. These have a mass matrix:

$$\begin{array}{c}
D \\
D^*
\end{array}
\begin{array}{c}
\bar{D}^* \\
\bar{D}
\end{array}
\begin{pmatrix}
m_D^2 & \lambda_3 N A_3 m + \lambda_2 \lambda_3 \nu \bar{\nu} \\
\lambda_3 N A_3 m + \lambda_2 \lambda_3 \nu \bar{\nu} & m_{\bar{D}}^2
\end{pmatrix}
\quad (51)$$

where

$$m_D^2 = m_{\tilde{D}}^2 + \lambda_3^2 N^2 + X_D (X_{\tilde{H}} \bar{\nu}^2 + X_H \nu^2 + X_N N^2) g'^2 - \cos 2\theta M_Z^2 \left(\frac{1}{3} \sin^2 \theta_w \right) \quad (52.a)$$

$$m_{\bar{D}}^2 = m_{\tilde{\bar{D}}}^2 + \lambda_3^2 N^2 + X_{\bar{D}} (X_{\tilde{H}} \bar{\nu}^2 + X_H \nu^2 + X_N N^2) g'^2 + \cos 2\theta M_Z^2 \left(\frac{1}{3} \sin^2 \theta_w \right) \quad (52.b)$$

and $\cos 2\theta = (\bar{v}^2 - v^2)/(\bar{v}^2 + v^2)$, m_D^r and $m_{\bar{D}}^r$ being the renormalized soft masses of D and $\bar{D}$. For these, one can easily obtain:

$$m_D^{2r} = \frac{2}{3} m^2 + \frac{1}{3} m_N^2 + e_D(t) \frac{1}{3} M^2 \quad (53.a)$$

$$m_{\bar{D}}^{2r} = \frac{2}{3} m^2 + \frac{1}{3} m_N^2 + e_{\bar{D}}(t) \frac{1}{3} M^2 \quad (53.b)$$

where m_N^2 is given by Eq. (39) and e_D , $e_{\bar{D}}$ only depend on gauge coupling constants and are shown in Appendix B. The eigenvalues of the mass matrix of the D-squarks are

$$m_{D, \bar{D}}^2 = \frac{1}{2} \left\{ (m_D^2 + m_{\bar{D}}^2) \pm \sqrt{(m_D^2 - m_{\bar{D}}^2)^2 + 4(\lambda_3 N A_3 m + \lambda_2 \lambda_3 \gamma \bar{v})^2} \right\} \quad (54)$$

When computing these masses in specific models, one very often finds that $N^2 \sim m_D^2$, $m_{\bar{D}}^2$ and one of the eigenvalues can be very low, lower than the mass of usual squarks. In fact, sometimes the lightest eigenstate has a negative (mass)² indicating that charge and colour would be broken. One has to restrict the parameters in such a way that this is not the case. The mass of the D-quarks is obviously

$$m_{\tilde{D}} = \lambda_3 N \quad (55)$$

which is usually heavier than the lightest D-squark. This implies that the lightest D-squark would be stable if there were no further couplings to other particles. For example, if there were couplings $\bar{D}QL$ and $D e_L^c u_L^c$ (but no QCD or $u_L^c d_L^c \bar{D}$ to avoid fast proton decay) this problem could be avoided. Notice that in this case, the D-squarks would not give rise to "monojets" at the CERN collider but to events with $\ell^+ \ell^- + \text{jets}$. The leptons may be different ($\mu^+ \ell^-$). If the above lepto-quark couplings are present with a large enough Yukawa coupling (why not?), the D-squarks could easily be produced at HERA as clear s-channel resonances. Of course, the D-squarks could be also observed at $e^+ e^-$ accelerators if they are light enough. If the lepto-quark couplings are absent but there are instead quark-quark-D couplings, the D-squark would be most easily produced

singly in P-P colliders but mostly in pairs in P- $\bar{P}$ colliders.

Concerning other particles of the third generation, the s-top's may also have substantial "off-diagonal" masses. The relevant mass matrix for them is

$$\begin{array}{cc} & \begin{array}{c} \tilde{t}_L \\ \tilde{t}_R \end{array} \\ \begin{array}{c} \tilde{t}_L^* \\ \tilde{t}_R^* \end{array} & \begin{pmatrix} m_{\tilde{t}_L}^2 & m_t (A_t m + \lambda_2 N/\omega) \\ m_t (A_t m + \lambda_2 N/\omega) & m_{\tilde{t}_R}^2 \end{pmatrix} \end{array} \quad (56)$$

where $\omega = \bar{v}/v$ and the diagonal entries are

$$m_{\tilde{t}_L}^2 = m_{\tilde{t}_L}^2 + X_Q g'^2 (X_{\bar{H}} \bar{v}^2 + X_H v^2 + X_N N^2) + \cos 2\theta M_{\frac{1}{2}}^2 \left(-\frac{1}{2} + \frac{2}{3} \sin^2 \theta_w\right) \quad (57.a)$$

$$m_{\tilde{t}_R}^2 = m_{\tilde{t}_R}^2 + X_U g'^2 (X_{\bar{H}} \bar{v}^2 + X_H v^2 + X_N N^2) - \cos 2\theta M_{\frac{1}{2}}^2 \left(\frac{2}{3} \sin^2 \theta_w\right) \quad (57.b)$$

The renormalized value of $m_{\tilde{H}}^2$ is shown in Eq. (37) and the functions $e_u(t)$ and $e_Q(t)$ only depend on gauge coupling constants and are given in Appendix B. The eigenvalues of the s-top mass matrix are

$$m_{\tilde{t}_{1,2}}^2 = m_t^2 + \frac{1}{2} \left\{ (m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2) \pm \sqrt{(m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2)^2 + 4 m_t^2 (A_t m + \lambda_2 N/\omega)^2} \right\} \quad (59)$$

Contrary to the case of the D-squark, in this case one usually has $m_t \ll N \sim M_{Z^0}$ so that the s-top's are never very light.

The spectrum of string models with an extra U(1) is quite baroque. Apart from the usual squarks and sleptons and the particles of the third generation discussed above (D, $\bar{D}$ and the physical Higgses H, $\bar{H}$ and N + SUSY partners), the first two generations do also contain extra fermions $\tilde{H}_i^+$, $\tilde{H}_i^0$, $\tilde{H}_i^-$, $\tilde{N}_i$, $\tilde{D}_i$, $\tilde{D}_i^+$, ν_{R_i} (with $i = 1, 2$ generations) and their supersymmetric scalar partners. In the case of the Y' hypercharge discussed in the previous section, the ν_{R_i} 's may be absent.

We have assumed in the renormalization group computations above that only the N , H , $\bar{H}$, D , $\bar{D}$ fields of the third generation may have large Yukawas triggering the symmetry breaking. However, the particles of the first two generations must have non-negligible couplings λ_{2i} , λ_{3i} to the third generation N , H and $\bar{H}$ in order that the fields $\tilde{H}_1^\pm$, $\tilde{H}_1^0$, $\tilde{\bar{H}}_1^0$, $\tilde{N}_1$, $\tilde{D}_1$, $\tilde{\bar{D}}_1$ do get masses consistent with experiment. These masses would be bounded above by the corresponding masses of the third generation. Thus, with the assumptions above, one expects for the first two generation fermions

$$m_{\tilde{D}_i} \leq \lambda_3 N \quad (60.a)$$

$$m_{\tilde{H}_i^\pm} = m_{\tilde{H}_i^0} = m_{\tilde{\bar{H}}_i^0} \leq \lambda_2 N \quad (60.b)$$

$$m_{\tilde{N}_i} \leq \lambda_2 \sqrt{y^2 + \bar{y}^2} \quad i = 1, 2 \quad (60.c)$$

Of course, these are just estimations; the real situation is more complicated. The existence of these non-negligible couplings amongst the first two generations and N , H , $\bar{H}$ means that in fact also the Higgses N_i , H_i , $\bar{H}_i$ ($i = 1, 2$) get induced VEVs. This means in turn that we get larger gaugino-chargino (4×4), gaugino-neutralino (11×11) and D-quark (3×3) mass matrices. However, Eqs. (60.a-c) are probably a good guess for the upper bounds of those masses. Notice in particular that the "singlinos" $\tilde{N}_i$ of the first two families (the one of the third generation is almost completely swallowed by the Z_0 -ino) can be quite light depending on the values of λ_{2i} . They can very easily be the LSP of the models with an extra $U(1)$.

With regard to the scalar partners of those first and second generation fermions, their masses do not depend strongly on the Yukawa couplings but on the soft SUSY-breaking terms. The masses for all the scalars of the first two generations are given in Appendix C for a general model with an extra $U(1)$. One point to remark is that one has to check in specific models that the charged scalars $H_i^\pm$ ($i = 1, 2$) are not too light [or even have negative $(\text{mass})^2$]. This happens sometimes because the $\frac{1}{2}(D')^2$ contribution is very often negative in these models, and one has to check that the rest of the contributions to the masses of $H_i^\pm$ ($i = 1, 2$) are large enough. The same is true for the scalars D_i , $\bar{D}_i$ ($i = 1, 2$) although in this case, due to the fact that they are coloured, the $\frac{1}{2}(D')^2$ term is in general not big enough to cause problems.

