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ASPECTS OF LEFT-RIGHT SYMMETRIC MODELS IN SUPERSTRING PHENOMENOLOGY

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ABSTRACT

The general features of low-energy, rank-six left-right symmetric models as inspired by the superstring are examined. Particular attention is paid to the mass-mixing matrix of the exotic- and down-type quarks. A 'no go' statement is presented and justified for a class of models that is based on $SU(3)_C \times SU(2)_L \times SU(2)_N \times U(1)_L \times U(1)_R$. Models based on the other left-right group, namely $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$, work rather well from the points of view investigated. In this case a parameter fitting is presented which is compatible with the lowering of the group rank by two units via radiative breaking, without conflicting either with weak universality or with lepton conservation. The unification scale comes in the 10^{17} GeV mass range, the intermediate (right-handed) scale is quite low, not exceeding the 10^4 GeV range, and the Weinberg angle is correct. Good prospects are also obtained for proton decay and neutrino masses.

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1. INTRODUCTION

It is by now widely accepted that the superstring [1,2] provides the only serious existing candidate for the unification of the strong, electroweak, and gravitational interactions with gauge group $E_8 \times E_8$ [3]. The requirement of compactification from ten to four dimensions in an effective $N = 1$ supersymmetric gauge theory suggests, according to Candelas et al. [4], that the extra six dimensions form a Ricci flat, Kähler manifold with $SU(3)$ as a holonomy group (Calabi-Yau) and one of the E_8 factors is subsequently broken down to E_6 , the second E_8 being coupled to ordinary matter through gravitational interactions. The E_6 group and E_6 model building have been widely explored [5] on general grounds. In relation to superstring unification, however, an exhaustive but preliminary analysis was done [6] concerning model building and, apart from a single exception [7], no model was developed and its phenomenological aspects investigated in any detail. From superstring requirements, the intermediate and low-energy superpotential [8] should contain N_G generations of matter fields transforming as the fundamental 27 representation with the possibility of additional pairs $27, \bar{27} : \delta, (\bar{27} + 27)$ for $\delta \geq 1$. The form of the superpotential is nevertheless to be fixed from phenomenology. The gauge-symmetry breaking can proceed via the usual Higgs mechanism. However, since we are restricted to the 27 whose singlets under the standard model are also singlets under $SU(5)$, this mechanism alone cannot account for the first breaking stage as it would lead to unacceptable baryon-number violation. This is instead performed by 'flux breaking', possible for non-simply connected manifolds [8,9]. In such cases it is possible to obtain a discrete symmetry group G acting freely [8] on the manifold, and non-trivial gauge configurations U_g ($g \in G, U_g \in E_6$) will then break E_6 down to some subgroup K satisfying

$$[K, U_g] = 0. \quad (1)$$

The possible patterns of such flux breaking have been classified [8]. We will adopt the notation of ref. [8], writing the U_g matrix in the basis of the maximal subgroup $SU(3)_C \times SU(3)_L \times SU(3)_R$ of E_6 as

$$U_g = 1 \otimes \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha^{-2} \end{pmatrix} \otimes \begin{pmatrix} \beta & & \\ & \gamma & \\ & & \delta \end{pmatrix} \quad (2)$$

where $\beta\gamma\delta = 1$. So we require that the unbroken group contains $SU(3)_C \times SU(2)_L$. Of course, the third factor in expression (2) may not be reducible to the diagonal form, in which case the E_6 rank will be lowered by one unit.

Left-right symmetric models, based on the group $SU(2)_L \times SU(2)_R$, have been extensively analysed in the conventional GUT [10] and SUSY GUT [11] scenarios. In particular, it was found

[11] that a relatively light W_R gauge boson can occur in a certain class of models. In superstring-inspired phenomenology, left-right symmetric models have not received much attention, apart from a few references in the literature [6,12].

In this paper we investigate, in some detail, a few aspects of left-right symmetric models based on the groups $SU(2)_L \times SU(2)_N$ and $SU(2)_L \times SU(2)_R$. Here $SU(2)_N$ represents the subgroup of $SU(3)_R$ which commutes with electric charge, corresponding to the γ and δ elements in expression (2). $SU(2)_R$ contains the electric charge and corresponds to the β and γ positions.

Each E_6 generation transforming as a 27 contains colour triplet, $SU(2)_L$ singlet states (g quarks), whose antiparticles ($\bar{g}$) have the same quantum numbers as the $\bar{d}$ -type quarks. So these states can mix and it is not possible *a priori* to know to which representation of the standard model they are assigned. The ambiguity can only be lifted once the mass matrix of the g and d quarks has been written and diagonalized, so that one can inspect which fields get mass from weak breaking ($\Delta I = 1/2$ masses) and which get mass from K-breaking ($\Delta I = 0$ masses). There are still two other possible interchanges in the 27, one involving the $Y = -1/2$ weak doublet H_d with the lepton doublet L and the other, the two standard model singlets S_1, S_2 , which combine to form ν_c [belonging to 16 of $SO(10)$] and N [belonging to a singlet of $SO(10)$]. In order to account for neutrino masses, the mass matrix of these fields must admit a vanishing eigenvalue.

This paper is divided as follows: in section 2 we expound the physical content of both models, $SU(2)_L \times SU(2)_N$ and $SU(2)_L \times SU(2)_R$. The g, d mass matrix, governed by the two scales $M_I (\Delta I = 0)$ and $\mu (\Delta I = 1/2)$, is diagonalized and a fitting of the Yukawa couplings is made for the simplest version of the models. Here we find a remarkable no-go statement for models with $SU(2)_N$. On the other hand, models with $SU(2)_R$ work rather well. The two scales M_I and μ cannot be too far apart here since, as will be briefly explained, the intermediate breaking does not preserve supersymmetry. The existence of a Yukawa coupling for both fields acquiring vacuum expectation values (v.e.v.'s) $O(M_I)$ makes it possible for these v.e.v.'s to be generated by dimensional transmutation [13], thus reducing the rank of the intermediate group by 2 into the standard model. In section 3 a one-loop renormalization group analysis is done. We wish to emphasize that whereas in conventional SUSY GUTs a low-lying W_R [11] or even a low right-handed scale [14] required artificially complicated scenarios, in the superstring-inspired models such a low scale appears promptly in the simplest versions. Finally, section 4 contains some conclusions and comments.

2. GENERAL FRAMEWORK AND MASSES

2.1 $SU(3)_C \times SU(2)_L \times SU(2)_N \times U(1)_L \times U(1)_R$

We start with a collection of N_G 27 representations plus a pair $27, \bar{27}$ at the compactification scale. The first survive flux breaking without acquiring a mass and we can use the $27, \bar{27}$ to break the intermediate group with a vanishing D-term, thus in a supersymmetric way. In fact, the whole of this

pair of representations will acquire mass from flux breaking except for those components of $\bar{27}$ left invariant [8] by the U_g matrix (2) and the corresponding copies in 27. In the case of this intermediate group we are free to choose the invariant components in $\bar{27}$ to be the standard model singlets $\bar{S}_{1,2}$ lying in the $(1, 3_3, \bar{3}_2)$ and $(1, 3_3, \bar{3}_3)$ directions [12] of $SU(3)_C \times SU(3)_L \times SU(3)_R$. Their mirror components $S_{1,2}$ in 27 will then also remain light [8]. So U_g will be the matrix

$$U_g = 1 \otimes \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha^{-2} \end{pmatrix} \otimes \begin{pmatrix} \alpha^4 & & \\ & \alpha^{-2} & \\ & & \alpha^{-2} \end{pmatrix} \quad (3)$$

leaving also invariant the group $SU(3)_C \times SU(2)_L \times SU(2)_N \times U(1)_L \times U(1)_R$. From the decomposition of the 27 given in table 1 we can write down the most general superpotential under the intermediate group resulting from its self-trilinear-coupling

$$W = \lambda_1 C_2 A_1 B_2 + \lambda_2 C_4 A_2 B_2 + \lambda_3 A_1 B_1 C_1 + \lambda_4 A_2 B_1 C_3 + \lambda_5 C_4 C_1 C_1 + \lambda_6 C_2 C_1 C_3 + \lambda_7 A_1 A_1 A_2 + \lambda_8 B_1 B_1 B_2. \quad (4)$$

The form of this superpotential is obviously the same for both the $SU(2)_N$ and the $SU(2)_R$ cases. However, the particle content of each representation will be different in each case. The hypercharge Y does not originate from $SU(2)_N$ so, as seen from table 1, in order to get the correct assignments for the hypercharge of known particles, we must define

$$Y = \frac{1}{5} Y_L - \frac{2}{5} Y_R. \quad (5)$$

Turning now to the particle contents we see that A_1 contains $\bar{g}$ - and $\bar{d}$ -type quarks. We denote by x_2^+ and x_2^- the up and down components of this $SU(2)_N$ doublet which combine to form the physical quarks. C_1 contains H_d , the weak doublet Higgs with $Y = -1/2$, and L , the lepton doublet. Similarly, rotating these, we will have C_1^+ and C_1^- . C_2 contains the two standard model singlets $S_{1,2}$ again corresponding to the up and down components of an $SU(2)_N$ doublet. Here are contained the ν_c and N mentioned earlier. The remaining assignments are unambiguous: A_2 contains the $\bar{u}$ -type quark, B_1 contains $Q \equiv (u,d)$, B_2 contains g , C_3 contains H_u and C_4 contains e^+ . All these assignments are shown in table 1.

