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Experiments on Weak Decays of Leptons and Quarks

Robert Klanner

ABSTRACT

After a short review of the standard model of electroweak interactions, recent experiments on weak decays of leptons and quarks and their relevance for the standard model are discussed.

1 Introduction

These lecture notes describe several of the experiments on weak decays of leptons and quarks, which form one of the pillars of the very successful standard model of electroweak interactions. The lectures should provide enough information to understand the underlying ideas and the most important results of these experiments, and give some hints on their technical realisation and experimental difficulties. The standard model of electroweak interactions, in its most simple version is taken as the frame to discuss motivation and relevance of these experimental efforts.

The outline of the lectures is as follows:

The basic ideas and the notations of the standard model are reviewed in the second chapter. Then experiments on the properties of leptons are discussed, in particular on lepton number conservation in muon decay, neutrino mixing and and neutrino masses. In the following experiments on the space-time structure of the weak charged current, the question of quark mixing and the present status of the determination of the quark mixing matrix is treated. The actual lectures also covered the field of CP-violation in the $K^-$-system, in particular the new experiments on the determination of the CP-violation parameters $c'/c$, which do not appear in the written version of the lectures.

In spite of the wide field, covered by these lectures, the notes are pretty short, and we refer to several excellent review articles, and the original publications, in which the various topics are treated in significantly more detail [1.1].

2 The Standard Model of Electroweak Interactions

These lectures summarize only a few aspects of the standard model in its most simple form. More details can be found in [2.1]. Within the frame work of a renormalizable quantum field theory, the standard model, describes the weak and electromagnetic interactions between the fundamental constituents of the model and the gauge fields.

The fundamental constituents are the spin 1/2 quarks and leptons which, as far as we know now, come in three families (table. 2.1a). The quarks come in three colours, whereas the leptons are colour singlets. The left-handed fermions are grouped in weak isospin doublets $(f, f')_L$, which are associated with the weak isospin group $SU_2L$ (table. 2.1b).
Table 2.1a. Spin 1/2 Fermions

<table>
<thead>
<tr>
<th>leptons</th>
<th>quarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e, \nu_e )</td>
<td>( d, u )</td>
</tr>
<tr>
<td>( \mu, \nu_\mu )</td>
<td>( s, c )</td>
</tr>
<tr>
<td>( \tau, \nu_\tau )</td>
<td>( b, t )</td>
</tr>
</tbody>
</table>

Table 2.1b. Weak Isospin Grouping of Fermions

<table>
<thead>
<tr>
<th>isospin doublets:</th>
<th>leptons</th>
<th>quarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (\nu_\mu, e)_L )</td>
<td>( (\nu_\tau, \mu)_L )</td>
<td>( (\nu_\tau, \tau)_L )</td>
</tr>
<tr>
<td>( (u, d')_L )</td>
<td>( (c, s')_L )</td>
<td>( (t, b')_L )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>isospin singlets:</th>
<th>leptons</th>
<th>quarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( e_R )</td>
<td>( \mu_R )</td>
<td>( \tau_R )</td>
</tr>
<tr>
<td>( (u, d')_R )</td>
<td>( (c, s')_R )</td>
<td>( (t, b')_R )</td>
</tr>
</tbody>
</table>

The right-handed fermions are SU2L singlets. In its most simple form, compatible with presently generally accepted experiments, neutrinos are massless, and there are no right-handed neutrinos. Both left-handed doublets and right-handed singlets carry weak hypercharge, associated with the group U1Y. Thus the group structure of the constituents of the standard model is:

\[
SU_3^C \cdot SU_2^L \cdot U_{1Y} \tag{2.1}
\]

The gauge bosons associated with the individual groups, and their couplings are:

- SU3C ......eight gluons with coupling \( g_S \),
- SU2L ......three W-bosons \( W^j \) with coupling \( g \),
- U1Y ......one B-boson with coupling \( g' \).

The latter two groups unify weak and electromagnetic interactions. The elementary gauge fields \( W^\mu \) and \( B^\mu \) are related to the the physical particles:

- photon .... \( A^\mu = B^\mu \cdot \cos \theta_W + W^\mu_3 \cdot \sin \theta_W \),
- neutral vector boson ... \( Z^\mu = -B^\mu \cdot \sin \theta_W + W^\mu_3 \cdot \cos \theta_W \),
- charged vector bosons ... \( W^\pm_\mu = (W^1_\mu \pm iW^2_\mu)/\sqrt{2} \).

The couplings \( g \) and \( g' \) are related to the coupling constants of weak and electromagnetic interactions via:

- the Fermi constant \( G_F = g^2/(\sqrt{32} \cdot m_W^2) \),
- the electron charge \( e = g \cdot \sin \theta_W \),
- the weak mixing angle \( \sin^2 \theta_W = g^2/(g^2 + g'^2) \).

The interaction between gauge bosons and fermion fields is described by the following invariant Lagrangian \( L \), which can be symbolically written for the most simple version of the standard model as:

\[
L = L_{em} + L_{CC} + L_{NC} \tag{2.2}
\]
with the electromagnetic, charged current and neutral current parts:

\[ L_{em} = g \cdot \sin \theta_W \cdot (J^\mu_{em} \cdot A_\mu + h.c.) \]

\[ J^\mu_{em} = q_f \cdot \bar{f} \gamma^\mu f \]  \hspace{1cm} (2.3)

\[ L_{CC} = g/\sqrt{8} \cdot (J^\mu_{CC} W^-_\mu + J^\mu_{CC} W^+_\mu + h.c.) \]

\[ J^\mu_{CC} = (\bar{u} \gamma^\mu U_{KM} \begin{pmatrix} d \\ s \end{pmatrix} + (\bar{\nu}_e \nu_\mu \nu_\tau) \gamma^\mu U_{KM} \begin{pmatrix} \mu \\ \tau \end{pmatrix}) \]

\[ f_L = (1 + \gamma_5)f \quad \text{and} \quad f_R = (1 - \gamma_5)f \]  \hspace{1cm} (2.4)

\[ L_{NC} = e \cdot (J^\mu_{NC} Z_\mu + h.c.)/(\sin \theta_W \cdot \cos \theta_W) \]

\[ J^\mu_{NC} = \epsilon(f_L) \cdot \bar{f} \gamma^\mu f_L + \epsilon(f_R) \cdot \bar{f} \gamma^\mu f_R \]

\[ \epsilon(f_L) = I^3 - q_f \cdot \sin^2 \theta_W \quad \text{and} \quad \epsilon(f_R) = -q_f \cdot \sin^2 \theta_W \]  \hspace{1cm} (2.3)

(I^3 is the third component of the weak isospin).

The first term of the Lagrangian \( L_{em} \), describes the coupling of the photon via the charge \( q_f \) to the fermions. The charged current term \( L_{CC} \) the coupling between the charged intermediate vector bosons to the left-handed fermions. In the way written, the coupling strength is described by a single universal coupling constant \( g \). The unitary mass-mixing matrix \( U_{KM} \) allows transitions between members of the quark families; its absence in the lepton sector expresses the experimental observation of lepton number conservation. The left-handed coupling of the charged current interaction is exhibited by the absence of terms with right-handed fermions. The neutral current coupling term one of the big successes of the unification of weak and electromagnetic interactions describes the interaction of the fermions with the neutral vector boson and the photon. Its coupling constant is related to the Fermi constant \( G_F \) and the electromagnetic coupling via the weak mixing angle \( \theta_W \). Its couplings \( \epsilon(f_L) \) and \( \epsilon(f_R) \) are uniquely predicted from the SU2L \cdot SU1Y group structure. Again the absence of a mixing matrix expresses the observed absence of neutral currents leading to transitions within the quark families or the lepton families.

3 Lepton Number Conservation

The experimental search for interactions, both in the neutral and charged current sector, which lead to transition between lepton families, is an active field since fifty years. Fig. 3.1 shows the upper limit for various muon-number violating decay modes as function of the year. We see that over the last forty years upper limits have typically decreased by ten orders of magnitude.

One particular beautiful recent experiment is the search for the decay mode \( \mu^+ \rightarrow e^+e^-\bar{\nu}_e \), performed by the SINDRUM-collaboration [3.1]. Fig. 3.2 shows a schematic drawing of the detector. A monochromatic (\( \approx29.5\text{MeV/c} \)) muon-beam, derived from
π-mesons stopping close to the surface of the production target, hit the target of the SINDRUM-spectrometer, where they are stopped. As the typical energy of electrons from the $\mu \rightarrow eee$ decay is only 35MeV, and the precise determination of directions and momenta of the electrons is the key for background rejection, the target material is minimized. It consists of a double cone of Rohacell of 11mg/cm² thickness. Momenta and directions of the decay electrons are measured in a solenoid with $\approx 2m^3$ field volume and < 1% field inhomogeneity, equipped with very low mass cylindrical multiwire proportional chambers. In the experiment a total of $(7.3 \pm 0.5)10^{12}$ muons were stopped in the SINDRUM target. Events with three electrons come from the allowed decay $\mu^+ \rightarrow e^+e^+e^-\nu\nu$ or from the decay $\mu^+ \rightarrow e^+\nu\nu$, with the positron scattered back from the coil and passing close to the production vertex. After topological cuts and requiring energy and momentum conservation, the experiment is left with zero background events yielding an upper limit for the branching ratio $BR(\mu^+ \rightarrow e^+e^+e^-) < 2.4 \cdot 10^{-12}$ at 90% confidence level.

Thus lepton number conservation, as assumed in the Lagrangian has been tested to a very high accuracy. This measurement can also be used to obtain more quantitative limits on interactions beyond the standard model. For example a lower mass limit of 50TeV can be derived for a horizontal neutral boson mediating the decay [3.2]. In composite models for lepton an upper limit for the size of the muon of $4 \cdot 10^{-20}$cm can be derived [3.3].

In its most simple version the standard model assumes zero mass for the neutrinos. The most accurate determinations of the mass of the electron neutrino come from measurements of the electron energy spectrum in tritium $\beta$-decay: $^3H \rightarrow ^3He + e^- + \nu_e$.

The shape of the energy spectrum close to the endpoint (18.556keV assuming zero neutrino mass), is complicated by the following effects:

- the binding energy of the electron in the $^3$He-ion,
- the binding of the $^3$H in the valine molecule, on which the most accurate measurements are done presently,
- the determination of the energy resolution function of the electrons.

The most precise experiments have been done at ITEP-Moscow [3.4]. The best fit to electron spectrum yields a mass for the electron neutrino of $(37.4\pm1.7)\text{eV}$, which, when taking into account systematic uncertainties of the energy spectrum from binding energy, yields a range between 20eV and 45eV at 90% confidence level. Objections raised by Berqkvist [3.5], concern the quality of the description of the energy spectrum by the fit and the experimental resolution function. Given the importance of a non-zero neutrino mass, it seems cautious to wait for a confirmation of the experiment, if possible on free tritium, before unambiguously claiming a finite value for the electron neutrino mass.

The question of a finite neutrino mass of the muon neutrino, has been studied in several precision experiments of the decay $\pi \rightarrow \mu\nu\mu$. An experiment at SIN [3.6], has measured with high precision the vector difference of $\pi$- and $\mu$-momentum for $\pi$-decay in flight, and compared it to the value expected from a zero muon-neutrino mass. An upper limit of 500keV has been found. This method has the advantage that it is insensitive to the exact knowledge of $\pi$-meson and muon mass.

Another experiment at SIN [3.7], has determined the muon momentum from the $\pi \rightarrow \mu\nu$ decay at rest to be $p(\mu) = (29.7873 \pm 0.0008)\text{MeV}$, which, together with the precisely known values of $\pi$- and $\mu$-masses, yields an upper limit of 490keV.
To study the mass of the \( \tau \)-neutrino, \( \tau \)-production and decay into \( \tau \rightarrow 4\pi + \nu \), has been studied in electron positron annihilation [3.8,3.9]. From a comparison of the \( 4\pi \)-mass spectrum to predictions assuming various values for the neutrino mass, the presently best upper limit of \( m(\nu_\tau) < 70 \text{MeV} \) at 90% confidence level has been obtained by the ARGUS collaboration.

Mass differences between the different types of neutrinos can lead to the phenomena of neutrino oscillations i.e. transitions between different neutrinos in neutrino beams. In this case the neutrino flavour eigenstates are different to the neutrino mass eigenstates. For the case of two neutrinos, the formalism describing neutrino oscillations is similar to transitions in the neutral K-system. An extension to more neutrinos is straightforward [3.10]. If we denote in the case of two neutrinos the flavour eigenstates \( \nu_1, \nu_2 \) and the mass eigenstates \( \nu_e \) and \( \nu_\mu \), we can write:

\[
\nu_e = \nu_1 \cdot \cos \theta + \nu_2 \cdot \sin \theta \\
\nu_\mu = -\nu_1 \cdot \sin \theta + \nu_2 \cdot \cos \theta
\]

with \( \theta \) being the mixing angle between the two neutrinos. If we have a pure neutrino beam of the type \( \nu_e \) at time \( t=0 \) (for example at the point \( L=0 \), where the neutrino has been produced via a decay), the time development in the rest frame is given by:

\[
\nu(t) = \nu_1 e^{-im_1t} \cos \theta + \nu_2 e^{-im_2t} \sin \theta \\
= (\cos^2 \theta e^{-im_1t} + \sin^2 \theta e^{-im_2t}) \cdot \nu_e + (\cos \theta \sin \theta (e^{im_2t} - e^{-im_1t})) \cdot \nu_\mu.
\]

From this we can evaluate for a neutrino beam of energy \( E \) the probabilities of observing an electron-neutrino or a muon-neutrino after a length \( L \):

\[
P(\nu_e \rightarrow \nu_e) = 1 - \sin^2(2\theta) \cdot \sin^2(\Delta/2) \\
P(\nu_e \rightarrow \nu_\mu) = \sin^2(2\theta) \cdot \sin^2(\Delta/2),
\]

with \( \Delta = (m_2^2 - m_1^2) \cdot L/2E \). The formula is valid for \( E \gg m_1, m_2 \) and \( \Delta \gg 1 \).

