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STATIC PROPERTIES OF HADRONS FROM SCALAR LATTICE QCD

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ABSTRACT

Wave functions for mesons and baryons are computed from lattice QCD. They are used to obtain the electromagnetic properties of the low lying mesons, including charge radii and electromagnetic form factors. Experimental data for the pion's and proton's electromagnetic form factor fall close to the theoretical lattice computation. Predictions are made for other states. The lepton width of the ρ , ϕ and J/Ψ are obtained. The wave functions are also used to obtain estimates for isospin splittings and proton decay.

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I. INTRODUCTION

Quantum chromodynamics has defied analytic treatment for more than 10 years. It seems unlikely that, in the near future, an analytic approach will be found for quantities which depend on the non-perturbative aspects of the theory. In light of this, one is left with numerical methods, that is, the Monte Carlo approach of lattice QCD [1]. The exciting thing about this approach is that it offers a means of calculating phenomenology from first principles.

Most previous computer simulations have focused on the hadron spectrum for u and d quarks [2–27]. Results have been accurate to the 10–15% level, which is competitive with the quark potential and bag models, although the latter entail some ad hoc assumptions. Since lattice QCD incorporates the “true” forces which bind the quarks together, one would hope that simulations will yield a spectrum accurate to better than 5%. In defense of lattice QCD, one should realize the difficult and ambitious nature of the computations. First of all, they are extremely taxing on computer resources. Even with today’s supercomputers, hundreds of CPU hours are needed to obtain meaningful results even when employing the quenched approximation [2,3]. Memory requirements are also severe. The code must often be structured so that the important variables reside in core memory [28–30]. Propagator data sets can be so huge that it becomes prohibitive to store them on tape. A second difficulty is the inability to work on top of the physical u and d quark masses. Calculations are carried out for several “heavy” values of the bare u quark mass so that the resulting hadrons are heavier than those observed in nature. Only after extrapolating the data is contact made with experiment. A typical project obtains pion masses of 1400 MeV, 1200 MeV, 1000 MeV, 800 MeV and 600 MeV and then extrapolates to the physical pion mass of 140 MeV. A similar procedure is done for each state. This can introduce errors of the order of 10%. Results can depend on the extrapolation method, making it less clear what is a true prediction. For example, with Wilson fermions one extrapolates linearly in all masses except the pion where one extrapolates quadratically. Hence chiral behavior in the pion mass is assumed. There are two reasons why it is difficult to work at the physical u quark mass. As the bare quark mass is made lighter, the propagator inversion algorithms (Gauss–Seidel or conjugate gradient) slow down to such a point that a prohibitive amount of computer time is needed. In addition, the size of the bound state grows and finite size effects enter. At $\beta = 6.0$ one needs a 32^4 lattice to avoid finite size effects for the physical pion, nucleon and rho states. A third problem is that scaling violations have been observed in the cross-over region [25, 31–36]. This has forced calculations to be done further and further into the weak coupling region. Since the lattice spacing in physical units decreases as the coupling, g^2 , decreases, finite size effects become more severe. Calculations performed at $\beta = 6.1$ require lattices larger than 32^4 for the light hadrons. Such a beta value is appropriate for J/Ψ which would nicely fit inside a 16^4 lattice. In contrast, at $\beta = 5.6$, where the lattice spacing is larger, a 16^4 lattice would suffice for the pion. Given a lattice of fixed size, stronger coupling values are more relevant for light quarks and weaker coupling values for heavy quarks. Since asymptotic freedom scaling violations [25, 31–36] have been

seen for $\beta < 6.0$, computations done at such stronger couplings cannot be confidently extrapolated to obtain continuum results. Forthly, there is the problem of putting fermions on the lattice [37–39]. When the Dirac equation is naively discretized a doubling of physical degrees of freedom appears. To avoid this, one can add higher derivative terms (Wilson’s method [37]) which explicitly break chiral symmetry or one can associate the replication of fermions with flavor symmetry which is the Kogut–Susskind approach [38]. With the latter, there is no clear-cut way of determining the quantum numbers of the hadronic bound state since parity, spin, and flavor have been intermingled. In short, lattice fermions are not exactly equivalent to continuum fermions which can cause the physics and the spectrum to be distorted. As an example, Wilson’s methods introduces an explicit anomalous magnetic moment for the quark which makes it smaller than its Dirac value. Lattice magnetic moment measurements [6,17,18] should be systematically smaller than particle data table results. Smaller magnetic moments also imply smaller spin–spin splittings so that lattice computations with Wilson fermions should give spin splittings between states (such as the nucleon and Δ and the ρ and the π) which are smaller than in nature (and this is what is observed).

The challenge of future calculations will be to overcome these theoretical and numerical difficulties. Some day Monte Carlo methods for QCD will be regarded in the same way as Feynman graph techniques are for QED.

It is useful to use an indirect approach in an effort to deal with the above mentioned difficulties. We will analyze a simpler analog system to learn as much as possible about the hadronic bound state. In the end we will try to make contact with phenomenology.

Since the strong interaction bound state problem is a difficult one, one should perhaps treat the simplest manageable system. In terms of physics and computer resources, lattice QCD has little problem in handling the gluon degrees of freedom. We thus retain the gluons but replace the quarks with something easier to deal with. We have chosen to study how scalars interact with gluons to form bound states. This system is not QCD but should retain some of its features. It is similar to what is done with the hydrogen atom where, instead of solving the Dirac equation, one uses the Schrödinger equation and incorporates spin effects perturbatively. The electron and proton are fermions but a bosonic equation, the Schrödinger equation, suffices for many purposes. Before discussing the physics of this “idealized spinless world”, we would like to indicate why this system, when combined with other techniques, overcomes the above mentioned obstacles and makes the lattice computations of the bound state problem feasible.

A. COMPUTER CONSIDERATIONS

There are computational advantages in using scalars. Compared to Wilson fermions, propagators can be inverted 20 times faster. Since 200 hours of CYBER 205

CPU time (spread out over a one year period) went into this project, the same project with the same high statistics would not have been possible with fermions. Scalars require 1/16th as much memory as Wilson fermions. Approximately 600 propagators were used in this project. They were stored on 15 computer tapes at 6250 bpi. For Wilson fermion some 240 tapes would have been needed. These computer considerations are of non-trivial importance. For more about the computer aspects of this project see ref.[30].

B. THE TWO – PARAMETER ACTION AND SCALING BEHAVIOR

Asymptotic scaling violations have forced lattice QCD computations to be performed further and further into the weak coupling region, a region more appropriate for heavy quark systems. These scaling violations have been understood in terms of the two – parameter fundamental plus adjoint coupling theory [40,41]. Plotting β_a , the adjoint coupling, on the vertical axis and β_f , the inverse fundamental or Wilson action coupling, on the horizontal axis, a rich phase structure is observed (see Figure 1 of ref. [41]). First order phase transitions appear above the x – axis, one of which approaches the Wilson theory in the cross – over region where Monte Carlo computations are performed. This nearby phase transition is the likely cause of the scaling violations. One would like to be sensitive to the weak coupling critical point where the continuum limit is achieved. Unfortunately, the physics is affected by the nearby phase transition. This manifests itself in the specific heat where a spectacular bump has been seen [42] in the cross – over region of the Wilson theory. It can be shown [36] that such a bump causes quantities like the string tension and the deconfining phase transition temperature to fall faster than asymptotic freedom predictions, which is what is observed. One way to overcome the problem is to use the Wilson action further into weak coupling and use very large lattices ($> 32^4$ at $\beta = 6.1$). Alternatively, the phase transition can be avoided by going below the Wilson axis. The latter has important advantages. One need not work at very weak coupling values. Hence enormous lattices are not required and one can work in the intermediate and stronger strong coupling regions which are more appropriate for light quark physics. Finally scaling violations are greatly diminished so that results can be confidently extrapolated to the continuum. We observed an improvement in scaling behavior. This was not unexpected since the bump in the specific heat diminishes below the axis [43].

Many hadron spectrum projects are carried out at one coupling constant and scaling is assumed. To avoid this assumption, computations were performed at three coupling constant values, $(\beta_f, \beta_a) = (7.52, -3.321), (7.84, -3.213)$ and $(8.16, -3.105)$ where β_f and β_a refer to the parameters in the following action [40]

$$A = \sum_{\square} ((\beta_f / 6) \text{tr}(U_{\square} + h.c.) + (\beta_a / 9) |\text{tr}(U_{\square})|^2) \quad (I.1)$$

Scaling was then checked in the following quantities: the static quark potential [44], the string tension (see Appendix A), heavy – light wave functions [44], meson wave functions (see Sect. II) for states between 1.3 and 2.5 GeV, hadron masses [45 – 47] and constituent masses (Sect. V). Figure II.2b of Sect. II and Table A.1 of Appendix A typify the excellent scaling behavior observed.

C. GETTING ON TOP OF PHYSICAL QUARK MASSES

The combination of scalar matter and modified action permitted us to avoid the problem of quark mass extrapolation. Calculations were performed on top of the physical quark mass values so that no data extrapolation was done. The Gauss – Seidel algorithm, which is guaranteed to converge for the scalar propagator, may have aided us in measuring light quark propagators. By working below the axis in the intermediate coupling constant region where the lattice spacing* in physical units was not too small, finite size effects were avoided. At the first coupling point $a = .22$ Fermi ($a^{-1} = 905$ MeV). At the second and third coupling points, the lattice spacings were .14 Fermi and .10 Fermi. At the first coupling point, the light hadrons sat nicely (see figures in Sect. II) within our 16^4 lattice. At $\beta = 6.0$ on the Wilson axis, we would not have been able to work at the physical quark masses due to finite size problems and extrapolation of the data would have been necessary. This would have created two difficulties: (A) It would have been problematic to work with states of two light flavors such as the kaon since a double extrapolation in u and s quark masses is very difficult and (B) it would have been impossible to obtain reliable wave functions since this would have involved extrapolating enormous data sets (many correlation functions were measured to get the wave functions). In short, the use of the two – parameter action (Eq.(I.1)) in this project was crucial.

D. AVOIDING THE LATTICE DISTORTION OF SPIN INTERACTIONS

Wilson fermions introduce an explicit anomalous magnetic moment which affects the spin – spin color hyperfine interaction. The lattice itself also distorts this interaction this short – ranged interaction which is well represented by a delta function (see Eq.(I.2) below). The lattice is very good at reproducing the physics at large distances (such as the confinement phenomenon) since from a far away viewpoint the lattice looks like a fine mesh. At short distances, however, “one sees the holes”. Short distance physics, like the color hyperfine interaction, is likely to be disturbed by the lattice cutoff. Minimizing the distortion requires as fine a lattice as possible, that is, working deep in the weak coupling region and using large lattices. This, of course, necessitates powerful computer resources. With spin 1/2 quarks there is no direct way of controlling the lattice distortion of spin degrees of freedom. “By taking the spin out of the quarks” and treating this degree of freedom by hand, the distortion to a large extent can be avoided. In first order perturbation theory in the

* The lattice spacing was fixed from the string tension measurements in Appendix A.

spin – spin interaction, splittings are proportional to $|\Psi(0)|^2$, the wave function at the origin [48]:

$$\Delta H = -\frac{\pi\alpha_s}{6m_1m_2}\delta^3(\vec{r})\lambda_1\cdot\lambda_2\sigma_1\cdot\sigma_2, \quad (1.2)$$

where the subscripts 1 and 2 denote two different quarks, α_s is the strong coupling constant, m_1 and m_2 are the constituent quark masses, the λ 's are the appropriate Gell – Mann SU(3) color generators and the σ 's are the Pauli spin matrices. By measuring the wave functions at all distance scales and extrapolating to short distances, a reliable determination of $\Psi(0)$ can be obtained. Then Eq.(1.2) can be used to obtain the spectrum for hadrons with spin. This was the method we used in references [45 – 47] to compute hadron masses. Basically by treating the spin interaction "by hand" [48], one overcomes the lattice distortion of the color hyperfine interaction and thereby gets good spin splittings from lattice calculations.

An indication of the reliability of this approach is given by the hadron spectrum, taken from references [45,46] and shown here in Tables I.1 and I.2. In these calculations, four parameters had to be fixed: the u, s and c quark masses and a Λ_{QCD} scale used to compute α_s in Eq.(1.2). Below we describe how the u, s and c quark masses were determined.

Since spin 1/2 quarks are not being used, there are certain strong interaction physics issues which we cannot address. An example is chiral symmetry. Quantities which depend crucially on chiral symmetry such as the pion decay constant, f_π , and the pion mass itself cannot be computed by our methods.