Let us now briefly discuss proton decay. As it stands, it can readily be seen that the superpotential (4) generates too fast proton decay through the interchange of g -type squarks resulting from combinations of $\lambda_1 C_2 A_1 B_2$, $\lambda_7 A_1 A_1 A_2$, and $\lambda_8 B_1 B_1 B_2$ couplings. The λ_1 coupling is essential for reasons that will become clear below and we may just suppress the λ_7 and λ_8 couplings using the following discrete symmetry

$$\begin{aligned}
C_1 &\rightarrow C_1 & A_1 &\rightarrow \eta A_1 \\
C_2 &\rightarrow \eta^{-2} C_2 & A_2 &\rightarrow \eta^{-1} A_2 \\
C_3 &\rightarrow \eta^2 C_3 & B_1 &\rightarrow \eta^{-1} B_1 \\
C_4 &\rightarrow C_4 & B_2 &\rightarrow \eta B_2
\end{aligned} \tag{6}$$

where, for example, $\eta = e^{2\pi i/3}$. Of course, this provides no ultimate satisfactory solution to the Yukawa suppression but, at this early stage, we must assume that a suitable Calabi-Yau manifold exists which provides this set of couplings [15]. Furthermore, as will become apparent from the discussion on neutrino masses later, the couplings λ_5, λ_6 must also be suppressed, unless unnatural fine tunings are made. To this end we will alter the discrete symmetry (6) into one which suppresses proton decay and neutrino masses.

Next we consider the g,d mass matrix. Given expression (4) and the particle content of the representations described above, we see that the couplings involved are λ_1 and λ_3 . Expanding these terms in $SU(3)_C \times SU(2)_L \times SU(2)_N \times U(1)_L \times U(1)_R$ representations we have

$$\lambda_1 C_2 A_1 B_2 = \lambda_1 S_1 g x_2^- + \lambda_1 S_2 g x_2^+ \tag{7a}$$

$$\lambda_3 A_1 B_1 C_1 = \lambda_3 x_2^- Q C_1^+ + \lambda_3 x_2^+ Q C_1^- \tag{7b}$$

where we are using the notation introduced after eq. (5). The physical fields H_d, L are of course related to C_1^+, C_1^- by some rotation

$$\begin{pmatrix} C_1^+ \\ C_1^- \end{pmatrix} = \begin{pmatrix} h_1 & h_2 \\ \ell_1 & \ell_2 \end{pmatrix} \begin{pmatrix} H_d \\ L \end{pmatrix} \tag{8}$$

and in the vacuum the right-hand side of expression (7b) contains

$$\lambda_3 \langle C_1^+ \rangle d x_2^- + \lambda_3 \langle C_1^- \rangle d x_2^+ = \lambda_3 h_1 \langle H_d \rangle x_2^- d + \lambda_3 \ell_1 \langle H_d \rangle x_2^+ d \tag{9}$$

Denoting $\langle H_d \rangle = \bar{v}$, $\langle S_1 \rangle = s_1$, $\langle S_2 \rangle = s_2$ [with $\bar{v} = O(M_W)$ and $s_1, s_2 = O(M_I)$, the intermediate scale] we have the following mass matrix:

$$(x_2^+ \ x_2^-) \begin{pmatrix} \lambda_1 \lambda_2 & \lambda' \bar{u} \\ \lambda_1 \lambda_1 & \lambda'' \bar{u} \end{pmatrix} \begin{pmatrix} g \\ d \end{pmatrix} \quad (10)$$

where $\lambda' = \lambda_3 \ell_1$ and $\lambda'' = \lambda_3 h_1$. Since this matrix M is not symmetric we will diagonalize it by means of two independent orthogonal rotations U, V on the left- and on the right-hand states, respectively. The quantity (10) can then be written as

$$(x_2^+ \ x_2^-) M \begin{pmatrix} g \\ d \end{pmatrix} = (x_2^+ \ x_2^-) U^T U M V^T V \begin{pmatrix} g \\ d \end{pmatrix} \quad (11)$$

The matrix UMV^T is by definition diagonal. It suffices now to find the exact form of U and V . To do this, let D denote the diagonal matrix

$$D = U M V^T \quad \text{or, transposing,} \quad D = V M^T U^T \quad (12)$$

Multiplying these equations we get

$$D^2 = U M V^T V M^T U^T = U M M^T U^T \quad (13)$$

or, equivalently,

$$D^2 = V M^T U^T U M V^T = V M^T M V^T \quad (14)$$

so the matrices U and V are the ones that diagonalize MM^T and $M^T M$, respectively. U rotates x_2^+, x_2^- into $\bar{g}, \bar{d}$ so the matrix amplitude displaying the masses and mixings between g and d quarks is

$$(\bar{g} \ \bar{d}) U M \begin{pmatrix} g \\ d \end{pmatrix} = (\bar{g} \ \bar{d}) \begin{pmatrix} M_{\bar{g}g} & \mu_{\bar{g}d} \\ \mu_{\bar{d}g} & m_{\bar{d}d} \end{pmatrix} \begin{pmatrix} g \\ d \end{pmatrix} \quad (15)$$

The calculational details are shown in the Appendix. Here we just write the matrix UM explicitly

$$UM = \begin{pmatrix} a_1 \lambda_1 s_2 + \frac{\alpha_1 - D_1}{D_2} a_1 \lambda_1 s_1 & a_1 \lambda'_1 \bar{u} + \frac{\alpha_1 - D_1}{D_2} a_1 \lambda''_1 \bar{u} \\ b_1 \lambda_1 s_2 + \frac{\alpha_2 - D_1}{D_2} b_1 \lambda_1 s_1 & b_1 \lambda'_1 \bar{u} + \frac{\alpha_2 - D_1}{D_2} b_1 \lambda''_1 \bar{u} \end{pmatrix} \quad (16)$$

where $D_1 = \lambda_1^2 + \lambda'^2 \epsilon^2$, $D_2 = \lambda_1^2 + \lambda' \lambda'' \epsilon^2$ are the coefficients of the top-row matrix elements of MM^T (A.2), α_1 and α_2 are the eigenvalues^{*)} of MM^T divided by M_I (A.3), $\epsilon = M_W/M_I \approx \bar{v}/s_{1,2}$ and a_1, b_1 correspond to the first components of the two normalized eigenvectors of MM^T and are given in Eq. (A.5).

We now attempt a parameter fitting. Looking at expression (15) we see that one should require the off diagonal elements to be several orders of magnitude smaller than the diagonal ones. From the explicit form (16) of the matrix UM one immediately realizes that this leads to a contradiction. Such a contradiction can only be avoided if we let the two left entries in the mass matrix M (10) be very different in magnitude (at least by a factor of 10^2). This means the two v.e.v.'s $\langle S_1 \rangle, \langle S_2 \rangle$ are totally separated in order of magnitude, so only one of them will be available to break the intermediate group which can therefore not be of rank greater than 5. However, the possible rank 5 groups do not admit a left-right symmetry [8,12]. Thus we find a no go statement for $SU(2)_L \times SU(2)_N$ models from the superstring and so these need not be discussed any further.

