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THE EFFECTIVE LAGRANGIAN OF SUPERSYMMETRIC
YANG-MILLS THEORY

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ABSTRACT

The one-loop effective action of the supersymmetric Yang-Mills theory in the presence of a covariantly constant Yang-Mills superfield is calculated. This is done with the help of Schwinger's proper time method. The propagator of the Yang-Mills superfield in the presence of a non-vanishing background superfield is derived in a closed form. The effect of an unstable mode is isolated.

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1. - INTRODUCTION AND RESULTS

The last few years have seen an enormous growth in the interest in supersymmetry. Almost all of the efforts to reconcile supersymmetry with the observable world indicate that supersymmetry will at best be phenomenologically relevant in the form of an effective, non-renormalizable low-energy field theory. However, in the most preferable case, such a low-energy theory should be calculable from first principles, i.e., from a fundamental supersymmetric theory, which has all the good features of a well-behaved quantum field theory. In order to implement these ideas, a clear understanding of the relations between the fundamental level and the effective level of supersymmetric field theories is required. There is, in particular, the need to develop calculational techniques which allow to derive supersymmetric effective actions in an economical fashion.

As far as superfield formulations are available, superfield techniques have proved to be in most respects superior to manipulations on the component level. Not only do they provide a convenient framework to construct supersymmetric Lagrangians, superfields are indispensable in doing perturbation theory. However, the calculation of an effective theory is a problem which, in general, goes beyond standard manipulations in perturbation theory.

There are some attempts in the literature¹⁾⁻³⁾ to transfer various techniques like the heat kernel method or the method originally used by Coleman and Weinberg to calculate effective potentials into the framework of superspace. Unfortunately, such a transfer is only possible at the price of breaking supersymmetry superficially by the approximation which has to be employed to perform such a transfer successfully.

In non-supersymmetric theories, a gauge covariant method to calculate effective actions non-perturbatively is Schwinger's operator technique⁴⁾. Starting from the supersymmetric proper time method⁵⁾, this technique has recently been generalized to supersymmetric QED (SQED)⁶⁾. This method supersedes previous attempts to calculate effective actions, in that it is fully supersymmetric. In the present paper, we shall apply Schwinger's method to supersymmetric Yang-Mills theory (SYM).

The purpose of this article is twofold. At first, we would like to demonstrate the flexibility of Schwinger's method when combined with superspace techniques. Secondly, by calculating the effective action of SYM we hope to gain

some understanding necessary to apply the method to theories which exhibit even larger gauge invariances, notably supergravity theories.

It will turn out that the calculation is less involved in the present case, compared with previous work⁶⁾. This is due to the fact that SYM is formulated in terms of a vector superfield, whereas SQED requires chiral matter superfields. Accordingly the superspace heat kernel of SYM is unconstrained. This fact allows to write it down explicitly. As a byproduct the propagator of SYM in the presence of a constant background field can be obtained in a closed form.

Due to the UV properties of SYM, the effective action does not require any UV subtractions. However, there are potential IR singularities. We shall show how these singularities can be avoided. It is well known that in non-supersymmetric Yang-Mills theory these IR singularities can be attributed to an unstable mode. It leads to an imaginary part of the effective action⁷⁾. We shall isolate the effect of the corresponding unstable mode of SYM and calculate the imaginary part which contributes to the effective action.

In the present paper, we shall employ the conventions of Ref. 6).

2. - BACKGROUND - QUANTUM SPLITTING

Let us begin by reviewing the derivation of the general form of the one-loop effective action which will be the starting point of our considerations in the following section. The procedure is well known⁸⁾, we shall therefore sketch its derivation only briefly.

The action of SYM is given by

$$S = \frac{1}{4} \text{tr} \int d^4x d^2\Theta W^2 + h.c. , \quad (2.1)$$

where

$$W^{\dot{a}} = -\frac{1}{4} [\bar{D}_{\dot{a}}, \{\bar{D}^{\dot{a}}, D_a\}] , \quad (2.2)$$

and

$$D_a := e^{-V} D_a e^{+V} , \quad \bar{D}_{\dot{a}} := e^{-U} \bar{D}_{\dot{a}} e^{+U} . \quad (2.3)$$

The prepotentials U and V take values in the fundamental representation of the gauge group. A gauge transformation acts on V like

$$e^V \rightarrow e^{V'} = e^{i\bar{\Lambda}} e^V e^{-iK} \quad \mathcal{D}_\alpha \bar{\Lambda} = 0, \quad (2.4)$$

and similarly for U.