E. THE IDEALIZED WORLD

The theory we are treating is not the real world since we use spinless constituents. It is useful to review what would the world look like if quarks had no spin. First of all, there would be fewer hadron states. The distinction between the π and the ρ would be absent. Both are s – wave states distinguished by whether the spins add up (the ρ) or cancel (the π). Without spin there would be a common s – wave bound state. The same would be true for the other lowest lying pseudo – vector and pseudo – scalar mesons: the K and the K^* , the D and D^* , the F and F^* , and the η_c and the J/ Ψ . Baryon states such as the delta and nucleon would also be degenerate. Returning to the pion – rho case, what would be the mass of the common spinless meson? While one cannot determine its value exactly, Eq.(1.2) provides a good estimate. It predicts, in first order perturbation theory, that

$$\begin{aligned} m^{0-} &= M_0 - \frac{24\pi\alpha_s}{9m_q^c m_0^c} |\Psi(0)|^2, \\ m^{1-} &= M_0 + \frac{8\pi\alpha_s}{9m_q^c m_0^c} |\Psi(0)|^2, \end{aligned} \quad (1.3)$$

where m^{0-} and m^{1-} are the masses of the pseudoscalar and pseudovector, m_q^c and m_Q^c are the constituent masses of the quarks and M_0 is the "spin-independent" mass. Turning Eq.(I.3) around one gets

$$M_0 = \frac{3}{4} m^{1-} + \frac{1}{4} m^{0-} \quad (I.4)$$

Our calculation measured, M_0 . We adjusted the bare mass of one of our spinless constituents, the one which played the role of the u quark, so that M_0 was 610 MeV. The same procedure was done for the s and c quarks using diagonal flavor mesons, $\bar{s}s$ and $\bar{c}c$. It took three or four attempts to fix each of the bare quark masses: A candidate quark mass was chosen; Propagators were measured for many lattices; Data analysis was carried out and one found the spinless meson mass to be different from that of Eq.(I.4). The bare quark mass was readjusted and then the whole procedure was repeated until, in the case of a u quark, a value for 610 MeV for M_0 was obtained. The mass of the d quark was chosen to be the same as the u quark so that the same set of propagators could be used. The u, d, s and c quark masses were, once and for all, fixed. Propagators were measured and appropriate linear combinations were taken to obtain the other s-wave mesons and baryons. Then the wave functions were measured. At this point, further phenomenological results were completely fixed: Since the quark masses were pinned down in the spectrum project, there was no way to readjust them to change the wave functions. In this sense, there were no adjustable parameters in our results for electromagnetic form factors, charge radii, lepton decay widths, etc.

References [45,46] (Tables I.1 and I.2) obtained the spectrum for hadrons with spin by perturbatively taking into account spin effects. This paper deals with the wave functions for which the spin corrections can only be obtained in second order perturbation theory and hence are computationally unfeasible to include. Each wave function is common to several hadron states (i.e. a common ρ and π wave function, a common Δ and nucleon wave function, etc.).

F. RESULTS

References [45 – 47] presented the mass spectrum results of our work. This paper is a continuation of that work in which wave functions and the corresponding phenomenology are presented. Methods of obtaining wave functions have been introduced in refs. [49] and [44]. The latter used a gauge invariant definition but treated only the system of a light quark bound to an infinitely heavy quark, a system appropriate for mesons involving a t quark. This paper obtains wave functions for light matter, both for mesons and baryons. The wave functions provide an instant picture of the bound state and can be used to address a wide range of phenomenology. With $|\Psi(0)|^2$, we compute the decay rate of the J/Ψ , ρ , and ϕ into lepton pairs (see Sect. II.F) and estimate the rate for the proton decay in GUT models (see Sect. VI). $|\Psi(0)|^2$ also entered in the hadron mass spin-spin splittings of refs. [45] and [46]. The agreement of all these computations with experiment in-

indicates that the values of $|\Psi(0)|^2$ are accurate to about 15%. Since the wave functions provide the probability that the quarks are in a given configuration, multiplication by the charges yields the charge distribution. We obtain these distributions and compute the charge radii $\langle r_{ch}^2 \rangle$ both for the low-lying s-wave mesons, the F, D, K and π (Sect. II.D), and the s-wave baryons, the proton, Σ , Λ , Ξ , Ω , Σ_c , Λ_c and Ω_c (Sect. III.B). In the cases where experimental data are available (the K_c^- , K_c^0 , π_c^- and the proton) agreement with experiment is good. In the other cases, predictions are provided. By taking the Fourier transform of the charge distribution, electromagnetic form factors of the above states are obtained for the momentum range up to 2 GeV. Agreement for the experimentally measured π^- and proton cases is fairly striking (See Figs. II.5f and III.4a of Sects. II and III). Predictions for the remaining hadrons are given. The wave functions are also used to obtain rough estimates of the electromagnetic self-energies of hadrons (Sect. IV). With additional assumptions, the mass splittings of states in the same isospin multiplet are estimated (Sect. IV). We also argue that these differences can be computed from first principles using standard lattice QCD. Sections V and VI discuss applications to constituent masses and proton decay.

G. METHODOLOGY

Our Monte Carlo and computer techniques have been described in [30] and [44], so we will be brief in what follows. We generated 75 16^4 lattices (25 at each of the three coupling constants) thoroughly thermalized according to the action in Eq. (I.1) using a hybrid [30,44] of the Cabibbo-Marinari technique [50]. These lattices represent typical important gluonic configurations to the path integral. When quantities are averaged over these configurations, the dynamics of the non-abelian SU(3) gauge theory is reproduced. We then introduced scalar matter and let it propagate throughout these background gluonic media. The probability that the a scalar moves from one site in the lattice to another, namely the propagators, was measured. They were computed by inverting the Klein-Gordon equation in the presence of a background gluonic field configuration. Hence these scalars move relativistically throughout the lattice experiencing the forces of the SU(3) non-abelian gauge theory. When widely separated, they feel a constant confining force; nearby, they presumably experience a logarithmically modified Coulomb force; and at intermediate distances, their dynamics is governed by some interpolating potential, the one given by QCD. Mesons were generated by using a quark and an antiquark; baryons by using three quarks. Masses were measured by producing the hadron at $t=0$, letting it propagate in time and observing how it decayed exponentially. Wave functions were measured by producing the hadron at $t=0$, letting it propagate in time so that excited states decay away and introducing an operator which annihilated the quarks at fixed locations. These provided us with the probability distribution of the quarks in a hadronic bound state. When we use the word "wave function", we mean these correlation functions.

Our calculation differs from a quark potential model or a bag model computation in that the matter moves relativistically under the influence of the forces of QCD. In the quark and bag models one must introduce these forces by hand. Although different models use different potentials and lead to different results [51,52], they do play an important role in understanding confining – like systems.

It is perhaps worth making precise the difference between spin 1/2 quark and scalar matter in the quenched approximation. In the continuum, one would like to solve the Dirac equation:

$$(i\not{D} - g\not{A} - m)\Psi \equiv (i\not{D} - m)\Psi = 0 \quad , \quad (1.5)$$

where A_μ is a background gluonic field. Unfortunately, because the Dirac equation is a linear differential operator, it cannot be naively discretized. In Wilson's method, one instead considers the discretized version of

$$(i\not{D} + \kappa D^2 - m)\Psi = 0 \quad , \quad (1.6)$$

where κ denotes the Wilson parameter which is usually set to 1 (in lattice units). It has been argued (and it can be shown in the free case) that this extra term is not important for sufficiently weak coupling and small quark mass, m (although one must be careful when measuring certain quantities (see ref.[53])). In the intermediate coupling constant range where Monte Carlo computations are performed, the Wilson term may affect results. Eq.(1.5) implies the equation:

$$(D^2 - m^2 + \frac{g}{2} \sigma^{\mu\nu} F_{\mu\nu})\Psi = 0 \quad . \quad (1.7)$$

The equation used with scalar matter is

$$(D^2 - m^2)\Psi = 0 \quad . \quad (1.8)$$

It differs from Eq.(1.7) in that the $\sigma^{\mu\nu} \cdot F_{\mu\nu}$ term has been dropped. For mass computations, this term has been treated perturbatively. If the effect of Wilson term is sufficiently large, then the using Eq.(1.6) is not as good as using Eq.(1.8). When sufficiently powerful supercomputers and special purpose processors become available, the use of large lattices at weak couplings with small quark masses will unquestionably make Eq.(1.6) the better equation to use since the κD^2 term is believed not to be important in the continuum limit.

Lattice finite size effects were handled as in ref.[44]. To test for gluonic finite size effects, Wilson lines were measured, and observed to be zero indicating that we

were in the confining region. For matter degrees of freedom, wave functions were measured and examined at distances of 8 lattice units ($1/2$ the spatial extent of the lattice). Small wave functions at such distances indicated little probability of constituents wrapping around the lattice.

Although with scalar matter, one has the option of not using the quenched approximation [2,3], the quenched approximation was used because the physics is easier to handle. If heavier states could decay into lighter states, we would not have had enough time to measure the static properties such as the masses and the wave functions.

To ensure that our wave functions were physical, scaling tests were carried out (see Sect. II.A). For a given value of the hadron mass in physical units, wave functions were measured at the three different coupling constants. The renormalization group provided us with scaling factors. The wave functions were then scaled by these factors, superimposed on top of each other and examined. We found that, in all cases, a common wave function (see Figure II.b). This means that our definition of the wave function is a physical one which can be extrapolated to the continuum. The verification of scaling was carried out for many bound states (some 500 propagators were involved in these tests). This was the most CPU intensive part of our project.

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II. MESON RESULTS

A. WAVE FUNCTIONS: TECHNIQUES

Many of the techniques used to compute the wave functions were given in ref. [44]. For this reason, we will be brief in what follows (for more details see Sect. 4 of ref. [44]): At the origin at time $t = 0$, one inserts on the lattice the operator $\bar{q}(\vec{0}, 0) Q(\vec{0}, 0)$, where $\bar{q}(\vec{x}, t)$ produces a quark, q , at time, t , and location, $\vec{x}$, and likewise $Q(\vec{x}, t)$ produces an antiquark, Q , at time, t , and position, $\vec{x}$. This operator couples to all mesons with the appropriate quantum numbers but as t increases, the excited states decay away leaving only the lowest s -wave mass meson state. One then inserts the operator,

$$O(\vec{x}, \vec{y}, t) = q(\vec{y}, t) \exp(i \int_P A_\mu dx^\mu) \bar{Q}(\vec{x}, t) \quad , \quad (II.1)$$

at a much later time t . This operator annihilates the quark, q , at $\vec{y}$ and the antiquark, Q , at $\vec{x}$ and thus measures whether the quarks are at the specified locations (see Fig. II.1). P is any path between the two quarks. The Wilson line, $\exp(i \int_P A_\mu dx^\mu)$, is inserted to make $O(\vec{x}, \vec{y}, t)$ gauge invariant. On the lattice, it is represented by a path ordered product of link variables, U_l . This line integral generates an artificial self energy term which must be removed to obtain physical results. In the continuum limit, it would become the Coulombic self energy of a static charge existing for a duration of time $= |\vec{y} - \vec{x}|$ and would blow up as the lattice spacing is sent to zero. $O(\vec{x}, \vec{y}, t)$ must thus be renormalized. How this is done is explained in Appendix A (basically, the self-energy of a Wilson line can be extracted by analyzing Wilson loop data). Summarizing, the interesting correlation functions to consider are

$$C(\vec{r}, t) = \langle \Omega | O(\vec{x}, \vec{y}, t) \bar{q}(\vec{0}, 0) Q(\vec{0}, 0) | \Omega \rangle \quad , \quad (II.2)$$

where $\vec{r} = \vec{y} - \vec{x}$ and $|\Omega\rangle$ is the vacuum state. Inserting a complete set of states in Eq.(II.2), one concludes that

$$C(\vec{r}, t) = \sum_s c_s \Psi_s(\vec{r}) \exp(-m_s t) \quad , \quad (II.3)$$

where s stands for the physical meson states, m_s is the mass of the meson and c_s is the overlap matrix element between the operator $\bar{q} Q$ and the physical meson state, $\langle \text{meson}_s | :$

$$c_s = \langle \text{meson}_s | \bar{q}(\vec{0}, 0) Q(\vec{0}, 0) | \Omega \rangle \quad (II.4)$$

$\Psi_S(x)$ in Eq.(II.4) is our definition of the wave function up to an overall normalization constant. The normalized wave function is obtained by requiring the integral of the square of the wave function to be one. In the heavy quark mass limit, the above definition reduces to the usual one corresponding to the non-relativistic Schrodinger equation. Regardless of the quark mass, it provides a measurement of the probability distribution of quarks in a meson.

Measuring wave functions is an ambitious undertaking, much more involved than extracting hadron masses (for example). To make headway, we adopted several simplifying procedures: First, to make the data analysis manageable, only straight line paths, P , along principal lattice axes were used in Eq.(II.1). Non-straight paths have the disadvantage that they are “kinky” (because of the lattice) and would have complicated renormalization properties. In formal terms, the restriction to straight paths is a truncation of the basis of states in the Fock Hilbert space of a meson. This truncation is particularly good when the quarks are far apart, since then the electric flux tube goes directly from one quark to the other. Having obtained results along principal directions, we assumed that rotational invariance gave us the wave function in all other directions. Since one is on the “good” side of the roughening phase transition, the side in which long distance rotational symmetry is restored, this assumption is probably quite good (see Ref. [54] for an excellent visual representation of rotational symmetry restoration). Finally, the infinite sum in Eq.(II.3) was represented by its first two terms, a procedure which works well for large t . The wave functions, $\Psi(x)$, were obtained by fitting the correlation function data in Eq.(II.2) to the first two terms in Eq.(II.3). The wave functions were first normalized to 1 at the origin for convenience (later, normalized wave functions were computed).