The spectrum of the unswallowed physical Higgs particles inside H , $\bar{H}$, N is also quite complicated. There is a charged Higgs with mass

$$m_{H^\pm}^2 = M_W^2 + m_{\bar{H}}^2 + m_H^2 \quad (61)$$

but the masses of neutral physical Higgses are more difficult to calculate. There are six neutral scalars inside H , $\bar{H}$, N and there is a 6×6 mass matrix for them²⁷⁾. There are two vanishing eigenvalues which correspond to the longitudinal degrees of freedom of Z^0 and Z'^0 . The other four states are physical neutral Higgses whose masses have to be calculated numerically.

All the analytic results presented in this section plus the formulae in Appendix B allow to compute the restrictions on the soft parameters A , m and M imposed by the appropriate scalar potential minimization conditions. This can be done for an arbitrary string model with an extra $U(1)$. Of course, the conditions are not enough to fix all the parameters. As we wrote above, there are nine free parameters and five constraints (for a given m_t). However, the arbitrariness is not as large as this might suggest since there are a good number of restrictions which should be obeyed. First, no coloured and/or charged scalar particle should get a negative (mass)² at the desired minimum. Second, no new charged particle should weigh less than ~ 20 GeV. Third, in order to avoid unwanted minima appearing because of having two large trilinear scalar couplings, one also should impose

$$A_t^2 m^2 \leq 3 (m_{\bar{t}_L}^2 + m_{\bar{t}_R}^2 + m_{\bar{H}}^2) \quad (62.a)$$

$$A_2^2 m^2 \leq 3 (m_H^2 + m_{\bar{H}}^2 + m_N^2) \quad (62.b)$$

$$A_3^2 m^2 \leq 3 (m_D^2 + m_{\bar{D}}^2 + m_N^2) \quad (62.c)$$

as well as having $|A| < 3$ at the GUT scale. Finally, the fact that no extra Z_0' has been observed does also restrict the possibilities. When one goes to specific models one finds that the parameters are more constrained than one could have thought. This is specially the case for the parameters λ_2 , λ_3 and $\omega \equiv \bar{v}/v$. Also one usually gets $M \gg m$ in order to get consistent breaking. This seems to be required to get the appropriate splitting of m_H^2 , $m_{\bar{H}}^2$ and m_N^2 when one renormalizes from the compactification scale down to low energies.

As we remarked in the previous section, the hypercharges Y' and Y''' defined there seem to be specially interesting. A model with an extra $U(1)$ corresponding to Y''' has been studied recently in Ref. 27). However, the authors assume particular boundary conditions at the compactification scale ($m = 0, A = 0$) and a particular mechanism for generating gaugino masses about which we have already expressed our scepticism in the second section. One cannot directly identify their results with a general model with $m = 0, A = 0$ and M arbitrary because they have a specific constraint for the gaugino mass.

It could be interesting to reanalyze numerically the model based on Y''' with arbitrary soft terms m, M, A . We have preferred in the present paper to concentrate on the model based on the Y' hypercharge where numerical computations are concerned. We do this because, as we remarked in Section 3, this $U(1)'$ symmetry allows the ν_L^c 's to acquire a large Majorana mass and solve the neutrino mass problem in a natural way. However, we expect that many of the qualitative results obtained for this model will be similar to the ones appearing in other models with an extra Z^0 , since the form of the different terms in the Lagrangian is very similar.

5. - A SPECIFIC MODEL WITH AN EXTRA Z^0 : THE HEAVY ν_R MODEL

We have already introduced this model¹⁹⁾ (Y' -hypercharge) in Chapter 3. As we discussed there, it could be obtained after Wilson-loop plus some extra symmetry breaking ($N_R \neq 0$) from an E_6 model. Alternatively, it may appear after Wilson-loop breaking of an $SO(10)$ model^{34),35)}. With the assumptions discussed in the previous chapter, the relevant superpotential is the one in Eq. (27) which leads to a scalar potential in the neutral directions

$$\begin{aligned} V = & \frac{g_1^2}{2} (2 |\bar{H}|^2 + 3 |H|^2 - 5 |N|^2)^2 + \frac{(g_1^2 + g_2^2)}{8} (|H|^2 - |\bar{H}|^2)^2 + \\ & + \lambda_2^2 (|N|^2 |H|^2 + |N|^2 |\bar{H}|^2 + |H|^2 |\bar{H}|^2) + m_N^2 |N|^2 + m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 \\ & + m A_2 \lambda_2 (N H \bar{H}) + h.c. \end{aligned} \quad (63)$$

We have solved numerically the minimization conditions Eqs. (31) for this model in order to look for constraints on the unknown parameters of the model. As we discussed in the previous chapter, for given M_{Z^0}' , $\bar{\nu}$, m_t , λ_2 and λ_3 , Eqs. (31)

give a system of three equations for A , M and m which we solved numerically. We also computed results for the masses of the unobserved particles existing in this type of model. We present some of these numerical results in this section.

In Figs. 2 and 3, we present results for the values of M , m and A consistent with the appropriate symmetry breaking as discussed in the previous section. For the sake of definiteness, the results we show correspond to $\lambda_2(0) = \lambda_3(0)$. There is no need for this equality to hold, but we take it to decrease the number of free parameters in order to avoid the proliferation of plots. We will discuss later on what happens when one relaxes this equality. We advance that most of the qualitative conclusions remain. Most of the numerical results we show correspond to top quark masses $m_t = 40$ GeV and $m_t = 50$ GeV, which could be consistent with a top quark interpretation of the UA1 data³⁷⁾. For $m_t = 40$ GeV, one typically obtains appropriate symmetry breaking for values

$$\lambda_3(0) \sim 0.04 - 0.10 \quad ; \quad 2 \gtrsim \omega \equiv \bar{\nu}/\nu \gtrsim 1 \quad (64)$$

One does not obtain Z_0 's lighter than ~ 200 GeV for $m_t = 40$ GeV. In any case, it is easier to get a relatively light Z_0 the heavier the top quark is. For $m_t = 50$ GeV, one generally has

$$\lambda_3(0) \sim 0.04 - 0.10 \quad ; \quad 4 \gtrsim \omega \gtrsim 1 \quad (65)$$

and the Z_0 may be as light as present experimental limits allow. In Fig. 2 we show results for m , M and A as a function of the extra Z_0 mass M'_{Z_0} (for $m_t = 50$ GeV). The points correspond to examples of values of A , m and M consistent with all the constraints. As one could expect, the values of m and $|M|$ grow with M'_{Z_0} whereas those for A do not show a strong variation. Another point to notice is that the required m is higher the lower ω is (for fixed λ_3). We must also remark that we have found no consistent solution for

$$\omega \lesssim 1 \quad (66)$$

This also seems to be true for $\lambda_2 \neq \lambda_3$ and higher top quark masses. Notice that looking at the $(D')^2$ term in the potential, one would expect $v > \bar{v}$ (i.e., $v \sim \sqrt{3/2} \bar{v}$). However, in the process of minimization, the fact that $m_H^2 \gg m_{\bar{H}}^2$ (although usually $m_{\bar{H}}^2 > 0$) due to the top quark Yukawa coupling is more important.

An interesting question is whether there are consistent solutions which correspond to the boundary conditions discussed in Section 2, i.e.:

$$m \approx 0, \quad A \approx 0 \quad (67)$$

This would imply that a situation in which supersymmetry breaking is triggered just by gaugino masses is consistent with appropriate breaking of the gauge symmetries. To see this we plot in Fig. 3a-c the quantity $m/|M|$ versus A for $m_t = 40, 50, 70$ GeV. Again, the dark points correspond to examples of $m/|M|$ and A values consistent with appropriate symmetry breaking. There are less points for $m_t = 40$ GeV because it is more difficult to find solutions. Since the present is just a one-loop computation which involves several other uncertainties, we will take as values consistent with the boundary conditions Eq. (67) any solutions with

$$|m/M| \lesssim 1/2 \quad ; \quad |A| \lesssim 1/2 \quad (68)$$

This region is marked in Fig. 3 with a dashed line. A first thing to remark is that for $m_t = 40-70$ GeV practically every solution has