2.2 $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$

In order to break E_6 down to this group via the flux mechanism the matrix U_g can be written as

$$U_g = \mathbf{1} \otimes \begin{pmatrix} \alpha & & \\ & \alpha & \\ & & \alpha^{-2} \end{pmatrix} \otimes \begin{pmatrix} \beta & & \\ & \beta & \\ & & \beta^{-2} \end{pmatrix} . \quad (17)$$

Mirror generations are not present here to break the intermediate group, implying that this breaking must not occur at energies greater than 10^4 GeV owing to the non-vanishing of D terms. On the other hand, light Higgs doublets will be available as before from the N_G generations of 27's. The superpotential has the same form as in the previous case, i.e.

^{*)} We emphasize that they satisfy $\alpha_2 = O(\epsilon^2)\alpha_1$ (see Appendix).

$$W = \lambda_1 C_2 A_1 B_2 + \lambda_2 C_4 A_2 B_2 + \lambda_3 A_1 B_1 C_1 + \lambda_4 A_2 B_1 C_3 + \lambda_5 C_4 C_1 C_1 + \lambda_6 C_2 C_1 C_3 \quad (18)$$

where we omitted the last two terms on grounds of the discrete symmetry (6). The hypercharge Y originates from $SU(2)_R$ and according to the 27 decomposition in table 1 we must define

$$Y = \pm T_{3R} + \frac{1}{5} Y_L + \frac{1}{5} Y_R \quad (19)$$

The particle content of each representation now differs from the $SU(2)_N$ case and is also shown in table 1. A_1 contains a $\bar{u}$ -type quark, whereas the $\bar{g}$ and $\bar{d}$ types correspond to linear combinations of an $SU(2)_R$ singlet x_1 ($x_1 \equiv A_2$) and the other component of the A_1 doublet x_2 ($x_2 \in A_1$). C_1 contains H_u (weak doublet with $Y = +1/2$) and y_2 . The latter combines with $y_1 \equiv C_3$ to form the physical lepton doublet L and H_d . C_2 contains e^+ and one singlet of the standard model S_1 (S_2), whereas the other singlet S_2 (S_1) coincides with C_4 . The fields N , ν_c are linear combinations of $S_{1,2}$. Finally B_1 and B_2 involve no ambiguities: they involve $Q \equiv (u, d)$ and g respectively (see table 1).

Let us now consider the g , d mass matrix. Given the superpotential (18) and the particle contents just described, the relevant couplings are $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, whose terms once expanded in $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ representations give

$$\begin{aligned} \lambda_1 C_2 A_1 B_2 &= \lambda_1 S_1 g x_2 + \lambda_1 e^+ g \bar{u} \\ \lambda_2 C_4 A_2 B_2 &= \lambda_2 S_2 g x_1 \\ \lambda_3 A_1 B_1 C_1 &= \lambda_3 x_2 Q y_2 + \lambda_3 \bar{u} Q H_u \\ \lambda_4 A_2 B_1 C_3 &= \lambda_4 x_1 Q y_1 \end{aligned} \quad (20)$$

where we have adopted the (+) sign in eq. (19) and assigned S_1 to C_2 and S_2 to C_4 . Again the physical fields H_d , L are related to y_1, y_2 by some orthogonal rotation

$$\begin{pmatrix} H_d \\ L \end{pmatrix} = \begin{pmatrix} \lambda_{a1} & \lambda_{a2} \\ \lambda_{a3} & \lambda_{a4} \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} \lambda_{a1} & \lambda_{a3} \\ \lambda_{a2} & \lambda_{a4} \end{pmatrix} \begin{pmatrix} H_d \\ L \end{pmatrix} \quad (21)$$

and in the third and fourth of eqs. (20) we have in the vacuum

$$\begin{aligned}\lambda_3 \langle y_2 \rangle d x_2 + \lambda_3 \langle H_u \rangle \bar{u} u &= \lambda_3 \lambda_{a_2} \langle H_d \rangle d x_2 + \lambda_3 \langle H_u \rangle \bar{u} u \\ \lambda_4 \langle y_1 \rangle d x_1 &= \lambda_4 \lambda_{a_1} \langle H_d \rangle d x_1\end{aligned}\quad (22)$$

because we are assuming as before $\langle L \rangle = 0$. We can then write

$$(x_1 \ x_2) \begin{pmatrix} \lambda_2 s_2 & \lambda'_4 \bar{v} \\ \lambda_1 s_1 & \lambda'_3 \bar{v} \end{pmatrix} \begin{pmatrix} g \\ d \end{pmatrix} \quad (23)$$

where we defined $\lambda'_3 = \lambda_3 \lambda_{a_2}$ and $\lambda'_4 = \lambda_4 \lambda_{a_1}$. As in the previous case, this matrix M is diagonalized through two different orthogonal transformations U and V on the left- and right-hand fields, respectively. The matrix U diagonalizes MM^T and V diagonalizes $M^T M$ [see eqs. (13), (14)]. The calculations proceed in the same manner as in the previous subsection and for details we refer to the Appendix. The matrix UM displaying the masses and mixings between g and d quarks is now

$$UM = \begin{pmatrix} M_{\bar{g}g} & \mu_{\bar{g}d} \\ \mu_{\bar{d}g} & m_{\bar{d}d} \end{pmatrix} = \begin{pmatrix} a_1 \lambda_2 s_2 + \frac{\alpha_1 - D_1}{D_2} a_1 \lambda_1 s_1 & a_1 \lambda'_4 \bar{v} + \frac{\alpha_1 - D_1}{D_2} a_1 \lambda'_3 \bar{v} \\ b_1 \lambda_2 s_2 + \frac{\alpha_2 - D_1}{D_2} b_1 \lambda_1 s_1 & b_1 \lambda'_4 \bar{v} + \frac{\alpha_2 - D_1}{D_2} b_1 \lambda'_3 \bar{v} \end{pmatrix} \quad (24)$$

where $D_1 = \lambda_2^2 + \lambda_4'^2 \epsilon^2$, $D_2 = \lambda_1 \lambda_2 + \lambda_3 \lambda_4 \epsilon^2$ are the coefficients of the top matrix elements of MM^T [eq. (A.15)] and the rest of the notation follows the same pattern as in the previous case [eqs. (A.16) to (A.18)]. The situation concerning parameter fitting is now essentially different from the $SU(2)_N$ case. Requiring the inequalities

$$M_{\bar{g}g} \gg \mu_{\bar{g}d} \sim \mu_{\bar{d}g} \quad \text{and} \quad m_{\bar{d}d} \gg \mu_{\bar{d}g} \quad (25)$$

is quite possible, as these differences will be provided by the differences between parameters λ_1 , λ_2 and between λ'_3 , λ'_4 . So $\langle S_1 \rangle$, $\langle S_2 \rangle$ may have the same order of magnitude M_I , and a parameter fit can now be attempted. We find

$$a_1 \sim 1, b_1 \sim 10^{-2}, \lambda_1 \sim 10^{-4}, \lambda_2 \sim 10^{-2}, \lambda'_3 \sim 10^{-2}, \lambda'_4 \sim 10^{-4}. \quad (26)$$

With these numbers one gets

$$\begin{aligned} M_{\bar{g}g} &\sim 10^{-2} M_I & \mu_{\bar{g}d} &\sim 10^{-4} \epsilon M_I \\ \mu_{\bar{t}g} &\sim 10^{-4} M_I & m_{\bar{t}d} &\sim 10^{-2} \epsilon M_I \end{aligned} \quad (27)$$

and for the matrices U, V which act on $(x_1 x_2)$ and $(g d)$,

$$\begin{pmatrix} \bar{g} \\ \bar{d} \end{pmatrix} = U \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} a_1 & a_2 \\ b_1 & b_2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \approx \begin{pmatrix} 1 & 10^{-2} \\ 10^{-2} & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad (28)$$

$$\begin{pmatrix} \tilde{g} \\ \tilde{d} \end{pmatrix} = V \begin{pmatrix} g \\ d \end{pmatrix} = \begin{pmatrix} \tilde{a}_1 & \tilde{a}_2 \\ \tilde{b}_1 & \tilde{b}_2 \end{pmatrix} \begin{pmatrix} g \\ d \end{pmatrix} \approx \begin{pmatrix} 1 & 10^{-2} \epsilon \\ 10^{-2} \epsilon & -1 \end{pmatrix} \begin{pmatrix} g \\ d \end{pmatrix} \quad (29)$$

Expression (29) is valid for $\epsilon \geq 10^{-2}$. Note that the matrix V satisfies $\tilde{a}_1 > 10 \tilde{a}_2$ and $\tilde{b}_2 > 10 \tilde{b}_1$ as required by weak universality [16]. This implies the ratio of $\Delta I = 0$ to $\Delta I = 1/2$ mass terms [16] to be > 10 .