To make sure that the above definition had physical content, scaling of the wave functions was checked. First the mass of a $q\bar{q}$ meson was fixed in physical units and then wave function measurements were made. Figure II.2a shows an example before scaling is taken into account. The three sets of data correspond to the three different coupling constants. Using the $1/E$ version of the renormalization group [36], scaling factors were computed in ref.[44] to be 1.50 (the factor between point 2 and 1) and 2.21 (the factor between point 3 and 1). These theoretically computed numbers tell us how the physics at one coupling constant is to be scaled to be comparable to the physics at another coupling constant. Such scaling should work only when one is sufficiently close to the continuum limit. Figure II.2b displays the same wave functions as in Figure II.2a but related by the above factors. The fact that the data points fall on top of each other is a striking example of scaling and indicates that these wave functions are physical quantities. They obey a renormalization group equation which allows one to extrapolate the results to the continuum limit. Figure II.2 was for a meson weighing 2 GeV but the scaling was an omnipresent phenomenon, being observed for all $q\bar{q}$ mesons with masses between 1.3 GeV and 2.6 GeV (the range of masses for which there were no finite size effects for all three couplings).

B. WAVE FUNCTIONS: PICTURES

We remind the reader that, in the "idealized world", the wave functions for the 0^- and 1^- states are identical. Hence we obtained common wave functions for the π and ρ , K and K^* , D and D^* , F and F^* , and J/Ψ and η_c . Incorporating spin effects perturbatively requires computing second order matrix elements and excited wave functions, an endeavor beyond the scope of the present work. One can, however, make some qualitative remarks. Since the spin–spin interaction is attractive for the 0^- and repulsive for 1^- , the 0^- states will be slightly narrower than our results and 1^- states will be slightly broader. In the quenched approximation, the ρ is nature's largest s –wave meson. A way of judging the magnitude of the spin effects is via the neutron's electromagnetic form factor (see Sect. III.C) from which one concludes that they are not too significant.

Figures II.3a – II.3d display the wave functions, normalized to 1 at the origin, for the $\eta_c = J/\Psi$, $D = D^*$, ϕ and $\pi = \rho$ as a function of the radial interquark distance in Fermi. The $q\bar{s}$ wave functions differ only slightly from the $q\bar{d}$ ones, indicating that SU(3) flavor symmetry is quite good for wave functions. The K (respectively, F) wave function is slightly narrower than the π (respectively, D) wave function. To save space, we omit the figures for these also degenerate cases. The wave function data were piecewise fit to $\exp(-m r) \cdot (A_0 + A_1 r + A_2 r^2)$. This form is convenient because expectations of polynomials in r as well as Fourier transforms can be done analytically. The fits are the solid curves in Figures II.3a – II.3d.

Near the origin, a pure exponential, $\exp(-m r)$, could not fit the data well (even for the J/Ψ), indicating a non–Coulombic behavior at short distances. The deviation from Coulombic behavior did, however, diminish as the mass increased as one can see in Figs. II.3a – II.3d (the $u\bar{u}$ wave function is "quite round" near the origin). At large distances the data were well parametrized by an exponential with m the order of the constituent mass of the lightest quark. Such behavior is expected on physical grounds (see Sect. IV of ref.[44]).

Figures II.3a – II.3d pictorially describe the $q\bar{q}$ bound state occurring in nature. The rest of this section uses these wave functions to obtain various phenomenological results.

C. THE CHARGE DISTRIBUTION

Having obtained the quark–antiquark wave functions, it is not too difficult to get the electric charge density distribution. However, one must know how to relate quark coordinates to the center–of–mass coordinates. This is straightforward for a non–relativistic system. For relativistic systems which we are treating, we employed the following ideas.

View the quark – antiquark bound state in the center of mass frame where the total momentum is zero. Figure II.4 shows the setup. The quark and antiquark rotate in their orbits at the same angular rate. If the velocities are denoted by β and $\bar{\beta}$ then $\beta/x = \bar{\beta}/\bar{x}$, where x ($\bar{x}$) is the distances between the quark (antiquark) and the center of mass, C, of the system (see Figure II.4). Since the magnitude of the momenta are equal and the quark momenta are the quark velocities times the energies, $E x = \bar{E} \bar{x}$. The average energy is the constituent mass, m^C , hence

$$m^C x \approx \bar{m}^C \bar{x} . \quad (II.5)$$

Equation (II.5) determines the center of the system which coincides with the non – relativistic result if ordinary masses are replaced by constituent masses. The above derivation is classical and expected to hold approximately in the quantum mechanical case. The constituent mass values of table 4 of ref.[46] were used.

If $|\Psi(\vec{r})|^2$ (with $\vec{r}$ as in Fig. II.4) represents the probability distribution, then the charge density, $\rho(\vec{y})$, is

$$\rho(\vec{y}) = |\Psi(\vec{r})|^2 (Q_q \delta^3(\vec{y} - \vec{x}) + Q_{\bar{q}} \delta^3(\vec{y} - \vec{\bar{x}})) , \quad (II.6)$$

when C is the origin of the coordinate system. Q_q and $Q_{\bar{q}}$ are the charges of the quarks and x and $\bar{x}$ are as in Fig. II.4.

D. THE CHARGE RADII

The charge radii were obtained from

$$\langle r_{ch}^2 \rangle = \int d^3 \vec{r} \rho(\vec{r}) r^2 , \quad (II.7)$$

and Eq.(II.6). Here are the results (in Fermi²) versus experiment [55 – 58]:

$\langle r_{\text{ch}}^2 \rangle$		
Meson	Our Computation	Experiment
F^-	-.11	---
D^0	-.27	---
D^-	-.17	---
K^0	-.06	$-.054 \pm .026$
K^-	-.25	$-.28 \pm .05$
π^-	-.27	$-.44 \pm .03$
		$-.31 \pm .04$
		$-.51 \pm .03$

The experimental results for the π^- come from direct measurements using elastic $e\pi$ scattering [57] and from single pion electroproduction [58] ($e N \rightarrow e \pi N$; which requires extrapolating to the pion pole). The latter gives the larger value in the above table. As one can see, results from different experiments are not in agreement with each other. Even though our computed electromagnetic form factor agrees well (see Sect. II.E) with electroproduction experiments, the corresponding charge radius computations disagree. Since the charge radius is obtained by measuring the second derivative of the form factor at the origin, it may be that these experiments were carried out for too large a value of Q (the momentum transferred). Our charge radius does agree with the $e \pi$ experiments. The agreement in the case of the K^0 and K^- is good.

E. ELECTROMAGNETIC FORM FACTORS

The electromagnetic form factor has a precise definition in terms of the matrix element of the electromagnetic current. In the low momentum region, the form factor is also given by the Fourier transform of the charge density:

$$f(Q^2) = \int d^3\vec{r} \exp(i \vec{Q} \cdot \vec{r}) \rho(\vec{r}) \quad . \quad (\text{II.8})$$

We have not attempted to compute the large Q behavior because of the short distance lattice cutoff. In that regime, perturbative QCD may be of some help [59]. Lattice calculations in conjunction with analytic perturbative methods might be able to yield the complete picture over the entire momentum range.

Equation (II.8) results in a sum of two Fourier transforms of our wave functions which were done analytically using the piecewise fits. Figures II.5a – II.5f show our form factors for the F^+ / F^{+*} , D^+ / D^{+*} , D^0 / D^{0*} , K^+ / K^{+*} , K^0 / K^{0*} and π^+ / ρ^+ . After obtaining these curves, we searched the literature for experimental measurements. The bars in Fig.II.5f reflect these experiments [57,58] in the case of the π . In principle, it would be preferable to compare these results to the rho case, but there is no available experimental data. Only the pion's electromagnetic form factor has been measured over any extended momentum range. The $Q < .5$ GeV data are the $e \pi \rightarrow e \pi$ experiments [57] for which 42 original data points were averaged in bins of 7 to make the data legible. The data closely follow our theoretical curve. The agreement between the lattice calculation and experiment is good. Phenomenologically, the pion's form factor is fit with about the same degree of accuracy by a pole formula coming from vector meson dominance [60,61]. The other Figures (Figs. II.5a – II.5e) are predictions from lattice methods for which experimental data are needed. Figures II.5a – II.5e can probably be used as good representations of the form factors until experimental determinations become available. In principle, one should be able to obtain the electromagnetic form factors by computing matrix elements in lattice QCD with spin 1/2 quarks. In fact, there has already appeared some preliminary work in this direction using the SU(2) gauge theory [62].

F. THE LEPTON WIDTH OF THE VECTOR MESONS

The purpose of this subsection is to calculate the electromagnetic decay of a 1^- $q\bar{q}$ meson into lepton pairs, $l^- l^+$, using the wave functions and the Van Royen – Weisskopf formula [63]:

$$\Gamma_{1^- \rightarrow l^+ l^-} = 16 \pi \alpha^2 e_q^2 |\Psi(0)|^2 (1 - \beta_q^2/3) / m_V^2, \quad (II.9)$$

where m_V is the mass of the 1^- meson, α is the fine structure constant, e_q^2 is the mean-squared charge of the quarks making up the meson, and β_q is the velocity of the quarks when they annihilate. Usually, the factor $(1 - \beta_q^2/3)$ is not included in the Van Royen – Weisskopf formula because the formula is derived in the non-relativistic approximation. Although there are other relativistic corrections, this one is one of the simplest to compute.

To use Eq.(II.9), an estimate of the quark velocity is needed. This velocity is the one which the quarks have when they annihilate. Certainly, for u and s quarks and hence for the ρ and ϕ , $\beta_q^2 = 1.0$. For the c quark the velocity at annihilation can be estimated using a model in conjunction with experimental data. A typical estimate can be found in ref. [52]. For the c quark, $\beta_q^2 \approx .6$ at annihilation. Modifying this velocity does not significantly effect the results.

There are also strong interaction corrections to Eq.(II.9). One such factor,

$1 - 16\alpha_s/3\pi$, was computed in ref.[64] and is quite large for the vector meson states. A careful reading of reference [64], however, reveals that this factor is the QCD correction to the Coulombic wave function at the origin, that is, the factor required change $|\Psi(0)|^2$ computed in the Coulomb potential into $|\Psi(0)|^2$ computed for the QCD corrected potential. It is understandable why this factor is large, since it is trying to transform $|\Psi(0)|^2$ for a non-confining theory into the value for a confining one. We should not multiply Eq.(II.9) by this factor since our wave functions include this and other perturbative (as well as non-perturbative) corrections.

Using Eq.(II.9), one obtains for the $\rho(769)$, $\phi(1020)$, and $J/\Psi(3097)$

$$\begin{aligned}\Gamma_{J/\Psi} &= 4.8 \pm 1.0 \text{ KeV} , \\ \Gamma_{\phi} &= 1.0 \pm .2 \text{ KeV} , \\ \Gamma_{\rho} &= 5.4 \pm 1.1 \text{ KeV} .\end{aligned}\tag{II.10}$$

These numbers should be compared to the experimental results [65]:

$$\begin{aligned}\Gamma_{J/\Psi \rightarrow e^-e^+} &= 4.7 \pm 1.4 \text{ KeV} , \\ \Gamma_{J/\Psi \rightarrow \mu^-\mu^+} &= 4.7 \pm 1.4 \text{ KeV} , \\ \Gamma_{\phi \rightarrow e^-e^+} &= 1.31 \pm .08 \text{ KeV} , \\ \Gamma_{\phi \rightarrow \mu^-\mu^+} &= 1.06 \pm .16 \text{ KeV} , \\ \Gamma_{\rho \rightarrow e^-e^+} &= 7.08 \pm .54 \text{ KeV} , \\ \Gamma_{\rho \rightarrow \mu^-\mu^+} &= 10.3 \pm 2.2 \text{ KeV} .\end{aligned}\tag{II.11}$$

Results for the decay to e^-e^+ should be the same as for the decay to $\mu^-\mu^+$ since the muon and electron masses can be neglected in the calculation and in the phase space factor. Hence for the ϕ and the ρ there is some experimental discrepancy.

Our calculation has an uncertainty of 15% due to the uncertainty in $|\Psi(0)|^2$. Probably the largest systematic error is the use of Eq.(II.9) which, apart from the

($1 - \beta_q^2/3$) factor was derived in the non-relativistic approximation and neglected those strong interaction effects which did not simply modify $|\Psi(0)|^2$. The fact that these effects diminish as the quark mass becomes heavy, probably accounts for the excellent agreement in the case of the J/Ψ . Because of the accuracy of our electromagnetic form factors and charge radii, we believe other systematic effects (such as the use of the quenched approximation) were not as important. For the ϕ and ρ the agreement is reasonable. They represent an order of magnitude improvement in accuracy over earlier lattice QCD computations [10].

