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PARITY VIOLATION AND THE EFFECTIVE GRAVITATIONAL ACTION IN THREE DIMENSIONS*

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ABSTRACT

A diagrammatic computation of the parity breaking gravitational effective action induced by massless fermions in three dimensions is presented. The lowest order approximation to the Chern-Simons action is obtained, in agreement with a general index-theoretic result.

I. INTRODUCTION

Redlich [1] has shown that the Chern-Simons action is induced by fermions coupled to an external gauge field in three dimensions. The result has been extended by general and mathematical methods to any odd dimension $2k+1$ ($k > 0$) and also to the case of a gravitational background field [2].

An explicit calculation has thus far been lacking in the gravitational case. It is the purpose of this Letter to report on such a computation in three space-time dimensions. It is not possible to obtain the complete Chern-Simons gravitational action from perturbation theory, since the action is not a polynomial in the field variables. This is in contrast with the gauge field case. However, at least to lowest order in the metric perturbation $h_{\mu\nu} \equiv g_{\mu\nu} - \eta_{\mu\nu}$, our expectation is confirmed, as we shall see.*

II. THE EFFECTIVE ACTION

The functional integral we are interested in is

$$Z = \int d\bar{\psi} d\psi \exp \left[i \int d^3x \det e_a^\mu \bar{\psi} \left(\gamma^a \bar{D}_\mu - \bar{D}_\mu \gamma^a \right) \psi \right] \quad (1)$$

where the forward and backward covariant derivatives are defined by

$$\begin{aligned} \bar{D}_\mu \psi &= (\partial_\mu + \omega_\mu) \psi, \\ \bar{\psi} \bar{D} &= \bar{\psi} (\bar{\partial}_\mu - \omega_\mu), \end{aligned} \quad (2)$$

and ω_μ is the spin connection:

$$\omega_\mu \equiv \frac{1}{2} \sigma^{bc} \omega_{b\mu c} \equiv \frac{1}{8} [\gamma^b, \gamma^c] e_{b\nu} \nabla_\mu e_c^\nu \quad (3)$$

* During the course of this investigation it was brought to my attention that a similar result has now been obtained independently [3].

Fig. 1: The lowest order Feynman graphs.

Our metric decomposition $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ induces an expansion for the dreibein $e_{a\mu} = \eta_{a\mu} + \frac{1}{\Lambda} h_{a\mu} + \text{higher order corrections}$.

To compute Z to second order in $h_{\mu\nu}$, there are three Feynman diagrams we must consider: they are displayed in Fig. 1. The Feynman rules are the same as in Ref. [4] (section 6), except that the chiral projection factor is omitted. Only the vacuum polarization graph (1c) contributes to the parity-breaking term. In fact, when we evaluate graphs (1a) or (1b), the external momenta $q^{(i)}$ factor out of the loop integral and consequently

$$I_{a,b}(h_{\mu\nu}, q^{(i)}, m, \Delta) = F(h_{\mu\nu}, q^{(i)}) P(m, \Delta)$$

where $P(m, \Delta)$ is a polynomial in the fermion mass m and the cutoff momentum Δ . Such a contribution will be cancelled in the Pauli-Villars subtraction scheme, which we employ to regulate our calculation.

The Feynman integral for graph (1c) is

$$\Pi(q) = \int \frac{d^3 p}{(2\pi)^3} \dots \frac{\text{Tr} \left\{ \frac{-i}{4} h^{\mu\nu}(q) \gamma_\mu (2p+q) i(\not{p}+m) \frac{-i}{4} h^{\alpha\beta}(-q) \gamma_\alpha (-2p-q) i(\not{p}+h+m) \right\}}{(p^2 - m^2 + i\epsilon)((p+q)^2 - m^2 + i\epsilon)} \quad (4)$$

The Dirac algebra is realized by Pauli matrices

$$\gamma^0 = \sigma_3, \quad \gamma^1 = i\sigma_1, \quad \gamma^2 = i\sigma_2, \quad (5)$$

and

$$\text{Tr} \gamma^\mu \gamma^\nu \gamma^\lambda = -2i\epsilon^{\mu\nu\lambda}. \quad (6)$$