Next we come to the discussion of neutrino masses. Owing to the unavailability of mirror generations, no see-saw mechanism can be provided [17] for these. So the neutrino mass problem must be investigated solely from the analysis of the 'neutralino' mass matrix. This is a symmetric matrix with 9×9 entries involving the fields $(\tilde{y}_1, \tilde{y}_2, \tilde{H}_u^0, \tilde{S}_1, \tilde{S}_2, \tilde{W}_L^3, \tilde{W}_R^3, \tilde{B}_{1L}, \tilde{B}_{1R})$. In order to have massless (or almost massless) neutrinos, this matrix must have a vanishing (or almost vanishing) mass eigenvalue. Recalling that the fields H_d, L combine to form y_1, y_2 with definite group assignments [see eq. (21)] we have for the neutralino mass matrix

	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{H}_u^0$	$\tilde{S}_1$	$\tilde{S}_2$	$\tilde{W}_{3L}$	$\tilde{W}_{3R}$	$\tilde{B}_{1L}$	$\tilde{B}_{1R}$
$\tilde{y}_1$	0	0	$\lambda_6 s_1$	$\lambda_6 v$	0	$\frac{g_{21} \bar{v}}{\sqrt{2}} \lambda_{a_1}$	0	$-\frac{5}{3} \frac{g_{11} \bar{v}}{\sqrt{2}} \lambda_{a_1}$	$\frac{5g_{18}}{3\sqrt{2}} \bar{v} \lambda_{a_1}$
$\tilde{y}_2$	0	0	$\lambda_5 s_2$	0	$\lambda_5 v$	$\frac{g_{21} \bar{v}}{\sqrt{2}} \lambda_{a_2}$	$\frac{g_{22} \bar{v}}{\sqrt{2}} \lambda_{a_2}$	$-\frac{5}{3} \frac{g_{11} \bar{v}}{\sqrt{2}} \lambda_{a_2}$	$\frac{5g_{18}}{3\sqrt{2}} \bar{v} \lambda_{a_2}$
$\tilde{H}_u^0$	$\lambda_6 s_1$	$\lambda_5 s_2$	0	$\lambda_6 \lambda_1 \bar{v}$	$\lambda_5 \lambda_{a_2} \bar{v}$	$-\frac{g_{21} v}{\sqrt{2}}$	$-\frac{g_{22} v}{\sqrt{2}} \lambda_{a_2}$	$-\frac{5}{3} \frac{g_{11} v}{\sqrt{2}}$	$\frac{5g_{18}}{3\sqrt{2}} v$
$\tilde{S}_1$	$\lambda_6 v$	0	$\lambda_6 \lambda_1 \bar{v}$	0	0	0	$\frac{g_{22} v}{\sqrt{2}} s_1$	$\frac{5\sqrt{2}}{3} g_{11} s_1$	$\frac{5g_{18}}{3\sqrt{2}} s_1$
$\tilde{S}_2$	0	$\lambda_5 v$	$\lambda_5 \lambda_{a_2} \bar{v}$	0	0	0	0	$\frac{5\sqrt{2}}{3} g_{11} s_2$	$-\frac{5\sqrt{2}}{3} g_{18} s_2$
$\tilde{W}_{3L}$	$\frac{g_{21} \bar{v}}{\sqrt{2}} \lambda_{a_1}$	$\frac{g_{21} \bar{v}}{\sqrt{2}} \lambda_{a_2}$	$-\frac{g_{21} v}{\sqrt{2}}$	0	0	M_{2L}	0	0	0
$\tilde{W}_{3R}$	0	$-\frac{g_{22} \bar{v}}{\sqrt{2}} \lambda_{a_2}$	$-\frac{g_{22} v}{\sqrt{2}} \lambda_{a_2}$	$\frac{g_{22} v}{\sqrt{2}} s_1$	0	0	M_{2R}	0	0
$\tilde{B}_{1L}$	$-\frac{5}{3} \frac{g_{11} \bar{v}}{\sqrt{2}} \lambda_{a_1}$	$-\frac{5}{3} \frac{g_{11} \bar{v}}{\sqrt{2}} \lambda_{a_2}$	$-\frac{5g_{11} v}{3\sqrt{2}}$	$\frac{5\sqrt{2}}{3} g_{11} s_1$	$\frac{5\sqrt{2}}{3} g_{11} s_2$	0	0	M_{1L}	0
$\tilde{B}_{1R}$	$\frac{5}{3} \frac{g_{18} \bar{v}}{\sqrt{2}} \lambda_{a_1}$	$\frac{5}{3} \frac{g_{18} \bar{v}}{\sqrt{2}} \lambda_{a_2}$	$\frac{5g_{18} v}{3\sqrt{2}}$	$\frac{5g_{18} s_1}{3\sqrt{2}}$	$-\frac{5\sqrt{2}}{3} g_{18} s_2$	0	0	0	M_{1R}

(30)

Here we have set $\langle S_1 \rangle = s_1$, $\langle S_2 \rangle = s_2$, $\langle H_u \rangle = v$, $\langle H_d \rangle = \bar{v}$, and the rest of the notation is standard. We now attempt to complete the fitting (26). Using the notation introduced earlier (21), the relevant couplings of the superpotential are

$$\lambda_5 C_4 C_1 C_1 = \lambda_5 S_2 H_u (\lambda_{a_2} H_d + \lambda_{a_4} L) \quad (31)$$

$$\lambda_6 C_2 C_1 C_3 = \lambda_6 S_1 H_u (\lambda_{a_1} H_d + \lambda_{a_3} L) + \lambda_6 e^+ (\lambda_{a_1} H_d + \lambda_{a_3} L) (\lambda_{a_2} H_d + \lambda_{a_4} L) \quad (32)$$

The couplings $\lambda_5 \lambda_{a_4} S_2 H_u L$ in expression (31) and $\lambda_6 \lambda_{a_3} S_1 H_u L$ in expression (32) violate lepton number and must therefore be suppressed. We recall however from (21) that the relative magnitudes of the λ_a 's are not totally arbitrary owing to the orthogonality of their matrix. In fact, we must have, for

example, $\lambda_{a_1} = \lambda_{a_4}$ and $\lambda_{a_2} = -\lambda_{a_3}$. Denoting by δ_a [18] the ratio of the lepton number violating (LNV) to the lepton number conserving (LNC) couplings ($0 < \delta_a < 1$), we can choose $\lambda_{a_4} = \delta_a \lambda_{a_2}$. This implies, however, $\lambda_{a_1} = -\delta_a \lambda_{a_3}$ owing to the orthogonality of the λ_a 's matrix which is the reverse of what we expected. This last relation can only be harmless if we impose

$$\frac{\lambda_5 \lambda_1}{\lambda_5 \lambda_2} \leq \frac{\lambda_{a_4}}{\lambda_{a_2}} \quad (33)$$