III. BARYONS

A. WAVE FUNCTIONS

For baryons we proceeded in a manner similar to the meson case. We introduced at $t=0$ the operator, $q_1 q_2 q_3$ where the subscripts refer to different flavors. The cases studied were when all quarks had the same mass (such as the proton) or when two of the quarks had the same mass and the third mass was different (such as the Ξ). The operator at $t=0$ produced three quarks which propagated throughout the lattice under the influence of the forces generated by the background gluonic fields. At a later time, t , the operator,

$$O(x_1, x_2, x_3, t) = \epsilon_{\alpha\beta\gamma} [q_1(x_1, t) \exp(i \int_{P_1} A_\mu dx^\mu)]_\alpha [q_2(x_2, t) \exp(i \int_{P_2} A_\mu dx^\mu)]_\beta [q_3(x_3, t) \exp(i \int_{P_3} A_\mu dx^\mu)]_\gamma, \quad (III.1)$$

was introduced (see Fig. III.1). α , β , and γ refer to color indices. It annihilated the quarks at the sites x_1 , x_2 and x_3 and thus measured the probability that the quarks were at these locations. For sufficiently large t , only the lowest mass baryon with the appropriate quantum numbers survives and the correlation functions

$$C(x_1, x_2, x_3, t) = \langle \Omega | O(x_1, x_2, x_3, t) \epsilon_{\alpha\beta\gamma} \bar{q}_1^{\alpha}(\vec{0}, 0) \bar{q}_2^{\beta}(\vec{0}, 0) \bar{q}_3^{\gamma}(\vec{0}, 0) | \Omega \rangle, \quad (III.2)$$

will contain information about the wave function for that state.

The lowest lying s -wave baryon bound states are much more complicated than their meson counterparts since three constituents are involved. Because the three body problem is more difficult, some simplifications were necessary. We were motivated by the situation for the non-relativistic system where the s -wave wave functions depend on two radial parameters, ρ and λ , shown in Fig. III.2. ρ is the distance between the two equal mass (i.e. equal flavor) quarks and λ is the distance from the third quark to the point half-way between the two equal mass quarks. For the relativistic system, we assumed [52] that the s -wave wave functions were still functions of ρ and λ only. We carried out measurements for a particular configuration corresponding to particular values of ρ and λ and used rotational symmetry to get the wave functions elsewhere. The correlation functions measured in Eq.(III.2) had the quarks in a "T" configuration with ρ and λ pointing along principal axes. Again, the paths in the line integrals in Eq.(III.2) were chosen to be straight, joining in the middle as in Fig. III.2.

The s -wave baryon which is antisymmetric in flavor (such as the Ω^- or the Δ^{++}) does not exist for three scalars, because of Bose statistics. To get around this, the following trick was used. We introduced two other scalar constituents of the

same mass and regarded them as different flavors. By choosing the wave function to be antisymmetric in flavor, the difficulty with statistics was avoided and a non-vanishing wave function was obtained. In a world with three equal mass quarks, SU(3) symmetry is perfect so the masses and spatial wave functions of the antisymmetric flavor state is the same as the symmetric one. In this way we were able to treat the Ω^- and Δ^{++} states.

The fitting and extrapolating techniques were similar to the ones used in the meson case. For example, we saturated the correlation functions in Eq.(III.2) with the lowest lying and first excited states. We also first normalized our wave functions to 1 at the origin. To obtain normalized wave functions, the integral under the square of the wave function had to be done. Unlike the meson case, it is awkward to fit the wave functions to two-parameter functional forms. We thus computed integrals and matrix elements numerically. As a check, we redid the meson calculations using numerical integration and found that the results were very close to the analytic treatment.

B. CHARGE DISTRIBUTIONS

To obtain the electric charge distribution, center of mass coordinates had to be determined from the quark coordinates. The center of mass coordinate for the two equal mass quarks is the point half way between them. These two quarks, more or less, act as one system with twice the quark mass. The overall center of mass is then determined by combining this system with the third quark in the same way as in the meson case. In the above, quark mass means the constituent mass. The net result is that the center of mass coordinate is determined as in the non-relativistic case using constituent masses for quarks. The charge distribution is then given by

$$\rho(\vec{y}) = |\Psi(\rho, \lambda)|^2 \sum_{i=1}^3 Q_{q_i} \delta^3(\vec{y} - \vec{x}_i) \quad , \quad (\text{III.3})$$

where y and the x_i are measured from the overall center of mass. Figures III.3a–III.3k display the charge distributions. The charge distributions for the Δ^{++} and Δ^- are respectively $+2/3$ and -1 times the one of the proton (Fig. III.3a). In these plots, we have had to greatly vary the scale of the y -axis in order to make them legible. For the neutron we get exactly zero since the three scalars all have the same mass and the average charge is zero. In nature a small non-zero charge distribution is generated via spin effects. These spin effects push the d quarks apart a bit and draw the u quark inward, thereby generating a small positive charge distribution at short distances and a small negative one at large distances. The non-zero charge distribution of the neutron is a measure of the importance of spin effects in the bound state.

These pictures conform qualitatively to one's expectations. For example, in the Σ^+ distribution (Fig. III.3c), the positive distribution of u quarks is partially cancelled at shorter distances by the heavier negatively charge $-1/3$ s quark, thus creating a "bump" in the figure. For the charmed states (Figs. III.3h – III.3k), one sees a very strong concentration of positive charge at short distances coming from the heavy $+2/3$ charged c quark. Other qualitatively, physically intuitive effects can be seen in the other distributions.

C. CHARGE RADII

We computed baryon charge radii using Eq.(II.7). Here are the results (in Fermi^2):

$\langle r_{\text{ch}}^2 \rangle$		
Baryon	Our Computation	Experiment
Δ^{++}	.81	---
$p - \Delta^+$	.54	$.66 \pm .02$
$n - \Delta^0$	.00	$-.116 \pm .003$
Δ^-	-.54	----
Σ^+	.36	---
$\Sigma^0 - \Lambda^0$	.05	---
Σ^-	-.25	----
Ξ^0	.07	----
Ξ^-	-.29	----
Ω^-	-.21	----
Σ_c^{++}	.46	----
$\Sigma_c^+ - \Lambda_c^+$	.13	----
Σ_c^0	-.19	----
Ω_c^0	-.14	----

The only experimental results are for the proton and the neutron [66–69]. Our value for the proton's charge radius is smaller than the experimental value. For the neutron, we automatically get zero. The negative charge radius of the neutron is an indication of the error introduced by neglecting spin effects, since if spin effects were not present the neutron's charge radius would be zero [51,70]. In the proton the spin effects repel the u quarks and draw the d quark inward. This should make the experimental value of $\langle r_{ch}^2 \rangle$ for the proton larger than our spinless computation, which is the case. The difference between our scalar calculation and experiment can be attributed to spin effects. For states involving heavier quarks, such effects should be less sizeable.

D. ELECTROMAGNETIC FORM FACTORS

Baryonic electromagnetic form factors in the low energy regime ($0 < Q < 2$ GeV) were computed using Eq.(II.8). Figures III.4a – III.4k display the results. For the neutral $\Sigma^0 - \Lambda^0$ (Fig. III.4c) and Ξ^0 (Fig. III.4e) states, the form factor is quite small and we have blown up the scale of the y-axis. Only the nucleon's electromagnetic form factor has been measured. In the case of the proton our calculation fits the experimental data [61,71,72] well although we are slightly above the data in the $Q \approx .6$ GeV range (Fig. III.4a). For the neutron we automatically get zero. Figure III.5 displays the neutron's electromagnetic form factor [61,68,69,72]. It provides an estimate of the error introduced by neglecting spin effects since the form factor would be zero if quarks were spinless. From Fig. III.5 we conclude that this error is of the order of 10% for states involving the u and d quarks. Vector meson dominance does not work for the proton [61]. Phenomenologically, the form factor is well represented by a dipole fit [61].

The neutron also provides a means of testing our assumptions about center of mass coordinates and the use of a "T" like configuration. Because the third quark is treated differently, an asymmetry is introduced in the system. A non-zero electromagnetic form factor is obtained if the charge $2/3$ u quark is used as the "asymmetric" third quark and likewise if the $-1/3$ d quark is used as the third quark (averaging over these possibilities generates zero; the Δ and nucleon are the only states where an averaging procedure is required since for all other states the third quark either has a mass different from the other two or has the same charge (i.e. the Ω)). Figure III.6 shows the result one would obtain treating the d quark as the third quark. From this, one concludes that the use of the "T" configuration introduces an error of 5–10% in the form factor results. By pretending that there exists a charged $+2/3$ quark with the same mass as the s quark, one can analyze the effect of the "T" configuration for s quarks. Figure III.7 shows that the effect is quite small (note that there is a factor of 10 in the scale of the y-axis in going from Fig. III.7 to Fig. III.6). As the mass of the quark increases the asymmetry introduced should go to zero. Apparently, this already happens in going from a d to an s quark.

Since the perturbative treatment of spin and the assumptions about the center of mass coordinates improve as the masses of the quarks increases and since we are already in good agreement with experiment for the lightest baryon, namely the proton, Figs. III.4b – III.4k probably represent quite accurately nature's baryonic electromagnetic form factors.

IV. ISOSPIN SPLITTING ESTIMATES

One purpose of ref. [44] was to see what non-abelian lattice gauge theories could tell us about the quark potential models [48,51,52,73–77]. These models make certain assumptions and approximations which are motivated by QCD but do not necessarily follow from QCD. The next three sections obtain results from “the quark model on the lattice”, namely that particular quark model whose interactions are determined by lattice gluons. We will find that certain features of the standard quark model need to be modified: for example, the notion of constituent mass (this is addressed in the next section). This section estimates the electrostatic self energy of a hadronic bound state. These calculations are based on the photon cloud model [51,78–83], valid only in the extremely heavy quark limit. For this reason our results should only be regarded as estimates. The purpose is to demonstrate that our hadronic wave functions give the right order of magnitude for the isospin splittings of isospin multiplets. It is also fruitful to see how these results compare with those of the quark potential models.

The electrostatic self energy of a bound state can be estimated by its Coulomb self energy [51,78–83]:

$$\int d^3x d^3y \rho(x) \rho(y) \frac{1}{|\vec{x}-\vec{y}|} \quad , \quad (\text{IV.1})$$

where $\rho(x)$ is the charge distribution. Sects. II and III have obtained $\rho(x)$, so that we can reliably compute the above integral. If Eq.(IV.1) were the exact formula for the electrostatic self energy, we would be able to accurately compute the electromagnetic mass contribution to hadrons. The formula, however, is only valid in the very heavy quark mass limit and thus provides only an estimate of the true electromagnetic contribution to the mass. Gluonic exchanges modify Eq.(IV.1). Many of these exchanges simply renormalize the quark charges and Eq.(IV.1) takes this into account, but other exchanges modify the functional form and generate other types of interactions. One should also consider other sources of electrostatic self energy such as the hyperfine term:

$$-\sum_{i < j} \frac{2\pi}{3} \frac{e_i e_j}{m_i m_j} \sigma_i \cdot \sigma_j \delta^3(x_i - x_j) \quad . \quad (\text{IV.2})$$

For the low lying s -wave hadrons, the second and third columns of Tables IV.1 and IV.2 display our computations of Eqs.(IV.1) and (IV.2).

The isospin splittings of multiplets arises from QED interactions and from the quark – u quark mass difference. To make contact with experiment, we have used a model to estimate the contribution coming from the latter. For the 0^- pseudoscalar mesons, we assume chiral behavior, namely that

$$M_{Q\bar{q}}^2 \sim c (m_Q + m_{\bar{q}}) \quad , \quad (\text{IV.3})$$

where $M_{Q\bar{q}}$ is the mass of the meson and $m_{\bar{q}}$ and m_Q are the current algebra masses of the quarks. Then the splitting between isospin members due to quark mass differences is given by

$$M_{d\bar{q}} - M_{u\bar{q}} = M_{u\bar{q}} \left[\sqrt{\frac{m_d + m_{\bar{q}}}{m_u + m_{\bar{q}}}} - 1 \right] . \quad (\text{IV.4})$$

For current masses, we used the established values [82] 5.1, 8.9, 175 and 1250 MeV for the u, d, s and c quarks. Although individual current masses are not precisely known, the ratios of current masses are (and this is what enters in Eq.(IV.4)) [61].

For the other mesons and baryons we assumed the d quark – u quark contribution was additive and used 2.7 MeV for this mass difference. Another contribution comes from the color hyperfine interaction in Eq.(I.2). The d – u mass difference makes the hyperfine interaction contribution different for u and d quarks [83]. Column 3 of Tables IV.3 and IV.4 displays our estimates of the isospin splittings arising from the d – u mass difference.

Column 4 of Tables IV.3 and IV.4 gives the final result for the isospin differences. Column 5 displays the experimental values and Column 6 shows a typical quark model calculation [83].