The prepotentials U and V are now split into a background piece $U^B(V^B)$ and a quantum piece $U^Q(V^Q)$ according to

$$e^V = e^{V^B} e^{V^Q}, \quad e^U = e^{U^B} e^{U^Q}. \quad (2.5)$$

Performing a gauge transformation with $K = -iU^Q$, we get

$$\mathcal{D}_\alpha = e^{-V'} \mathcal{D}_\alpha^B e^{V'}, \quad \bar{\mathcal{D}}_i = \bar{\mathcal{D}}_i^B, \quad (2.6)$$

where

$$e^{-V'} = e^{U^Q} e^{-V^Q},$$

and the covariant derivatives $\mathcal{D}_\alpha^B$, and $\bar{\mathcal{D}}_i^B$ contain only the background gauge connections.

In order to quantize the theory, the action (2.1) has to be augmented by gauge fixing and ghost terms. To maintain invariance under gauge transformations of the background field, we add a gauge fixing term which is covariantized with respect to the background field

$$S_{G\bar{F}} = \frac{1}{16} + \tau \int d^2z \left\{ \mathcal{D}_\beta^A, [\mathcal{D}_\alpha^B, V^Q] \right\} \left\{ \bar{\mathcal{D}}_i^B, [\bar{\mathcal{D}}_\beta^A, V^Q] \right\}. \quad (2.7)$$

In the quadratic approximation the ghosts contribute a piece to the effective action which is identical to the one already computed in Ref. 6) for SQED. Therefore, we shall simply drop all the ghost contributions in the present work.

Inserting the decomposition (2.6) into (2.1) and keeping only contributions quadratic in the quantum field $\hat{V}$ we obtain

$$S + S_{G\bar{F}} = \frac{1}{2} \int d^2z \hat{V}(z) \square \hat{V}(z), \quad (2.8)$$

where

$$\square := -\frac{1}{2} \mathcal{D}_a^2 + \frac{1}{2} \bar{W} \bar{\mathcal{D}} - \frac{1}{2} W \mathcal{D} \quad , \quad (2.9)$$

and

$$\mathcal{D}_a := -2i [\mathcal{D}_a, \bar{\mathcal{D}}_a] \quad . \quad (2.10)$$

All the covariant derivatives and fields appearing in (2.9) have to be taken in the adjoint representation of the gauge group.

As V' is a vector superfield, it is readily integrated out to yield the effective action

$$\Gamma = -\frac{1}{2} i \text{Tr} \log \square \quad . \quad (2.11)$$

The trace in the foregoing expression has to be taken over the full superspace.

3. - THE SUPERSYMMETRIC HEAT KERNEL

The effective action Γ is related to the superspace heat kernel $h(z_1, z_2; \tau)$, taken at coinciding arguments $z := z_1 = z_2$, via the proper time integral

$$\text{Tr} \log \square = \text{Tr} \int_{-\infty}^0 \frac{d\tau}{\tau} h(z, z; \tau) \quad . \quad (3.1)$$

The heat kernel $h(z_1, z_2; \tau)$ is defined by

$$\left(i \frac{d}{d\tau} - \square \right) h(z_1, z_2; \tau) = 0 \quad , \quad (3.2a)$$

$$h(z_1, z_2; 0) = \delta^8(z_1 - z_2) \quad . \quad (3.2b)$$

The Feynman ie prescription $\square \rightarrow \square + i\epsilon$ is understood in (3.1) and (3.2). The information encoded in $h(z_1, z_2; \tau)$ does not only allow to calculate Γ , $h(z_1, z_2; \tau)$ is closely related to the propagator associated with $\square$

$$(-\square - i\epsilon)^{-1} = i \int_{-\infty}^0 d\tau h(z_1, z_2; \tau) \quad (3.3)$$