From above formulae we see, that experiments on neutrino mixing yield simultaneously information on the neutrino mixing angles \( \theta_{ij} \) and the squared mass differences \( \Delta m_{ij}^2 = m_i^2 - m_j^2 \). Both have to have values different from zero to observe neutrino oscillations. It can also be seen, that two types of experiments can be done:

- disappearance experiments, in which the decrease in flux of one type of neutrino is measured; these experiments are sensitive to the inclusive transitions to all possible other neutrino types,

- appearance experiments, which search for the increase in flux of a particular neutrino type.

As an example for an appearance experiment, we quote the BNL-Columbia experiment at FNAL [3.11], which is sensitive to transitions \( \nu_\mu \rightarrow \nu_e \) and \( \nu_\mu \rightarrow \nu_\tau \). Protons from the FNAL accelerator are dumped in a target. The neutrinos come from the decay of \( \pi \)-and K-mesons in the 400m long decay channel following the target. All particles, but neutrinos are dumped at the end of the channel. The dominant component of the beam are muon-neutrinos, with a small admixture of electron-neutrinos from \( K \rightarrow e\pi\nu \) decay.
The 15 foot bubble chamber, 1200m downstreams of the decay channel is used as detector. The flux of electron neutrinos is measured via the charged current reaction $\nu_{e} \rightarrow e$, with the electron being identified in the heavy liquid of the bubble chamber. A total of 69000 events of the type $\nu_{\mu} \rightarrow \mu$ and no excess of events of the type $\nu_{e} \rightarrow e$ has been observed beyond the the expectation from the $\nu_{e}$-component of the beam. An upper limit of $3 \cdot 10^{-3}$ for the transition $\nu_{\mu} \rightarrow \nu_{e}$ is obtained. The corresponding regions of $\sin^{2}(2\theta)$ and $\delta m^{2}$, which can be excluded by this experiment, together with data from other experiments are shown in fig. 3.3. The same experiment also gives limits on transitions $\nu_{\mu} \rightarrow \nu_{\tau}$, as charged current $\nu_{e}$- interactions yield $\tau$-leptons, which after a decay length too short to be detected in this experiment, decay into electrons with a $(16.5 \pm 0.9)$% branching ratio. These results, together with a more sensitive search for the direct observation of $\tau$-decays using nuclear emulsions [3.12] are shown in fig. 3.4.

Disappearance experiments have been done using the electron-neutrino flux from nuclear reactors, from the sun and muon-neutrino beams from proton accelerators. The accelerator experiments have yielded only upper limits so far [3.13]. The reactor experiments [3.14, 3.15] measure the $\bar{\nu}_{e}$-flux at various distances from the reactor, by detecting the reaction $\bar{\nu}_{e}p \rightarrow e^{+}n$, using scintillator sandwiches to determine the energy of the positron and proportional chambers filled with $^{3}$He for neutron detection via the capture reaction $n + ^{3}$He $\rightarrow ^{3}$H + p. The LAPP, ISN-Annecy, Grenoble experiment compares the electron neutrino flux at 13.6m and 18.3m from the Bugey reactor, and after correction for the distance find a flux ratio of $R_{13.6}/R_{18.3} = (1.102 \pm 0.014 \pm 0.028)$, where the last error estimates the systematic uncertainties. This yields the allowed region in the $\sin^{2}(2\theta)$ versus $\delta m^{2}$ plot of fig. 3.5.

A CALTECH, SIN, TU-Munich group has performed a similar experiment at distances of 37.9m, 45.3m and more recently at 64.7m from the Goesgen reactor [3.15]. The measured energy spectra of the electrons agrees with the predictions of absence of neutrino mixing, thus excluding oscillations as shown in fig. 3.5, in disagreement with the results of the Bugey experiment.

The experiment of Davis et.al. [3.16] has been measuring the flux of electron neutrinos from the sun and comparing it to the flux expected from the different reaction taking place inside of the sun [table 3.1].

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Energy(MeV)</th>
<th>Theor. Flux($10^{10} cm^{-2} sec^{-1}$)</th>
<th>SNU($^{37}$Cl)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p + p \rightarrow D + e^{+} + \nu_{e}$</td>
<td>0.0-0.4</td>
<td>6.1</td>
<td>0</td>
</tr>
<tr>
<td>$p + p + e^{-} \rightarrow D + \nu_{e}$</td>
<td>1.4</td>
<td>0.015</td>
<td>0.23</td>
</tr>
<tr>
<td>$^{7}$Be + $e^{-} \rightarrow ^{7}$Li + $\nu_{e}$</td>
<td>0.86/0.34</td>
<td>0.34</td>
<td>1.03</td>
</tr>
<tr>
<td>$^{8}$B $\rightarrow ^{8}$Be + $e^{+} + \nu_{e}$</td>
<td>0-1.4</td>
<td>0.006</td>
<td>6.48</td>
</tr>
<tr>
<td>$^{13}$N $\rightarrow ^{13}$C + $e^{+} + \nu_{e}$</td>
<td>0-1.2</td>
<td>0.045</td>
<td>0.07</td>
</tr>
<tr>
<td>$^{15}$O $\rightarrow ^{15}$N + $e^{+} + \nu_{e}$</td>
<td>0-1.7</td>
<td>0.035</td>
<td>0.23</td>
</tr>
</tbody>
</table>

The electron neutrinos are detected via the reaction $\nu_{e} + ^{37}$Cl $\rightarrow ^{37}$Ar + e in a large tank filled with C$_{2}$Cl$_{4}$. The $^{37}$Ar is removed from the vessel by purging it with He and subsequent detection via K-capture. As the threshold for $^{37}$Ar production is 0.814MeV, only a small fraction of the $\nu_{e}$-flux of the sun can be detected. This is also shown in table 3.1. Expressed in units of $10^{-36}$ $\nu_{e}$-captures per sec and $^{37}$Cl nucleus (1 SNU), the
expected rate is $8.3 \pm 3.3$, whereas $2.2 \pm 0.4$ are observed. Given the big uncertainties of the predicted flux it seems premature to take this experiment as unambiguous evidence for neutrino oscillations. In the future the experiment will be repeated using $^{71}$Ga instead of $^{37}$Cl, which has a threshold of 0.236 MeV resulting in an expected rate of 102 SNU($^{71}$Ga), with much reduced uncertainty.

We should conclude this chapter by summarizing that so far lepton number conservation, absence of coupling between different neutrinos and zero neutrino mass has been confirmed to high precision in a big number of difficult, beautiful and sometimes very sensitive experiments.

4 Lepton Universality

Lepton universality expresses the experimental observation of equal interaction and coupling of the leptons from the different families to the gauge bosons. It has been studied in many experiments, of which we will discuss only two recent ones, which compare the coupling of the $\tau$-lepton with the coupling of the lighter leptons.

4.1 Lifetime of the $\tau$-Lepton

Fig 4.1 shows the diagram of the purely leptonic $\tau$-decay. Assuming the left-handed structure of the charged current, zero neutrino masses and universal coupling $G_F$, gives for the decay width of a lepton $\ell$ with mass $m_\ell$:

$$\Gamma(\ell \to e\nu\bar{\nu}) = G_F^2 \cdot m_\ell^5 / 192\pi^3.$$  \hspace{1cm} (4.1)

For the $\tau$-lepton with mass $(1784.0 \pm 3.2)$ MeV, a branching ratio in the mode $e\nu\bar{\nu}$ of $(17.6 \pm 1.1)\%$ and the precisely known values of mass and lifetime of the muon, the lifetime of the $\tau$-lepton can be predicted:

$$\tau_\tau = (m_\mu/m_\tau)^5 \cdot \tau_\mu \cdot BR(\tau \to e\nu\bar{\nu}) = (2.82 \pm 0.18) \cdot 10^{-13}\text{sec.}.$$  \hspace{1cm} (4.2)

Experiments at the storage rings PEP and PETRA use precision vertex detectors to measure the $\tau$-lifetime, either via the length of the decay path or via the projected impact parameter (i.e. the minimum distance by which decay tracks miss the production vertex, when extrapolated backwards). The experiments are particularly difficult, as the inaccuracy due to measurement precision, multiple scattering in the beam pipe and beam spread, exceeds the average decay length or impact parameter. Presently the most accurate results come from the MARK-2 detector at PEP which finds

$$\tau_\tau = (2.86 \pm 0.16 \pm 0.25)10^{-13}\text{sec.}.$$  \hspace{1cm}

Fig 4.2 compares the prediction with the results from the different experiments - all measurement agree with the prediction, giving a test for lepton universality at the 10% level.

4.2 Measurement of $\tau \to K\nu$ and $\tau \to \pi\nu$

The measurement of the branching fractions $\tau \to \pi\nu$ and $\tau \to K\nu$ allows a sensitive test of lepton universality as can be seen from fig. 4.3. Assuming left-handed coupling, and lepton universality the ratio $BR(\tau \to K\nu)/BR(\tau \to \pi\nu)$ can be predicted. Using its good
particle identification over a large solid angle the DELCO detector at PEP has managed to identify 21 events from $K\nu$-decay over a background of 5.1 events yielding

$$BR(\tau \to K\nu) = (0.59 \pm 0.18)\%,$$

agreeing with the expected value of $(0.71 \pm 0.10)\%$. Combining these results with a similar analysis of measurements of the decays $\pi \to e\nu$ and $\pi \to \mu\nu$ show, that within the experimental uncertainty of $\leq 10\%$, the charged current coupling of $e$, $\mu$, $\tau$ to $u$-, $d$- and $s$-quarks is universal.

5 Space Time Structure of the Weak Current

In chapter 2 we have discussed the experimental observation that the charged weak current couples to left-handed fermions only. This has been verified in many experiments - the most precise one in the decay of polarized muons. In the kinematic region, where the electron in muon decay is emitted with maximum momentum in the direction opposite to the polarization of the muon, the left handed coupling supresses the events, and a high sensitivity for other terms can be achieved (fig. 5.1). In this region the rate is given by:

$$d^2\Gamma/dxd\cos \theta \propto x^2(1 - P \cdot \cos \theta). \quad (5.1)$$

The normalized electron momentum $p_e/p_e(max)$ is called $x$, the muon polarization $P$, and $\theta$ is the angle between electron direction and the direction opposite to muon polarization vector.

The experiment [5.1] uses a polarized, monochromatic (29.5MeV/c) muon beam, derived from $\pi$-mesons stopping close to the surface of a production target. The very pure polarization is due to the left-handed coupling in the $\pi \to \mu\nu$ decay. The muons are stopped in high purity metal targets of Al, Cu, Ag and Au. The muon polarization is preserved by a longitudinal magnetic field. Direction and momenta of electrons emitted opposite to the muon polarization are measured by wire chambers and an analysing magnet. The response of the spectrometer can be checked using decays from muons, randomly polarized by an additional transverse magnetic field at the target, which precesses the spin. Fig. 5.2 shows momentum spectra of electrons close to the maximum electron momentum ($x=1$), for fixed muon spin and precessing muon spin. For the data with fixed muon spin the rate approaches zero towards $x=1$ as expected from purely left-handed coupling, and the response function of the detector. If the spectrum close to $x=1$ and $\theta = 180^\circ$ is parametrized by

$$d^2\Gamma/dxd\cos \theta \propto (1 - A \cdot P \cdot \cos \theta) \quad (5.2)$$

a value of $A \cdot P = (0.9989 \pm 0.0015 \pm 0.0018)$ is found. The value is compatible with one. This experiment is by a factor of ten more precise than previous ones. It can be used to set various upper limits for deviations from left-handed coupling in muon decay. As an example at 90\% confidence level, the amplitude for right-handed coupling has to be below 2.9\% of the left-handed coupling if the mass of the neutrinos is below 10MeV. Similar limits can be derived for scalar, tensor and pseudoscalar couplings. Another way to express the same measurement uses a model in which two charged vector bosons $W_L$ and $W_R$ mediate the charged current interactions. If the observed difference in left- and
right-handed coupling is solely due to the mass difference of the two vector bosons, a lower limit of 400 GeV for the mass of $W_R$ can be derived at 90% confidence level. Evidence for the deviations from left-handed coupling has also been searched for in purely leptonic $\tau$-decays. The momentum spectrum of the charged lepton is given by

$$\frac{d\Gamma}{dx} = G^2_F m^2/(384\pi^4) \cdot x^3\left[3(1-x) + 2\rho(4x-3)/3 + \alpha/(4\pi \cdot C)\right]. \quad (5.3)$$

$C$ describes radiative corrections, which are small for muons, but can be as high as 50% for high electron momenta. The Michel-parameter $\rho$ has a value of $3/4$ for purely left-handed coupling, and a value of zero for right-handed coupling.

DELCO at PEP [5.2] and CLEO at CESR [5.3] have measured the decay spectra and find after applying radiative corrections:

$$\rho(e) = 0.66 \pm 0.11$$

$$\rho(\mu) = 0.81 \pm 0.14,$$

values obviously compatible with pure left-handed coupling.

6 Quark Mixing

Charged weak currents lead to transitions between members of different quark families, whereas no such transitions have been observed for neutral current reactions. In the standard model with three quark families, all with equal couplings, these transitions are parametrized by the unitary $3 \times 3$ matrix $U_{KM}$, as discussed in chapter 2. There are many different ways in which the four real parameters of this matrix (e.g. three rotation angles and one phase) can be parametrized, but in this lecture we will concentrate only on the absolute values of the different matrix elements. It should be noted, that so far there are no theoretical predictions for the matrix elements, in spite of quite good experimental determinations. So far the constraints derived from unitarity are realised experimentally, but experimental errors do not provide very stringent tests.