In the future, direct measurements of the isospin splittings should be possible with lattice QCD. One needs to introduce, in addition to the background gluon fields, a photon field. Propagators would have to be inverted in these background fields for a set of configurations. The $\pi^+ - \pi^0$ mass difference is difficult to obtain since the CPU intensive quark annihilation diagrams must be taken into account. However, the $K^+ - K^0$ mass difference can be used to fix the masses of the u and d quarks. Then one could compute the mass differences of $D^+ - D^0$, proton – neutron, $\Sigma^+ - \Sigma^0$, etc. Even though these numbers are small, using the same set of background gauge configurations should make the computation possible. The calculation is only slightly more computer intensive than the standard quenched hadron mass projects since the effect of introducing a photon field is not significant in CPU time.

V. ON THE CONSTITUENT MASSES OF QUARKS

In a previous paper, the quark model [51,52] was studied on the lattice [44] in order to gain insight into the standard quark model. Lattice QCD might dictate that certain features of the quark model require modification. Our goal here is to use the lattice scalar quark approach to measure the constituent masses of quarks and evaluate the additive quark model from this viewpoint.

A. MESONS

In the “idealized spinless” world discussed in the Introduction, we took linear combinations of states so that spin effects (at least to first order) dropped out. For each quark, let us assign a varying constituent mass, chosen such that the mass of any “spinless hadron” is exactly reproduced as the sum of the individual quark constituent masses. The constituent mass is to alter from state to state in order to reproduce the spectrum of the idealized world. It can be more or less uniquely fixed by requiring that its variation from state to state be minimized. The degree to which the constituent mass varies is an indication of the reliability of the additive quark model. If it worked perfectly, the constituent mass of a quark would be the same for all states. This would be the case for a system of particles (such as nucleons in a nucleus) in which the binding energies are considerably smaller than the masses of the constituents. What is remarkable is that, although this condition is not satisfied for the u , d and s quarks, the low energy spectrum can be reproduced by such considerations [48,51,52,77–79]. The purpose of this section is to show that, when one considers a wider range of meson masses, the additive quark model breaks down for the u , d and s quarks. It will be shown that the constituent mass of a light quark increases significantly with the mass of the partner antiquark. This result will be supported by numerical evidence from lattice QCD where we are able to vary greatly the masses of our quarks. We shall provide some support from the present experimental mesons mass table. When $t\bar{t}$, $t\bar{u}$ and $t\bar{s}$ mesons are observed the phenomenon should be quite pronounced.

The key concept in the quark model, the notion of a constituent mass, is loosely speaking, the average energy which a quark has in a bound state. This includes the bare mass¹, the kinetic energy, and the potential energy. For light quarks, most of the contribution comes from the latter two. For heavy quarks, it comes from the bare mass. By light quarks, we mean quarks with bare masses less than Λ_{QCD} , i.e. the u , d , and s quarks. By heavy quarks, we mean the c , b , and t quarks.

¹By bare quark mass, we mean essentially the current algebra mass, that is, the mass of a quark including short distance self-energy effects, but neglecting contributions coming from interactions with other quarks.

In the additive quark models, one tries to assign one set of constituent quark mass values which reproduces the spectrum. Although these values can vary from model to model, most models yield approximately the same numbers. Within the context of these models, the notion of a constituent mass becomes a physical concept, once a precise definition of it is given. For example, the quark model predicts that the mass of an S-wave $q\bar{q}$ system is given as in Eq.(I.3) so that a spinless meson mass which takes out these effects can be define as in Eq.(I.4). The $u\bar{u}$ spinless meson weighs $3/4 m_\rho + 1/4 m_\pi \approx 610$ MeV so that the constituent mass of the u quark is 305 MeV for the π -rho system. The same can be carried out for the diagonal $q\bar{q}$ mesons. The definition includes binding energy effects, but since they represent potential energy, they should be included. We will call this definition the "light definition of the constituent mass" because for u, d, and s, it is the constituent mass in the presence of an equally light antiquark. Using the particle tables [65], one finds

$$\begin{aligned} m_u^c &\approx m_d^c \approx 305 \text{ MeV} , \\ m_s^c &\approx 480 \text{ MeV} , \\ m_c^c &\approx 1535 \text{ MeV} . \end{aligned} \tag{V.1}$$

Another definition of the constituent quark mass is the mass that a quark, q, has in the presence of a very heavy antiquark, $\bar{Q}$. Let $M^{\bar{Q}q}$ denote the mass of a meson made from a $\bar{Q}$ and a q. Since, as $m_{\bar{Q}}$ goes to infinity, the spin-spin splitting terms vanish, $m_{\bar{Q}}^c$ can be precisely (in this limit, all definitions of $m_{\bar{Q}}^c$ will coincide) determined via:

$$m_{\bar{Q}}^c = M^{\bar{Q}Q} / 2 . \tag{V.2}$$

Again, since spin splittings can be neglected,

$$M^{\bar{Q}q} = m_q^c + m_{\bar{Q}}^c . \tag{V.3}$$

Equation (V.3) defines what we call "the heavy definition of the constituent mass" since it is the energy of a quark in the presence of a heavy antiquark.

The quantities in Eq.(V.3) can be computed using lattice QCD as was shown in ref. [44]. There, the following correlation function was considered

$$\langle \bar{Q}(0,t)q(0,t) Q(0,0)\bar{q}(0,0) \rangle . \tag{V.4}$$

$\bar{q}(0,0)$ produces a q quark at the spatial origin at $t=0$ and $q(0,t)$ destroys it at time $t=t$. Likewise, $Q(0,0)$ produces an antiquark $\bar{Q}$ at $t=0$ which is destroyed at $t=t$ by $\bar{Q}(0,t)$.

In the heavy $\bar{Q}$ – mass limit, the antiquark, $\bar{Q}$ is static and just sits at the origin. Eq.(V.4) can then be replaced by

$$\langle q(0,t) P \exp (i \int_0^t A_0(0,s) ds) \bar{q}(0,0) \rangle = z_Q^t , \quad (V.5)$$

where z_Q , the hopping parameter for the Q quark, is related to the unrenormalized bare Q quark mass, m_Q^0 , by $z_Q = \exp(-m_Q^0)$ (in the heavy mass limit). The line integral in Eq.(V.5), which on the lattice is a product of link variables, generates a self energy contribution, δm to the m_Q mass:

$$m_Q = m_Q^0 + \delta m . \quad (V.6)$$

The correlation function in Eq.(V.5), should behave, for large t , like $\exp(-t(m_q^c + m_Q))$. It is desirable to know δm so that m_q^c can be evaluated from Eq.(V.3). Since it involves the self energy of a Wilson line, it should be computable from Wilson loop data in the pure glue sector. Appendix A describes how to extract δm .

We have carried out a Monte Carlo calculation of Wilson loops to obtain δm . By removing the self – energy effects, we obtain an m_q^c with physical significance. We find

$$\begin{aligned} m_u^c &\approx m_d^c \approx 510 \pm 60 \text{ MeV} , \\ m_s^c &\approx 620 \pm 70 \text{ MeV} . \end{aligned} \quad (V.7)$$

In computing Eq.(V.7), the bare quark masses (that is, the hopping parameters) were fixed from the meson spectrum as described in the Introduction. The constituent masses scaled to about 15% between different coupling points and this is why our error in Eq.(V.7) is somewhat large.

These heavy definition mass values seem large vis – a – vis Eq.(V.1). Apparently, the constituent mass of the u , d , and s quarks is greater when paired with a heavy antiquark. Lattice QCD predicts that the constituent mass of a light quark varies significantly with the mass of the partner antiquark.

The result can be understood physically. The constituent mass is the total energy of a quark in a bound state, that is, the sum of the bare quark mass, the average potential energy, and the average kinetic energy. When the quark and antiquark are far apart, they move slowly, so that most of the energy is stored in potential energy. When they approach each other, this potential energy is converted into kinetic energy. In the case of the light quarks, the bare mass makes an insignificant contribution. Practically, all the energy is kinetic or potential energy. For the equal mass light mesons (e.g. ρ , π , and ϕ), the quark and antiquark share the energy equally. Now contrast this system with a heavy – light system such as the B meson. Since the

b quark is very heavy, its kinetic energy is fairly small. It contributes to the B meson mass via its bare mass. How is the potential energy divided among the b and u quarks when the two are far apart? A priori, it is perhaps not clear how to partition it. However, when the u quark approaches the b quark, it will gain speed and the potential energy will be converted into the u quark's kinetic energy. Thus, the potential energy should be associated with the u quark. Since one expects that the potential energy stored in the $b\bar{u}$ system is not substantially different from that of the $u\bar{u}$ system, and it is shared in the latter case, the constituent mass of a light quark will be larger in the presence of a heavy antiquark.

One can crudely estimate this effect in the case of a massless quark. Consider first the heavy–light system. Assume that the light quark oscillates between $-r$ and r in a linear potential and let p be its momentum when it is at the origin. The Heisenberg uncertainty relation (or Bohr–Sommerfeld quantization) implies $r \cdot p \approx 1$. At maximum separation, the potential energy, PE, is $\sigma \cdot r$. This PE must equal p by energy conservation. This implies $r \approx 1/\sqrt{\sigma}$ and $PE = m_q^c \approx \sqrt{\sigma}$. Now consider the light–light case. If the quark moves a distance, x , to the right, the antiquark will be at $-x$, and the distance of separation is twice that of the heavy case. The physics is identical to the heavy case if σ is formally replaced by 2σ . Compared to the previous case, the quark ventures $1/\sqrt{2}$ th of the distance from the origin. The distance between quark and antiquark is twice this or $\sqrt{2}$ times larger. The total potential energy at maximum separation (= mass of the meson) is also $\sqrt{2}$ times larger but since the constituent mass of a quark is half of this, the light definition of the constituent mass is $\approx 1/\sqrt{2}$ times the heavy value. Eqs.(V.1) and (V.7) give $.6 \pm .1$ versus the above estimate of $\approx .7$. The above argument predicts that the constituent mass of a massless quark should be $\sqrt{\sigma}/\sqrt{2} \approx 297$ MeV which is quite close to Eq.(V.1) for the u quark.

The reason for the large variation in the constituent mass is that the potential energy is apportioned differently in the heavy–light and light–light cases.

The phenomenon is evident in the Monte Carlo simulations. Figure V.1 plots the heavy definition versus the light definition of the constituent mass. If the two were the same, the data would follow the $y=x$ line. The two arrows indicate the u and s quark cases. As can be seen the constituent mass of the u quark is significantly greater in the $\bar{Q}u$ meson than in the $\bar{u}u$ meson. The same is true of the s quark.

As the quark mass becomes larger, potential energy and kinetic energy considerations diminish. Then, most of the meson mass comes from the bare quark mass, and the two definitions should coincide. This can be seen in Figure V.1. As the mass increases, the data points approach the $y=x$ line.

The prediction made here is experimentally testable. When the t quark is discovered, the spectrum of the $t\bar{t}$ states will permit a precise determination of the t quark mass, m_t^c . The mass of the T meson, the $t\bar{u}$ state, should then be $m_t^c + 510 \pm 60$ MeV or 205 MeV larger than $m_t^c + 305$ MeV. For the $t\bar{s}$ state, the "quark

model on the lattice” predicts a mass of $m_t^c + 620 \pm 70$ MeV rather than $m_t^c + 480$ MeV.

There is some experimental evidence for this phenomenon. Consider the case of charmed mesons. The charmed quark constituent mass, m_c^c , is 1535 MeV. The (spin) averaged m^{uc} mass is (using Eq.(I.4)) 1975 MeV which implies that the constituent mass of a u quark in a charmed meson is 440 MeV. Using the Upsilon and B meson data, we find that for beautiful mesons $m_u^c = 550$ MeV which is even larger than Eq.(V.7). Finally, the F and F^* masses imply that $m_s^c = 565$ MeV in the presence of a charmed quark.

When one computes the meson spectrum in quark potential models [73–77], one does observe an increase in the constituent mass of light quarks, although for charmed systems the effect is not too dramatic. What our Monte Carlo simulations are saying is that, in the case of even heavier systems, the effect will be quite pronounced. It will be interesting to see if these effects are observed in mesons with truth.

B. BARYONS

The quark model treatment of baryons is similar to mesons:

$$M_{q_1 q_2 q_3} = m_{q_1}^c + m_{q_2}^c + m_{q_3}^c + \text{“spin – spin splitting term”} . \quad (V.8)$$

An appropriate linear combination of baryon masses eliminates the spin – spin terms and leaves one with the “spinless baryon mass”. We then choose the constituent masses so that their sum is the spinless baryon mass. The spinless baryon mass for u – d quarks is $(M_{N(940)} + M_{\Delta(1232)})/2 = 1085$ MeV. Typically, the quark model requires a larger constituent mass than in the meson case to reproduce the experimental baryon data. For u and d quarks, one needs $m_d^c \approx m_u^c \approx 360$ MeV, which is greater than the 305 MeV in Eq.(V.1). A clue as to what is happening comes from the inequality [85]

$$\text{“baryon mass”} \geq 3/2 \text{ “meson mass”} , \quad (V.9)$$

valid if (large N) factorization holds.