Thus we ensure that in the superpotential (31) + (32) the ratio of LNV to LNC couplings is δ_a . Further, if we are to have a coefficient $O(M_W)$ for the $H_u H_d$ term in expression (32), we must have

$$\lambda_5 \sim \varepsilon, \lambda_6 \sim \delta_a \varepsilon, \lambda_{a_1} = \lambda_{a_4} = \delta_a \lambda_{a_2}, \lambda_{a_2} = -\lambda_{a_3} \leq 1 \quad (34)$$

which completes the fitting (26). According to ref. [18] the ratio δ_a may range between 1/150 (for the e family) to 1 (for the τ family). All this reasoning would be acceptable if it were not for vanishing neutrino masses. Let us now discuss this aspect. Rotating S_1, S_2 into N, ν^c we have for expressions (31), (32)

$$\lambda_5 C_4 C_1 C_1 = \lambda_5 H_u (\lambda_{S_2} N + h_{S_2} \nu^c) (\lambda_{a_2} H_d + \lambda_{a_4} L) \quad (35)$$

$$\lambda_6 C_2 C_1 C_3 = \lambda_6 H_u (\lambda_{S_1} N + h_{S_1} \nu^c) (\lambda_{a_1} H_d + \lambda_{a_3} L) + \dots \quad (36)$$

with obvious notation. Vanishing Dirac neutrino mass means

$$\lambda_\nu \nu^c H \quad \text{with} \quad \lambda_\nu \neq 0 \quad (37)$$

the λ_ν being identified from (35), (36) as

$$\lambda_\nu = \lambda_5 h_{S_2} \lambda_{a_4} + \lambda_6 h_{S_1} \lambda_{a_3} \quad (38)$$

For this expression to vanish, a non-natural fine tuning must be done. Moreover, the neutralino mass matrix (30) does not admit a vanishing eigenvalue unless some miraculous fine tuning occurs. If we let however $\lambda_5 = \lambda_6 = 0$ both a zero mode will appear in (30)*), as can be easily checked, and $\lambda_\nu = 0$. The suppression of λ_5, λ_6 together with λ_7, λ_8 which prevent fast proton decay can be realized through the following discrete symmetry of the superpotential (4)

*) On condition that $g_{1R} = g_{1L}$ which always happens exactly in a 'true' left-right symmetric model. For one of the two classes of models analysed in Section 3 this equality holds within 3% only.

$$\begin{array}{lll}
B_1 \rightarrow B_1 & A_1 \rightarrow \eta A_1 & C_1 \rightarrow \eta^{-1} C_1 \\
C_2 \rightarrow C_2 & A_2 \rightarrow \eta A_2 & C_3 \rightarrow \eta^{-1} C_3 \\
C_4 \rightarrow C_4 & B_2 \rightarrow \eta^{-1} B_2 &
\end{array} \quad (39)$$

where, for instance, $\eta = e^{i\pi/2}$. The symmetry (6) is thus replaced by (39) which prevents the proton from decaying too fast and neutrinos from acquiring mass at the tree level. One consequence of (39) is that the soft supersymmetry breaking terms proportional to

$$H_u H_d S_1, H_u H_d S_2 \quad (40)$$

are removed from the superpotential. Such terms will have to be introduced by hand as in conventional supersymmetric grand unified theories.

To conclude this section it is appropriate to consider the radiative breaking of the group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ to the standard model with the v.e.v.'s $\langle S_1 \rangle$ and $\langle S_2 \rangle$. Both these fields have a Yukawa coupling: λ_1 and λ_2 , respectively. Owing to their positive contribution to the mass renormalization group equations, these couplings can generate [13] negative masses for S_1, S_2 at a hierarchically small scale as compared to the Planck scale (dimensional transmutation). Thus S_1 and S_2 can acquire v.e.v.'s implying gauge symmetry breaking. The dimensional transmutation scenario, which has been widely used [13,19], is the only existing candidate for generating a scale hierarchy. In our case the equations involved are

$$\frac{dm_{S_1}^2}{d(\ln Q)} = \frac{1}{8\pi^2} \left(-3M_{2R}^2 g_{2R}^2 - \frac{20}{3} M_{1L}^2 g_{1L}^2 - \frac{5}{3} M_{1R}^2 g_{1R}^2 + X_{S_1} \lambda_1^2 \right) \quad (41)$$

$$\frac{dm_{S_2}^2}{d(\ln Q)} = \frac{1}{8\pi^2} \left(-\frac{20}{3} M_{1L}^2 g_{1L}^2 - \frac{20}{3} M_{1R}^2 g_{1R}^2 + X_{S_2} \lambda_2^2 \right) \quad (42)$$

where [13]

$$\begin{aligned} \chi_{S_1} &= 3 \left(m_{\tilde{g}}^2 + m_{\tilde{q}}^2 + m_N^2 + A_{\lambda_1}^2 \right) \\ \chi_{S_2} &= 3 \left(m_{\tilde{g}}^2 + m_{\tilde{q}}^2 + m_N^2 + A_{\lambda_2}^2 \right) \end{aligned} \quad (43)$$

and the A 's are the trilinear couplings in the scalar potential, e.g. $\lambda_2 A_{\lambda_2} S_2 g x_1$. Since we can have two v.e.v.'s breaking the intermediate group, we find the possibility of this group having rank 6 as entirely self-consistent. Also keeping in mind eqs. (43), the fittings (26), (34), and the gauge-coupling relation $g_{2R}^2 \gg g_{1R}^2$, it is clear that the positive contribution on the right-hand side of eqs. (41), (42) is greater in the case of S_2 , leading us to expect $s_1 < s_2$.

3. RENORMALIZATION GROUP ANALYSIS

Given the framework developed in the previous section, we now perform a one-loop-level calculation for the gauge-coupling constants. This is subject to a few ambiguities, as seen below, but these do not affect our predictions in any significant way. We start with the renormalization group equations for the $SU(3)_C$, $SU(2)_L$, $U(1)_Y$ couplings. We denote by m_s the mass splitting within each supermultiplet due to supersymmetry breaking [20]

$$m_s = O \left(\frac{M_s^2}{M_p} \right) \quad (44)$$

where M_s is the primordial supersymmetry breaking scale. We let m_s run in the range

$$10^2 \text{ GeV} < m_s < 10^4 \text{ GeV} \quad (45)$$

as required by lower mass bounds on supersymmetric particles on one hand and the gauge hierarchy preservation on the other. Two cases must be treated separately: $m_s < M_I$ and $m_s > M_I$. As for the first, specially, not much freedom is allowed, since the intermediate scale cannot be significantly larger than m_s as we have seen. Then, for $m_s < M_I$ we have

$$\begin{aligned} \frac{1}{\alpha_3} &= \frac{1}{\alpha_G} - \frac{b_3^-}{2\pi} \ln \frac{m_s}{\mu} - \frac{b_{3s}^-}{2\pi} \ln \frac{M_I}{m_s} - \frac{b_{3s}^+}{2\pi} \ln \frac{M_x}{M_I} \\ \frac{1}{\alpha_{2L}} &= \frac{1}{\alpha_G} - \frac{b_{2L}^-}{2\pi} \ln \frac{m_s}{\mu} - \frac{b_{2LS}^-}{2\pi} \ln \frac{M_I}{m_s} - \frac{b_{2LS}^+}{2\pi} \ln \frac{M_x}{M_I} = \frac{\sin^2 \theta_w}{\alpha} \\ \frac{1}{\alpha_1} &= \frac{5}{3} \frac{1}{\alpha_G} - \frac{5}{3} \frac{b_1}{2\pi} \ln \frac{m_s}{\mu} - \frac{5}{3} \frac{b_{1s}}{2\pi} \ln \frac{M_I}{m_s} - \frac{5}{3} \frac{1}{2\pi} \left(b_{2RS} + \frac{1}{25} b_{1LS} + \right. \\ &\quad \left. + \frac{1}{25} b_{1RS} \right) \ln \frac{M_x}{M_I} = \frac{\cos^2 \theta_w}{\alpha} \end{aligned} \quad (46)$$

where the notation is standard^{*)} and we are using the definition (19) for the hypercharge. We recall that at the low-energy scale we assume the ordinary fermions plus the two weak doublets in each generation of 27 and take $N_G = 3$. The g-type quarks are essentially associated with scale M_I and we relate the fermionic $(e^+, \tilde{S}_1)$ $SU(2)_R$ doublet to scale μ , whereas its bosonic counterpart $(\tilde{e}^+, S_1)$ will be associated with scale M_I . Of course, the arbitrariness here is considerable but irrelevant as far as our predictions for $\sin^2 \theta_w$, M_X , α_G are concerned. With these definitions and assumptions, it is now straightforward to obtain the coefficients of the β functions

$$\begin{aligned} b_1 &= -\frac{43}{10}, \quad b_{2L}^- = \frac{17}{6}, \quad b_3^- = 7, \quad b_{1S} = -\frac{93}{10}, \quad b_{2LS}^- = -3, \quad b_{2LS}^+ = -3, \\ b_{2RS} &= -3, \quad b_{1LS} = -75, \quad b_{3S}^- = 3, \quad b_{1RS} = -75, \quad b_{3S}^+ = 0. \end{aligned} \quad (47)$$

Inserting $\mu = 100$ GeV, $\alpha_3 = 0.101$, $\alpha = 0.00784$, this leads to the following equation among the unification, the intermediate and the supersymmetry breaking scales

$$\ln M_X = \frac{7}{36} \ln m_s - \frac{17}{36} \ln M_I + 41.19. \quad (48)$$

For M_I , m_s in the allowed order of magnitude ($m_s < M_I$) M_X , $\sin^2 \theta_w$ come within a perfectly permissible range (see table 2). The unification coupling is $\alpha_G \approx 0.080$.