The matrix element $U_{ud}$, responsible for $\beta$-decay without change in strangeness, is most precisely determined from the comparison of the rate of various $0^+ \rightarrow 0^+$ nuclear $\beta$-transitions with muon decay. From the analysis of eight such transitions, and taking into account corrections for nuclear binding and electroweak effects, the presently most precise value is [6.1]:

$$|U_{ud}| = 0.9730 \pm 0.004(\pm 0.0020).$$

The matrix element $U_{us}$ has been determined from semileptonic decays of strange mesons and baryons. The determination from the K-meson decay rates $K^+ \rightarrow \pi^+ e^+ \nu$ and $K_L^0 \rightarrow \pi^- e^- \nu$ yields $|U_{us}| = 0.219 \pm 0.003$ [6.2], whereas the systematic measurement of the $\beta$-decays of $\Lambda, \Sigma, \Xi^+$ and $\Xi^-$ yields $|U_{us}| = 0.231 \pm 0.003$ [6.3]. The origin of this marginal disagreement is not clear at present.

As of now, there does not exist a direct measurement of the element $|U_{cd}|$, responsible for charm decays which do not lead to strangeness in the final state. The best determination should come from the measurement of semileptonic branching ratios like $D \rightarrow \pi \ell \nu$ and $D \rightarrow \rho \nu$. The reconstruction of these rare decay modes, which are not fully analysable due to the evasive $\nu$, is experimentally difficult and has not been achieved so far. The
interpretation of purely hadronic modes like $\pi\pi$ or $K\pi$, which have been measured, is complicated by not understood strong interaction effects. Qualitatively one observes that charm decays without strangeness in the final state are suppressed by about an order of magnitude - compatible with $|U_{cd}| \approx |U_{cs}|$, as expected from unitarity.

So far the best determination comes from a comparison of charm production in $\nu_\mu$- and $\bar{\nu}_\mu$- beams. Charm production is tagged via muonic charm decay, which gives the dominant source of oppositely charged muon pairs. At the quark level the following reactions are measured:

$$\nu_\mu + d \rightarrow \mu^- + c(\rightarrow \mu^+ s\nu_\mu)$$  \hspace{1cm} (6.1)  

$$\nu_\mu + s \rightarrow \mu^- + c(\rightarrow \mu^+ s\nu_\mu),$$  \hspace{1cm} (6.2)

and similar for $\bar{\nu}_\mu$-beams. The rate of (6.1) is proportional to $|U_{cd}|^2$ and (6.2) to $|U_{cs}|^2$. Using the fractional momentum distributions $U,D,S$ and $\bar{U},\bar{D},\bar{S}$ for the quarks in the target, the cross-sections for charm production in $\nu_\mu$- and $\bar{\nu}_\mu$-beams for an isoscalar target can be written:

$$d^2\sigma(\nu)/dxdy \propto |U_{cd}|^2 \cdot (U+D)+|U_{cs}|^2 \cdot 2S$$  \hspace{1cm} (6.3)  

$$d^2\sigma(\bar{\nu})/dxdy \propto |U_{cd}|^2 \cdot (\bar{U}+\bar{D})+|U_{cs}|^2 \cdot 2\bar{S}.$$  \hspace{1cm} (6.4)

Assuming $\bar{S} = S$ and using $U,D,\bar{U},\bar{D},\bar{S}$ as determined from single muon data, and an effective branching ratio for semileptonic charm decay by weighing measured branching ratios with the respective cross sections yields [6.4]:

$$|U_{cd}| = 0.26 \pm 0.03,$$

$$|U_{cs}| > 0.59 \quad (90\% CL).$$

The value of $|U_{cs}|$ can also be estimated from semileptonic D-meson decay. The semileptonic width can be expressed as the sum over the transitions from the charm quark $c$ to the lighter quarks $q$:

$$\Gamma(D \rightarrow \ell\nu X) = G_F \cdot m_\ell^5/(192\pi^3) \cdot \Sigma[|U_{cq}|^2 \cdot f(m_q/m_c)].$$  \hspace{1cm} (6.5)

with the quark masses $m_q, m_c$ and the function $f(m_q/m_c)$, taking into account QCD-corrections. Within big errors - mainly due to the dependence on the poorly known value of $m_c$, a value $U_{cs} = 0.86 \pm 0.15$ is found.

Experimental information on transitions from the third quark family $b$ and $t$ exists on $U_{ub}$ and $U_{cb}$ only. The results come from the measurement of the semileptonic decay of $B$-particles. The measurement of the lepton spectrum close to its maximum value, allows a determination of the ratio $|U_{ub}|/|U_{ub}|$, as the recoiling system containing a charm quark is heavier, than the system containing lighter quarks only. For the analysis the three-body decays of the $B$-particle $b \rightarrow u\ell\nu$ and $b \rightarrow c\ell\nu$ are assumed [6.5]. Bound state effects of the $(b\bar{q})$-system are taken into account via a Fermi momentum distribution between the quarks with an average value of 150 MeV/c. To conserve energy and momentum, the mass $m_b$ has to be varied on an event to event basis. The data come from the experiments CLEO [6.6] and CUSB [6.7], working at the $\Upsilon(4S)$-resonance at the storage ring CESR. Combining the data from both experiments yields:

$$\Gamma(b \rightarrow u\ell\nu)/\Gamma(b \rightarrow c\ell\nu) < 3.5\%.$$
The many assumptions made in the analysis make this limit however quite uncertain. For the ratio of matrix elements the following upper limit results:

\[ |U_{ub}| / |U_{cb}| < 0.14 \text{ at } 90\% \text{ CL.} \]

The semileptonic decay width provides - as already discussed for the charm decays - additional information on the matrix elements \(U_{ub}\) and \(U_{cb}\). The higher quark mass makes the result in this case more reliable. The decay width can be written as

\[ \Gamma(B \to \ell\nu X) = G_F \cdot m_b^5 / (192\pi^3) \cdot \Sigma[|U_{qb}|^2 \cdot f(m_q/m_b)]. \tag{6.6} \]

The value of \(\Gamma(B \to \ell\nu X)\) is obtained from the measurement of the branching fraction \(BR(B \to \ell\nu X)\) and the lifetime \(\tau(B)\). Practically all experiments at high energy electron positron storage rings have measured these branching fractions and the average values for the electron- and muon-modes are:

\[ BR(B \to e\nu X) = (11.3 \pm 0.9)\% \text{ and } BR(B \to \mu\nu X) = (12.0 \pm 1.0)\%. \]

The measurement of the lifetimes \(\tau(B)\) is a field of major experimental activity. A pair of B-particles has been observed directly and unambiguously in a photographic emulsion by the WA-75 group at CERN \[6.8\]. The two decays have life times of

\[ \tau(B^+) = 0.8 \cdot 10^{-13}\text{sec} \text{ and } \tau(B^-) = 5 \cdot 10^{-13}\text{sec}. \]

All other results come from experiments working at the storage rings PEP and PETRA using high precision vertex detectors. The experiments obtain an enriched sample of events containing B-decays, either by selecting leptons with high transverse momenta, or by selecting event topologies expected from the decay of a pair of heavy mass objects. The lifetime is either obtained from the measurement of the average impact parameter of the decay tracks, or an estimated decay length. Typically the measurement errors on the individual events exceed the effect of the finite life-time, so that a detailed and envolved study of the systematic errors is essential. In addition the results depend on the understanding of the enrichment cuts, which need sophisticated Monte Carlo calculations. The results from seven experiments are shown in fig. 6.1. In spite of the difficult analyses, the results of the different experiments agree, and after averaging - which probably should not be done in view of systematic uncertainties - a value of

\[ \tau(B) = (1.08 \pm 0.16) \cdot 10^{-12}\text{sec} \]

is obtained. It also should be noted, that as in the case of the charmed hadrons, the different B-particles may have different leptonic branching ratios, and the individual experiments may not have the same mixture after their data selection. From these data the following values for the matrix elements are obtained:

\[ |U_{cb}| = 0.058 \pm 0.005 \]
\[ |U_{ub}| < 0.008. \]

To summarize this chapter we give the values of the individual elements of the mass mixing matrix as measured

\[
U_{KM} = \begin{pmatrix}
0.9737 \pm 0.001 & 0.231 \pm 0.003 & < 0.008 \\
0.26 \pm 0.03 & 0.86 \pm 0.15 & 0.058 \pm 0.005 \\
- & - & -
\end{pmatrix}
\]
as well as the values obtained, if one takes into account the unitary constraint assuming three families of quarks.

\[ U_{KM} = \begin{pmatrix} 0.9737 \pm 0.001 & 0.231 \pm 0.003 & < 0.008 \\ 0.231 \pm 0.003 & 0.972 \pm 0.002 & 0.058 \pm 0.005 \\ < 0.02 & 0.058 \pm 0.01 & > 0.998 \end{pmatrix} \]

It can be seen that the present data are just sufficient to have a quite precise determination of the individual elements, they are however not yet sufficient precise to allow a stringent test of the model. Finally it is worth noting, the the matrix exhibits the following pattern: the mixing between generations becomes weaker with the generation number, and in addition with the distance between generations.

## 7 Conclusions

At the end of these lectures on weak decays of leptons and quarks, we would like to summarize:

- the huge amount of experimental information on the decays of leptons and quarks is described by the standard model of electroweak interactions. So far this model has given the correct answers, whenever it was tested experimentally - some of the most important tests are:
  (i) the properties of the gauge bosons W and Z,
  (ii) the precision experiments on the leptons and light quarks testing the space-time structure of the currents, lepton number conservation and universality of the coupling,
  (iii) the first, but however not yet very accurate experiments on heavy quark decay.

- there are many open questions, which the standard model cannot answer, as they are just put into the model as assumptions. These are for example the origin of the family structure, the masses of the fermions, the values of the mixing matrix, the observed right-left asymmetry. Thus the standard model in spite of its many successes is far from being a final theory.

- The experiments in the near future will continue to obtain more precise determinations of the many free parameters of the standard model, and try detailed tests of its predictions and constraints, with the aim to find the way from the standard model to the underlying theory.
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Fig. 3.1 Upper limit of lepton number violating muon decays versus year of publication.

Fig. 3.2 Layout of SINDRUM-spectrometer
C...chambers - M...magnet coil
H...scintillator hodoscope - T...target.
Fig. 3.3 Regions in $\delta m^2$ vs $\sin^2(2\theta)$-plane excluded for $\nu_\mu \to \nu_e$-transitions, data from A[3.13], B[3.11], C[3.13], D[3.13].

Fig. 3.4
Regions in $\delta m^2$ vs $\sin^2(2\theta)$-plane excluded for $\nu_\mu \to \nu_\tau$-transitions, data from A[3.12], B[3.11].
Fig. 3.5
Regions in $\delta m^2$ vs $\sin^2(2\theta)$-plane excluded for $\bar{\nu}_e \rightarrow X$-transitions, data from A B[3.14], C[3.15].

Fig. 4.1
Feynman diagram for leptonic $\tau$-decay

Fig. 4.2
Results on lifetime measurements of the $\tau$-lepton.
Fig. 4.3
Feynman diagrams contributing to BR(τ → Kν)/BR(τ → πν).

Fig. 5.1
Polarized μ-decay at x = 1 and θ = 180°. Momenta are indicated by slim, helicities by open arrows. Left-handed coupling (V−A) suppresses the event rate.

Fig. 6.1
Experimental results on the lifetime of B-particles.
Fig. 5.2
Electron momentum spectrum at $x=0$ and $\theta = 180^\circ$ for polarized and precessing muons.
LEPTON–HADRON INTERACTIONS

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1. INTRODUCTION

Lepton-nucleon deep inelastic scattering has yielded many informations on the structure of elementary particles\([1-8]\). The first experiments were performed with electron beams; then followed experiments with muon and muon neutrino beams. The basic reactions studied are of the type

\[
l + N \rightarrow l' + X
\]

(1.1)

where \(l\) and \(l'\) are leptons, the nucleon \(N\) may be a proton or a neutron and the hadronic system \(X\) may be either observed or unobserved.

One could say that the most important result of the study of lepton-hadron deep inelastic interaction has been the discovery that lepton production of hadrons may be simply interpreted as due to the elastic or quasi-elastic scattering of the lepton with a point-like constituent (a parton) of the nucleons. The partons were later identified as quarks and gluons. This gave the first dynamical understanding of the quark substructure of nucleons. Before this, one only had a static understanding of the quark substructure, based on the regularities of the hadron multiplets.

The four basic lepton-proton inelastic reactions are:

- deep electron inelastic scattering
  \[
e^- p \rightarrow e^- X^+
\]
  (1.2)
- deep muon inelastic scattering
  \[
\mu^- p \rightarrow \mu^- X^+
\]
  (1.3)
- neutrino CC scattering
  \[
\nu_{\mu} p \rightarrow \mu^- X^{++}
\]
  (1.4)
- neutrino NC scattering
  \[
\nu_{\mu} p \rightarrow \nu_{\mu} X^+
\]
  (1.5)

The first two reactions proceed via the exchange of a photon (also the vector boson \(Z^0\) may be exchanged: this leads to small interference effects). The third reaction proceeds via the exchange of the charged intermediate vector bosons (\(W^\pm\)); the last reaction proceeds via the neutral intermediate vector boson (\(Z^0\)). The four reactions are illustrated in Fig. 1.1.

---

![Fig. 1.1 - Comparison of various deep hadron probes.](image-url)
The best known way to study an object is to examine the interaction of photons with that object. The scale on which structure can be observed is inversely related to the energy of the photons used. The scattering of electrons or of muons on atoms, nuclei and nucleons is thought to proceed via the exchange of a virtual photon, as illustrated in Fig. 1.1. Thus the scattering of charged leptons has a more familiar appearance and corresponds to scatter photons. As the available lab energy of the incoming leptons increased from few MeV to many GeV, the momentum transfer squared, $Q^2$, which characterizes the scattering, had a corresponding increase and one could probe smaller and smaller distances (down to $\approx 0.01$ fm).