$$|m/M| < 1 \quad (69)$$

and the lower the top quark mass is, the smaller is $|m/M|$. So, for $m_t \lesssim 70$ GeV, there is a tendency for the gaugino mass to be the dominant source of supersymmetry breaking. It is easy to understand why this is the case. For a relatively small top quark Yukawa coupling (small m_t), large gluino masses are required to generate the difference in masses $m_H^2 \gg m_{\bar{H}}^2$ through radiative corrections. In fact, something similar happened in the "minimal low-energy supergravity model", in which for $m_t = 60-70$ GeV the gaugino mass has a tendency to be large precisely because of the above reason [see, e.g., Fig. 7 in Ref. 31)]. Looking at Fig. 3 in more detail, we observe that there are consistent solutions with $|m/M|, |A| \lesssim 0.5$ for $m_t = 40$ GeV. For $m_t = 50$ GeV, there are still solutions within the conditions in Eq. (68) but as one increases m_t it is more difficult. Notice that for $m_t = 70$ GeV (Fig. 3-c) most of the solutions have $|m/M| \gtrsim 0.5$. So, as we discussed above, there is a tendency for large gaugino masses for $m_t \lesssim 70$ GeV. There is no special tendency for the solutions to prefer a small $|A|$. We should remark at this point that for $\lambda_2(0) \neq \lambda_3(0)$ (e.g., $\lambda_2(0) = 0.5 \lambda_3(0)$), one gets examples of solutions with different $A - m/|M|$ distributions but the qualitative

features remarked above remain true. We conclude from this analysis that the boundary conditions suggested in Chapter 2 for A , m and M are consistent with appropriate gauge symmetry breaking if

$$m_t \lesssim 70 \text{ GeV} \quad (70)$$

Notice that this result is non-trivial; we find no way to have consistent breaking in the opposite situation, $|M/m| \ll 1$.

We come now to the discussion of the mass spectrum in this model based on the $U(1)'$ hypercharge. Some typical spectra^{*)} consistent with the minimization conditions are shown in Table 2. We show results for $m_t = 40, 50, 70$ and 100 GeV and extra Z_0' masses from 100 - 600 GeV. It is easier to obtain solutions with relatively light Z_0' 's with a heavy top quark. Notice also that examples c, d and g in Table 2 are consistent with the gaugino mass being the dominant source of supersymmetry breaking.

The s-quarks are in general quite heavy since, as we discussed, M is usually large. A good approximate formula for the mean squark-mass is

$$\begin{aligned} m_{\tilde{q}}^2 &\simeq m^2 + 2 M^2 \tilde{\alpha}(0) \frac{4}{3} f_3 = m^2 + \frac{4}{3n} \alpha(0) \log\left(\frac{M_c^2}{Q^2}\right) M^2 \\ &\simeq m^2 + 3.7 M^2 \end{aligned} \quad (71)$$

so that, for small m , one has $m_{\tilde{q}} \sim 2M_{\tilde{g}}$ (recall that the gluino mass gets no renormalization in one loop and $M_{\tilde{g}} = |M|$). The sleptons are lighter and a similar approximate formula for them is

$$m_{\tilde{l}_i}^2 \simeq m^2 + 0.45 M^2 \quad (72)$$

so that for small m one has $m_{\tilde{l}_L} \sim 0.67 M_{\tilde{g}}$ and $m_{\tilde{l}_R} \sim 0.33 m_{\tilde{q}}$. Concerning the six neutralinos, eigenvectors of the matrix in Eq. (50), two of them have an average mass $\sim M_{Z_0}$ and two others have an average mass $\sim M_{Z_0}'$ and are "zino-like" objects.

The other two eigenvectors usually have smaller masses and correspond mostly to a photino $\tilde{\gamma}$ and a Higgsino $\tilde{H}_0$, although, of course all the eigenvectors are mixtures. The photino-like eigenvector has a mass $\sim \alpha_{e.m.}/\alpha_3 |M| \sim (5.5)^{-1} |M|$.

*) Those numerical results correspond to $\lambda_2(0) = \lambda_3(0)$ but the conclusions do not change for $\lambda_2(0) \neq \lambda_3(0)$.

Very often the Higgsino is lighter than the photino (examples b,c,g,i,j in Table 2) and it is a candidate to be the lightest supersymmetric particle (LSP). The reason for this is the tendency to have large M (for $m_t \lesssim 70$ GeV) in these models. However, there are other candidates to be the LSP in this type of model. In particular, the "singlinos" $\tilde{N}_{1,2}$ of the first two generations may have rather small masses [see Eq. (60.c) in the previous chapter] depending on the sizes of their Yukawa couplings to the Weinberg-Salam doublets. In fact, the examples a, b and d shown in Table 2 have good chances of having a "singlino" $\tilde{N}_{1,2}$ as the LSP. One expects this property of having Higgsinos or singlinos as the LSP's to be a general feature of stringy models with an extra $U(1)$. Notice that many of the expected experimental signatures of supersymmetry would change in this case with respect to the photino = LSP hypothesis. Accordingly the present experimental limits on SUSY particles would also change. Another interesting property to remark is that the lightest of the two "charginos" in Eq. (46), $\tilde{W}_\lambda$, may be also relatively light as shown in examples f,i and j. One could observe it as a heavy lepton in Z^0 decays or in e^+e^- colliders.

The D-squarks of the third generation (i.e., the ones with bigger couplings to N) often have very different masses between them as a result of the large off-diagonal terms in the mass matrix Eq. (51). The lightest D_L may have a low mass as shown in examples a,e,i and j and could, in fact, be produced at presently planned accelerators. The other eigenvalue D_H is usually much heavier and the same happens in general (but not always) with the fermionic partner, the D-quark $\tilde{D}$. As we explained in Chapter 4, in order to avoid D_L (or $\tilde{D}$) being absolutely stable, some couplings like, e.g., $DQ\bar{Q}$ or $\bar{D}Q\bar{L}$ are needed so that D_L can decay. These required couplings may lead to interesting phenomenology, as already explained in Chapter 4.

The five last entries in Table 2 correspond to the extra fermions and charged scalars of the first two generations present in the model (this is why we put a 1,2 subindex). Due to the large contribution of the $(D')^2$ term in the potential, some of the charged scalars $H_{1,2}^+$, $\bar{H}_{1,2}^+$ may get some large negative contribution to their $(\text{mass})^2$ [see Eqs. (1.k-l) in Appendix C] and may be relatively light. These scalars are in general the lightest charged scalars in the theory (see examples a,h and i) and could even be detectable at present accelerators. The corresponding supersymmetric fermionic partners form Dirac particles

$\tilde{H}_{1,2}^{\pm}$ and may also be relatively light. The values of the masses for the last five entries in Table 2 are given only as an estimation of their maximum value since, as we indicated in Chapter 4, we do not know the size of the coupling of these particles to N and the Weinberg-Salam doublets $H, \bar{H}$.

Summarizing the results for the spectrum of new particles, we have shown that the new Z_0' may be relatively light, but it is easier to get a relatively light Z_0' for heavier t -quark masses. The squarks, sleptons and gluino are in general rather heavy and the charged particles with the best chances of being light are the lightest chargino $\tilde{W}_\lambda$, the scalars $H_{1,2}^{\pm}, \bar{H}_{1,2}^{\pm}$ of the first two generations (along with their partners $\tilde{H}_{1,2}^{\pm}$) and the coloured-triplet scalar D_L . The lightest supersymmetric particle is usually either a Higgsino or a "singlino" $\tilde{N}_{1,2}$ of the first two generations (although it is sometimes the photino $\tilde{\gamma}$).

All the numerical results in this section apply in principle only to the "heavy ν_R " model¹⁹⁾ characterized by the $U(1)'$ hypercharge introduced in Section 3. Results which depend crucially on the hypercharge assignments (such as, e.g., the scalar potential) will be different for models with a different extra $U(1)$. Thus the constraints for λ_2, λ_3 and ω would in general be different since they depend strongly on the form of the $\frac{1}{2}(D')^2$ term in the scalar potential. The same will happen with the masses of the scalars (like $D_L, H_{1,2}^{\pm}, \bar{H}_{1,2}^{\pm}$, etc.) which depend on the D -term contribution, which is model-dependent. There are, however, a number of features which we expect to hold true for other extra $U(1)$ models. We expect $M \gg m$ for relatively light t -quarks, since this fact remarked above does not depend on the extra $U(1)$ assignments. We also expect the states $\tilde{W}_\lambda, D_L, H_{1,2}^{\pm}, \bar{H}_{1,2}^{\pm}$ and $\tilde{H}_{1,2}^{\pm}$ to be the lightest charged new particles and the Higgsino or a singlino $\tilde{N}_{1,2}$ to often be the LSP. The approximate relations for m_q, m_λ and M_g in Eqs. (71) and (72) will hold true for a large class of models (all those with essentially three $\underline{27}$'s of E_6 light).