Turning now to the case $m_s > M_I$ we have

$$\begin{aligned} \frac{1}{\alpha_3} &= \frac{1}{\alpha_G} - \frac{b_3^-}{2\pi} \ln \frac{M_I}{\mu} - \frac{b_3^+}{2\pi} \ln \frac{m_s}{M_I} - \frac{b_{3S}^+}{2\pi} \ln \frac{M_X}{m_s} \\ \frac{1}{\alpha_{2L}} &= \frac{1}{\alpha_G} - \frac{b_{2L}^-}{2\pi} \ln \frac{M_I}{\mu} - \frac{b_{2L}^+}{2\pi} \ln \frac{m_s}{M_I} - \frac{b_{2LS}^+}{2\pi} \ln \frac{M_X}{m_s} = \frac{\sin^2 \theta_w}{\alpha} \\ \frac{1}{\alpha_1} &= \frac{5}{3} \frac{1}{\alpha_G} - \frac{5}{3} \frac{b_1}{2\pi} \ln \frac{M_I}{\mu} - \frac{5}{3} \frac{1}{2\pi} (b_{2R} + \frac{1}{25} b_{1L} + \frac{1}{25} b_{1R}) \cdot \\ &\quad \cdot \ln \frac{m_s}{M_I} - \frac{5}{3} \frac{1}{2\pi} (b_{2RS} + \frac{1}{25} b_{1LS} + \frac{1}{25} b_{1RS}) \ln \frac{M_X}{m_s} = \frac{\cos^2 \theta_w}{\alpha} \end{aligned} \quad (49)$$

^{*)} See for instance ref. [11].

with

$$b_{2R} = \frac{7}{3}, b_{1L} = -\frac{625}{12}, b_{1R} = -\frac{125}{3}, b_{2L}^+ = \frac{43}{12}, b_3^+ = 5 \quad (50)$$

and the remaining b coefficients as before (47). We now obtain

$$\ln M_x = -\frac{67}{324} \ln M_I - \frac{23}{324} \ln m_s + 41.19 \quad (51)$$

Numerical values are shown in table 3. Again $M_x, \sin^2 \theta_w$ come in the correct range. For the unification coupling we find $\alpha_G \approx 0.086$.

We have also investigated the effect of adding an arbitrary number of generations of 27 with the same mass pattern. The results are hardly sensitive to an increase in the parameter N_G . Taking $N = N_G - 3$, eqs. (48), (51) are modified, respectively, to

$$\ln M_x = \frac{21-15N}{108+20N} \ln m_s - \frac{51-3N}{108+20N} \ln M_I + \frac{1112}{27+5N} + \frac{N}{54+10N} \ln \mu \quad (52)$$

($m_s < M_I$)

$$\ln M_x = \frac{319N-138}{1944+360N} \ln m_s + \frac{5N-402}{1944+360N} \ln M_I + \frac{1112}{27+5N} + \frac{N}{54+10N} \ln \mu \quad (53)$$

($m_s > M_I$)

whereas for $m_s < M_I$

$$\sin^2 \theta_w = \frac{\alpha}{\alpha_3} + \frac{\alpha}{2\pi} \left[(b_{30}^- - b_{2L_0}^- + \frac{1}{6}N) \ln \frac{m_s}{\mu} + \right. \\ \left. + (b_{3S_0}^- - b_{2LS_0}^- + N) \ln \frac{M_I}{m_s} + (b_{3S_0}^+ - b_{2LS_0}^+) \ln \frac{M_x}{M_I} \right] \quad (54)$$

and for $m_s > M_I$

$$\sin^2 \theta_w = \frac{\alpha}{\alpha_3} + \frac{\alpha}{2\pi} \left[(b_{30}^- - b_{2L_0}^- + \frac{1}{6}N) \ln \frac{m_s}{\mu} + \right. \\ \left. + (b_{30}^+ - b_{2L_0}^+ + (5 - \frac{43}{12})N) \ln \frac{M_I}{m_s} + (b_{3S}^+ - b_{2LS}^+) \ln \frac{M_x}{M_I} \right] \quad (55)$$

Here the subscript 0 refers to the case $N = 0$ previously calculated.

Synthesizing the results of this section, we may say that left-right symmetric models from superstring phenomenology based on the group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ seem to be in good shape as far as the unification scale and the Weinberg angle are concerned.

4. SUMMARY AND OUTLOOK

We have investigated the prospects for left-right symmetric models from superstring phenomenology. It is worth pointing out that the superstring imposes restrictions on the possibilities for E_6 phenomenology which make clearer the way to either accept or reject specific models.

The breaking of E_6 parametrized by the matrix U_g allows for the two left-right possible E_6 subsymmetries, namely $SU(3)_C \times SU(2)_L \times SU(2)_N \times U(1)_L \times U(1)_R$ and $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$. We find an interesting no go result for models of the first type, which follows when constraining the mass matrix of the g (exotic) quark and d quark to agree with phenomenology. The argument implies that the orders of magnitude of the v.e.v.'s breaking the intermediate group must be very different ($> 10^2$), thus excluding the possibility of lowering its rank by two units. On the other hand, the second model works rather well from the points of view we analyzed: here we proposed a fitting of the Yukawa couplings involved. This fitting is consistent with the dynamical generation of mass for the two standard model singlets in 27. These can therefore acquire v.e.v., thus lowering the rank by two units. This model also predicts correct values of M_X , $\sin^2 \theta_W$ and α_G . The intermediate breaking scale M_I must not exceed the 10^4 GeV range, owing to the absence of mirror generations. It is remarkable that this right-handed scale could most naturally appear at such a low range in the simplest version of the model, as opposed to conventional GUTs and SUSY GUTs.

Items to be investigated include minimizing the Higgs potential, exploring the supersymmetry breaking scheme, and attempting more definite predictions for the sparticle masses.

Open problems in this model are common to other models based on superstring unification. They include finding the correct compactification scheme and manifold consistent with the set of Yukawa couplings we have. Recently it appeared [21] that the Calabi-Yau manifold solutions [4], although correct in their essentials, have to a certain extent to be perturbed, as regards one of their properties, namely the Ricci flatness. It has in fact been argued [22] that in general there exist non-local transformations to non-Ricci flat metrics for these manifolds which are solutions to the equations of motion. This is supported by the observation [23] that the central charge of the Virasoro algebra is not modified through four loops.

To sum up, the model we have investigated satisfies a few basic requirements for a candidate theory which stems from superstring unification being, interestingly enough, compatible with a low right-handed scale.