Fig. 1.2 illustrates the basic reasoning in deep inelastic lepton scattering as viewed in the naive quark parton model. The electron scatters emitting a hard virtual photon, $\gamma$, which interacts with a constituent (a valence quark) of the proton. In this simple picture one neglects other types of constituents, that is sea quarks and gluons. Fig. 1.3 shows a more complete picture of the whole scattering process. The inelastic collision between a lepton and a nucleon is viewed as a two-stage process. The first stage consists of the quasi-elastic scattering of the lepton by one of the partons, carrying a fraction $x$ of the nucleon four-momentum. The corresponding structure function $F(x, q^2)$ describes the density distribution of constituents of fractional momentum $x$ and determines the scattering cross-section. The second stage of the process consists of the fragmentation of the partons to form the secondary hadrons. These may be subdivided in hadrons of the so-called “current-jet”, coming from the fragmentation of the struck parton, and of the

![Diagram](image)

Fig. 1.2 - Deep inelastic lepton-proton scattering in the naive quark parton model.

![Diagram](image)

Fig. 1.3 - Deep inelastic lepton-nucleon hadroproduction as a two-stage process.
"target-jet" hadrons, formed from the spectator partons. The current jet particle distribution, called the fragmentation function \( D(x, q^2) \), gives the probability that a particular hadron carries a fraction \( z \) of the energy of the struck parton.

If the first and second stage of the scattering process were really independent, the single particle inclusive cross section would factorize:

\[
d\sigma/dx \, dq^2 \, dz = F(x, q^2) \, D(x, q^2)
\]

But factorization does not really hold, since current and target jets are not completely separated in rapidity; thus there is some "cross talk".

Historically the classical work of electron scattering on nuclei probed the charge distribution of nuclei. Later the deep inelastic scattering of electrons revealed the parton structure of nucleons. Subsequently an alternative charge lepton projectile became available: the muon. At high energies the effective lifetime of the muon is practically infinite in comparison to its transport time from source to experiment. Since protons could be accelerated to high energies more easily than electrons and muon beams of good "optical" properties were made, the study of charged lepton-nucleon scattering was dominated by muon scattering. Later on, high energy neutrino beams were made; this fact together with the development of good neutrino detectors led to the study of neutrino-nucleon interactions.

Though the emphasis of these lectures will be on neutrino-nucleon interactions, the relations of \( \nu N \) with \( eN \) and \( \mu N \) interactions will be considered. In Section 2 and 3 will be defined the variables and the general theoretical framework. Electron-proton elastic scattering is discussed in Section 4. Experimental details of deep inelastic scattering will be considered in Section 5. Neutrino-nucleon charged current interactions and muon-nucleons interactions are discussed in Section 6, while neutral current interactions are considered in Section 7. Miscellaneous items concerning the hadronic system are considered in Section 8.

2. KINEMATICAL NOTATIONS

The inclusive reaction of order one is defined as

\[
a + b \rightarrow c + X,
\]

where "a" is the incident particle, "b" is the target particle; "c" is the produced particle, which is identified and whose momentum and production angles are measured. The hadron system \( X \) (also referred to as "anything") may or may not be observed. Reaction (2.1) is kinematically defined by three variables, for example the center-of-mass energy, \( \sqrt{s} \), the c.m. polar angle, \( \theta^* \), and the momentum, \( p^* \), of particle \( c \). The azimuthal angle, \( \phi^* \), of particle \( c \) is usually ignorable because polarization effects are not measured. There are many different choices for the three independent variables, for the cross sections and for the scaling laws. Moreover, the conventional notation changes appreciably for inclusive reactions due to strong, electromagnetic or weak interactions\(^5\). In the last two cases, inclusive reactions are better known as deep inelastic processes. Table 2.1 gives a list of definitions used in describing the three types of interactions.
Table 2.1 - Notation for inclusive measurements[^5].

**Weak or Electromagnetic Interactions**

\[ q^2 = 4E_0E_h \sin^2 \frac{\theta}{2} \mu \]

\[ p = \text{momentum four-vector} \]

\[ p, \theta = \text{laboratory momentum and scattering angle} \]

\[ p^*, \theta^* = \text{center-of-mass (c.m.) momentum and scattering angle} \]

\[ s = E_{\text{c.m.}}^2 = (p_a + p_b)^2 \]

\[ t = (p_a - p_c)^2 \]

\[ p_\perp = p \sin \theta = p \sin \theta^* \]

\[ p_\parallel = p \cos \theta^* \]

\[ p_\perp^* = p_\perp \cos \theta^* \]

\[ x_\perp = \frac{p_\perp}{p_\perp^{\text{max}}} = \frac{2p_\perp}{\sqrt{s}} \]

\[ x_\parallel = \frac{x}{x + \frac{1}{x}} = \frac{2}{\sqrt{s}} \]

\[ x_\perp^* = \sqrt{x_\perp^2 + x_\parallel^2} \]

**Strong Interactions**

\[ s = E_{\text{c.m.}}^2 = (p_a + p_b)^2 \]

\[ t = (p_a - p_c)^2 \]

\[ p_\perp = p \sin \theta = p \sin \theta^* \]

\[ p_\parallel = p \cos \theta^* \]

\[ p_\perp^* = p_\perp \cos \theta^* \]

\[ x_\perp = \frac{p_\perp}{p_\perp^{\text{max}}} = \frac{2p_\perp}{\sqrt{s}} \]

\[ x_\parallel = \frac{x}{x + \frac{1}{x}} = \frac{2}{\sqrt{s}} \]

\[ x_\perp^* = \sqrt{x_\perp^2 + x_\parallel^2} \]

Rapidity = \[ y^* = \frac{1}{2} \ln \frac{E_\perp^* + p_\parallel^*}{E_\perp^* - p_\parallel^*} \]

Pseudorapidity = \[ \eta^* = -\ln \tan \frac{\theta^*}{2} \]

(y^* = \eta^*, for m_c small)

Lab rapidity = \[ y_{\text{lab}} = y_{\text{beam}} - y^* \]

MM^2 = M_X^2 = (p_a + p_b - p_c)^2

= s(1 - \kappa)

Invariant Cross Section,

\[ f = \frac{d^3 \sigma}{d^2 \sigma \ dp} = \frac{E}{p^2} \frac{d^2 \sigma}{dp \ du \ dp} \]

= f(s, p_\perp, p_\parallel)
Another classification of inclusive reactions is based on the transverse momentum value of the observed particle "c". One speaks of low-\( p_T \) and large-\( p_T \) phenomena. The latter have always been associated with interactions at small distances. It may be worthwhile to remember that the average transverse momentum in processes due to strong interactions is small (about 350 MeV/c) and only slightly dependent on c.m. energy. On the contrary, the average transverse momentum squared in processes due to weak interactions grows linearly with the laboratory energy of the incoming particle.

The Lorentz invariant quantities, \( s, t, u \), may be defined in c.m. and lab frames (the target particle "b" is assumed to be at rest in the lab system).

\[
s = \text{total c.m. energy squared} \quad \text{(metric chosen \( \vec{p}_i^2 = E_i^2 - \vec{p}_i^2 = m_i^2 \)), where \( p_i \) is a four-vector and \( \vec{p}_i \) a three-vector; \( p_0 = 0, E_b = m_b \}).
\]

\[
s = (p_a + p_b)^2 = p_a^2 + p_b^2 + 2\vec{p}_a \cdot \vec{p}_b + 2E_a E_b =
\]

\[
= m_a^2 + m_b^2 + 2\vec{p}_a \cdot \vec{p}_b + 2E_a E_b = m_a^2 + m_b^2 + 2E_a m_b
\]  \hspace{1cm} (2.2a)

Specializing to reaction (2.4) (\( m_a = m_e = 0, m_b = m_p, E_{\nu} > m_p \)):

\[
s = m_p^2 + 2E_{\nu} m_p \approx 2E_{\nu} m_p
\]  \hspace{1cm} (2.2b)

\[
t = \text{four momentum transfer (between a and c) squared}
\]

\[
t = (p_a - p_c)^2 = m_a^2 + m_c^2 - 2\vec{p}_a \cdot \vec{p}_c - 2E_a E_c
\]  \hspace{1cm} (2.3a)

Specializing to reaction (2.4):

\[
t = m_p^2 - 2p_\nu p_\mu \cos \theta_\mu - 2E_\nu E_\mu =
\]

\[
= -4E_\nu E_\mu \sin^2 \frac{\theta_\mu}{2} \approx -(E_H + E_\mu) E_\mu \theta_\mu^2 = -Q^2
\]  \hspace{1cm} (2.3b)

where \( E_\nu = E_H + E_\mu, W = \nu = E_H \) = total energy of the hadronic system X.
Among the various variables we shall recall the Bjorken scaling variables \( x \) and \( y \):

\[
x = \frac{Q^2}{2m_p^2} = \frac{4E_\mu E_\nu \sin^2(\theta/2)}{2m_p(E_\nu - E_\mu)} \cong \frac{E_H}{2m_p E_H} E_\mu \theta^2
\]

\[
y = \frac{\nu}{E_\nu} = \frac{E_H}{E_H + E_\mu}
\]

(2.9) (2.10)

For other kinematical variables see Table 2.1.

Fig. 2.2 shows the ranges of \( Q^2 - y \) available for muon inclusive measurements at 280 GeV lab energy.

![Graph](image)

Fig. 2.2 - Ranges of \( Q^2, E_H = \nu, \text{ etc vs } y \), for inclusive muon-proton interactions at \( E_o = 280 \) GeV[2].

3. THE STANDARD MODEL

The standard model is characterized by the
- group structure \( SU(2) \times U(1) \times SU(3) \)
- fermion representations.
- spontaneous symmetry breaking and Higgs representation[1].

3.1 The building blocks

On a distance scale of \( \sim 0.01 \) fm the fundamental building blocks of matter are the spin 1/2 fermions. There are many of them; they may be ordered in a "periodic system", as shown in Table 3.1. All known particles are classified vertically in families or generations (of which the first one
is explained in more detail in Table 3.2) and horizontally in groups of equal electric charge. There is a striking symmetry between leptons and quarks. Neutrinos will be assumed to be massless. So far no reaction induced by a $\nu_f$ has been observed. The top quark, $t$, may have been seen, but confirmation is needed\[10\]. There are no experimental indications of fermions beyond the ones in the three families and the Standard Model does not tell us how many families exist. The three generations are replicas of each other regarding their symmetry properties. It is therefore sufficient to show the multiplet structure of the first generation under the gauge groups $SU(2)$ and $SU(3)\[11\].

Table 3.1 - "Periodic System" of the fundamental spin $1/2$ fermions.

<table>
<thead>
<tr>
<th>Particles</th>
<th>Forces</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu_e$</td>
<td>$\nu_\mu$</td>
</tr>
<tr>
<td>$\nu_\tau$</td>
<td>only weak</td>
</tr>
<tr>
<td>e</td>
<td>$\mu$</td>
</tr>
<tr>
<td>$\tau$</td>
<td>weak and electromagnetic</td>
</tr>
<tr>
<td>u</td>
<td>c</td>
</tr>
<tr>
<td>t</td>
<td>weak, electromagnetic</td>
</tr>
<tr>
<td>d</td>
<td>s</td>
</tr>
<tr>
<td>b</td>
<td>and strong</td>
</tr>
</tbody>
</table>

Table 3.2 - Multiplet structure under $SU(2)$ and $SU(3)\[11\].

\[
\begin{align*}
(\nu_e)_L &\quad (u_\nu^u)_{L_L} (u_\nu^d)_{L_L} \\
\nu_R &\quad u_R^u_R u_R^d_R \\
\mu_R &\quad d_R^u_R d_R^d_R \\
SU(3) &\quad SU(3) \\
singlet &\quad triplet
\end{align*}
\]

Left- and right-handed fermions behave differently under $SU(2)$. Each fermion can be decomposed uniquely into a left- and a right-handed spinor. Should experiments prove the neutrino to be massive, then there would be also a $\nu_R$.

Higgs mesons are needed in order to break down in a "spontaneous" way the symmetry of the unified interaction when the energy is decreased. In the minimal form one needs a doublet of Higgs mesons, $\varphi^+, \varphi^0$, which have $I = 1/2$, $I_3 = \pm 1/2$ and hypercharge 1.

3.2 The forces

Neglecting gravity, three types of forces act between the pointlike, spin $1/2$ fermions (see table 3.3). The three forces are assumed to arise from a local gauge symmetry and are mediated by vector gauge bosons. An important feature is the locality of the gauge symmetry satisfied by the Lagrangian. Contrary to $U(1)$ the algebra of $SU(2)$ and $SU(3)$ is nonabelian, i.e. not commutative. Therefore, the 3 weak and the 8 strong intermediate spin 1 bosons have the property of coupling
Table 3.3 - The three types of forces.

<table>
<thead>
<tr>
<th>Local Gauge Symmetry</th>
<th>Force</th>
<th>Intermediate Vector Bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>SU(2)</td>
<td>weak</td>
<td>$W^+, W^-, Z^0$</td>
</tr>
<tr>
<td>U(1)</td>
<td>electromagnetic</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>SU(3)</td>
<td>strong</td>
<td>$g_1, g_2, \ldots, g_8$</td>
</tr>
</tbody>
</table>

to themselves. The electromagnetic force is of infinite range, because the photon is massless. The weak force has short range because of massive mediators and thus has a broken symmetry. The situation is different in the case of the strong force which is assumed to be mediated by massless gluons and nevertheless of finite range (confinement).