Note that in contrast to the SQED heat kernel of Ref. 6), the heat kernel $h(z_1, z_2; \tau)$ is defined on the full superspace. This fact implies that here we

have much more liberty in formulating a supersymmetric constant field approximation. We shall require that

$$\mathcal{G}_\alpha{}^\beta := \{D_\alpha, W^\beta\} \neq 0 \quad (3.4a)$$

$$\bar{\mathcal{G}}_{\dot{\alpha}}{}^{\dot{\beta}} := \{\bar{D}_{\dot{\alpha}}, \bar{W}^{\dot{\beta}}\} \neq 0 \quad (3.4b)$$

whereas

$$\partial_\alpha W_\beta = \partial_\alpha \bar{W}_{\dot{\beta}} = 0 \quad (3.5)$$

as well as the derivatives of $\mathcal{G}_\alpha{}^\beta$ and $\bar{\mathcal{G}}_{\dot{\alpha}}{}^{\dot{\beta}}$ vanishing. Using the Jacobi identity in the same way as in Ref. 9), one may show that in this approximation the superfields W and $\bar{W}$ anticommute among each other, i.e., are effectively Abelian.

We shall next calculate the heat kernel $h(z_1, z_2, \tau)$ almost along the same lines as in Ref. 6). In contrast to Ref. 6), here we shall only need the Heisenberg representation $\mathbb{D}_a(\tau)$ of the covariant derivative $\mathbb{D}_a$

$$i \frac{d}{d\tau} \mathbb{D}_a(\tau) = [\square, \mathbb{D}_a] \quad , \quad \mathbb{D}_a(0) = D_a \quad (3.6)$$

whereas $\mathbb{D}_\alpha$ and $\bar{\mathbb{D}}_{\dot{\alpha}}$ are not transformed into the Heisenberg picture. Equation (3.6) is solved by

$$\mathbb{D}_a(\tau) = (e^{-i\tau R})_a{}^b D_b \quad . \quad (3.7)$$

In order to substitute the $\mathbb{D}_a^2$ part of $\square$ in (3.2a) by an ordinary function, we have to make use of the Heisenberg representation of the distance $(z_1 - z_2)$. It turns out to be convenient to introduce the supersymmetric invariant distances

$$\xi_a := (x_1 - x_2)_a + 2i \Theta_\alpha^1 \bar{\Theta}_{\dot{\alpha}}^2 - 2i \Theta_\alpha^2 \bar{\Theta}_{\dot{\alpha}}^1 \quad , \quad (3.8)$$

$$\Delta_\alpha := \Theta_\alpha^1 - \Theta_\alpha^2 \quad , \quad \bar{\Delta}_{\dot{\alpha}} := \bar{\Theta}_{\dot{\alpha}}^1 - \bar{\Theta}_{\dot{\alpha}}^2 \quad . \quad (3.9)$$

Defining the Heisenberg representation of ξ_a , Δ_α and $\bar{\Delta}_{\dot{\alpha}}$ in the same manner as for $\mathbb{D}_a$ we get

$$\xi_a(\tau) - \xi_a = 2 [R^{-1}(e^{-i\tau R} - 1)]_a^b \mathcal{D}_b - \chi_a(\tau) \quad , \quad (3.10)$$

$$\Delta^a(\tau) = (W \mathcal{Q}^{-1}(e^{\frac{1}{2}i\tau \mathcal{Q}} - 1))^a + \Delta^a \quad , \quad (3.11)$$

$$\bar{\Delta}_i(\tau) = ((e^{-\frac{1}{2}i\tau \tilde{\mathcal{Q}}} - 1) \tilde{\mathcal{Q}} \bar{W})_i + \bar{\Delta}_i \quad , \quad (3.12)$$

where

$$\tilde{\mathcal{Q}}_i^{\beta} = \bar{\mathcal{Q}}_i^{\beta} \quad , \quad (3.13)$$

$$\chi_a(\tau) = \int_0^\tau d\tau' \chi'_a(\tau') \quad , \quad (3.14)$$

$$\chi'_a(\tau) = \bar{W}_i(\tau) \Delta_a(\tau) - W_a(\tau) \bar{\Delta}_i(\tau) \quad . \quad (3.15)$$