6. - THE MINIMAL RANK = 4 "STRINGY" MODEL

As we discussed at the end of Chapter 3, one may also obtain the standard model gauge group $SU(3) \times SU(2) \times U(1)$ upon compactification of the string. This is probably the most interesting possibility, since in this case there is no need to have at low energies any of the extra new particles (D 's, $H_{1,2}$'s, ν_R 's, etc.)

which lead to problems in the models with an extra $U(1)$. The minimal model with gauge group $SU(3) \times SU(2) \times U(1)$ should contain, apart from the standard quark-lepton superfields, the Higgs doublets H and $\bar{H}$ and a singlet N . This singlet N was not considered in the "minimal low-energy supergravity" model²⁸⁾⁻³²⁾, but here it is necessary to include it in order to give a VEV to H and avoid the existence of a dangerous Peccei-Quinn axion. This singlet N should couple to the doublets through a term in the superpotential

$$W_N = \lambda_2 N H \bar{H} \quad (73)$$

The scalar potential (along the neutral direction) in this minimal model is

$$\begin{aligned} V = & \frac{(g_1^2 + g_2^2)}{8} (|H|^2 - |\bar{H}|^2)^2 + \lambda_2^2 |H|^2 |\bar{H}|^2 + \lambda_2^2 |N|^2 (|H|^2 + |\bar{H}|^2) + \\ & + m_N^2 |N|^2 + m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 + m_{A_2} \lambda_2 (N H \bar{H}) + h.c. \end{aligned} \quad (74)$$

If the last term in this potential was absent (i.e., $A_2 = 0$ or $\lambda_2 = 0$), there would be a Peccei-Quinn symmetry in this model which would be broken by the VEV $\langle \bar{H} \rangle \neq 0$, giving rise to a phenomenologically unacceptable axion. In the minimal low-energy supergravity model, there is an explicit Higgs mass term $\mu H \bar{H}$, and its corresponding soft term (usually denoted by $\mu_3^2 H \bar{H} + h.c.$) breaks explicitly the Peccei-Quinn symmetry. In a superstring model you do not expect small bilinear terms in the superpotential, particularly since they are forbidden by the original E_6 symmetry (a $27 \times \overline{27}$ coupling is not of the type we want, since $\overline{27}$ does not couple to quarks and leptons, at least with renormalizable couplings). Any bilinear coupling between H and $\bar{H}$ has to appear through the VEV of a singlet N and that is why we have to include the coupling (73).

The topic of massless Goldstone bosons in this type of "minimal" model is, in fact, a bit more subtle. The existence of the coupling in Eq. (73) makes possible the definition of a global supersymmetry-preserving $U(1)$ symmetry (not an R-symmetry) such as, e.g., one of the extra $U(1)$'s considered in the previous sections. When N gets a VEV, one obtains a massless Goldstone superfield which would have been eaten by the corresponding Z^0 if the $U(1)$ symmetry had been gauged. After including the effect of the soft SUSY-breaking terms, a massless Goldstone still remains. However, in the superstring models under consideration, this global $U(1)$ was in fact originally gauged (inside E_6) and spontaneously broken either upon compactification or maybe at an intermediate scale. Under

these circumstances, no really massless Goldstone boson will be generated; what will happen is that the real swallowed Goldstone boson will get a small admixture ($\sim \langle N \rangle / M_c$) from the light field N . Still, in order to understand the origin of the small VEV generated for N , Eq. (74) gives us the appropriate low-energy scalar potential.

The scalar potential above looks similar to the one of "minimal low-energy supergravity" (MLES), but it has several important differences due to the existence of the $\lambda_2 N H \bar{H}$ coupling. In particular, it has no flat direction ($\langle H \rangle = \langle \bar{H} \rangle$) for $m_H^2 \approx m_{\bar{H}}^2 \approx \mu_3^2$ as happened in the MLES case. This was important in that case because one could have radiative $SU(2) \times U(1)$ breaking with a low t -quark mass, since a large difference between m_H^2 and $m_{\bar{H}}^2$ was not needed³⁰⁾⁻³²⁾. The term which makes all the difference is $\lambda_2^2 |H|^2 |\bar{H}|^2$ since the others are the consequence of an effective μ -parameter $\mu = \lambda_2 \langle N \rangle$. In the present case one needs a negative $m_{\bar{H}}^2$ to be generated since there is no flat direction, and an extra positive term in the potential ($\lambda_2^2 |H|^2 |\bar{H}|^2$) has to be overwhelmed in order to get a stable minimum. This has the important consequence that a relatively heavy top quark ($m_t \gtrsim 70$ GeV) is required to trigger $SU(2) \times U(1)$ breaking, as happened in the original versions of the MLES model. The minimization conditions are however rather more involved, since there is a new dynamical variable, N ,

$$\frac{(g_1^2 + g_2^2)}{4} (\nu^2 - \bar{\nu}^2) + \lambda_2^2 (\bar{\nu}^2 + N^2) + m_H^2 + m \lambda_2 A_2 \frac{N \bar{\nu}}{\nu} = 0 \quad (75.a)$$

$$- \frac{(g_1^2 + g_2^2)}{4} (\nu^2 - \bar{\nu}^2) + \lambda_2^2 (\nu^2 + N^2) + m_{\bar{H}}^2 + m \lambda_2 A_2 \frac{N \nu}{\bar{\nu}} = 0 \quad (75.b)$$

$$\lambda_2^2 (\nu^2 + \bar{\nu}^2) + m_N^2 + m \lambda_2 A_2 \frac{\nu \bar{\nu}}{N} = 0 \quad (75.c)$$

This makes the numerical analysis of this minimal "stringy" model more complicated than the MLES model. Notice that the free parameters are

$$m, M, A ; N, \nu, \bar{\nu} ; \lambda_2, h_t \quad (76)$$

and there are four constraints (75.a-c) and Eq. (43). There is a fifth constraint for a given t -quark mass, $h_t \bar{\nu} = m_t$. The renormalized values of m_H^2 , $m_{\bar{H}}^2$ in terms of A , m , M and h_t were explicitly computed in Ref. 32), assuming a negligible effect of λ_2 in the renormalization group equations. For the other relevant parameters one finds

$$m_N^2 = m^2 \quad (77.a)$$

$$\gamma_2(t) = \gamma_2(0) E_2(t) / (1 + 6 \gamma_1(0) F_t(t))^{1/2} \quad (77.b)$$

$$A_2(t) = A \left(1 - \frac{3 \gamma_1^0 F_t}{(1 + 6 \gamma_1^0 F_t)} \right) + \frac{M}{m} \left(H_2^2 - \frac{3 \gamma_1^0 H_3^2}{(1 + 6 \gamma_1^0 F_t)} \right) \quad (77.c)$$

where the functions E_2 , F_t , H_2^2 and H_3^2 were defined in Eqs. (33.c), (34), (B.1c) and (B.1d) (one has to set also $g' = 0$ in those formulae). In general, to obtain the relevant renormalization group equations in this model, set $g' = \lambda_3 = 0$ in the formulae of Appendices A and B. After putting the renormalized values of m_N^2 , m_H^2 , $m_{\bar{H}}^2$, λ_2 , and A_2 in Eqs. (75) in terms of the parameters A , m , M , λ_2 and h_t , one gets a set of three quadratic equations on the variables A , m and M [for fixed values of $\lambda_2(0)$, $\langle N \rangle$ and m_t]. We have solved these numerically to study the constraints existing amongst the parameters in order to obtain appropriate $SU(2) \times U(1)$ breaking. We now present some of these numerical results.

As we advanced above, in this model we only find consistent solutions for

$$m_t \gtrsim 70 \text{ GeV} \quad (78)$$

just because of the existence of no approximate flat direction $v = \bar{v}$ in the potential. Symmetry breaking is generated radiatively through a negative $m_{\bar{H}}^2$ which implies in turn that

$$\omega = \bar{v}/v \gtrsim 1 \quad (79)$$

as usual. One gets symmetry breaking for large values of ω consistent with Eq. (79). To get some idea of the values of A , m and M allowed by consistent symmetry breaking, we plot $m/|M|$ versus A for $m_t = 80$ and $m_t = 100$ GeV in Fig. 4. One finds solutions for a large range of values for λ_2 [typical values include $\lambda_2(0) \sim 0.03-0.15$], but for $m_t \lesssim 70$ GeV it is already difficult to get consistent solutions. We have also drawn a line corresponding to the region

$$|m/M| \lesssim 0.5, \quad |A| \lesssim 0.5 \quad (80)$$

in order to see if there are solutions consistent with supersymmetry breaking triggered just by gaugino masses, as we did with models with an extra $U(1)$. One observes that for $m_t \lesssim 100$ GeV, usually $m/|M| \lesssim 1.0$ and gaugino masses tend to be bigger than the soft parameter m . On the other hand, no special preference for small $|A|$ is apparent, although in principle there are plenty of solutions with both small $m/|M|$ and $|A|$. We conclude from this analysis that it is easy to obtain consistent $SU(2)_L \times U(1)$ breaking, with supersymmetry breaking being triggered just by gaugino masses if $m_t \lesssim 100$ GeV (particularly for $m_t \approx 70-80$ GeV).