3.3 The electroweak lagrangian

In the Standard Model weak and electromagnetic phenomena are treated on the same footing and both exhibit a local gauge structure. However, a new principle had to be introduced: spontaneous symmetry breaking and the Higgs mechanism. The Lagrangian can be decomposed into a kinetic term, into a term responsible for electromagnetic interactions ($\gamma$), three terms for weak interactions ($W^+, W^-, Z$), and finally into a term containing the Higgs sector:

$$L^{GSW} = L_0 + L_{em} + L_{CC} + L_{NC} + L_H$$

(3.1)

free $\gamma$ $W^\pm$ $Z$ Higgs

with

$$L_{em} = g \sin\theta_w J_{\lambda}^{em} A^{\lambda}$$

$$L_{CC} = \frac{g}{2\sqrt{2}} \left( J_{\lambda}^{CC} W^\lambda + \text{h.c.} \right)$$

(3.2)

$$L_{NC} = \frac{g}{4\cos\theta_w} J_{\lambda}^{NC} Z^\lambda$$

The three interaction Lagrangians have the same structure:

(coupling constant) (current) (gauge boson field)

(3.3)

One identifies $g \sin\theta$ with the electromagnetic coupling $e$: $e = g \sin\theta_w$. This may be called the unification equation. Weak and electromagnetic phenomena are not truly unified, since then only a single coupling constant would appear. Nevertheless, it is justified to talk about "electroweak" phenomena, since the three coupling constants can be expressed in terms of two, e.g. the electromagnetic coupling constant, $e$, and the weak angle $\theta_w$. 

The Higgs sector contains two terms

\[ L_H = \left( \frac{g v}{2} \right)^2 W^+ \lambda W^\lambda + \frac{1}{2} \left( \frac{g v}{2 \cos \theta_w} \right)^2 Z_\lambda Z^\lambda + \ldots \] (3.4)

They show which quantities are to be identified with the masses of the weak gauge bosons:

\[ m_W = \frac{g v}{2} \quad \text{and} \quad m_Z = \frac{m_W}{\cos \theta_w} \] (3.5)

The constant \( v \) is the vacuum expectation value of the Higgs field \( \phi \), defined as

\[ v = \frac{1}{\sqrt{2}} \langle 0 | \phi | 0 \rangle. \]

All other terms appearing in \( L_H \) describe the Higgs coupling to itself, to \( Z, W^\pm \) and to the fundamental fermions.

### 3.4 The currents

The explicit structure of the three currents is summarized in the following three equations:

\[ J^\text{em}_{\lambda} = \sum_f Q_f \overline{\psi}_f \gamma_\lambda \psi_f \] (3.6a)

\[ J^\text{CC}_\lambda = \sum_q \overline{\psi}_q \gamma_\lambda \left( 1 + \gamma_5 \right) \psi_q + \sum_{q,q'} \overline{\psi}_q \gamma_\lambda \left( 1 + \gamma_5 \right) U_{qq'} \psi_q \] (3.6b)

\[ J^\text{NC}_\lambda = \sum_f \psi_f \gamma_\lambda \left( \sigma_{\lambda}^I + \sigma_{\lambda}^A \gamma_5 \right) \psi_f \] (3.6c)

where

- \( f \) is the flavor index, i.e. it runs over \( \nu_e, \nu_\mu, \nu_\tau, e, \mu, \tau, u, d, c, s, t, b; \)
- \( l=(e, \mu, \tau), q=(u, c, t), q'=(d, s, b); Q_f \) is the electric charge in units of \( e(>0) \); \( U= \) flavor mixing quark matrix (Kobayashi - Maskawa matrix); \( U \) is unitary, i.e. \( UU^+ = 1 \); \( g_{V,A}^I, g_{V,A}^A \) = vector and axial-vector coupling constants of fermion \( f \) to the neutral weak gauge boson \( Z \), which depends only upon \( \sin^2 \theta_w \).

The electromagnetic current is a pure vector current (V), whereas the weak charged current is of pure (V-A) type (the minus sign is conventional), i.e. only the left-handed component of \( \psi_f \) is active in interactions due to \( W^\pm \). The weak neutral current is in general neither pure V nor pure A.

### 3.5 The current x current form

Low energy weak phenomena are known to be well described by the following effective Lagrangian:

\[ L^{\text{eff}} = \frac{G}{\sqrt{2}} \left[ J^\text{CC}_{\lambda} \left( J^\text{CC}_{\lambda} \right)^+ + \frac{\rho}{2} J^\text{NC}_{\lambda} J^\text{NC}_{\lambda} \right] \] (3.7)

where \( G \) is the Fermi coupling constant and \( \rho \) a parameter measuring the overall strength of the weak neutral current with respect to the weak charged current. This effective form can be
Table 3.4 - The Standard Model.

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Strong</th>
<th>Electroweak</th>
</tr>
</thead>
<tbody>
<tr>
<td>Local symmetry</td>
<td>SU(3)</td>
<td>SU(2)xU(1)</td>
</tr>
<tr>
<td>Coupling</td>
<td>$g_s$</td>
<td>$g$</td>
</tr>
</tbody>
</table>

"Matter" field

- Quarks
- Gluons

"Gauge" fields

- Color charge
- Baryon number
- Electromagnetic

Conserved quantities

- $q, l, H$
- $W^+, W^-, Z^0$
- Electronic number
- Muon number
- Taunic number

- Electric charge
- $\gamma$

Derived in the Standard Model in the limit $|q^2| << m^2$, where $m$ is the relevant gauge boson mass. This consideration may help explain what the terms "low energy" and "weak" really mean: the masses of the weak gauge bosons set the scale for weak phenomena. So, all lepton-hadron deep inelastic experiments are to be considered low energy experiments, since e.g. for a wide band neutrino experiment at the CERN-SPS one has $<Q^2> \approx (7 GeV)^2$ which is small compared to $m_W^2 \approx (82 GeV)^2$. Weak interactions get 'weak' due to the suppression factor $1/m_W^2$, which is the remainder of the $W$ or $Z$ propagator. At sufficiently high energies weak and electromagnetic phenomena occur at comparable rates. Furthermore one has

$$\frac{G}{\sqrt{2}} = \frac{g^2}{8m_W^2}, \quad \frac{\rho G}{2\sqrt{2}} = \frac{g^2}{16cos^2\theta_w m_Z^2}$$

(3.8)

and thus

$$\rho = \frac{m_W^2}{m_Z^2 cos^2\theta_w}$$

(3.9)

In the Standard Model the parameters $\rho$ equals 1 in lowest order as a consequence of the simplest choice of the Higgs representation. Table 3.4 summarizes pictorially the properties of the Standard Model.

3.6 The quark-parton model

In order to describe lepton-nucleon scattering we have to describe the nucleon as composite of quarks (and gluons) and extend the electroweak theory to lepton-quark scattering.
Fig. 3.1 - Simple picture of the nucleon in the infinite momentum frame (see text).

The simplest model of hadrons is the naive quark-parton model, which pictures the nucleon as composite of three quasi-free, point-like charged quarks. At sufficiently high $Q^2$ the intermediate vector boson interacts incoherently with one of the quarks. In the infinite momentum frame the proton is visualized as consisting of three quarks. A quark has three momentum $x\mathbf{p}$, where $\mathbf{p}$ is the momentum of the proton (see fig. 3.1).

If $q_\mathbf{x}$ is the probability of finding a quark of type $i$ with momentum fraction $x$ and charge $e_i$, energy-momentum conservation requires

$$\int_0^1 dz \sum_i xq_i(x) = 1 - g$$

(3.10)

where $g$ should be the fraction of the nucleon momentum carried by partons, others than quarks and neutral. These are identified with gluons and early lepton-nucleon experiments indicated that about 50% of the nucleon momentum is carried by gluons.

The simple picture of a nucleon made of three quarks (Fig. 3.2a) becomes more involved: one should add gluons and quark-antiquark pairs (the "sea" quarks and antiquarks). see Fig. 3.2b. Furthermore gluons may interact with one another (c) and it is possible that quarks may form units, like a diquark[11], similar to $\alpha$-particles in nuclei (d).

Fig. 3.2 - Description of a nucleon as composite. The figure illustrates progressively more complicated approximations, starting from the simplest picture of the nucleon made of 3 valence quarks (a), the inclusion of gluons and sea-quarks (b), the interactions among gluons (c) and more complex pictures, like the presence of diquarks in the nucleon.
4. ELECTRON – PROTON ELASTIC SCATTERING

In this Section will be briefly recalled the main features of Coulomb scattering, applied to electron-proton elastic scattering, in the one-photon exchange approximation. We shall also briefly define and discuss the proton form factors as they are measured in \( ep \) elastic scattering\(^\text{[7]} \).

The computation of the differential cross-section of \( ep \) elastic scattering may be performed using the Feynman prescriptions in a series of approximations (Equations 4.1-4.6).

As illustrated in Table 4.1 the simplest computation concerns the scattering of a point spinless electron of charge \(-e\) and mass \(m\) from an infinitely heavy point charge \(Ze\). This case may be referred to as Rutherford scattering and is described by the Rutherford differential formula \( (d\sigma/d\Omega)_R \), eq. 4.1, indicated in Table 4.1. In the table is also sketched the corresponding Feynman diagram.

The next approximation is the treatment of the electron with spin (it may be called the Dirac electron). The corresponding scattering formula is the Mott formula, eq. 4.2. The inclusion of rescattering effects arising from the finite mass \(M\) of the proton leads to a modification of the Feynman graph and to the \( \sigma_{NS} \) differential formula, eq. 4.3. If the proton spin is also considered one obtains the formula indicated in the table with \( \sigma \), eq. 4.4.

Table 4.1 - Classification of Coulomb scattering\(^\text{[7]} \).

| Rutherford | Scattering of a point spinless electron of charge \(-e\) and mass \(m\) on an infinitely heavy point charge \(Ze\). |
| Dirac electron | |
| Mott | Dirac electron on an infinitely heavy point charge \(Ze\). |
| Target recoil | |
| \( \sigma_{NS} \) | Dirac electron on a point-spinless proton of mass \(M\). |
| Dirac proton | Dirac electron on a Dirac proton. |
| Proton form factor | Dirac electron on a finite-size proton with spin. |
| Rosenbluth | |

\[
\frac{d\sigma}{d\Omega} = \frac{e^4}{4\pi^2 \sin^4 \theta/2} \quad (4.1)
\]

\[
\frac{d\sigma}{d\Omega} = \frac{(d\sigma)}{(d\Omega)} \left(1 - \frac{\beta^2 \sin^2 \theta}{2} \right) \frac{d\sigma}{d\Omega}_R \cos^2 \frac{\theta}{2} \quad (4.2)
\]

\[
\frac{d\sigma}{d\Omega} = \frac{1}{1 + (Ze/M) \sin^2 (\theta/2)} \quad (4.3)
\]

\[
\frac{d\sigma}{d\Omega} = \frac{1}{1 + \frac{\sigma^2}{2}} \left(\frac{\sigma^2}{2} \theta - \frac{\theta}{2}\right) \quad (4.4)
\]

\[
\frac{d\sigma}{d\Omega} = \frac{e^4}{4\pi^2} \left[1 + \frac{\sigma^2}{2} \left(\frac{\sigma^2}{4\pi^2} \left[1 + 2\sigma^2 \frac{1}{\sigma^2} \left(1 + 2\sigma^2 \frac{1}{\sigma^2} \right)\right] \frac{\sigma^2}{2}\right)\right] \quad (4.5)
\]

\[
\frac{d\sigma}{d\Omega} = \frac{e^4}{4\pi^2} \left[1 + \frac{\sigma^2}{2} \left(\frac{\sigma^2}{2} \theta^2 - \frac{\theta}{2}\right)\right] \quad (4.6)
\]

For an extended object, like the proton, one introduces form factors by replacing the point charge \(e\) by a charge distribution

\[
\rho(z) = e \ f(z) \quad (4.7)
\]

with the normalization \( \int f(z) dz = 1 \). The distribution function \( f(x) \) may be interpreted as a classical spatial extension of the particle or as a statistical probability distribution of more
elementary, point-like constituents. The form factor is defined as the Fourier transform of the spatial distribution function

\[ F(q) = \int e^{iqx} f(x) dx \]  \hspace{1cm} (4.8)

Examples of form factors of different spatial distributions are given in the table below:

<table>
<thead>
<tr>
<th>Charge distribution</th>
<th>Form factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>point ( f(r) = \delta(r - r_0) )</td>
<td>( F(q^2) = 1 ) unity</td>
</tr>
<tr>
<td>exponential ( f(r) = \frac{a^3}{8\pi} e^{-ar} )</td>
<td>( F(q^2) = \left( \frac{1}{1 + q^2/a^2} \right)^2 ) dipole</td>
</tr>
<tr>
<td>Yukawa ( f(r) = \frac{a^2}{4\pi} r e^{-ar} )</td>
<td>( F(q^2) = \frac{1}{1 + q^2/a^2} ) pole</td>
</tr>
<tr>
<td>Gaussian ( f(r) = \left( \frac{a^3}{2\pi} \right)^{3/2} e^{-a^2 r^2/2} )</td>
<td>( F(q^2) = e^{-(q^2/a^2)} ) Gaussian</td>
</tr>
</tbody>
</table>

We finally note that the spatial interpretation of the form factor is unsatisfactory in the scattering of high-energy particles, since the probe particle (the electron) does not see a static charge distribution, but an accelerated one. Thus the form factors will have to be introduced in a more formal way.

The neutron and proton are each described by two (Dirac and Pauli) form factors: \( F_1^p, F_2^p \), \( F_1^n, F_2^n \). With the introduction of these form factors, electron-proton scattering is described by the Rosenbluth formula, eq. 4.5. It is convenient to define the

**Electric form factor**

\[ G_E(q^2) = F_1(q^2) - \frac{q^2}{4M^2} F_2(q^2) \]  \hspace{1cm} (4.9a)

**Magnetic form factor**

\[ G_M(q^2) = F_1(q^2) + F_2(q^2) \]  \hspace{1cm} (4.9b)

The "electric" and "magnetic" form factors normalize to the total charge and the total magnetic moment of each particle:

\[
\begin{align*}
G_E^{\text{proton}}(q^2) & \text{ normalization: } G_E^{\text{proton}}(0) = 1 \\
G_M^{\text{proton}}(q^2) & G_M^{\text{proton}}(0) = 2.79 \\
G_E^{\text{neutron}}(q^2) & G_E^{\text{neutron}}(0) = 0 \\
G_M^{\text{neutron}}(q^2) & G_M^{\text{neutron}}(0) = -1.91
\end{align*}
\]  \hspace{1cm} (4.10)

The introduction of the "G" form factors in the Rosenbluth formula gives expression (4.6), where there is no interference between the two form factors.