In (3.15) the Heisenberg representations of W and $\bar{W}$ appear

$$W^a(\tau) = (W e^{\frac{1}{2}i\tau \mathcal{Q}})^a \quad , \quad \bar{W}_i(\tau) = (e^{-\frac{1}{2}i\tau \tilde{\mathcal{Q}}} W)_i \quad . \quad (3.16)$$

It is obvious that

$$\xi_a(\tau) h(z_1, z_2; \tau) = 0 \quad . \quad (3.17)$$

We can now invert (3.10) to express $\mathcal{D}_a$ in terms of ξ_a and $\chi_a(\tau)$. Making use of this representation of the action of $\mathcal{D}_a(0)$ on $h(z_1, z_2; \tau)$ one gets

$$\square h(z_1, z_2; \tau) = \left\{ \hat{\square} + \frac{1}{2} \bar{W} \bar{\mathcal{D}} - \frac{1}{2} W \mathcal{D} \right\} h(z_1, z_2; \tau) \quad , \quad (3.18)$$

with

$$\begin{aligned} \hat{\square} &= \frac{1}{4} \text{tr} [R c \text{th}(\frac{1}{2}i\tau R)] \\ &- \frac{1}{32} (\xi - \chi(\tau))_a [R^2 \text{sh}^{-2}(\frac{1}{2}i\tau R)]^a_b (\xi - \chi(\tau))^b \end{aligned} \quad (3.19)$$

With the help of the relation (3.18) the heat equation (3.2a) is integrated formally

$$h(z_1, z_2; \tau) = U(\tau) \exp \left\{ -i \int_0^\tau d\tau' U(-\tau') \hat{\square}(\tau') U(\tau') \right\} G(z_1, z_2), \quad (3.20)$$

in terms of the differential operator

$$U(\tau) := \exp \left\{ -\frac{1}{2} i \tau (\bar{W} \bar{\mathcal{D}} - W \mathcal{D}) \right\}. \quad (3.21)$$

Analyzing the behaviour of (3.20) for small values of τ , one recognizes that the initial condition (3.2b) is satisfied, provided

$$G(z_1, z_2) \Big|_{\tau=0} = \left(\frac{i}{4\pi} \right)^2 \Delta^2 \bar{\Delta}^2. \quad (3.22)$$

Though (3.20) is the general solution of the heat equation after substituting (3.18), it is not a solution of (3.2a) as long as the function $G(z_1, z_2)$ remains unspecified. This peculiar situation arises because the substitution (3.18) is valid only for solutions of the heat equation (3.2). All that we have to do to obtain a solution of (3.2) is to find a suitable function $G(z_1, z_2)$ such that (3.17) is valid. Using the same method as in Ref. 6), it is straightforward to show that (3.17) is satisfied, provided the function $G(z_1, z_2)$ respects the relation

$$\mathcal{D}_a G(z_1, z_2) = -\frac{1}{4} \oint^b R_{ba} G(z_1, z_2). \quad (3.23)$$

To solve (3.23) one first factorizes a path dependent phase factor

$$\exp \left\{ - \int_C d\xi_A \phi^A(\xi) \right\} \quad (3.24)$$

from $G(z_1, z_2)$ where the path C is a straight line in superspace running from z_2 to z_1 . This procedure transforms the covariant derivative $\mathcal{D}_a$ appearing on the left-hand side of (3.23) into the Schwinger-Fock gauge, where a representation of the gauge connection in terms of the field strength can be given. This has been worked out in Ref. 10). The resulting differential equation is solved easily, giving

$$G(z_1, z_2) = \exp \left\{ - \int_C d\xi_A \phi^A(\xi) \right\} \left(\frac{i}{4\pi} \right)^2 \Delta^2 \bar{\Delta}^2. \quad (3.25)$$