Let us now examine how the typical SUSY spectrum is in this "minimal stringy model". One can easily compute the masses of the sparticles by putting $\lambda_3 = g' = 0$ in the formulae given in the previous chapter and Appendix C. We show some significant examples for $m_t = 70, 80, 100$ and 130 GeV in Table 3. The SUSY spectra are in general lighter than in the extra $U(1)$ case since in the latter the overall SUSY-breaking scale is $\sim M'_{Z_0}$. For the average squark and slepton masses one has [see, e.g., Ref. 32)]

$$m_{\tilde{q}}^2 \simeq m^2 + 7.4 M^2 \simeq m^2 + 0.82 M_g^2 \quad (81.a)$$

$$m_{\tilde{e}}^2 \simeq m^2 + 0.4 M^2 \simeq m^2 + 0.04 M_g^2 \quad (81.b)$$

up to D^2 -term contributions. For small m (which is the usual case if $m_t \lesssim 90$ GeV), one thus has $m_{\tilde{q}} \approx 0.9 M_g$ and $m_{\tilde{e}} \approx 0.23 m_{\tilde{q}}$. The charged sleptons may be relatively light (see, e.g., examples b,f and j in Table 3) in many instances. When this is the case, the sneutrino is quite light and may even be the lightest supersymmetric particle. This is because, as first pointed out in Ref. 3), the sneutrinos $\tilde{\nu}_L$ get an additional negative contribution to their $(\text{mass})^2$ from the D^2 -term which is^{31),38)}

$$- \left(\frac{\omega^2 - 1}{\omega^2 + 1} \right) \frac{1}{2} M_{Z_0}^2 \quad (82)$$

which may be substantial and overwhelm the positive radiative contribution $m_{\tilde{\nu}_L}^2$ for ω slightly bigger than 1. This happens very often in this "minimal stringy model" where the sneutrino has a good chance of being the LSP (examples f,h,i and j in Table 3). In fact, many possible minima of the scalar potential are ruled out because they lead to a negative $(\text{mass})^2$ for the sneutrino and thus to a VEV $\langle \tilde{\nu}_L \rangle \neq 0$. This would be a problem because lepton number is violated, the charged

leptons mix with the "charginos" and one expects unacceptable rates for decays like $\mu \rightarrow e\gamma$, etc. Light sneutrinos allow for the observation of the decays

$$W^\pm \rightarrow \tilde{e}_L \tilde{\nu}_L \quad ; \quad Z^0 \rightarrow \tilde{e}^+ \tilde{e}^-, \tilde{\nu}_L \tilde{\nu}_L \quad (83)$$

at $P\bar{P}$ colliders.

There is a 5×5 mass matrix of neutralinos in this model which can be obtained from that in Eq. (50) by setting $M' = g' = 0$. Two of the five eigenvectors are Zeeno-Higgsino objects with masses in the range 50-170 GeV. There are two other states which are singlino-Higgsino ($\tilde{N}-\tilde{H}^0, \tilde{\tilde{H}}^0$) objects with masses $\sim \lambda_2 v$ and $\lambda_2 \tilde{v}$, as one could expect from the matrix in Eq. (50). There is finally a photino-like eigenvector with a mass $m_{\tilde{\gamma}} \approx \alpha_{\text{e.m.}} / \alpha_3 m_{\tilde{g}} \approx \frac{1}{2} |M|$. The photino is very rarely the lightest supersymmetric particle and this only happens for large top quark masses. Very often, one sneutrino [the three of them are not exactly degenerate³⁸⁾] is the LSP, as we have seen above, but also quite often the LSP is one of the "singlino"-Higgsino objects (examples a,b,c,d,e).

Let us finally comment that one of the two charginos ($\tilde{W}_L$) may be rather light and could be accessible to present or planned accelerators (examples c,d,e,g,h in Table 3). In fact, example g is ruled out because the chargino $\tilde{W}_H$ would be the LSP, which is cosmologically unacceptable since it is charged.

We think that this minimal "stringy" model is very attractive because of its simplicity, especially if we compare it with the models with extra $U(1)$'s that we studied in previous sections.

Let us recall, however, that the specific numerical results presented in this section assume the existence of a "big desert" up to the compactification scale. If an intermediate symmetry-breaking scale exists (e.g., at $\sim 10^{10-14}$ GeV) the boundary conditions of renormalization group equations are modified. Our equations still hold below the intermediate scale and it is a straightforward exercise to include the renormalization effects above that scale.

7. - CONCLUSIONS

In this paper we have studied in some detail the constraints on a large class of superstring-inspired supersymmetric models coming from imposing appropriate low-energy gauge symmetry breaking. We considered generic soft

supersymmetry breaking parameters m , M and A since a complete understanding of the supersymmetry breaking mechanism is not yet available at present. We explored, however, the possibility of having the supersymmetry breaking in the observable sector triggered by gaugino masses ($|A| \ll 1$, $m/|M| \ll 1$). We argued that models with two extra Z^0 's are problematic. There is on the other hand a one-parameter family of models with a single extra Z^0 [Eq. (24)]. There are two particularly interesting extra $U(1)$ models (hypercharges Y' and Y'' in the text). One of them (Y') is the only one which allows for the existence of a superheavy right-handed neutrino ν_R . These models have necessarily a bunch of new particles present around the weak scale ($D, \bar{D}, H_{1,2}, \bar{H}_{1,2}$, etc.) which lead to several problems: fast proton decay and flavour-changing neutral currents. One has to assume that some Yukawa couplings are absent to avoid those problems. We solved analytically the renormalization group equations for the mass and coupling parameters in an arbitrary model with an extra $U(1)$, expressing all the results in terms of a few quadratures. We also give general formulae for the mass spectrum of a model with an arbitrary extra $U(1)$. Contrary to some claims in the literature, one can obtain the desired pattern of symmetry breaking with no problem. Numerical results are presented for the heavy ν_R model [$U(1)'$ hypercharge]. One obtains consistent radiative symmetry breaking

$$SU(2)_L \times U(1)_Y \times U(1)' \longrightarrow U(1)_{e.m.} \quad (84)$$

for certain ranges of the parameters. For t -quark masses $m_t \lesssim 70$ GeV, one usually has $|M| \gg m$ and solutions with small $|A|$ are also possible in that case. The extra Z^0 boson may be as light as experiment allows, but it is difficult to get $m_{Z^0} \lesssim 150$ GeV unless $m_t \gtrsim 50$ GeV. The lightest supersymmetric particle is usually the Higgsino $\tilde{H}^0$ or one of the fermionic "singlinos" $\tilde{N}_{1,2}$ of the first two generations.

We finally considered a model with no extra Z^0 boson, the only "stringy remnant" being the existence of a singlet N coupling to the minimal Weinberg-Salam Higgs set. This "minimal stringy model" is very similar to the "low-energy supergravity" model studied in the last few years, although the existence of the singlet N leads to important differences. In order to generate radiative symmetry breaking one needs $m_t \gtrsim 70$ GeV and very often one obtains relatively light sleptons. The sneutrino is often the lightest supersymmetric particle and the decay $W^\pm \rightarrow \tilde{e}\tilde{\nu}$ and $Z^0 \rightarrow \tilde{\ell}^+\tilde{\ell}^-$, $\tilde{\nu}\tilde{\nu}$ are then possible. Sometimes the LSP is a Higgsino-singlino state ($\tilde{H}^0 - \tilde{N}$), but very rarely the photino. For $m_t \lesssim 100$ GeV usually $m/|M| \ll 1.0$ and, there are plenty of solutions consistent with SUSY

breaking triggered just by gaugino masses (i.e., $|A| \ll 1$, $m/|M| \ll 1$), especially for $m_t \approx 70-90$ GeV.

There is, of course, no experimental evidence in favour of any of the string-inspired models we have studied in this paper. But one must admit that the models with an extra Z^0 are much more artificial and unnatural than the "minimal stringy model" whose gauge group is just $SU(3)_C \times SU(2)_L \times U(1)_Y$. Extra $U(1)$ models require a number of lucky coincidences to happen: i) some Yukawas are absent in order to avoid fast proton decay; ii) other B-violating couplings are present in order to avoid that the D-fields are stable; iii) the extra $SU(2)$ doublets in the models have negligible couplings to quarks in order to avoid FCNC; iv) $\nu_L - \nu_R - \bar{H}$ Yukawas are absent in order to avoid unobserved neutrino masses [this is solved for the $U(1)$ ' hypercharge]. Furthermore, the range of parameters leading to the desired pattern of symmetry breaking is much smaller than the one in the model with no extra Z^0 . We are afraid that one could say about the models with an extra $U(1)$ the same that Alfonso X of Castille, a famous XIIIth century astronomer, said about the Ptolemaic system: "If God had called me into his Council, the machine of the world would have been much less complex"³⁹⁾.