We have measured the “baryon mass” in scalar lattice QCD over a wide range of quark masses. We find that the inequality in Eq.(V.9) is not saturated. Lattice QCD naturally accounts for the larger than expected baryon mass. Figure V.2 plots the baryon to meson mass ratio versus the (light) meson definition of constituent mass for the case where all quark masses are equal. Data were obtained for many values of the hopping parameter and then fit to a functional form. The statistical uncertainty in the computation is 5%. The arrow in Fig. V.2 indicates the u and d

quark case where this ratio is $1.90 \pm .08$ which is larger than the experimental value of 1.78. Because, for light quarks, the slope in Fig. V.2 is steep, our result is quite sensitive to the meson mass. If we increase it 5% from 610 MeV to 640 MeV, the baryon to meson mass ratio becomes the experimental value, 1.78. This may, in part, account for why our proton mass turned out to be too heavy in Table I.2. Note that as m_q gets large, the ratio approaches 3/2. This can be shown to follow from asymptotic freedom: Consider the equal mass case for simplicity. As $m_q \rightarrow$ infinity, perturbative QCD may be used to obtain the spectrum. The mass of the meson will be $2 m_q - \alpha_s^2 m_q/4$. The baryon mass will be $3 m_q - C \cdot \alpha_s^2 m_q$ where C is a geometrical factor related to the three-body wave function. α_s should be evaluated at a scale proportional to m_q . Since α_s goes to zero with increasing energy, the binding energy effects percentagewise will contribute an ever decreasing amount so that the inequality in Eq.(V.9) will be saturated. The ratio should go logarithmically in quark mass to 1.5.

Summarizing, the lattice quantum chromodynamic interactions give rise to baryons masses which are automatically heavier than what would be obtained from an additive quark model with the constituent masses being determined by the meson spectrum.

VI. PROTON DECAY ESTIMATES

The possible instability of the proton provides a means of detecting whether the strong, weak and electromagnetic interactions are unified into one grand unified theory [86]. By examining branching ratios, proton decay can distinguish different GUT groups, thereby helping to decide which of the proposed fundamental theories (simple GUTS, supersymmetric GUTS, string theories) is correct. For this and other reasons, it is important to obtain a reliable computation of its rate. Several methods have been used (see ref. [87] for a review). Among them are the current algebra approaches [88–92], bag model estimates [93,94] and quark model calculations [95–97]. The biggest uncertainty is in M_X , the mass of the mediating X boson. Present estimates [96,98,99] indicate a value in the range of

$$M_X = (2-8) \times 10^{14} \text{ GeV} \quad . \quad (\text{VI.1})$$

If for some unforeseeable reason, M_X is increased by a factor of 6, proton decay will be experimentally undetectable. The present experimental lifetime [99,100] for the $\pi^0 e^+$ channel is

$$1/\Gamma(p \rightarrow e^+ \pi^0) \geq 10^{32} \text{ years} \quad . \quad (\text{VI.2})$$

Theoretical calculations [87–99] give roughly

$$1/\Gamma(p \rightarrow e^+ \pi^0) \approx (M_X/4 \times 10^{14} \text{ GeV})^4 10^{30} \text{ years} \quad (\text{theory}) \quad , \quad (\text{VI.3})$$

so that with the above estimates of M_X , most GUT models are ruled out.

There are other theoretical uncertainties in the calculations besides M_X such as the computation of matrix elements. In this regard, lattice QCD wave functions can be of some use particularly in the case of the quark model approach. There are three stages involved in such a calculation. The first consists of determining the effective four-fermion interactions obtained by integrating out the X boson. Next, the interactions must be recast in a form where quark model methods are applicable. This typically involves writing Dirac fields in terms of large and small components and explicitly computing the spin-flavor matrix elements thereby reducing the problem to one involving spatial wave functions. The final stage computes the overlap integrals of these spatial wave functions. Either estimates or ad hoc wave functions frequently enter in this final stage. By using wave functions directly obtained from lattice QCD, the uncertainty in the overlap integrals can be significantly reduced. Unfortunately, the second stage of the above procedure involves several approximations such as the neglect of higher order strong interaction effects and the neglect of certain relativistic effects. For this reason, the quark model approach yields only estimates of proton decay rates. With sufficient computer power, lattice QCD should be able to directly compute these processes [101].

We will first consider the two – quark fusion process of Figure VI.1a and use the calculation of ref.[97] (reviewed in ref. [99]) to illustrate how lattice QCD wave functions can be of use. Ref. [97] computed spin – flavor matrix elements and reduced the problem to one involving the following interaction operator:

$$V(x_1, x_2, x_3, x_1', x_2') = \delta^3(x_1 - x_1') \delta^3(x_1 - x_2) \delta^3(x_3 - x_3') \exp(i k_{\text{lepton}} \cdot x_1') , \quad (\text{VI.4})$$

where x_1, x_2 and x_3 denote the positions of the three quarks in the proton (as in Figure III.1) with the third quark acting as the spectator; x_1' and x_3' are the positions of the antiquark and quark in the meson; and k_{lepton} is the momentum of the lepton which, in the rest frame of the proton (and hence center – of – mass frame), is $-k_{\text{meson}}$. In physical terms, the first two delta functions in Eq.(VI.4) require that quarks 1 and 2 be on top of each other in order for the process, quark + quark $\rightarrow$ antiquark + lepton, to proceed. The last delta function says that the spectator quark in the baryon is to be identified with the final state quark in the meson. The relevant matrix element is

$$\langle k_{\text{meson}}, k_{\text{lepton}} | V | k_{\text{proton}} \rangle = (2\pi)^3 \delta^3(k_{\text{proton}} - k_{\text{meson}} - k_{\text{lepton}}) \phi(k) , \quad (\text{VI.5})$$

where

$$\phi(k) = \int d^3x \psi_B(0, x) \psi_M(x) \exp(m_3/(m_1 + m_3) k_{\text{lepton}} \cdot x) , \quad (\text{VI.6})$$

is a kind of overlap form factor. In Eq.(VI.6) $\psi_B(\rho, \lambda)$ is the proton wave function with coordinates ρ and λ as in Fig. III.2, ψ_M is the meson wave function and m_1 and m_3 are the constituent masses of the antiquark and quark in the meson (the factor in the exponent arises from the same assumptions about center of mass coordinates as in Sects. II and III). To evaluate Eq.(VI.6), ref.[97] used harmonic oscillator wave functions both for the proton and meson. This is potentially dangerous since such wave functions cannot globally represent these states: If one fixes the scale in the wave function using the experimentally known values of the charge radii, one typically finds that $|\Psi(1=2)|^2$ is a factor 3 too small for the proton, pion and kaon. The wave functions in Figs. II.3a – II.3d would be poorly represented by gaussian functions. Ref. [97] then evaluated Eq.(VI.6) and argued that it may be replaced by $|\Psi_B(1=2)|$. Since we can compute Eq.(VI.6), we can test this assumption and eliminate the uncertainty in this part of that calculation. In fact, numerically, the replacement turns out not to be too bad. Table VI.1 displays our value for $\phi(k)$ for various processes in the SU(5) model as well as the ratio $\phi(k)^2/|\Psi_B(1=2)|^2$ (with $|\Psi_B(1=2)|^2 = 2.6 \times 10^{-3} \text{ GeV}^3$, the result obtained from our wave functions). This ratio, when multiplied by Eq.(55) of ref. [97], gives the decay rates with

Eq.(VI.6) evaluated by lattice wave functions. Table VI.1 shows that the assumption of replacing this matrix element by $|\Psi_B(1=2)|$ introduced, in the worse case, a 30% correction. Also displayed are the branching ratios (with three and higher body decays neglected) given in ref.[97] and improved by our computation. For that part of $p \rightarrow \pi^0 e^+$ involving only the two – quark fusion process, we get.

$$1/\Gamma(p \rightarrow \pi^0 e^+) = (M_X/4.0 \times 10^{14} \text{ GeV})^4 \times 5.7 \times 10^{30} \text{ years} \quad . \quad (\text{VI.7})$$

For the total proton lifetime (two body decays and two quark fusion processes only), we obtain

$$\tau_{\text{proton}} = (M_X / 4.0 \times 10^{14} \text{ GeV})^4 \times 1.9 \times 10^{30} \text{ years} \quad , \quad (\text{VI.8})$$

that is, about 20% longer than ref. [97]. We therefore conclude that the assumption of ref. [97] of replacing $\phi(k)$ by $|\Psi_B(1=2)|$ is not a bad approximation. The role of the above calculation is to remove the uncertainty in this assumption (which some authors frequently assign a factor of 3).

Since $|\Psi_B(1=2)|^2$ for the proton appears in several phenomenological applications, it is perhaps worth quoting the value obtained from the lattice calculation:

$$|\Psi_B(1=2)|^2 = 2.6 \times 10^{-3} \text{ GeV}^2 \quad , \quad (\text{VI.9})$$

with a 15% numerical uncertainty. Phenomenological estimates for this quantity have ranged from $1.1 \times 10^{-3} \text{ GeV}^2$ to $11.5 \times 10^{-3} \text{ GeV}^2$ [102]. Values used in proton decay estimates have varied from $2 \times 10^{-3} \text{ GeV}^2$ to $8 \times 10^{-3} \text{ GeV}^2$ [87].

Let us now consider other proton decay mechanisms. The direct decay of three quarks into a lepton can happen if the proton first emits a pion [89,91,103] or if the proton is inside a nucleus so that nearby nucleons can absorb momentum [104] (see Figure VI.1b). The latter process appears as $p \rightarrow e$. Since all three quarks must be on top of each other, the relevant quantity is $|\Psi_B(1=2=3)|^2$. This decay has the nice theoretical feature that gluonic corrections occur only in the initial state and in this respect it is similar to the vector meson decay into lepton pairs discussed in Sect. II. Since most of the strong interaction corrections to quark model formulas are already included in the wave functions, this process can be reliably calculated (as was the case for vector meson decay). Unfortunately, there are additional theoretical uncertainties coming from the nuclear physics. Ref. [104] has computed this process:

$$\tau_{\text{proton via 3-quark fusion}} = (M_X/4.0 \times 10^{14} \text{ GeV})^4 \times 2.2 \times 10^{29} \text{ years} \quad , \quad (\text{VI.10})$$

when $A = \text{atomic number} = 16$ (relevant for water experiments) and $N = 2$ (see ref. [104]). Although large theoretical uncertainties were admitted, ref. [104] con-

cluded at that time (1981) that this decay mechanism was more important by a factor of 3 compared to other mechanisms (i.e. the two quark fusion processes discussed above). Comparing Eq.(VI.10) to Eq.(VI.8) it would now appear dominant by a factor of 9. Ref. [104], however, used a value for $|\Psi_B(1=2=3)|^2$ of $6.4 \times 10^{-5} \text{ GeV}^6$ obtained by assuming $|\Psi_B(1=2=3)|^2 = |\Psi_B(1=2)|^4$ with $|\Psi_B(1=2)|^2 = 8 \times 10^{-3} \text{ GeV}^3$ (a value about 3 times bigger than our calculation). We have computed $|\Psi_B(1=2=3)|^2$ and find

$$|\Psi_B(1=2=3)|^2 = 1.3 \times 10^{-5} \text{ GeV}^6 . \quad (\text{VI.11})$$

Using this, we obtain instead of Eq.(VI.10)

$$\tau_{\text{proton via 3-quark fusion}} = (M_X / 4.0 \times 10^{14} \text{ GeV})^4 \times 1.1 \times 10^{30} \text{ years} . \quad (\text{VI.12})$$

In short, the result for $|\Psi_B(1=2=3)|^2$ obtained from the lattice reduces the rate by about a factor of 5 for the nuclear matter induced decay, $p \rightarrow e$, compared to a previous calculation which had used a "guessed" value for this quantity. Nevertheless, this decay mechanism still seems important for proton decay experiments conducted in water. Compared to Eq.(VI.17) below, it is not the dominant process, although it is significant enough so that it must still be included.