Inserting this expression into (3.20) we obtain the heat kernel

$$h(z_1, z_2; \tau) = \left(\frac{1}{4\pi}\right)^2 \exp\left\{-\int_0^\tau d\tau' \phi^A(\tau')\right\} \times \exp\left\{-i \int_0^\tau d\tau' U(\tau-\tau') \hat{\square}(\tau') U(\tau'-\tau)\right\} \Delta^2(\tau) \bar{\Delta}^2(\tau) U(\tau) \quad (3.26)$$

where the covariant derivatives appearing in $U(\tau)$ have to be taken in the Schwinger-Fock gauge. The derivatives embodied in $U(\tau)$ appearing outside the $d\tau'$ integral are easily isolated by rewriting $U(\tau)$ as

$$U(\tau) = \exp\left\{-\frac{1}{2}i \int_0^\tau d\tau' U(\tau') (\bar{W}\bar{\phi} - W\phi) U(\tau')\right\} \times \exp\left\{-\frac{1}{2}i\tau (\bar{W}\bar{D} - W D)\right\} \quad (3.27)$$

The second exponential does not contribute in (3.26). Explicit expressions for ϕ and $\bar{\phi}$ have been given in Ref. 10).

Equation (3.26) together with (3.27) provides an explicit representation of the Green's function associated with $\square$ via (3.3).

4. - THE EFFECTIVE ACTION

The effective action Γ is determined by the coincidence limit of the heat kernel $h(z_1, z_2; \tau)$. As the coincidence limit of $\Delta^2(\tau)$ ($\bar{\Delta}^2(\tau)$) is proportional to $W^2(\bar{W}^2)$, W and $\bar{W}$ can be set to zero everywhere else in (3.26). In this way the effective action becomes

$$\Gamma = -\frac{1}{2} \left(\frac{1}{4\pi}\right)^2 \int d^4\Theta d^4X \int_{-\infty}^0 \frac{d\tau}{\tau} \exp\left\{-i \int_0^\tau d\tau' \hat{\square}(\tau')\right\} \times [W \tilde{q}^{-1} (e^{\frac{1}{2}i\tau \tilde{q}} - 1)]^2 [(e^{-\frac{1}{2}i\tau \tilde{q}} - 1) \tilde{q}^{-1} \bar{W}]^2 \quad (4.1)$$

This expression may be rewritten as

$$\begin{aligned}
 \Gamma = & -\frac{1}{(16\pi)^2} \int d^4\Theta d^4x W^2 \bar{W}^2 \times \\
 & \times \int_{-\infty}^0 \frac{d\tau}{\tau} \left\{ \left[\bar{T}\tau G^2 - \bar{T}\tau \bar{G}^2 \right] \left[\bar{T}\tau \left(\text{ch} \frac{1}{2} i\tau \bar{G} \right) - \bar{T}\tau' \left(\text{ch} \frac{1}{2} i\tau G \right) \right]^{-1} \right. \\
 & \times \bar{T}\tau' \left[\left(G^2 - \frac{1}{4} D^2 \right)^{-1} \left(\text{ch} \left(\frac{1}{4} i\tau D \right) - \text{ch} \left(\frac{1}{2} i\tau G \right) \right) \right] \times \\
 & \times \bar{T}\tau' \left[\left(\bar{G}^2 - \frac{1}{4} D^2 \right)^{-1} \left(\text{ch} \left(\frac{1}{4} i\tau D \right) - \text{ch} \left(\frac{1}{2} i\tau \bar{G} \right) \right) \right] \left. \right\} \quad (4.2)
 \end{aligned}$$

where G , $\bar{G}$ and D have been defined in Ref. 6).

In the proper time formalism UV singularities show up as singularities in the proper time integral for small values of τ . As is readily seen from (4.2), Γ is UV finite. However, this does not imply that the β -function of SYM vanishes to one loop, as we have dropped the ghost contributions which are UV singular¹¹⁾.

To show that (4.2) represents a well-defined, real action, one would like to rotate the $d\tau$ -contour into $\tau = is$, $0 \leq s \leq +\infty$. At this stage, one encounters a difficulty typically for massless Yang-Mills theories⁷⁾. The contour cannot be deformed immediately as the integrand does not vanish at infinity. In Ref. 7) it was shown that in the non-supersymmetric case an unstable mode of the analogue of the operator $\square$ is responsible for this behaviour. This mode must be treated separately. In the non-supersymmetric case, it leads to an imaginary contribution to Γ , even in the presence of a pure magnetic background field.