An essential feature of the Rosenbluth formula is that if one plots the cross-section for different incident momenta and different scattering angles, such that \( q^2 \) remains fixed, a linear dependence on \( t q^2(\theta/2) \) should be obtained.
\[
\left( \frac{d\sigma}{d\Omega} \right) / \left( \frac{d\sigma}{d\Omega} \right)_{Mott} = A(q^2) + B(q^2) \tan^2 \frac{\theta}{2}
\] (4.11)

The verification of a linear dependence on \( t \tan^2 \frac{\theta}{2} \) proves that the scattering is mediated by single virtual photon exchange.

Fig. 4.1 shows the nucleon form factors determined from ep elastic-scattering. The results may be parametrized with the following empirical expressions, containing a "scaling low" and the dipole formula:

\textbf{Scaling laws:}

\[
G(q^2) = G_E^p(q^2) = \frac{G^p_M(q^2)}{\mu_p} = \frac{G^n_M(q^2)}{|\mu_n|} \quad (4.12)
\]

\[
G_E^p(q^2) = 0 \quad (4.13)
\]

\textbf{Dipole formula:}

\[
G(q^2) = \left( \frac{1}{1 + (q^2/0.71)} \right)^2 \quad [q^2 \text{ in } (GeV/c)^2] \quad (4.14)
\]

\[
= \left( \frac{1}{1 + (q^2/18.1)} \right)^2 \quad [q^2 \text{ in } \text{fermi}^{-2}] \quad (4.15)
\]

\textbf{Fig. 4.1 - Comparison of the magnetic and electric form factors of neutron and proton. They are consistent with the scaling law.}

(a) Proton magnetic form factor;
(b) proton electric form factor;
(c) neutron magnetic form factor (Weber 1967).
The Fourier transform of the dipole form factor yields

\[ f(r)_{\text{dipole}} = 3.06 \exp(-4.25r) \] (4.16)

For small momentum transfers one has

\[ F_E^p(q^2) = F(q)\left[1 - \frac{1}{6}q^2 <r^2>\right] \] (4.17)

with \(<r^2> \frac{1}{2} = 0.81\) fm. At high momentum transfers, the elastic form factors are very small, and inelastic scattering of the incident electron is much more probable than elastic scattering.

5. EXPERIMENTAL

5.1 Introduction

In the last decade neutrino beams of relatively high quality and of high intensities were constructed at each of the high energy proton synchrotrons. Massive electronic detectors and large bubble chambers were used in conjunction with these beams. As a consequence the weak interactions of the "elusive" neutrinos could be studied in details. At the same accelerators, intense muon beams of good optical quality were developed and used in conjunction with large electronic detectors.

5.2 Neutrino beams

In order to produce high-energy beams of neutrinos (of muons), protons are accelerated in a protosynchrotron to the maximum energy (450 GeV in the CERN-SPS, 900 GeV at the Fermilab Tevatron). At the end of the accelerator cycle (every \(\approx 12\) seconds at the SPS) protons are extracted and sent to a beryllium target, approximately 50-100 cm long, where they produce \(\pi\) and \(K\) mesons: \(p + N \rightarrow \pi^\pm + K^\pm + X\). The produced mesons are formed into beams, either almost monochromatic for the narrow-band beam or broadband, see Fig. 5.1. Then the mesons

![Diagram](image)

Fig. 5.1 - Schematic layout of the CERN-SPS neutrino beams: (a) narrow band beam, (b) wide band beam\(^5\).
are sent into a long decay region, where they decay. Behind this decay region is located a shield for all hadrons and for all the muons which accompany the neutrinos. Finally, the neutrinos reach the experimental area, where many different detectors may be placed one after the other.

At the CERN-SPS about $10^{15}$ protons per pulse are fast extracted (either in 20 microseconds or in a few milliseconds) and sent to the neutrino beam line in the West Experimental Area. This primary proton beam, which has transverse dimensions of approximately 2 mm wide and 1 mm high, strikes a beryllium target (typically 60 or 110 cm long for $\nu$ and $\bar{\nu}$ beams respectively, 3 mm wide and 2 mm high) where pions and kaons are produced.

For the Narrow Band Beam (NB) the beryllium target is immediately followed by a set of quadrupoles (in order to have the largest acceptance possible) and then by bending magnets for momentum analysis (and charge selection); a second set of quadrupoles forms a parallel beam which is appropriate for the decay region. The accepted momentum band is typically $\Delta p/p \approx \pm 10\%$. The advantages of a narrow band beam are: (i) the neutrino energy spectrum is of the flat box type (see Fig. 5.2); (ii) it is possible to determine the neutrino energy for each event on the basis of the radial location of the interaction point; (iii) the monitoring of the beam intensity is relatively easier than for other types of neutrino beams. Its chief disadvantage is the lower flux, which makes it difficult to use with bubble chambers.

In order to obtain the wide band beam (WB) the charged particles pass through a magnetic horn, that is a system with a current sheet, having rotational symmetry, which produces a toroidal magnetic field. Particles of one sign are focused towards the beam axis, while particles of opposite

---

Fig. 5.2 - SPS Neutrino (solid lines) and antineutrino (dashed lines) fluxes at the CDHS detector for (a) wide band beam for $p_{\text{proton}} = 400$ GeV/c and (b) for the narrow band beam, $p_{\text{proton}} = 400$ GeV/c, $p_{\nu,K} = 200$ GeV/c.
Fig. 5.3 - Integrated charged current event rate in deuterium for the neutrino quadrupole triplet beam (solid lines) and single horn beam (dashed lines) for a primary proton beam of 1000 GeV at the Fermilab tevatron.

sign are defocused. A second horn, called reflector, is placed downstream of the first one in order to improve focusing conditions. The WB beam is the neutrino beam with highest intensity: the focusing allows the collection of most medium energy neutrinos, so that the mean energy of the neutrinos is low. Relatively high intensities, and larger mean energies, have been achieved by other systems, like the quadrupole triplet beam at Fermilab (Fig. 5.3).

The secondary beam of charged particles, properly focused into a parallel beam is sent to the 300 m long evacuated decay channel of the CERN-SPS. The overwhelming majority of π decays (99.98%) and most K decays (63.6%) occur through two body decays: $\pi^\pm \rightarrow \mu^\pm + \nu_\mu (\bar{\nu}_\mu)$, $K^\pm \rightarrow \mu^\pm + \nu_\mu (\bar{\nu}_\mu)$. If the parent particle is monochromatic, the laboratory energy spectrum of the neutrinos is simple, as shown in Fig. 5.2.

Behind the decay region is located a massive shielding to absorb all hadrons and muons. The length of the shielding is determined by the muon energy and its energy loss (about 1.2 GeV per meter of iron). In practice the length of the shielding is about equal to the decay length. A section of magnetized iron, acts as an "active" shielding, removing from the beam line those muons produced by neutrino interactions in the shielding.

The neutrino flux and its energy spectrum may be inferred from: (i) the measurement of the pions and kaons produced in the forward direction at various secondary momenta; (ii) the precise
knowledge of the beam and detector layout; (iii) the use of a Montecarlo program containing all
the above informations; (iv) monitoring of the protons sent to the neutrino target; (v) monitoring
of the muons in the shielding.

At CERN, the measurement of the muon flux was performed with solid state detectors placed
in gaps in the early part of the shielding. From the comparison of the predictions from the
Montecarlo and the muon measurements, one may establish an iteration procedure, yielding the
final answer.

The $\pi \to \nu, K \to \nu$ decay points are known only to within the length of the decay tunnel.
This leads to a smearing in the relation between the neutrino energy and the radial position of
the interaction point in the neutrino detector. Neutrinos from pion decay are still concentrated
at small radii, while neutrinos from K-decays are at large radii. But the measured radial position
of the interaction point determines the neutrino energy with reasonable precision ($\pm 10\%$) only for
neutrinos from K-decay.

5.3 Neutrino detectors

Neutrino detectors must have the following general features:
(i) The target has to be very massive in order to offset the smallness of the cross section and the
relatively small flux of neutrinos. Neutrinos have a mean free path of $\lambda = 2.6 \cdot 10^9$ Km of Fe at 10
GeV, $2.6 \cdot 10^8$ Km of Fe at 100 GeV. This means that the fraction of 10 GeV neutrinos interacting
in one meter of Fe is only $3 \cdot 10^{-11}$; considering the total neutrino flux of about $10^{10}$ neutrinos per $10^{15}$ incident protons, one has about 0.3 interaction per beam pulse in one meter of iron.
(ii) The target should be at the same time a sensitive volume, because in general one wants to
study the hadrons produced in the interactions. Bubble Chambers satisfy well this requirement.
electronic detectors use sampling calorimeters of different types.
(iii) Muon identification (and electron identification) is important in order to be able to separate
charged Current (CC) interactions from Neutral Current (NC) ones.

5.3.1 Electronic detectors

Most electronic detectors are massive detectors consisting of a target, a calorimeter to measure
the total hadron energy and a muon identifier to detect a muon and measure its momentum (Fig.
5.4).

Hadron energies are measured by sampling calorimeters, made of plates of iron, marble, etc.,
sandwiched between planes of ionization detectors (scintillation counters, proportional tubes, etc.).
The energy and angle resolutions of the CHARM-1 detector were the following\[8\]:

$$\frac{\Delta E_H}{E_H} = \sqrt{\frac{0.28}{E_H(GeV)}} \times \pm 4\% \quad \text{at} \quad 10\text{GeV}$$

(5.1)

$$\Delta \theta_H = [4 + \frac{600}{E_H(GeV)}] \text{mrad}, \quad \frac{\Delta p_\mu}{p_\mu} \approx \pm 10\%$$

(5.2)
The following detectors yielded important results in neutrino-hadron interaction at the highest accelerator energies: HPWF (70t), CITF, CCFRR (120t) and FMM E594 (340t) at Fermilab, CDHSW (1240t) and CHARM (180t) at CERN (the numbers in parenthesis refer to the total tonnage of each experiment)\cite{5,6}.

5.3.2 Bubble chambers

Several large bubble chambers have been used for neutrino experiments. Apart from size, the most important parameter is the liquid used in the chamber. $H_2$ and $D_2$ fillings allow the study of reactions on free protons and free neutrons. This is not possible in heavy liquids, which on the other hand have the advantages of good electron identification, complete $\gamma$ conversion and large mass (of the order of 10 tons). The Big European Bubble Chamber (BEBC) had a diameter of 3.7 m and a total volume of 30 m$^3$ (the fiducial volume was 18.9 m$^3$, corresponding to a mass of 1.19 tons of liquid hydrogen or 2.65 tons of deuterium). The chamber was immersed in a 35 Kgauss uniform magnetic field produced by a superconducting magnet.

The bubble chamber does not allow muon identification. Therefore most bubble chambers used for neutrino physics operated with an External Muon Identifier (EMI), consisting of a system

Table 5.1 - Main features of the largest bubble chambers used for neutrino physics.

<table>
<thead>
<tr>
<th>Name</th>
<th>Lab.</th>
<th>Dimensions</th>
<th>Volume</th>
<th>Fiducial volume</th>
<th>Magnetic field</th>
<th>Liquids</th>
</tr>
</thead>
<tbody>
<tr>
<td>BEBC</td>
<td>CERN</td>
<td>$\odot$ 3.7x4</td>
<td>30</td>
<td>(19)</td>
<td>35</td>
<td>Ne/H,H$_2$,D$_2$</td>
</tr>
<tr>
<td>15ft</td>
<td>FNAL</td>
<td>$\odot$ 3.3 sphere</td>
<td>30</td>
<td>(18)</td>
<td>30</td>
<td>Ne/H,H$_2$,D$_2$</td>
</tr>
</tbody>
</table>
Table 5.2 - Some properties of the liquids used in bubble chambers.

<table>
<thead>
<tr>
<th>Liquid</th>
<th>Density (g/cm³)</th>
<th>Collision length (cm)</th>
<th>Absorption length (cm)</th>
<th>Radiation length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_2$</td>
<td>0.063</td>
<td>680</td>
<td></td>
<td>970</td>
</tr>
<tr>
<td>$D_2$</td>
<td>0.14</td>
<td>320</td>
<td>400</td>
<td>890</td>
</tr>
<tr>
<td>H/Ne(21%)</td>
<td>0.27</td>
<td>100</td>
<td></td>
<td>115</td>
</tr>
</tbody>
</table>

Fig. 5.5 - Layout of BEBC bubble chamber with the External Muon Identifier and the Picket Fence counter system at the CERN-SPS[13].

of planes of proportional wire chambers located behind the bubble chamber (Fig. 5.5). In between there was the iron of the return path for the magnetic lines of force of the superconducting magnet. A set of proportional counters 10 cm wide and 4 cm long were added immediately outside the BEBC bubble chamber proper. These "picket fence" counters allow a better determination of the muon trajectory (within the short time resolution of the counters and of the proportional chambers) and are important for a detailed study of neutral current events. The sensitive volume of the BEBC bubble chamber was viewed by 5 cameras. The 15 ft bubble chamber (Fig. 5.6) at Fermilab has also a holographic system.
The following bubble chamber experiments yielded important informations on high energy neutrino-nucleon scattering: WA21 (νp, νp), WA25 (νD, νD), WA47 and WA59 (νNe, νNe) using BEBC at CERN, exposures with the Gargamelle Heavy liquid bubble chamber at CERN, and experiments E28, E45 (νp), E31 (νp), E545 (νD), E28, E53, E380, E536, E546 (νNe), E172, E388, (νNe) using the 15 ft chamber at Fermilab.