On the other hand, the model with no extra Z^0 (which is a straightforward generalization of the "minimal low-energy supergravity model") may avoid all of the above problems and within it radiative $SU(2) \times U(1)$ breaking occurs for very large ranges of the parameters. This being the case, we think that the construction of string compactification schemes leading directly to an $SU(5)$ or $SU(3) \times SU(2) \times U(1)$ gauge group is very important. The work in Refs. 34) and 35) could be a step in that direction. Still, we have to emphasize that even in models with initial E_6 or $SO(10)$ symmetries there can be an intermediate scale symmetry breaking leading to the standard model, as we comment in Chapter 3. We think this is preferable to a situation with a low-energy extra Z^0 , although, of course this is at present (unfortunately) just a matter of taste.

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APPENDIX A

Here we collect the renormalization group equations of the soft SUSY breaking as well as Yukawa couplings of a general model with an extra $U(1)$ interaction and a superpotential as in Eq. (27). For the scalar masses one has

$$\frac{dm_N^2}{dt} = \frac{\alpha'}{\hbar} (X_N)^2 M_1'^2 - 3\gamma_3 (m_{\bar{D}}^2 + m_D^2 + m_N^2 + A_3^2 m^2) - 2\gamma_2 (m_{\bar{H}}^2 + m_H^2 + m_N^2 + A_2^2 m^2) \quad (1.a)$$

$$\frac{dm_{\bar{H}}^2}{dt} = \frac{\alpha'}{\hbar} (X_{\bar{H}})^2 M_1'^2 + \frac{3\alpha_3}{4\hbar} M_2^2 + \frac{\alpha_1}{4\hbar} M_1'^2 - 3\gamma_t (m_{\bar{Q}}^2 + m_U^2 + m_{\bar{H}}^2 + A_t^2 m^2) - \gamma_2 (m_{\bar{H}}^2 + m_H^2 + m_N^2 + A_2^2 m^2) \quad (1.b)$$

$$\frac{dm_H^2}{dt} = \frac{\alpha'}{\hbar} (X_H)^2 M_1'^2 + \frac{3\alpha_3}{4\hbar} M_2^2 + \frac{\alpha_1}{4\hbar} M_1'^2 - \gamma_2 (m_{\bar{H}}^2 + m_H^2 + m_N^2 + A_2^2 m^2) \quad (1.c)$$

$$\frac{dm_{\bar{D}}^2}{dt} = \frac{\alpha'}{\hbar} (X_{\bar{D}})^2 M_1'^2 + \frac{4\alpha_3}{3\hbar} M_3^2 + \frac{\alpha_1}{9\hbar} M_1'^2 - \gamma_3 (m_{\bar{D}}^2 + m_D^2 + m_N^2 + A_3^2 m^2) \quad (1.d)$$

$$\frac{dm_D^2}{dt} = \frac{\alpha'}{\hbar} (X_D)^2 M_1'^2 + \frac{4\alpha_3}{3\hbar} M_3^2 + \frac{\alpha_1}{9\hbar} M_1'^2 - \gamma_3 (m_{\bar{D}}^2 + m_D^2 + m_N^2 + A_3^2 m^2) \quad (1.e)$$

$$\frac{dm_{\bar{Q}}^2}{dt} = \frac{\alpha'}{\hbar} (X_{\bar{Q}})^2 M_1'^2 + \frac{4\alpha_3}{3\hbar} M_3^2 + \frac{3\alpha_2}{4\hbar} M_2^2 + \frac{\alpha_1}{36\hbar} M_1'^2 - \gamma_t (m_{\bar{Q}}^2 + m_U^2 + m_{\bar{H}}^2 + A_t^2 m^2) \quad (1.f)$$

$$\frac{dm_U^2}{dt} = \frac{\alpha'}{\hbar} (X_U)^2 M_1'^2 + \frac{4\alpha_3}{3\hbar} M_3^2 + \frac{4\alpha_1}{9\hbar} M_1'^2 - 2\gamma_t (m_{\bar{Q}}^2 + m_U^2 + m_{\bar{H}}^2 + A_t^2 m^2) \quad (1.g)$$

where

$$\gamma_i \equiv \left(\frac{\lambda_i}{4\hbar}\right)^2; \quad \gamma_t \equiv \left(\frac{h_t}{4\hbar}\right)^2; \quad t = 2 \log(M_c/Q) \quad (2)$$

M_c being the compactification scale. For the relevant Yukawa couplings one has:

$$\frac{d\gamma_t}{dt} = \gamma_t \left(\frac{4\alpha_3}{3\hbar} + \frac{3\alpha_2}{4\hbar} + \frac{13\alpha_1}{36\hbar} + \Omega_t \frac{\alpha'}{\hbar} - 6\gamma_t - \gamma_2 \right) \quad (3.a)$$

$$\frac{d\gamma_3}{dt} = \gamma_3 \left(\frac{4\alpha_3}{3\hbar} + \frac{\alpha_1}{9\hbar} + \Omega_3 \frac{\alpha'}{\hbar} - 5\gamma_3 - 2\gamma_2 \right) \quad (3.b)$$

$$\frac{d\gamma_2}{dt} = \gamma_2 \left(\frac{3\alpha_2}{4\hbar} + \frac{\alpha_1}{4\hbar} + \Omega_2 \frac{\alpha'}{\hbar} - 3\gamma_3 - 3\gamma_t - 4\gamma_2 \right) \quad (3.c)$$

where Ω_i are defined

$$\Omega_t = -(X_u X_u + X_u X_{\bar{u}} + X_u X_{\bar{u}}) \quad (4.a)$$

$$\Omega_3 = -(X_{\bar{d}} X_d + X_N X_{\bar{d}} + X_N X_d) \quad (4.b)$$

$$\Omega_2 = -(X_{\bar{u}} X_u + X_N X_{\bar{u}} + X_N X_u) \quad (4.c)$$

and X_i are the extra $U(1)$ charges of the corresponding particles. For the relevant trilinear scalar couplings of the third generation one gets

$$\frac{dA_t}{dt} = \frac{4\alpha_3}{3\hbar} \frac{M_3}{m} + \frac{3\alpha_2}{4\hbar} \frac{M_2}{m} + \frac{13\alpha_1}{36\hbar} \frac{M_1}{m} + \Omega_t \frac{\alpha'_1 M'_1}{\hbar m} - 6\gamma_t A_t - \gamma_2 A_2 \quad (5.a)$$

$$\frac{dA_3}{dt} = \frac{4\alpha_3}{3\hbar} \frac{M_3}{m} + \frac{\alpha_1}{9\hbar} \frac{M_1}{m} + \Omega_3 \frac{\alpha'_1 M'_1}{\hbar m} - 5\gamma_3 A_3 - 2\gamma_2 A_2 \quad (5.b)$$

$$\frac{dA_2}{dt} = \frac{3\alpha_2}{4\hbar} \frac{M_2}{m} + \frac{\alpha_1}{4\hbar} \frac{M_1}{m} + \Omega_2 \frac{\alpha'_1 M'_1}{\hbar m} - 3\gamma_3 A_3 - 3\gamma_t A_t - 4\gamma_2 A_2 \quad (5.c)$$

APPENDIX B

We collect here some formulae and definitions required for the computation of the renormalized parameters in an arbitrary string model with an extra $U(1)$. Some of the functions used but not defined in Chapter 4 are the following

$$H_2^t = \tilde{\alpha}(0) \left(\frac{16}{3} h_3 + 3 h_2 + \frac{13}{15} h_1 + 4a \Omega_t h' \right) \quad (1.a)$$

$$H_2^3 = \tilde{\alpha}(0) \left(\frac{16}{3} h_3 + \frac{4}{15} h_1 + 4a \Omega_3 h' \right) \quad (1.b)$$

$$H_2^2 = \tilde{\alpha}(0) \left(3 h_2 + \frac{3}{5} h_1 + 4a \Omega_2 h' \right) \quad (1.c)$$

$$H_3^t = t E_t - F_t \quad (1.d)$$

$$H_3^3 = t E_3 - F_3 \quad (1.e)$$

$$H_4^t = H_2^t F_t - H_3^t \quad (1.f)$$

$$H_4^3 = H_2^3 F_3 - H_3^3 \quad (1.g)$$

$$G_1 = L_t - \frac{1}{2} (H_2^t)^2 \quad (1.h)$$

$$\hat{G}_1 = L_3 - \frac{3}{5} (H_2^3)^2 \quad (1.i)$$

$$L_t = \tilde{\alpha}(0) \left(\frac{3}{2} f_2 + \frac{3}{10} f_1 + 2a X_H^2 f' \right) \quad (1.j)$$