Let us analyze the effect of the unstable mode in the present case. For simplicity, we shall restrict our considerations to the case of a "pure magnetic" background superfield. We have in general

$$R_{ab} R^{ab} = \frac{1}{2} (\underline{E}^2 - \underline{B}^2) \quad , \quad (4.3)$$

and

$$R_{ab} \tilde{R}^{ab} = \underline{E} \cdot \underline{B} \quad , \quad (4.4)$$

where $\tilde{R}^{ab}$ is the dual of R_{ab} . $\underline{E}$ and $\underline{B}$ are the supersymmetric generalizations of

the electric and the magnetic field. Setting $\underline{E}$ to zero we get

$$(G^2)_a{}^\beta = -4 \delta_a{}^\beta B^2 \quad , \quad (4.5a)$$

$$(\tilde{G}^2)_i{}^{\dot{\beta}} = -4 \delta_i{}^{\dot{\beta}} B^2 \quad . \quad (4.5b)$$

In the presence of a "pure magnetic" background superfield the effective action becomes

$$\begin{aligned} \Gamma = & -\frac{1}{(16\pi)^2} \int d^4\Theta d^4x W^2 \bar{W}^2 \times \\ & \times \int_{-\infty}^0 \frac{d\tau}{\tau} \left\{ +\frac{2i}{\tau} B \left(\frac{1}{16} D^2 - B^2 \right)^{-2} \text{sh}^{-1}(iB\tau) \times \right. \\ & \left. \times \left[\text{ch}\left(\frac{1}{4}i\tau D\right) - \text{ch}(i\tau B) \right]^2 \right\} . \end{aligned} \quad (4.6)$$

We can now analyze the behaviour of (4.6) for large values of s . Subtracting and adding the expression

$$\frac{1}{8} e^{i\tau B} \left[e^{-\frac{1}{4}i\tau |D|} - e^{-i\tau B} \right]^2 \quad (4.7)$$

where $|D|$ refers to the lowest component of D , inside the curly bracket of (4.6) we obtain

$$\Gamma = \Gamma_{stable} + \Gamma_{unstable} \quad . \quad (4.8)$$

In Γ_{stable} we can rotate the contour of integration to obtain

$$\begin{aligned} \Gamma_{stable} = & -\frac{2}{(16\pi)^2} \int d^4\Theta d^4x W^2 \bar{W}^2 B \left(\frac{1}{16} D^2 - B^2 \right)^{-2} \times \\ & \times \int_0^\infty \frac{ds}{s^2} \left\{ \text{sh}^{-1}(Bs) \left[\text{ch}\left(\frac{1}{4}Ds\right) - \text{ch}(Bs) \right]^2 \right. \\ & \left. - \frac{1}{8} e^{-Bs} \left[e^{\frac{1}{4}s|D|} - e^{Bs} \right]^2 \right\} . \end{aligned} \quad (4.9)$$

$\Gamma_{unstable}$ is given by

$$\Gamma_{unstable} = -\frac{1}{4} \left(\frac{1}{16\pi}\right)^2 \int d^4\Theta d^4x W^2 \bar{W}^2 \times \int_0^\infty \frac{d\tau}{\tau} \frac{1}{\tau} B \left(\frac{1}{16} D^2 - B^2\right)^{-2} e^{i\tau B} \left[e^{i\tau \left(\frac{1}{4}|D| - B\right)} - 1 \right]^2. \quad (4.10)$$

Due to the Feynman ie prescription, this expression is well defined. It can be evaluated to give

$$\Gamma_{unstable} = -\frac{1}{4} \left(\frac{1}{16\pi}\right)^2 \int d^4\Theta d^4x W^2 \bar{W}^2 \times B \left(\frac{1}{16} D^2 - B^2\right)^{-2} \times \left[B \log \frac{|D| - 2B}{2B} + \frac{1}{2} |D| \log \frac{|D|}{2|D| - 4B} \right]. \quad (4.11)$$

One recognizes that in the present case, the unstable mode does not lead to an imaginary contribution to the effective action if $|D| - 2B > 0$ for the lowest components of these superfields.

NOTE ADDED

After the completion of this work, the author became aware of Ref. 12) where the effective Lagrangian of SYM has been computed for a pure magnetic background field. The results of the present paper indicate that the effective Lagrangian changes substantially once a full superfield is adopted as background.

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