The three quark fusion process can also be important in the decay, $p \rightarrow \pi^0 e^+$ if the proton first emits the pion as in Figure VI.1c. Several groups have tried to estimate this process [89,91,103]. To be concrete, we will consider how our wave functions would change the results of ref. [103] for the decay $p \rightarrow \pi^0 e^+$. Ref. [103] obtained the following matrix element:

$$M = -i G_{\text{GUT}} A B \bar{u}_e (\beta + \alpha \gamma_5) u_p , \quad (\text{VI.13})$$

$$B = B^{\text{pole}} + \eta B^{\text{spec}} ,$$

where $A \approx 3.4$ is the renormalization enhancement factors [95,96,105], η is the relative phase between pole and spectator processes (estimated for $\pi^0 e^+$ decay to be 1 in ref. [103]), β and α are numbers which depend on the GUT group (for SU(5) $\alpha = -1/2$ and $\beta = 3/2$) and B^{pole} and B^{spec} are the pole (Fig. VI.1c) and spectator (Fig. VI.1a) contributions to the $\pi^0 e^+$ decay:

$$B^{\text{spec}} = \phi(k) |\Psi_B(1=2)|^2 \sqrt{3} (2 \omega_k)^{1/2} , \quad (\text{VI.14})$$

$$B^{\text{pole}} = [g_A / (2f_\pi) u(k R_B)] [k / (\omega_k + k^2 / (2M_p))] 6\sqrt{2} |\Psi_B(1=2=3)| ,$$

where $\phi(k)$ is the overlap in Eq.(VI.6), ω_k is the energy of the pion (≈ 479 MeV), k (≈ 459 MeV) is the momentum of the pion (or electron), g_A (≈ 1.26) is the nucleon axial vector strength, f_π is the pion decay constant (≈ 93 MeV), M_p is the mass of the proton and $u(k R_B)$ is the πNN form factor computed in the bag model with bag radius, R_B . For $u(k R_B)$ ref.[103] used .9 which corresponds to $R_B = .43$ Fermi. For the more common value of $R_B = .8$ Fermi one gets $u(k R_B) = .69$. The first term in brackets represents the effect of the pion – nucleon – nucleon vertex in Fig. VI.1c, the second bracket represents the propagator and $|\Psi_B(1=2=3)|$ comes from the three quark fusion vertex. Ref. [103] used harmonic oscillator wave functions to evaluate $|\Psi_B(1=2)|$ and $|\Psi_B(1=2=3)|$ obtaining:

$$\begin{aligned} |\Psi_B(1=2)| &= (1/(\sqrt{2\pi} R_p))^3, \\ |\Psi_B(1=2=3)| &= (1/(\sqrt{3} \pi R_p^2))^3, \end{aligned} \quad (VI.15)$$

where R_p is a parameter appearing in the proton's gaussian wave function (corresponding roughly to the proton's radius). A gaussian wave function for the pion was folded in to compute $\phi(k)$.

Although the order of magnitude of R_p may be restricted on phenomenological grounds (see Sect. II of ref.[103] for a discussion), results are quite sensitive to its precise value. The rate for the pole decay depends on R_p to the -6 power, so that a small change in its numerical value makes an enormous change in the absolute rate. This is reflected in Table 1 of ref.[103] included here for completeness. The first five rows of Table VI.2 represent the results of ref.[103] for varying values of R_p . The fifth row is a combination which the authors preferred as their favored prediction. The value of B^{pole} changes by a factor of 24 ($= 585$ in the rate) in Table VI.2. The value of ρ , the ratio of the pole term to the spectator term, is a measure of the importance of considering the three quark fusion process. It varies by a factor of 3.5. We have added a sixth column representing the value of ρ for $u(k R_B) = .69$. For such a value, the terms in column 4 get multiplied by $.69/.9 \approx .77$. Ref. [103] concluded that the pole contribution should dominate over the spectator process by a factor of 2.2 and since it seems to add coherently, the rate would increase by a factor of $3.2^2 \approx 10$. $1/\Gamma(p \rightarrow \pi^0 e^+)$ in Eq.(VI.7) would get divided by 10 which dramatically shortens the proton's lifetime. The relative phase, η , for other processes determines whether they are suppressed or enhanced. Since the π decay has the largest branching ratio, the proton's total lifetime would probably be decreased by at least a factor of 5 in Eq.(VI.8). In the last row of Table VI.2, we have placed the results determined by using the lattice wave functions. For $u(k R_B) = .69$ we find $\rho = .78$, for $u(k R_B) = .90$, $\rho = 1.01$ so that the pole term is of comparable magnitude to the spectator term. Ref. [91] calculated the pole term using current algebra in combination with harmonic oscillator wave functions and concluded $\rho \approx .8$, which is in agreement with our computation. This means that the rate for $p \rightarrow \pi^0 e^+$ increases by about 3.5 when the pole term is included. The pole term is thus quite important. Including both three and two quark fusion processes, we estimate

$$1/\Gamma(p \rightarrow \pi^0 e^+) = (M_X/4.0 \times 10^{14} \text{ GeV})^4 \times 1.6 \times 10^{30} \text{ years} \quad (VI.16)$$

Since pole terms will enhance or suppress processes according to whether they add coherently, branching ratios in Table VI.1 will be greatly affected. This probably explains [91] the discrepancy between branching ratios obtained from the quark model (which neglected pole terms) and current algebra methods. Since the pion mode is dominant, the overall effect of adding three fusion processes to the quark spectator contribution will be to decrease the proton's lifetime by a factor of around 3:

$$\tau_{\text{proton}} \approx (M_X / 4.0 \times 10^{14} \text{ GeV})^4 \times 5 \times 10^{29} \text{ years} \quad . \quad (\text{VI.17})$$

Summarizing, the pole terms are of comparable magnitude to the spectator terms. Previous calculations of their relative importance have varied greatly and by using lattice wave functions this uncertainty can be eliminated.

Our computations of $\phi(k)$, $|\Psi_B(1=2)|^2$ and $|\Psi_B(1=2=3)|^2$ are probably accurate to 25% when systematic effects such as the quenched approximation are included so that the main obstacle to an accurate computation of proton decay is what we called stage 2 above, namely reducing the fundamental four-fermion interaction to an effective quark model interaction. The advantage of using lattice QCD wave functions is to greatly reduce the uncertainty in computing the overlap matrix elements of this effective interaction.

VII. CONCLUSION

We have used lattice QCD to understand how the strong force binds matter into bound states both for mesons and baryons. In particular, we have obtained quark – antiquark wave functions which describe the quark probability distributions. By multiplying these by the quark charges, the charge distributions have been obtained, from which charge radii and electromagnetic form factors were computed. Our theoretical computations are in good agreement with experiment in cases where such data are available. We have obtained the lepton widths of the ρ , ϕ , and J/Ψ and in the case of the latter we are in excellent agreement with experiment. The wave functions were also used to compute the photon cloud contribution to the electromagnetic self energy of hadrons and to reanalyzed the computation of matrix elements occurring in proton decay, thereby clearing up some of the uncertainties in these calculations.

We have tried to increase the utility of lattice QCD by doing several things differently from most other lattice QCD projects. First we worked below the axis in the two – parameter model where the physics is more amenable to Monte Carlo simulations. There, we were farther from the phase transition which tends to disturb the physics and scaling behavior. Excellent scaling was seen in many different quantities, Fig. II.2b being an example for wave functions. Secondly, we worked on top of the physical quark masses thereby avoiding extrapolations in the quark masses. By using charm and strange quarks as well as u and d quarks a large number of states were treated. We carried out a high statistics study on a large lattice and worked at those coupling constants relevant to low energy phenomenology. We tried to obtain a deeper understanding of the bound state problem by measuring the distribution of quarks and used these results to make direct contact with experiment.

Phenomenology can be broken up into three categories: (i) processes experimentally measurable but presently inaccessible to Monte Carlo methods (such as nucleon – nucleon scattering), (ii) results for which lattice QCD can be immediately compared to experiment (e.g. hadron spectroscopy), and (iii) phenomenology not possible with present experimental capabilities. The second category is important for building confidence in one's methods. The third is, perhaps, the most interesting as far as lattice QCD is concerned. Not only can the lattice can be used to predict results which someday may be produced in the laboratory such as the masses of "flavorful" hadrons [45,47], charge radii, electromagnetic form factors, feeble proton decay rates, etc. but lattice QCD can also be used to compute quantities which are not likely to be experimentally measured (e. g. the electromagnetic form factor of the Ω^-) In such cases, simulation with a supercomputer appears to be the only means of measurement.

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APPENDIX A

This Appendix computes the self energy of a Wilson line using Wilson loop data. The computation of self energy effects must be removed before a proper calculation of wave functions can be obtained from Eqs.(II.2) and (III.3). We also present, at the end of this Appendix, the scaling of the string tension measurements made at the three coupling constants.

We will assume that the Wilson loop data can be parameterized by an area law plus terms motivated from perturbation theory:

$$WL \approx \exp(-\sigma \cdot \text{area} - C_2 \cdot \text{perimeter} + \text{const} \cdot \sum_{l \neq l'} \frac{g_{\mu_l \mu_{l'}}}{(d_l - d_{l'})^2}) \quad (A.1)$$

$d_l - d_{l'}$ denotes the distance between links l and l' . The first term is the non-perturbative confining term. The remaining two terms represent perturbative Coulombic interactions. In the continuum in the Feynman gauge (since the Wilson loops are gauge invariant, any gauge may be used),

$$\langle A_\mu(x) A_{\mu'}(x') \rangle \approx -\text{const} \frac{g_{\mu\mu'}}{(x - x')^2} \quad (A.2)$$

In perturbation theory, "const" is proportional to g^2 . On the lattice, $U_\mu(x) \approx \exp(i A_\mu(x))$. Thus,

$$\langle U_l(x) U_{l'}(x') \rangle \approx \exp(+\text{const} \frac{g_{\mu_l \mu_{l'}}}{(x - x')^2}) \quad (A.3)$$

where μ_l is the direction of the link, l . Equation (A.3) produces non-singular terms when $l \neq l'$ and singular terms when $l = l'$. The former comprise the third term in Eq.(A.1), whereas the latter comprise the second term (at short distances, the lattice propagator differs from Eq.(A.3) so that actually the terms corresponding to $l = l'$ are finite). The crossover region is some distance from zero coupling so that there are corrections to Eq.(A.3). Some of these renormalize g^2 to an effective g_{eff}^2 . For this reason, we use the Monte Carlo data to compute the "const" in Eq.(A.1).

In Feynman gauge, the last term in Eq.(A.1) contributes only to Wilson lines which are parallel. In the case of square Wilson loops, this is either the self-same Wilson line or two different lines on opposite sides of the square. It is convenient to separate these two:

$$\begin{aligned} \text{last term in Eq.(A.1)} &\approx -C_3 \cdot \sum_{\substack{l \neq l' \\ \text{with } l \text{ and } l' \\ \text{in different lines}}} \frac{g_{\mu_l \mu_{l'}}}{(d_l - d_{l'})^2} \\ &- \frac{1}{2} C_4 \cdot \sum_{\substack{l \neq l' \\ \text{with } l \text{ and } l' \\ \text{in same line}}} \frac{g_{\mu_l \mu_{l'}}}{(d_l - d_{l'})^2} \end{aligned} \quad (A.4)$$

The 1/2 factor eliminates double counting. We fit the Wilson loop data to Eqs.(A.1) and (A.4) and determined the coefficients σ , C_2 , C_3 , and C_4 numerically at each of the coupling points.

If the two terms in Eq.(A.4) come from the same Coulombic interaction, C_3 would equal $-C_4$ ($g_{\mu\ell}^{\mu\ell'} = -1$ if ℓ and ℓ' point in opposite directions). This was approximately true.

The self energy of a Wilson line is then defined to be the contribution coming from the C_2 and C_4 terms. For a line of length L in lattice units, it is

$$\text{self energy} = C_2 L + C_4 \sum_{k=1}^{L-1} \sum_{j=1}^k \frac{1}{j^2} \quad (\text{A.5})$$

The numerical values of C_2 were .138, .150 and .154 and the values of C_4 were .115, .107 and .108 at, respectively, coupling points 1, 2, and 3.

The remaining coefficients represent the physical potential:

$$V(r) = -c/r + \sigma r, \quad (\text{A.6})$$

where $c = -\pi C_3$.

A consistency test of the above analysis is to see if the potential in Eq.(A.6) coincides with the potential obtained by other methods. Table A.1 summarizes the results for σ and c (a) as determined from Wilson lines, (b) by doing a Barkai, Moriarty, Rebbi fit [32] with our Wilson loop data, and (c) by using the above method of a propagator plus area law fit. The three techniques agree well.

The fourth row in Table A.1 is an average of the three methods. If the potential is parametrized as in Eq.(A.6), c should be universal, that is, the same for each of the three coupling points. The three values agree to 3 %. The fifth row in Table A.1 represents the string tensions scaled onto the first coupling point via the 1/E method [36]. They agree to a few per cent.

Summarizing, at the first coupling point

$$\begin{aligned} \sigma &= .215 \pm .005, \\ c &= .290 \pm .010. \end{aligned} \quad (\text{A.7})$$

The above represents a very precise determination of the string tension. It was very important to do this so that the lattice spacing could be accurately determined. If $\sqrt{\sigma} = 420$ MeV then the inverse lattice spacing at coupling point 1 is 905 MeV, that is, the lattice spacing is .22 Fermi.

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FIGURE CAPTIONS

Figure II.1: A Lattice Method of Measuring the Meson Quark Wave functions

Figure II.2 : Scaling of the Wave function Data

- (a) The data at the three coupling points before scaling is taken into account.
- (b) The data of (a) after scaling with theoretically computed scaling factors.

Figure II.3: Wave functions of the Low – Lying Mesons

- (a) The $c\bar{c}$ case (the J/Ψ and η_c).
- (b) The $u\bar{c}$ case (the D and D^*).
- (c) The $s\bar{s}$ case (the ϕ).
- (d) The $u\bar{u}$ case (the π and ρ).