$$L_3 = \tilde{\alpha}(0) (2a X_N^2 f') \quad (1.k)$$

$$G_2 = 6 J_t - K_t - 6 H_2^t H_4^t + 3 F_t (H_2^t)^2 - 3 H_6^t \quad (1.l)$$

$$J_t = F_t L_t - \int_0^t dt E_t L_t \quad (1.m)$$

$$J_3 = F_3 L_3 - \int_0^t dt E_3 L_3 \quad (1.n)$$

$$K_t = \tilde{\alpha}(0) \int_0^t dt E_t \left(16 f_3 + \frac{4}{5} f_1 + 6a (X_a^2 + X_v^2 - X_H^2) f' \right) \quad (1.o)$$

$$K_3 = \tilde{\alpha}(0) \int_0^t dt E_3 \left(16 f_3 + \frac{4}{5} f_1 + 6a (X_0^2 + X_6^2 - \frac{2}{3} X_N^2) f' \right) \quad (1.p)$$

$$\hat{G}_2 = 5 J_3 - K_3 - 6 H_2^3 H_4^3 + 3 F_3 (H_2^3)^2 - 3 H_6^3 \quad (1.q)$$

$$H_6^t = \int_0^t dt (H_2^t)^2 E_t \quad (1.r)$$

$$H_6^3 = \int_0^t dt (H_2^3)^2 E_3 \quad (1.s)$$

$$g(t) = \tilde{\alpha}(0) \left(\frac{3}{2} f_2 + \frac{3}{10} f_1 + 2a X_H^2 f' \right) \quad (1.t)$$

$$e_D(t) = \tilde{\alpha}(0) \left(8f_3 + \frac{2}{5} f_1 + 6a (X_D^2 - \frac{1}{3} X_W^2) f' \right) \quad (1.u)$$

$$e_{\bar{D}}(t) = \tilde{\alpha}(0) \left(8f_3 + \frac{2}{5} f_1 + 6a (X_{\bar{D}}^2 - \frac{1}{3} X_W^2) f' \right) \quad (1.v)$$

$$e_U(t) = \tilde{\alpha}(0) \left(4f_3 - \frac{3}{2} f_2 + \frac{1}{2} f_1 + 3a (X_U^2 - \frac{2}{3} X_H^2) f' \right) \quad (1.w)$$

$$e_A(t) = \tilde{\alpha}(0) \left(8f_3 + 3f_2 - \frac{1}{5} f_1 + 6a (X_A^2 - \frac{1}{3} X_H^2) f' \right) \quad (1.x)$$

where $\tilde{\alpha}(0) = \alpha_{\text{GUT}}/(4\pi)$ and "a" is a normalization factor of the extra U(1),

$$\alpha'(0) = a \alpha(0) \quad (2)$$

For U(1)', $a = 1/40$ and for U(1)''' $a = 1/15$. The functions f_i and h_i were introduced in Refs. 31) and 32) and are defined by

$$f_i(t) = \frac{1}{\beta_i} \left(1 - \frac{1}{(1 + \beta_i t)^2} \right) ; \quad h_i(t) = \frac{t}{(1 + \beta_i t)} \quad (3)$$

where $\beta_i = \tilde{\alpha}(0) b_i = \alpha_i(0) b_i / (4\pi)$. The b_i are the one-loop beta-function coefficients. Their values for both U(1)' and U(1)''' are

$$b_3 = 0 ; \quad b_2 = 3 ; \quad b_1 = 15 ; \quad b' = 9/a \quad (4)$$

Notice that all the functions defined above are very simple functions of the gauge coupling constants and the scale $t = 2\log(M_c/Q)$; Yukawa couplings do not appear. There are only six quadratures to be done, corresponding to the functions J_t , J_3 , K_t , K_3 , H_6^t and H_6^3 . The terms in the formulae corresponding to the extra U(1) gauge interactions can easily be computed reading the charge assignments of the different particles X_i from Table 1. The quantities Ω_t , Ω_3 and Ω_2 are defined in Appendix A and for the case of the U(1)' hypercharge they are

$$\Omega_t = 3 \quad ; \quad \Omega_3 = 19 \quad ; \quad \Omega_2 = 19 \quad (5)$$

whereas for the $U(1)'''$ case one gets

$$\Omega_t = 3 \quad ; \quad \Omega_3 = \frac{21}{4} \quad ; \quad \Omega_2 = \frac{21}{4} \quad (6)$$

APPENDIX C

Here we give formulae for the masses of the scalar particles of the first two generations (as well as the $\tilde{b}_L$ squark) whose Yukawa couplings are neglected. One has, in the notation introduced in the text

$$m_{\tilde{u}_L}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{4}{3} f_3 + \frac{3}{4} f_2 + \frac{1}{60} f_1 + X_a^2 f' \right) + \cos 2\theta M_z^2 \left(\frac{2}{3} \sin^2 \theta_w - \frac{1}{2} \right) + X_a D' \quad (1.a)$$

$$m_{\tilde{d}_L}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{4}{3} f_3 + \frac{3}{4} f_2 + \frac{1}{60} f_1 + X_a^2 f' \right) + \cos 2\theta M_z^2 \left(\frac{1}{2} - \frac{1}{3} \sin^2 \theta_w \right) + X_a D' \quad (1.b)$$

$$m_{\tilde{u}_R}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{4}{3} f_3 + \frac{4}{15} f_1 + X_u^2 f' \right) - \cos 2\theta M_z^2 \left(\frac{2}{3} \sin^2 \theta_w \right) + X_u D' \quad (1.c)$$

$$m_{\tilde{d}_R}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{4}{3} f_3 + \frac{1}{15} f_1 + X_d^2 f' \right) + \cos 2\theta M_z^2 \left(\frac{1}{3} \sin^2 \theta_w \right) + X_d D' \quad (1.d)$$

$$m_{\tilde{e}_L}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{3}{4} f_2 + \frac{3}{20} f_1 + X_L^2 f' \right) + \cos 2\theta M_z^2 \left(\frac{1}{2} - \sin^2 \theta_w \right) + X_L D' \quad (1.e)$$

$$m_{\tilde{\nu}_L}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{3}{4} f_2 + \frac{3}{20} f_1 + X_L^2 f' \right) - \cos 2\theta M_z^2 \frac{1}{2} + X_L D' \quad (1.f)$$

$$m_{\tilde{e}_R}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{3}{5} f_1 + X_e^2 f' \right) + \cos 2\theta M_z^2 \sin^2 \theta_w + X_e D' \quad (1.g)$$

$$m_{\tilde{\nu}_R}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) X_{\nu}^2 f' + X_{\nu} D' \quad (1.h)$$

$$m_{\tilde{\nu}}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{4}{3} f_3 + \frac{1}{15} f_1 + X_0^2 f' \right) - \cos 2\theta M_z^2 \left(\frac{1}{3} \sin^2 \theta_w \right) + X_0 D' \quad (1.i)$$

$$m_{\tilde{\nu}}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{4}{3} f_3 + \frac{1}{15} f_1 + X_0^2 f' \right) + \cos 2\theta M_z^2 \left(\frac{1}{3} \sin^2 \theta_w \right) + X_0 D' \quad (1.j)$$

$$m_{H^\pm}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{3}{4} f_2 + \frac{3}{20} f_1 + X_H^2 f' \right) + \cos 2\theta M_z^2 \left(\frac{1}{2} - \sin^2 \theta_w \right) + X_H D' \quad (1.k)$$

$$m_{H^\pm}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{3}{4} f_2 + \frac{3}{20} f_1 + X_H^2 a f' \right) - \cos 2\theta M_Z^2 \left(\frac{1}{2} - \sin^2 \theta_w \right) + X_H D' \quad (1.l)$$

$$m_{H^0}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{3}{4} f_2 + \frac{3}{20} f_1 + X_H^2 a f' \right) - \cos 2\theta M_Z^2 \frac{1}{2} + X_H D' \quad (1.m)$$

$$m_{\bar{H}^0}^2 = m^2 + 2 M^2 \tilde{\alpha}(0) \left(\frac{3}{4} f_2 + \frac{3}{20} f_1 + X_H^2 a f' \right) + \cos 2\theta M_Z^2 \frac{1}{2} + X_H D' \quad (1.n)$$

$$m_N^2 = m^2 + 2 M^2 \tilde{\alpha}(0) X_N^2 a f' \quad (1.o)$$

where $\cos 2\theta = (\omega^2 - 1)/(\omega^2 + 1) = (\bar{v}^2 - v^2)/(\bar{v}^2 + v^2)$, θ_w is the Weinberg angle and $M_Z^2 \equiv (g_1^2 + g_2^2)(v^2 + \bar{v}^2)/2$. The functions f_i and the quantity a are defined in Appendix B. $\tilde{\alpha}(0) = \alpha_{\text{GUT}}/(4\pi)$ and the charges X_i can be read out from Table 1 for the different U(1)'s. Notice that for the models based on U(1)' and U(1)''' one has

$$\alpha(0) = \alpha_{\text{GUT}} \simeq \alpha_3(M_w) \simeq 0.12 \quad (2)$$

since $b_3 = 0$. The quantity D' is the value of the extra U(1) D-term:

$$D' = g'^2 (X_H \bar{v}^2 + X_H v^2 + X_N N^2) \quad (3)$$

Notice that to the masses in Eqs. (1.a-o) one has to add the $(\text{mass})^2$ of the corresponding fermionic partner. These are only (presumably) substantial for the D, $\bar{D}$, $\bar{H}$, $\bar{H}$ and N fields. There will also be in general some "off-diagonal masses" for the latter but one expects the contributions shown above to be the most important.