5.4 Corrections, resolutions, uncertainties.

The standard procedure for taking data and for the analysis of neutrino bubble chamber experiments is the following: (i) Taking of pictures; (ii) Double scanning of the pictures; (iii) Measurements and remeasurements of the events; (iv) Geometrical and kinematical reconstruction of the events; (v) Making of a Data Summary Tape (DST); (vi) Application of corrections (mostly by Montecarlo methods); (vii) Physics analysis.

**Determination of neutrino energy:** For the wide band neutrino beam the energy of each interacting neutrino must be determined from the measurement of the particles produced. But, since there are some unseen neutrals, a correction has to be made. Several methods have been proposed and
used in the literature. In bubble chambers the correction is based on the transverse momentum imbalance. Best results have been obtained with the so called Heilmann's method\(^{15,16}\), which is based on the following formula:

\[
E_\nu = P_n^H + P_H^H \left(1 + \frac{\sum_{n=1}^{N} p_{n}\perp}{\sum_{n=1}^{N} P_H^H \perp} \right)
\]

where \(N_H\) is the total number of observed particles, excluding the muon, and \(N\) is the number of neutral hadrons; this number is not known, so one takes for \(\sum_{n=1}^{N} p_{n}\perp\) the missing transverse momentum \(\Delta p_{\perp}\). This approximation determines a slight underestimate of the hadronic energy. The basic assumption in this method is that:

\[
\frac{P_n\text{neutrals}}{P_H\text{neutrals}} = \frac{P_{\nu\text{ch}}}{P_{\nu\text{ch}}}\perp
\]

The method was checked by recomputing the neutrino energy after removing one charged track, then 2 charged tracks, etc. The stability of the solution was good.

For charged current events, a sizable fraction of the energy is carried away by the muon, which is well measured. Therefore the missing energy is small. It is 8% for \(\bar{\nu}D\) interactions and 15% for \(\nu D\) interactions. Monte Carlo studies have shown that the neutrino energy distribution determined with this method has a spread of \(\pm 10\%\). The resulting energy spectrum of the interaction is in good agreement with the predictions of flux analyses and the knowledge of CC total cross sections.

**Smearing corrections:** The distributions in \(x\), which strongly decrease with increasing \(x\), are smeared considerably by various resolutions, the most important of which is the uncertainty in the neutrino energy. There is essentially no correction in \(y\).

### 5.5 Muon experiments

Muon beams of good optical quality were built at CERN and at Fermilab. The following experiments, using general purpose detectors yielded important informations in muon-nucleon scattering at the highest energies: EMC and BCDMS at CERN and BFP at Fermilab\(^{13,14}\).

![Fig. 5.7 - Layout of the EMC (NA2) muon-nucleon experience at the CERN-SPS\(^{12}\).](image-url)
Fig. 5.7 shows the layout of the EMC (European Muon Collaboration) experiment. Fig. 5.8 shows the layout of primary and secondary beams at the Fermilab Tevatron-II accelerator (800 GeV primary protons on fixed targets). Notice the neutrino and muon beams. Experiments E632, E649, E652, E733, E744 and E745 use neutrino beams, while experiments E665 uses a muon beam.

Fig. 5.8 - Fermilab Tevatron-II secondary beams and the locale of experiments.

6. DEEP INELASTIC SCATTERING.

In terms of $Q^2$, $\nu$ the cross section for a general inelastic process mediated by one photon may be written, in analogy with the Rosenbluth formula, as

$$
\frac{d^2\sigma}{dQ^2 d\nu} = \frac{4\pi\alpha^2}{Q^4} \frac{E'}{E_m} \left| W_2(Q^2, \nu) \cos^2 \frac{\theta}{2} + 2W_1(Q^2, \nu) \sin^2 \frac{\theta}{2} \right| (6.1)
$$

where $E$ and $E'$ are the incident and emergent electron energies ($E, E' >> m_p c^2$). $W_1, W_2$ are arbitrary structure functions corresponding to the two possible polarization states, transverse and longitudinal, of the mediating photon.

6.1 Cross sections for $\nu N$ charged current interactions.

For the charged current inclusive reaction, $\nu_\mu p \rightarrow \mu^- X^{++}$, one obtains the following cross section formula:

$$
\frac{d^2\sigma^{\nu p}}{dQ^2 d\nu} = \frac{G^2}{2\pi m_p^2 E_\nu} \left\{ \cos^2 \frac{\theta}{2} W_2(Q^2, \nu) + 2\sin^2 \frac{\theta}{2} W_1(Q^2, \nu) \pm \frac{E_\nu + E_\mu}{m_p} \sin^2 \frac{\theta}{2} W_3(Q^2, \nu) \right\} (6.2)
$$

where $G = 10^{-5}/m_p^2$ in the Fermi constant. The $+$ sign is for $\nu p$ scattering while the $-$ sign is for $\bar{\nu} p$. The cross section is written in terms of three structure functions, $W_i(Q^2, \nu)$. They correspond,
in ep scattering, to: \( W_1 \rightarrow \) magnetic scattering, \( W_2 \rightarrow \) electric scattering, \( W_3 \rightarrow \) parity violating term due to V-A interference. The cross section (6.2) can be written in terms of \( x \) and \( y \):

\[
\frac{d^2\sigma^{\nu \bar{\nu}}}{dz \cdot dy} = \frac{G^2 m_p E_\nu}{\pi} \left\{ \left( 1 - y - \frac{m_p z y}{2E_\nu} \right) \frac{\nu W_2}{m_p} + z y^2 W_1 \pm z y \left( 1 - \frac{y}{2} \right) \frac{\nu W_3}{m_p} \right\}
\]  

(6.3)

For \( E_\nu \gg m_p \) the term \( m_p z y/2 E_\nu \) can be neglected. The constant at the beginning of the RHS has the value:

\[
\sigma_o = \frac{G^2 m_p}{\pi} = 1.52 \cdot 10^{-58} \text{cm}^2
\]

(6.4)

**Bjorken scaling.** Bjorken has shown that in the limit of \( Q^2 \rightarrow \infty, \nu \rightarrow \infty \), the structure functions become functions of the variable \( x \) only:

\[
W_1(Q^2, \nu) \rightarrow F_1(x), \quad \frac{\nu}{m_p} W_2(Q^2, \nu) \rightarrow F_2(x), \quad \frac{\nu}{m_p} W_3(Q^2, \nu) \rightarrow F_3(x)
\]

(6.5)

Equation (6.3) thus becomes:

\[
\frac{d^2\sigma^{\nu \bar{\nu}}}{dz \cdot dy} = \sigma_o E_\nu \left\{ (1 - y) F_2(x) + y^2 z F_1(x) \pm y \left( 1 - \frac{y}{2} \right) z F_3(x) \right\}
\]

(6.6)

As a consequence of Bjorken scaling one has

\[
\sigma \approx E_\nu, \quad \frac{d^2\sigma}{dz \cdot dy} \approx E_\nu \cdot \text{[Term indep. of } E_\nu]\]

(6.7)

Bjorken scaling may be understood in terms of a dimensional argument, assuming that the neutrino scatters from point-like objects (partons). The dimensions of the Fermi coupling constant \( G \) are:

\[
[G] = [\text{erg cm}^3] \quad \Rightarrow \quad \left[ \frac{1}{\text{MeV}^2} \right]
\]

(6.8)

**Callan-Gros relation.** If the point-like constituents have spin 1/2, and if they have no primordial transverse momentum inside the nucleon, then the Callan-Gross relation holds:

\[
F_2(x) = 2x \cdot F_1(x)
\]

(6.9)

Therefore one obtains the following cross section formulae:

\[
\frac{d^2\sigma^{\nu \bar{\nu}}}{dz \cdot dy} = \frac{\sigma_o}{2} E_\nu \left\{ \left[ F_2(x) \pm z F_3(x) \right] (1 - y)^2 + \left[ F_2(x) \mp z F_3(x) \right] \right\}
\]

(6.10)

One has two structure functions for each of the four scattering processes \( \nu_p, \nu n, \bar{\nu}_p, \bar{\nu}_n \) (8 in total; there are 12 if the Callan-Gross relation is not valid). For flavor conserving charged weak currents and from proton-neutron isospin symmetry one has

\[
F_i(\nu n) = F_i(\bar{\nu} p), \quad F_i(\nu p) = F_i(\bar{\nu} n)
\]

(6.11)

for \( i=1, 2 \). Relations (6.11) are exact for \( F_2 \), and only approximate for \( F_3 \) (one has to neglect the strange sea quarks in the nucleon). One is thus left with four independent structure functions,
for instance $F_2(\nu p)$, $F_3(\nu p)$, $F_2(\bar{\nu}p)$, $F_3(\bar{\nu}p)$. But it is clearly worthwhile to check the validity of charge symmetry.

The structure of eqs. (6.10) is of the type:

$$\frac{d^2\sigma}{dz \cdot dy} = A(x)(1-y)^2 + B(x)$$  \hspace{1cm} (6.12)

Thus one may fix an x-range and fit the resulting y-distribution obtaining values for $A$ and $B$; from these values one may compute $F_2$ and $F_3$ from the relations for $\bar{\nu}p$, $A \sim F_2 - xF_3$, $B \sim iF_2 + xF_3$.

The procedure may be repeated for various x-values, thus obtaining the complete dependences on x for $F_2$ and $xF_3$.

A test of the Callan-Gross relation may be performed by fitting the form:

$$\frac{d^2\sigma}{dz \cdot dy} = A(1-y)^2 + B + C(1-y)$$  \hspace{1cm} (6.13)

One should get $C=0$.

The integration over y of formulae (6.10) yields (remember that $0 \leq y \leq 1$ and $\int_0^1 (1-y)^2 dy = 1/3$):

$$\frac{d\sigma(\nu p)}{dx} = \sigma_0 E_\nu \left[ \frac{2}{3} F_2^{\nu p}(x) - \frac{1}{3} x F_3^{\nu p}(x) \right], \quad \frac{d\sigma(\bar{\nu}p)}{dx} = \sigma_0 E_\nu \left[ \frac{2}{3} F_2^{\bar{\nu}p}(x) - \frac{1}{3} x F_3^{\bar{\nu}p}(x) \right]$$  \hspace{1cm} (6.14)

The integration over x of formulae (6.14) yields the total cross sections.

The four independent structure functions may be expressed in terms of the quark functions $u$, $d$, $s$, $c$ and of the corresponding antiquarks ones:

$$F_2^{\nu p} = 2x(\bar{u} + \bar{d} + s + c)$$  \hspace{1cm} (6.15)

$$F_2^{\bar{\nu}p} = 2x(u + \bar{d} + s + c)$$  \hspace{1cm} (6.16)

$$xF_3^{\nu p} = 2x(\bar{u} + \bar{d} + s - c)$$  \hspace{1cm} (6.17)

$$xF_3^{\bar{\nu}p} = 2x(u - \bar{d} - s + c)$$  \hspace{1cm} (6.18)

The flavor non-conserving charged currents and the neutral ones will not be discussed here.

For an isoscalar target (with nuclei having the same number of protons and neutrons) one has $F_2(\nu N) = F_2(\bar{\nu}N)$ and $F_3(\nu N) = F_3(\bar{\nu}N)$; thus there are only two independent structure functions, for instance $F_2(\nu N)$ and $F_3(\nu N)$. For an isoscalar target eq. (6.18) gives:

$$F_2^{\nu N} = \frac{1}{2}(F_2^{\nu p} + F_2^{\nu n}) = \frac{1}{2}(F_2^{\bar{\nu} n} + F_2^{\bar{\nu} p}) = F_2^{\bar{\nu} N}$$  \hspace{1cm} (6.19)

If eqs. (6.15 - 6.18) are inserted in (6.19) and the sums of quark and antiquark contributions are introduced together with the "sea" properties it follows:

$$F_2^{\nu N} = F_2^{\bar{\nu} N} = x(q + \bar{q}) \simeq \sigma^{\nu} + \sigma^{\bar{\nu}} + \text{corrections}$$  \hspace{1cm} (6.20)

$$xF_3^{\nu N} = xF_3^{\bar{\nu} N} = x(q - \bar{q}) = x(u_+ + d_+) \simeq \sigma^{\nu} - \sigma^{\bar{\nu}} + \text{corrections}$$  \hspace{1cm} (6.21)
6.2 Cross sections for muon - proton inclusive scattering

The differential cross section for muon-proton scattering, assuming one photon exchange and neglecting the muon mass is given by\cite{20}:

\[
\frac{d^2\sigma}{dQ^2 dx} = \frac{4\pi\alpha^2}{Q^4} \left[ \left(1 - y - \frac{m_p z y}{2E}\right) \frac{F_2(z,Q^2)}{x} + y^2 F_1(z,Q^2) \right] = \frac{4\pi\alpha^2 F_2(z,Q^2)}{Q^4 x} \left[ 1 - y - \frac{m_p z y}{2E} + \frac{y^2 \left(1 + \frac{4m_p^2 z^2}{Q^2}\right)}{2(1 + R(z,Q^2))} \right].
\]

(6.22)

(6.23)

\(R(z,Q^2)\) is the ratio of the absorption cross sections \(\sigma_L\) and \(\sigma_T\) for longitudinally and transversely polarised virtual photons and can be written in terms of the proton structure functions \(F_1\) and \(F_2\) as

\[
R = \frac{\sigma_L}{\sigma_T} = \left[ \left(1 + \frac{4m_p^2 z^2}{Q^2}\right) F_2 - 2zF_1 \right]/2zF_1
\]

(6.24)

so that

\[
F_1 = \frac{(2m_p\nu - Q^2)}{8\pi^2}\sigma_T
\]

(6.25)

and

\[
F_2 = \frac{(2m_p\nu - Q^2)}{8\pi^2} \frac{Q^2}{(Q^2 + \nu^2)} \frac{\nu}{m_p} (\sigma_T + \sigma_L).
\]

(6.26)

An experimental knowledge of \(R(z,Q^2)\) is necessary to determine the structure function \(F_2(z,Q^2)\).