The desired term (i.e. the term containing a trace of three γ 's) can now be extracted from Eq. (4). Denoting the parity conserving and parity breaking terms by P_C and P_B , respectively, we have

$$I_{\text{eff}} = I_{\text{eff}}^{PC} + I_{\text{eff}}^{PB} = -i \ln Z,$$

where

$$\begin{aligned} I_{\text{eff}}^{PB}(h_{\mu\nu}, m) &= \\ &= -i \int \frac{d^3 q}{(2\pi)^3} \left[-\frac{1}{16} h^{\mu\nu}(q) h^{\alpha\beta}(-q) (-2im\epsilon_{\mu\alpha\gamma} q^\gamma) \times \right. \\ &\quad \left. \int \frac{d^3 p}{(2\pi)^3} \frac{(2p+q)_\nu (2p+q)_\beta}{(p^2 - m^2 + i\epsilon)((p+q)^2 - m^2 + i\epsilon)} \right] \end{aligned} \quad (7)$$

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The integration over the loop momentum can be easily performed using a Feynman parametrization and it yields

$$\begin{aligned} \int \frac{d^3 p}{(2\pi)^3} A^{-1} B^{-1} &= \frac{i}{16\pi|m|} J \\ \int \frac{d^3 p}{(2\pi)^3} p_\nu A^{-1} B^{-1} &= \frac{-i}{32\pi|m|} J q_\nu \\ \int \frac{d^3 p}{(2\pi)^3} p_\nu p_\beta A^{-1} B^{-1} &= \frac{-i}{6\pi^2} \eta_{\nu\beta} \Delta \end{aligned} \quad (8)$$

where $A = p^2 - m^2 + i\epsilon$, $B = (p+q)^2 - m^2 + i\epsilon$, Δ is a momentum cutoff and

$$\begin{aligned} J &= \int_{-1}^1 dy \left(1 - \frac{q^2}{4m^2} (1-y^2) \right)^{-1/2} \\ &= 2\sqrt{\frac{4m^2}{-q^2}} \tan^{-1} \sqrt{\frac{-q^2}{4m^2}} \end{aligned}$$

We notice that $J \rightarrow \text{constant}$, $\frac{|m|}{q}$ for $m \rightarrow 0$ and $J \rightarrow 2$ for $m \rightarrow \pm\infty$.

Because of the factor m on the right-hand side of Eq. (7), $I_{\text{eff}}^{PB}(h_{\mu\nu}, m)$ would seem to vanish in the limit $m \rightarrow 0$. However, this is not all of I_{eff}^{PB} because I_{eff} is two regulator masses* M_i ($i = 1, 2$) and coefficients c_i ($c_0 = 1$, $M_0 = m$) with the sum rules

$$\sum_{i=0}^2 c_i |M_i|^k = 0 \quad (k = 0, 1, 2)$$

Then, in the limit $m \rightarrow 0$ and $M_i \rightarrow \infty$ or $M_i \rightarrow -\infty$ for $i = 1, 2$

$$\begin{aligned} \sum_{i=0}^2 c_i I_{\text{eff}}^{PB}(h_{\mu\nu}, M_i) &= \\ &= -i \int \frac{d^3 q}{(2\pi)^3} \left\{ \frac{1}{128\pi} \frac{M_i}{|M_i|} h^{\mu\nu}(q) h^{\alpha\beta}(-q) \epsilon_{\mu\alpha\gamma} q^\gamma [q_\nu q_\beta - \eta_{\nu\beta} q^2] \right\} \\ &= \frac{1}{128\pi} \frac{M}{|M|} \int d^3 x h^{\mu\nu}(x) \epsilon_{\mu\alpha\gamma} \partial^\gamma [\partial_\nu \partial_\beta h^{\alpha\beta}(x) - \square h_\nu^\alpha(x)] \end{aligned} \quad (9)$$

* Actually, one regulator mass would be enough for regulating I_{eff}^{PB} , but we need another one to regulate I_{eff}^{PC} , which is cubically divergent.

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One may not immediately recognize this expression, but a functional derivation with respect to $h^{\mu\nu}(x)$ and symmetrization of the indices yields the linearized form of the three-dimensional Weyl tensor [5]. I_{eff}^{PB} is therefore equal to the (lowest order approximation of the) Chern-Simons action plus a possible surface term. This is consistent with the result in Ref. [2].

It is our intention to discuss the relationship between the induced Chern-Simons action and the zero modes of the Dirac equation in a separate publication [6].

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