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Table 1

The decomposition and assignments of the 27 in each model we consider.
 The A_i 's, B_i 's and C_i 's correspond, respectively, to the representations A, B, and C
 under the maximal $SU(3)_C \times SU(3)_L \times SU(3)_R$ subgroup of E_6

Subgroup of E_6	Decomposition of 27
$SU(3)_C \times SU(3)_L$ $\times SU(3)_R$	$A(\bar{3}, 1, \bar{3}) + B(3, 3, 1) + C(1, \bar{3}, 3)$
$SU(3)_C \times SU(2)_L$ $\times SU(2)_N \times U(1)_L$ $\times U(1)_R$	$A_1(\bar{3}, 1, 2, 0, -5/6)_{\bar{g}, \bar{d}} + A_2(\bar{3}, 1, 1, 0, 5/3)_{\bar{u}} + B_1(3, 2, 1, 5/6, 0)_{u, d}$ $+ B_2(3, 1, 1, -5/3, 0)_{\bar{g}} + C_1(1, 2, 2, -5/6, 5/6)_{H_{d, L}} + C_2(1, 1, 2, 5/3, 5/6)_{N, \nu_C}$ $+ C_3(1, 2, 1, -5/6, -5/3)_{H_u} + C_4(1, 1, 1, 5/3, -5/3)_{e^+}$
$SU(3)_C \times SU(2)_L$ $\times SU(2)_R \times U(1)_L$ $\times U(1)_R$	$A_1(\bar{3}, 1, 2, 0, -5/6)_{x_2, \bar{u}} + A_2(\bar{3}, 1, 1, 0, 5/3)_{x_1} + B_1(3, 2, 1, 5/6, 0)_{u, d} +$ $+ B_2(3, 1, 1, -5/3, 0)_{\bar{g}} + C_1(1, 2, 2, -5/6, 5/6)_{y_2, H_u} + C_2(1, 1, 2, 5/3, 5/6)_{e^+, S_1(e^+, S_2)}$ $+ C_3(1, 2, 1, -5/6, -5/3)_{y_1} + C_4(1, 1, 1, 5/3, -5/3)_{S_1(S_2)}$

Table 2

The values of M_X , $\sin^2 \theta_W$ in the
 $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ case
 for $M_I \geq m_S$. Units are in GeV

M_I	m_S	M_X	$\sin^2 \theta_W$
10^4	10^3	3.82×10^{16}	0.215
10^3	10^3	1.13×10^{17}	0.211
10^5	10^3	1.29×10^{16}	0.202
10^5	10^4	2.02×10^{16}	0.216

Table 3

The values of M_X , $\sin^2 \theta_W$ in the
 $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ case
 for $M_I \leq m_S$. Units are in GeV

M_I	m_S	M_X	$\sin^2 \theta_W$
500	500	1.38×10^{17}	0.210
500	10^3	1.31×10^{17}	0.209
500	10^4	1.11×10^{17}	0.204
10^3	10^4	9.63×10^{16}	0.206
10^4	10^4	5.98×10^{16}	0.212

APPENDIX

Here we explain in some detail the diagonalization of the g,d mass matrix M for each of the two scenarios analysed.

A.1 $SU(3)_C \times SU(2)_L \times SU(2)_N \times U(1)_L \times U(1)_R$

We start with the mass matrix implied by the superpotential (7) [expression (10) in the main text]

$$(x_2^+ \quad x_2^-) \begin{pmatrix} \lambda_1 s_2 & \lambda'_1 \bar{u} \\ \lambda_1 s_1 & \lambda''_1 \bar{u} \end{pmatrix} \begin{pmatrix} g \\ d \end{pmatrix} \quad (A.1)$$

The matrices $M^T M$ and $M M^T$ are symmetric and have equal eigenvalues. We have ($\epsilon = M_W/M_I \simeq \bar{v}/s_{1,2}$)

$$M^T M = M_I^2 \begin{pmatrix} 2\lambda_1^2 & \lambda_1(\lambda'_1 + \lambda''_1)\epsilon \\ \lambda_1(\lambda'_1 + \lambda''_1)\epsilon & (\lambda_1'^2 + \lambda_1''^2)\epsilon^2 \end{pmatrix} \equiv M_I^2 \begin{pmatrix} A & B\epsilon \\ B\epsilon & C\epsilon^2 \end{pmatrix}$$

$$M M^T = M_I^2 \begin{pmatrix} \lambda_1^2 + \lambda_1'^2 \epsilon^2 & \lambda_1^2 + \lambda'_1 \lambda''_1 \epsilon^2 \\ \lambda_1^2 + \lambda'_1 \lambda''_1 \epsilon^2 & \lambda_1^2 + \lambda_1''^2 \epsilon^2 \end{pmatrix} \equiv M_I^2 \begin{pmatrix} D_1 & D_2 \\ D_2 & D_3 \end{pmatrix} \quad (A.2)$$

the eigenvalues, divided by M_I , being

$$\alpha_1 = 2\lambda_1^2 + \frac{\lambda_1'^2 + \lambda_1''^2}{2} \epsilon^2 + \lambda'_1 \lambda''_1 \epsilon^2 + \frac{(\lambda_1'^2 + \lambda_1''^2)^2}{8\lambda_1^2} \epsilon^4 + O(\epsilon^6)$$

$$\alpha_2 = \frac{\lambda_1'^2 + \lambda_1''^2}{2} \epsilon^2 - \lambda'_1 \lambda''_1 \epsilon^2 - \frac{(\lambda_1'^2 + \lambda_1''^2)^2}{8\lambda_1^2} \epsilon^4 + O(\epsilon^6) \quad (A.3)$$

and we defined

$$\begin{aligned}
 A &= 2\lambda_1^2 & D_1 &= \lambda_1^2 + \lambda_1'^2 \varepsilon^2 \\
 B &= \lambda_1 (\lambda_1' + \lambda_1'') & D_2 &= \lambda_1^2 + \lambda_1' \lambda_1'' \varepsilon^2 \\
 C &= \lambda_1'^2 + \lambda_1''^2 & D_3 &= \lambda_1^2 + \lambda_1''^2 \varepsilon^2.
 \end{aligned} \tag{A.4}$$

The eigenvectors of $M^T M$ and MM^T are, respectively,

$$\bar{v}_1 = \begin{pmatrix} \tilde{a}_1 \\ \frac{\alpha_1 - A}{B\varepsilon} \tilde{a}_1 \end{pmatrix}, \quad \bar{v}_2 = \begin{pmatrix} \tilde{b}_1 \\ \frac{\alpha_2 - A}{B\varepsilon} \tilde{b}_1 \end{pmatrix}$$

and

$$\bar{u}_1 = \begin{pmatrix} a_1 \\ \frac{\alpha_1 - D_1}{D_2} a_1 \end{pmatrix}, \quad \bar{u}_2 = \begin{pmatrix} b_1 \\ \frac{\alpha_2 - D_1}{D_2} b_1 \end{pmatrix}. \tag{A.5}$$

It is straightforward to check the orthogonality between the two vectors $\bar{v}_1, \bar{v}_2$ and between $\bar{u}_1, \bar{u}_2$. Normalization for $\bar{v}_1, \bar{v}_2$ implies

$$\tilde{a}_1 = \frac{\lambda_1 (\lambda_1' + \lambda_1'') \varepsilon}{\sqrt{\lambda_1^2 (\lambda_1' + \lambda_1'')^2 \varepsilon^2 + \left[\frac{1}{2} (\lambda_1'^2 + \lambda_1''^2) \varepsilon^2 + \lambda_1' \lambda_1'' \varepsilon^2 + \frac{(\lambda_1'^2 + \lambda_1''^2)^2}{8\lambda_1^2} \varepsilon^4 \right]^2}} \tag{A.6}$$

$$\tilde{b}_1 = \frac{\lambda_1 (\lambda_1' + \lambda_1'') \varepsilon}{\sqrt{\lambda_1^2 (\lambda_1' + \lambda_1'')^2 \varepsilon^2 + \left[-2\lambda_1^2 + \frac{1}{2} (\lambda_1'^2 + \lambda_1''^2) \varepsilon^2 - \lambda_1' \lambda_1'' \varepsilon^2 - \frac{(\lambda_1'^2 + \lambda_1''^2)^2}{8\lambda_1^2} \varepsilon^4 \right]^2}}$$

and normalization for $\bar{u}_1, \bar{u}_2$ implies

$$\begin{aligned}
 a_1 &= \frac{\lambda_1^2 + \lambda_1' \lambda_1'' \varepsilon^2}{\sqrt{(\lambda_1^2 + \lambda_1' \lambda_1'' \varepsilon^2)^2 + \left[\lambda_1^2 - \lambda_1'^2 \varepsilon^2 + \frac{1}{2} (\lambda_1'^2 + \lambda_1''^2) \varepsilon^2 + \lambda_1' \lambda_1'' \varepsilon^2 + \frac{(\lambda_1'^2 + \lambda_1''^2)^2}{8\lambda_1^2} \varepsilon^4 \right]^2}} \\
 b_1 &= \frac{\lambda_1^2 + \lambda_1' \lambda_1'' \varepsilon^2}{\sqrt{(\lambda_1^2 + \lambda_1' \lambda_1'' \varepsilon^2)^2 + \left[-\lambda_1^2 - \lambda_1'^2 \varepsilon^2 + \frac{1}{2} (\lambda_1'^2 + \lambda_1''^2) \varepsilon^2 - \lambda_1' \lambda_1'' \varepsilon^2 - \frac{(\lambda_1'^2 + \lambda_1''^2)^2}{8\lambda_1^2} \varepsilon^4 \right]^2}}
 \end{aligned} \tag{A.7}$$