Figure II.4: Relativistic Center of Mass Coordinates

Figure II.5: Electromagnetic Form Factors for the Low – Lying Mesons

- (a) The $s\bar{c}$ case (the F^+ and F^{*+}).
- (b) The $d\bar{c}$ case (the D^+ and D^{*+}).
- (c) The $u\bar{c}$ case (the D^0 and D^{*0}).
- (d) The $s\bar{u}$ case (the K^+ and K^{*+}).
- (e) The $d\bar{s}$ case (the K^0 and K^{*0}).
- (f) The $d\bar{u}$ case (the π^+ and ρ^+). The circles are experimental data [57,58].

Figure III.1: A Lattice Method of Measuring the Baryonic Quark Wave functions

Figure III.2: Wave function Coordinates for the Baryon

Figure III.3: Baryonic Charge Distributions

- (a) The uud case (the proton and Δ^+). The "*" 's are experimental data [61,71,72].
- (b) The uus case (the Σ^+).
- (c) The uds case (the Σ^0 and Λ^0).
- (d) The dds case (the Σ^-).
- (e) The uss case (the Ξ^0).
- (f) The dss case (the Ξ^-).
- (g) The sss case (the Ω^-).
- (h) The uuc case (the Σ_c^{++}).
- (i) The udc case (the Σ_c^+ and Λ_c^+).
- (j) The ddc case (the Σ_c^0).
- (k) The ssc case (the Ω_c^0).

Figure III.4: The Baryonic Electromagnetic Form Factors

- (a) The uud case (the proton and Δ^+).
- (b) The uus case (the Σ^+).
- (c) The uds case (the Σ^0 and Λ^0).
- (d) The dds case (the Σ^-).
- (e) The uss case (the Ξ^0).
- (f) The dss case (the Ξ^-).
- (g) The sss case (the Ω^-).
- (h) The uuc case (the Σ_c^{++}).
- (i) The udc case (the Σ_c^+ and Λ_c^+).
- (j) The ddc case (the Σ_c^0).

(k) The ssc case (the Ω_c^0).

Figure III.5: The Neutron's Experimental Electromagnetic Form Factor

Figure III.6: Testing the Center – of – Mass Coordinates for u and d Quarks

Figure III.7: Testing the Center – of – Mass Coordinates for s Quarks

Figure V.1: The Heavy Definition of the Constituent Mass Versus the Light Definition

Figure V.2: The Baryon to Meson Mass Ratio

Figure VI.1: Proton Decay Processes

- (a) The "spectator" process in which the third quark does not participate.
- (b) Nucleon induced proton decay.
- (c) The pole process in which the proton first emits a meson and then disintegrates into an lepton.

TABLE CAPTIONS

Table I.1: The Meson Mass Results of Reference [46]

Table I.2: The Baryon Mass Results of References [45,47]

Table IV.1: Electromagnetic Contributions to Meson Masses
Numbers are in MeV.

Table IV.2: Electromagnetic Contributions to Baryon Masses
Numbers are in MeV.

Table IV.3: Meson Isospin Splittings Versus Experiment and a Quark Model
*Calculated from $K^+ - K^0$ mass difference and Dashen's theorem [84]. The Quark Model in Column 6 is from reference [83].

Table IV.4: Baryon Isospin Splittings Versus Experiment and a Quark Model
The Quark Model in Column 6 is from reference [83].

Table VI.1: The Spectator Quark Model Proton Calculation
The branching ratios in Column 4 are those of reference [97]. The last column is the branching ratios as computed from the lattice wave functions.

Table VI.2: Spectator Contribution Versus Pole Contribution
The first six rows are Table 1 of reference [103]. The last row is the result obtained from lattice wave functions.

Table A.1: The Scaling of the String Tension and Coulomb Coefficient (Eq.(A.6))

Figure II.1

Figure II.2(a)

Figure II.2(b)

Figure II.3(a)

Figure 11.3(b)

Figure II.3(c)

Figure 11.3(d)

Figure II.4

Figure II.5(a)

Figure II.5(b)

Figure II.5(c)

Figure 11.5(d)

Figure II.5(e)

Figure 11.5(f)

Figure III.1

Figure III.2

Figure III.3(a)

Figure III.3(b)

Figure III.3(c)

Figure III.3(d)

Figure III.3(e)

Figure III.3(f)

Figure III.3(g)

Figure III.3(h)

Figure III.3(i)

Figure III.3(j)

Figure III.3(k)

Figure III.4(a)

Figure III.4(b)

Figure III.4(c)

Figure III.4(d)

Figure III.4(e)

Figure III.4(f)

Figure III.4(g)

Figure III.4(h)

Figure III.4(i)

Figure III.4(j)

Figure III.4(k)

Figure III.5

Figure III.6

Figure III.7

Figure V.1

Figure V.2

Figure VI.1

Meson	Mass (in MeV)	Experiment
π	19 ± 150	137
ρ	809 ± 60	769
K	476 ± 90	496
K^*	905 ± 25	892
η_s	791 ± 25	—
ϕ	1016 ± 10	1020
D	1854 ± 20	1867
D^*	2000 ± 15	2009
F	1989 ± 20	1971
F^*	2119 ± 15	2140
η_c	2996 ± 20	2981
J/ψ	3096 ± 20	3097

Table I.1

Particle	Quark Content	Mass (in MeV)	Experiment
Proton	uud	1072 ± 45	939
$\Delta(1232)$	uuu	1261 ± 45	1232
Σ	uus	1227 ± 40	1193
$\Sigma^*(1385)$	uus	1443 ± 50	1385
Λ	uds	1182 ± 60	1116
Ξ	uss	1402 ± 55	1318
$\Xi^*(1530)$	uss	1552 ± 45	1535
Ω	sss	1694 ± 35	1672
Σ_c	uuc	2414 ± 20	2450
$\Sigma_c^*(\frac{3}{2}^+)$	uuc	2500 ± 45	not yet observed
Λ_c	udc	2306 ± 60	2282
$\Omega_c(\frac{1}{2}^+)$	ssc	2717 ± 25	2740
$\Omega_c^*(\frac{3}{2}^+)$	ssc	2767 ± 35	not yet observed
Σ_b	uub	5970 ± 20	not yet observed
$\Sigma_b^*(\frac{3}{2}^+)$	uub	5988 ± 25	not yet observed
Λ_b	udb	5901 ± 35	?5500?
$\Omega_b(\frac{1}{2}^+)$	ssb	6301 ± 30	not yet observed
$\Omega_b^*(\frac{3}{2}^+)$	ssb	6309 ± 30	not yet observed

Table I.2

Particle	Coulomb Energy	Hyperfine Term	Total
π^+	.48	.43	.91
π^0	-.60	-.54	-1.14
K^+	.49	.27	.77
K^0	-.25	-.13	-.38
ρ^+	.48	-.14	.34
ρ^0	-.60	.18	-.42
$K^{*+}(892)$	.49	-.09	.40
$K^{*0}(892)$	-.25	.04	-.20
ϕ^0	-.27	.04	-.23
D^+	.60	.15	.74
D^0	-1.19	-.29	-1.49
$D^{*+}(2010)$	.60	-.05	.55
$D^{*0}(2010)$	-1.19	.10	-1.10
F^+	.69	.17	.85
$F^{*+}(2140)$	.69	-.06	.63
η_c	-2.15	-.41	-2.56
J/Ψ	-2.15	.14	-2.02
B^+	.65	.07	.72
B^0	-.32	-.04	-.36
B^{*+}	.65	-.02	.62
B^{*0}	-.32	.01	-.31
F_b^0	-.37	-.05	-.42
F_b^{*0}	-.37	.02	-.36

Table IV.1

Particle	Coulomb Energy	Hyperfine Term	Total
proton	.00	-.41	-.41
neutron	-.62	-.31	-.92
Λ^0	-.75	-.30	-1.06
Σ^+	-.15	-.51	-.66
Σ^0	-.75	.02	-.73
Σ^-	.79	.10	.89
Δ^{++}	2.46	-.41	2.05
Δ^+	.00	-.00	-.00
Δ^0	-.62	.10	-.51
Δ^-	.62	-.10	.51
Ξ^0	-.73	-.24	-.97
Ξ^-	.72	.08	.80
$\Sigma^{*+}(1385)$	-.15	-.05	-.20
$\Sigma^{*0}(1385)$	-.75	.14	-.61
$\Sigma^{*-}(1385)$	.79	-.13	.66
$\Xi^{*0}(1530)$	-.73	.08	-.65
$\Xi^{*-}(1530)$	.72	-.08	.64
Ω^-	.83	-.09	.73
$\Lambda_c^+(2282)$	.19	-.35	-.17
$\Sigma_c^{*+}(2450)$	3.79	.09	3.89
$\Sigma_c^+(2450)$	.19	.20	.39
$\Sigma_c^0(2450)$	-1.13	-.22	-1.35
Σ_c^{*++}	3.79	-.40	3.39
Σ_c^{*+}	.19	.08	.26
Σ_c^{*0}	-1.13	.02	-1.11
Ω_c^0	-1.10	-.16	-1.26
Ω_c^{*0}	-1.10	.03	-1.07

Table IV.2

Part- cles	EM Term	u-d Mass Term	Total	Exper- ment	A Quark Model
$\pi^+ - \pi^0$	2.05	.00	2.05	4.60	5.4*
$K^+ - K^0$	1.15	-5.22	-4.07	$-4.01 \pm .13$	input
$K^{*+} - K^{*0}$	.60	-1.82	-1.21	-6.7 ± 1.2	-1.3
$D^+ - D^0$	2.23	2.87	5.10	$4.7 \pm .3$	4.7
$D^{*+} - D^{*0}$	1.65	2.47	4.12	2.9 ± 2.1	3.3
$B^+ - B^0$	1.08	-2.28	-1.20	-3.4 ± 3.0	-2.3
$B^{*+} - B^{*0}$	.93	-2.64	-1.70	—	-2.0

Table IV.3

Part- cles	EM Term	u-d Mass Term	Total	Exper- ment	A Quark Model
$p - n$	.51	-2.21	-1.69	-1.29	input
$\Sigma^+ - \Sigma^0$	.07	-2.89	-2.82	-3.10	input
$\Sigma^0 - \Sigma^-$	-1.62	-2.89	-4.51	-4.88	input
$\Xi^0 - \Xi^-$	-1.76	-3.47	-5.23	$-6.4 \pm .6$	-6.7
$\Sigma^{*+} - \Sigma^{*0}$	.41	-2.05	-1.64	$.3 \pm 2.5$	-1.6
$\Sigma^{*+} - \Sigma^{*-}$	-.86	-4.11	-4.97	$-5.1 \pm .5$	-4.9
$\Xi^{*0} - \Xi^{*-}$	-1.29	-2.32	-3.60	$-3.2 \pm .7$	-3.6
$\Sigma_c^{++} - \Sigma_c^+$	3.50	-2.62	.88	—	1.1
$\Sigma_c^+ - \Sigma_c^0$	1.74	-2.62	-.88	—	-.7
$\Sigma_c^{*++} - \Sigma_c^{*+}$	3.13	-2.36	.77	—	.8
$\Sigma_c^{*+} - \Sigma_c^{*0}$	1.37	-2.36	-.99	—	-1.0

Table IV.4

process	$\Phi(k)$ (in 10^{-3} GeV^3)	$\frac{\Phi(k)}{ \Psi(1=2) ^2}$	BR (GYOPR)	BR
$p \rightarrow e^+ \pi^0$	1.83	.70	37%	33%
$p \rightarrow e^+ \eta^0$		$(1.00)^*$	7%	9%
$p \rightarrow e^+ \rho^0$	2.49	.95	2%	3%
$p \rightarrow e^+ \omega^0$	2.50	.96	18%	22%
$p \rightarrow \bar{\nu}_e^0 \pi^+$	1.83	.70	15%	12%
$p \rightarrow \bar{\nu}_e^0 \rho^+$	2.49	.95	1%	1%
$p \rightarrow \mu^+ K^0$	2.31	.88	19%	21%
$p \rightarrow \bar{\nu}_\mu^0 K^+$	2.29	.88	0%	0%

Table VI.1

$R_p(\text{fm})$	$\Phi(k)$	$B_{\text{spec}}(\text{GeV}^2)$	$B_{\text{pole}}(\text{GeV}^2)$	ρ	ρ ($R_B = .80 \text{ fm}$)
.87	.68	.063	.037	0.59	0.45
.56	.85	.153	.137	0.90	0.69
.35	.94	.341	.560	1.64	1.26
.30	.96	.436	.890	2.04	1.57
hybrid	.68	.247	.560	2.27	—
this work	.84	.145	.146	1.01	0.78

Table VI.2

Method	Point 1		Point 2		Point 3	
	σ	c	σ	c	σ	c
Wilson Lines	.208	.272	.084	.310	not available	
BMR Fit of Creutz Ratios	.218	.274	.097	.284	.048	.254
This Paper Propagator Fit Plus Area Term	.200	.330	.101	.302	.041	.320
Average	.212	.295	.096	.298	.044	.287
Scaled onto Point 1 By 1/E Method	↓		↓		↓	
	.212		.216		.215	

Table A.1