The only third-generation scalar masses not shown in the text are the ones for $\tilde{\tau}$, $\tilde{\nu}_\tau$, $\tilde{b}_L$ and $\tilde{b}_R$. The masses for $\tilde{\tau}$, $\tilde{\nu}_\tau$ and $\tilde{b}_R$ are as the ones of the corresponding first-generation scalars (approximately). For $\tilde{b}_L$, one gets

$$m_{\tilde{b}_L}^2 = m_{\tilde{\tau}_L}^2 + \cos 2\theta M_Z^2 \left(\frac{1}{2} - \frac{1}{3} \sin^2 \theta_w \right) + X_Q D' \quad (4)$$

where $m_{\tilde{\tau}_L}^2$ is given by Eq. (58.b) in the text. Notice that the contribution to the scalar masses of the D' term can be huge if $N \gg v, \bar{v}$ (as one expects if $M_{Z_0} \gg M_{Z_0}$). The sign of this contribution will depend on the charge assignments as given in Table 1.

Table 1

Possible extra U(1) generators inside E_6 . The appropriate factor to get the same normalization as the SU(3) and SU(2) generators is indicated in brackets.

	Q_L	U_L^c	e_L^c	d_L^c	L	H	$\bar{D}$	$\bar{H}$	D	ν_L^c	N
$T_L (\times \frac{1}{2\sqrt{3}})$	-1	0	-2	0	1	1	0	1	2	-2	-2
$T_R (\times \frac{1}{\sqrt{3}})$	0	-1	1	1/2	-1/2	-1/2	1/2	1	0	-1/2	-1/2
T_N	0	0	0	1/2	1/2	-1/2	-1/2	0	0	-1/2	1/2
$Y (\times \sqrt{\frac{3}{5}})$	1/6	-2/3	1	1/3	-1/2	-1/2	1/3	1/2	-1/3	0	0
$Y' (\times \frac{1}{\sqrt{40}})$	-1	-1	-1	-2	-2	3	3	2	2	0	-5
$Y'' (\times \frac{1}{\sqrt{40}})$	-1	-1	-1	3	3	-2	-2	2	2	-5	0
$Y''' (\times \frac{1}{\sqrt{15}})$	-1	-1	-1	1/2	1/2	1/2	1/2	2	2	-5/2	-5/2
$Y'_1 (\times \frac{1}{4\sqrt{6}})$	-2	-2	-2	4	4	-2	-2	4	4	-8	-2
$Y''_1 (\times \frac{1}{4\sqrt{6}})$	-2	-2	-2	-2	-2	4	4	4	4	-2	-8

Table 2

Examples of possible low-energy spectra corresponding to the extra Z^0 model discussed in Chapter 5. The masses are given in GeV.

	a	b	c	d	e	f	g	h	i	j
m	22	121	174	59	148	134	158	35	41	95
M	-130	-254	-356	-173	-133	238	-339	62	-136	-96
A	2.6	-0.76	0.31	0.30	-0.86	-1.9	0.48	-2.1	-1.8	-0.1
λ_2	0.07	0.05	0.08	0.05	0.10	0.10	0.07	0.12	0.07	0.08
ω	1.2	1.4	1.4	1.8	1.2	1.2	1.8	1.2	4.7	4.7
m_t	40	40	40	50	50	50	50	70	70	100
M_{Z^0}	210	310	610	210	310	410	510	135	210	160
$\tilde{q}$	263	525	750	352	316	505	705	131	277	214
χ	82	182	292	111	184	206	260	57	99	119
$\tilde{\gamma}$	23	46	79	31	24	37	70	11	25	17
$\tilde{H}_0$	46	24	27	31	24	102	19	75	9	13
$\tilde{W}_\lambda$	84	95	132	71	75	28	121	44	41	34
$\tilde{D}$	222	248	741	165	438	584	554	206	222	185
D_H	432	734	1330	483	674	913	1130	289	444	358
D_L	93	161	203	159	134	413	183	149	115	80
$H_{1,2}^+$	$\lesssim 47$	$\lesssim 165$	$\lesssim 172$	$\lesssim 106$	$\lesssim 143$	$\lesssim 165$	$\lesssim 182$	$\lesssim 60$	$\lesssim 77$	$\lesssim 112$
$H_{1,2}^\pm$	$\lesssim 69$	$\lesssim 182$	$\lesssim 251$	$\lesssim 109$	$\lesssim 168$	$\lesssim 203$	$\lesssim 231$	$\lesssim 63$	$\lesssim 70$	$\lesssim 119$
$\tilde{H}_{1,2}$	$\lesssim 62$	$\lesssim 68$	$\lesssim 208$	$\lesssim 45$	$\lesssim 125$	$\lesssim 167$	$\lesssim 154$	$\lesssim 60$	$\lesssim 61$	$\lesssim 51$
$\tilde{D}_{1,2}$	$\lesssim 222$	$\lesssim 248$	$\lesssim 741$	$\lesssim 165$	$\lesssim 438$	$\lesssim 584$	$\lesssim 554$	$\lesssim 206$	$\lesssim 222$	$\lesssim 185$
$\tilde{N}_{1,2}$	$\lesssim 28$	$\lesssim 21$	$\lesssim 32$	$\lesssim 21$	$\lesssim 38$	$\lesssim 38$	$\lesssim 28$	$\lesssim 44$	$\lesssim 28$	$\lesssim 31$

Table 3

Examples of possible low-energy spectra corresponding to the minimal model with no extra Z^0 ' discussed in Chapter 6. The masses are given in GeV.

	a	b	c	d	e	f	g	h	i	j
m	7	11	11	15	15	18	1	27	40	23
M	-70	-34	-153	-120	-85	-46	-95	79	-47	17
A	-1.1	-0.35	1.4	-1.75	-0.32	-0.38	-1.9	-2.15	-1.0	-1.5
λ_2	0.06	0.03	0.04	0.03	0.06	0.08	0.12	0.07	0.12	0.07
ω	1.8	1.2	9.2	4.7	3.1	1.4	4.7	4.7	2.0	1.2
$\langle N \rangle$	114	60	300	250	71	53	48	19	41	60
m_t	70	70	70	70	80	80	80	100	100	130
$\tilde{q}_R$	184	90	407	314	224	123	250	210	129	50
$\tilde{g}$	210	101	460	354	253	139	283	238	141	52
$\tilde{e}_R$	43	25	75	65	54	36	56	59	56	30
$\tilde{e}_L$	63	33	123	100	77	48	83	79	65	33
$\tilde{\nu}_L$	20	7	91	61	22	4	28	9	12	5
zeenos	126, 68	108, 81	171, 56	151, 60	132, 67	115, 76	137, 65	128, 71	115, 76	98, 90
H-N- inos	19, 11	6, 8	10, 9	9, 6	13, 17	17, 22	31, 28	17, 16	26, 31	12, 16
$\tilde{\gamma}$	35	17	73	58	42	23	46	39	24	9
$\tilde{W}_H$	118	97	170	150	131	105	137	131	112	93
$\tilde{W}_L$	53	69	21	25	33	63	24	20	50	70

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FIGURE CAPTIONS

Fig. 1 : Terms related by supersymmetry to the Green-Schwarz anomaly-cancelling terms give rise to gaugino masses after truncation to four dimensions²⁶⁾.

Fig. 2 : Values for m , M and A as a function of M_{Z^0} , for $m_t = 50$ GeV. The results shown correspond to $\lambda_2(0) = \lambda_3(0) = 0.05-0.10$ and $\omega = 2.4, 1.8, 1.4$ and 1.2 . The points show examples consistent with appropriate gauge symmetry breaking.

Fig. 3 : $m/|M|$ versus A in the model with an extra Z^0 described in Chapter 5. Results are shown for $m_t = 40, 50$ and 70 GeV. In these examples we take $\lambda_2(0) = \lambda_3(0) = 0.05-0.10$ and $5 \gtrsim \omega \gtrsim 1$. Black dots are consistent examples of solutions.

Fig. 4 : $m/|M|$ versus A in the model with no extra Z^0 of Chapter 6. Results are given for $\lambda_2(0) = 0.03-0.12$, $\langle N \rangle = 3-300$ GeV and $9 \gtrsim \omega \gtrsim 1.2$. Black dots correspond to examples leading to consistent $SU(2)_L \times U(1)_Y$ breaking.

Truncation

Susy rotation

Fig. 1

Fig. 2

Fig. 3

Fig. 4