The matrix U which diagonalizes MM^T is therefore

$$U = \begin{pmatrix} a_1 & \frac{\alpha_1 - D_1}{D_2} a_1 \\ b_1 & \frac{\alpha_2 - D_1}{D_2} b_1 \end{pmatrix} \quad (\text{A.8})$$

and the matrix V which diagonalizes M^TM is

$$V = \begin{pmatrix} \tilde{a}_1 & \frac{\alpha_1 - A}{B\epsilon} \tilde{a}_1 \\ \tilde{b}_1 & \frac{\alpha_2 - A}{B\epsilon} \tilde{b}_1 \end{pmatrix} \quad (\text{A.9})$$

It is straightforward to verify that indeed UMM^TU^T and VM^TMV^T are diagonal. In fact, for the first of these

$$\begin{aligned} UMM^TU^T &= \\ &= \begin{pmatrix} a_1^2 \left[D_1 + 2(\alpha_1 - D_1) + \frac{(\alpha_1 - D_1)^2}{D_2^2} D_3 \right] & a_1 b_1 \left[D_1 + (\alpha_1 - D_1) + (\alpha_2 - D_1) + \frac{(\alpha_1 - D_1)(\alpha_2 - D_1)}{D_2^2} D_3 \right] \\ a_1 b_1 \left[D_1 + (\alpha_2 - D_1) + (\alpha_1 - D_1) + \frac{(\alpha_1 - D_1)(\alpha_2 - D_1)}{D_2^2} D_3 \right] & b_1^2 \left[D_1 + 2(\alpha_2 - D_1) + \frac{(\alpha_2 - D_1)^2}{D_2^2} D_3 \right] \end{pmatrix} \end{aligned} \quad (\text{A.10})$$

with off-diagonal elements

$$\begin{aligned} & a_1 b_1 \left[\alpha_1 + \alpha_2 - D_1 + \frac{\alpha_1 \alpha_2 + D_1^2 - D_1(\alpha_1 + \alpha_2)}{D_2^2} D_3 \right] = \\ & = a_1 b_1 \left[-D_1 + D_1 + D_3 + \frac{D_1 D_3 - D_2^2 - D_1(D_1 + D_3) + D_1^2}{D_2^2} D_3 \right] = 0. \end{aligned} \quad (\text{A.11})$$

For the second, VM^TMV , the off-diagonal term gives

$$\tilde{a}_1 \tilde{b}_1 \left[A + (\alpha_1 - A) + (\alpha_2 - A) + \frac{C}{B^2} (\alpha_1 - A)(\alpha_2 - A) \right] = 0 \quad (\text{A.12})$$

as can easily be seen. In deriving eqs. (A.11), (A.12) we made use of the eigenvalue equations

$$\det |MM^T - \alpha I| = 0 \quad \text{and} \quad \det |M^T M - \alpha I| = 0. \quad (\text{A.13})$$

The matrix UM given in eq. (16) of the main text can be obtained from (A.8) and (A.1). The final check of the diagonality of UMV^T is now straightforward, if rather tedious.

A.2 $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$

The starting point is now eq. (21) of the main text

$$(x_1, x_2) \begin{pmatrix} \lambda_2 s_2 & \lambda'_4 \bar{u} \\ \lambda_1 s_1 & \lambda'_3 \bar{u} \end{pmatrix} \begin{pmatrix} g \\ d \end{pmatrix} \quad (\text{A.14})$$

We now have

$$M^T M = M_I^2 \begin{pmatrix} \lambda_1^2 + \lambda_2^2 & (\lambda_2 \lambda'_4 + \lambda_1 \lambda'_3) \varepsilon \\ (\lambda_2 \lambda'_4 + \lambda_1 \lambda'_3) \varepsilon & (\lambda_3'^2 + \lambda_4'^2) \varepsilon^2 \end{pmatrix} \equiv M_I^2 \begin{pmatrix} A & B\varepsilon \\ B\varepsilon & C\varepsilon^2 \end{pmatrix}$$

$$M M^T = M_I^2 \begin{pmatrix} \lambda_2^2 + \lambda_4'^2 \varepsilon^2 & \lambda_1 \lambda_2 + \lambda'_3 \lambda'_4 \varepsilon^2 \\ \lambda_1 \lambda_2 + \lambda'_3 \lambda'_4 \varepsilon^2 & \lambda_1^2 + \lambda_3'^2 \varepsilon^2 \end{pmatrix} \equiv M_I^2 \begin{pmatrix} D_1 & D_2 \\ D_2 & D_3 \end{pmatrix} \quad (\text{A.15})$$

with common eigenvalues (divided by M_I)

$$\alpha_1 = \lambda_1^2 + \lambda_2^2 + \frac{(\lambda_2 \lambda'_4 + \lambda_1 \lambda'_3)^2}{\lambda_1^2 + \lambda_2^2} \varepsilon^2 + \frac{1}{4} \frac{(\lambda_3'^2 + \lambda_4'^2)^2}{\lambda_1^2 + \lambda_2^2} \varepsilon^4 + O(\varepsilon^6)$$

$$\alpha_2 = (\lambda_3'^2 + \lambda_4'^2) \varepsilon^2 - \frac{(\lambda_2 \lambda'_4 + \lambda_1 \lambda'_3)^2}{\lambda_1^2 + \lambda_2^2} \varepsilon^2 - \frac{1}{4} \frac{(\lambda_3'^2 + \lambda_4'^2)^2}{\lambda_1^2 + \lambda_2^2} \varepsilon^4 + O(\varepsilon^6) \quad (\text{A.16})$$

and the identifications

$$\begin{aligned}
 A &= \lambda_1^2 + \lambda_2^2 & D_1 &= \lambda_2^2 + \lambda_4'^2 \varepsilon^2 \\
 B &= \lambda_1 \lambda_3' + \lambda_2 \lambda_4' & D_2 &= \lambda_1 \lambda_2 + \lambda_3' \lambda_4' \varepsilon^2 \\
 C &= \lambda_3'^2 + \lambda_4'^2 & D_3 &= \lambda_1^2 + \lambda_3'^2 \varepsilon^2.
 \end{aligned} \tag{A.17}$$

The eigenvectors of $M^T M$ and MM^T are formally given by (A.5) but with $\tilde{a}_1$, $\tilde{b}_1$, a_1 , b_1 as follows:

$$\begin{aligned}
 \tilde{a}_1 &= \frac{(\lambda_1 \lambda_3' + \lambda_2 \lambda_4') \varepsilon}{\sqrt{(\lambda_1 \lambda_3' + \lambda_2 \lambda_4')^2 \varepsilon^2 + (\alpha_1 - \lambda_1^2 - \lambda_2^2)^2}} \\
 \tilde{b}_1 &= \frac{(\lambda_1 \lambda_3' + \lambda_2 \lambda_4') \varepsilon}{\sqrt{(\lambda_1 \lambda_3' + \lambda_2 \lambda_4')^2 \varepsilon^2 + (\alpha_2 - \lambda_1^2 - \lambda_2^2)^2}} \\
 a_1 &= \frac{\lambda_1 \lambda_2 + \lambda_3' \lambda_4' \varepsilon^2}{\sqrt{(\lambda_1 \lambda_2 + \lambda_3' \lambda_4' \varepsilon^2)^2 + (\alpha_1 - \lambda_2^2 - \lambda_4'^2 \varepsilon^2)^2}} \\
 b_1 &= \frac{\lambda_1 \lambda_2 + \lambda_3' \lambda_4' \varepsilon^2}{\sqrt{(\lambda_1 \lambda_2 + \lambda_3' \lambda_4' \varepsilon^2)^2 + (\alpha_2 - \lambda_2^2 - \lambda_4'^2 \varepsilon^2)^2}}
 \end{aligned} \tag{A.18}$$

The rest of the procedure is entirely analogous to case A.1.