In the approximation that \(E >> m_p\) and \(Q^2 >> m_p^2\), equation (6.24) may be rewritten as

\[
\frac{d^2\sigma}{dQ^2 dx} \approx \frac{4\pi\alpha^2}{Q^4 x} \frac{F_2}{x} \left[ 1 - y + \frac{y^2}{2(1 + R)} \right]
\]

The neutron quark content is (udd), while p=(uud). Proton and neutron are related by isospin transformation u - d; therefore

\[
u^n = d^p = d; \quad d^n = u^p = u
\]

(6.27)

where the indices n and p mean neutron and proton, respectively. The sea contributions are by definition independent of the valence quark composition:

\[
s_n = s_p = s = \bar{s}; \quad c_n = c_p = c = \bar{c}
\]

(6.28)

The functions u and d can be split into valence and sea parts, the former being responsible for the quantum numbers:

\[
u = u_v + u_s; \quad d = d_v + d_s
\]

(6.29)

For the sea parts one has:

\[
u_s = \bar{u}; \quad d_s = \bar{d}
\]

(6.30)
In order to reproduce the proton quantum numbers the quark distribution functions have to satisfy certain normalization conditions:

\[
\int_0^1 (u - \bar{u}) \, dx = \int_0^1 u_v \, dx = 2 \\
\int_0^1 (d - \bar{d}) \, dx = \int_0^1 d_v \, dx = 1 \\
\int_0^1 (s - \bar{s}) \, dx = \int_0^1 (c - \bar{c}) \, dx = 0
\]  \hspace{1cm} (6.31)

The structure functions \( F_2 \) for muon scattering off free protons or neutrons can be expressed in terms of quark distributions:

\[
F_2^{\mu p} = x \left\{ \frac{4}{9} (u + \bar{u}) + \frac{1}{9} (d + \bar{d}) + \frac{1}{9} (s + \bar{s}) + \frac{4}{9} (c + \bar{c}) \right\} \\
F_2^{\mu n} = x \left\{ \frac{1}{9} (u + \bar{u}) + \frac{4}{9} (d + \bar{d}) + \frac{1}{9} (s + \bar{s}) + \frac{4}{9} (c + \bar{c}) \right\}
\]  \hspace{1cm} (6.32) \hspace{1cm} (6.33)

Scattering off isoscalar nuclear targets yields

\[
F_2^{\mu N} = \frac{1}{2} (F_2^{\mu p} + F_2^{\mu n})
\]  \hspace{1cm} (6.34)

neglecting a possible mutual influence of the nucleons inside the nucleus. Inserting (6.33) and (6.32) gives:

\[
F_2^{\mu N} = \frac{5}{18} x \left\{ u + \bar{u} + d + \bar{d} + \frac{2}{5} (s + \bar{s}) + \frac{8}{5} (c + \bar{c}) \right\}
\]  \hspace{1cm} (6.35)

Introducing the sums of quark and antiquark contributions

\[
g = u + d + c + s
\]  \hspace{1cm} (6.36)

\[
\bar{g} = u + d + \bar{c} + \bar{s}
\]  \hspace{1cm} (6.37)

yields a simpler representation for the structure function of the isoscalar target.

\[
F_2^{\mu N} = \frac{5}{18} x \left\{ g + \bar{g} - \frac{3}{5} (s + \bar{s} - c - \bar{c}) \right\}
\]  \hspace{1cm} (6.38)

The comparison of (6.38) with the neutrino counterpart yields the quark parton model relation:

\[
F_2^{\mu N} = \frac{5}{18} F_2^{\nu N} - \frac{1}{6} x (s + \bar{s} - c - \bar{c})
\]  \hspace{1cm} (6.39)

The differences between different sea quark contributions are small and can be neglected for \( x \geq 0.4 \).
6.3 Neutrino-nucleon total charged current cross sections

The simplest demonstration of the constituent picture of nucleons is provided by the linear energy dependence of the total Charged Current (CC) cross section for reactions of the type

\[ \nu_\mu + N \rightarrow \mu^- + \text{hadrons}, \quad \bar{\nu}_\mu + N \rightarrow \mu^+ + \text{hadrons} \]  \hspace{1cm} (6.40)

The experimental determination of the CC cross section presents several problems connected with the non-monochromaticity of the neutrino beams, the determination of the neutrino flux, the coarseness of the neutrino detectors and a number of technical problems connected with acceptances. Table 6.1 gives a list of the corrections applied in a \( \nu D, \bar{\nu} D \) experiment using BEBC.

Table 6.1 - Average correction factors applied to the raw number of events for the \( \bar{\nu} \) and \( \nu \) deuterium cross-section calculation\[^{16}\].

<table>
<thead>
<tr>
<th></th>
<th>( \bar{\nu} )</th>
<th>( \nu )</th>
</tr>
</thead>
<tbody>
<tr>
<td>passing rates</td>
<td>1.056</td>
<td>1.075</td>
</tr>
<tr>
<td>radiative corrections</td>
<td>1.010</td>
<td>1.009</td>
</tr>
<tr>
<td>geometrical acceptance of the EMI</td>
<td>1.012</td>
<td>1.028</td>
</tr>
<tr>
<td>scanning losses</td>
<td>1.110</td>
<td>1.034</td>
</tr>
<tr>
<td>EMI electronic inefficiency</td>
<td>1.040</td>
<td>1.050</td>
</tr>
<tr>
<td>accidental EMI hits</td>
<td>0.986</td>
<td>0.984</td>
</tr>
<tr>
<td>muon momentum cut</td>
<td>1.056</td>
<td>1.127</td>
</tr>
<tr>
<td>1-prong events</td>
<td>1.12</td>
<td>-</td>
</tr>
<tr>
<td>energy smearing</td>
<td>1.000</td>
<td>1.006</td>
</tr>
</tbody>
</table>

Fig. 6.1 shows a compilation of the ratios \( \sigma_t/E_\nu = a_\nu \) for neutrino and antineutrino interactions. Both ratios are essentially constant, independent of energy. The experimental determinations of the constants group around two different numbers. We shall use for our purposes the following overages

\[ \frac{\sigma_t}{E_\nu} = a_{\nu N} = 0.62 \pm 0.03, \quad a_{\bar{\nu} N} = 0.33 \pm 0.02 \]  \hspace{1cm} (6.41)

in units of \( 10^{-38} \text{cm}^2/\text{GeV} \).

Experiment WA25, which studied the interactions of \( \nu, \bar{\nu} \) in D, determined the ratios of the interaction cross sections on neutrons and protons. Combining their results with the average values (6.41), one has the coefficients for the interactions on free protons and free neutrons

\[ a_{\nu p} = 0.40 \pm 0.04, \quad a_{\nu n} = 0.84 \pm 0.07, \quad a_{\bar{\nu} p} = 0.44 \pm 0.03, \quad a_{\bar{\nu} n} = 0.22 \pm 0.02. \]  \hspace{1cm} (6.42)

The linear rise of the \( \nu N \) cross sections is what one would expect for the scattering of a neutrino on a point like constituent. It would lead to pure S-wave scattering, which depends on the Fermi
Fig. 6.1 - $a_\nu = \sigma_T/E_\nu$ for the muon neutrino and antineutrino charged-current total cross sections as a function of neutrino energy\textsuperscript{[17.]} The error bars include both statistical and systematic errors. The straight lines are averages for the CCFRR and CDHS measurements. Note the change in the energy scale between 20 and 50 GeV. The data points on the right give averages for other high energy measurements.

coupling constant $G$ and on phase space. Phase space grows as $p^2$, the c.m. momentum of the two colliding point particles; but $p^2=mE_\nu/2$ where $m$ is the mass of the point like constituent. Therefore $\sigma \sim G^2E_\nu$. So the linear increase of the cross section is due to phase space for $S$-wave scattering. The linear dependence should be valid at "low" energies, where "low" means low with respect to the masses of the charged intermediate vector bosons (or at much lower energy than that corresponding to the unitarity limit) ($E_{cm} < m_W \simeq 82 GeV$, t.i. $E_\nu < \text{few thousand GeV}$).

As already stated, the linear rise is connected with elastic scattering on quasi-free, point-like constituents. But at the same time these constituents must be tightly bound together because no free quarks seem to easily escape from nucleons. The successes of the quark-parton model and the deviations from its successes lead to important information on the nature of quark-quark scattering.

6.4 $y$-dependence.

Fig. 6.2 shows the $y$-distribution from the CDHSW experiment in the energy range $30 < E_\nu < 200\text{GeV}$$^\text{[6,18.]}$. The neutrino distribution is consistent with being the sum of a flat distribution with a small $(1-y)^2$ admixture from scattering off antiquarks. The antineutrino distribution is predominantly $(1-y)^2$ with a small constant contribution. The shapes of these two distributions do not change appreciably with neutrino energy.
6.5 Neutron and proton structure functions (x-dependence).

From the measurement of the four reactions

\[ \nu n \rightarrow \mu^- X^+, \quad \nu p \rightarrow \nu^- X^{++}, \quad \bar{\nu} n \rightarrow \mu^+ X^-, \quad \bar{\nu} p \rightarrow \mu^+ X^o \]  

one may obtain the complete set of structure functions provided one assumes the validity of the Callan-Gros relation [or if one uses a definite value for the ratio \( R = (F_2 - 2xF_1)/2xF_1 \)]. In particular one may obtain the fractional momentum distributions for the valence quarks, \( xu_v \), \( zd_v \), as well as for the antiquarks.

From linear combinations of the measured differential cross sections, one has

\[ \widetilde{F}_2^{\nu n} = \frac{d\sigma^{\nu n}}{dx} + d\sigma^{\bar{\nu}n}/dx = F_2^{\nu n} + \frac{1}{4}(zF_3^{\nu p} - zF_3^{\bar{\nu} p}) = 2x\left( d + \bar{u} + \frac{3}{2}s \right) \]  

\[ \widetilde{F}_2^{\nu p} = \frac{d\sigma^{\nu p}}{dx} + d\sigma^{\bar{\nu}p}/dx = F_2^{\nu p} + \frac{1}{4}(zF_3^{\nu n} - zF_3^{\bar{\nu} n}) = 2x\left( u + \bar{d} + \frac{3}{2}s \right) \]  

\[ z\widetilde{F}_3^{\nu n} = \frac{d\sigma^{\nu n}}{dx} - d\sigma^{\bar{\nu}n}/dx = \frac{1}{2}(zF_3^{\nu n} + zF_3^{\bar{\nu} n}) = 2x(u - \bar{d}) \]  

\[ z\widetilde{F}_3^{\nu p} = \frac{d\sigma^{\nu p}}{dx} - d\sigma^{\bar{\nu}p}/dx = \frac{1}{2}(zF_3^{\nu p} + zF_3^{\bar{\nu} p}) = 2x(d - \bar{u}) \]

where the \( \widetilde{F} \) are sometimes referred to as "experimental" structure functions. The procedure is the following: from the cross section differences one first determines the \( \widetilde{F} \) and then the various combinations of quark functions. For the present analysis we shall neglect the small \( Q^2 \)-dependence and concentrate on the \( x \)-dependence for data at \( <Q^2> \sim 10 \text{ GeV}^2 \).
The fractional momentum distributions for the valence quarks and for the sea antiquarks obtained by experiment WA25 are shown in Fig. 6.3. There are no theoretical predictions on the form of the structure functions over the entire x range. There are, however, estimates of the valence quark dependence at small x, based on Regge behaviour, and at large x, based on quark counting arguments. The quark and antiquark distributions were fitted, following those suggestions, to the forms

\[ xq_v = Ax^\alpha (1 - z)^\beta \]  

(6.48)

and

\[ x\bar{q} = B(1 - z)^\gamma \]  

(6.49)

From the results, presented in table 6.2, it appears that these parametrizations give a good description of the data, that the up valence quark has a broader x distribution than the down quark (the slope parameter \( \beta \) is greater by about one unit for \( d_v \) than for \( u_v \)).

The ratio \( r = x_{d_v}/x_{u_v} \) shown in Fig. 6.4 is clearly consistent with the naive QPM expectation \( u=2d \) at \( u=0 \), but it falls to much smaller values for \( z \to 1 \). There are predictions concerning the value of that ratio at \( x=1 \): QCD calculations predict \( r=0.2 \); isospin arguments by Field and Feynman and SU(6) breaking effects give \( r=0 \) at \( x=1 \). The experiment results suggests a value of \( r \) for \( z \to 1 \) consistent with the above predictions.
Table 6.2 - Fits of the quark and antiquark momentum distributions to formulae (6.48) and (6.49). The errors on the integrated values include statistical and systematic errors[19].

<table>
<thead>
<tr>
<th>Quark combinations</th>
<th>$Fitted\ parameters$</th>
<th>$X^2/D_0F$</th>
<th>Integrals over $x$</th>
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</thead>
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<tr>
<td>$xu_v(z)$</td>
<td>$0.78\pm0.06$</td>
<td>3.3$\pm$0.1</td>
<td>0.9</td>
</tr>
<tr>
<td>$xd_v(z)$</td>
<td>$0.90\pm0.18$</td>
<td>4.4$\pm$0.5</td>
<td>0.7</td>
</tr>
<tr>
<td>$z\bar{u}(x)+3zs(x)/4$</td>
<td>$4.0\pm0.7$</td>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td>$x\bar{d}(x)+3zs(x)/4$</td>
<td>$3.9\pm1.0$</td>
<td></td>
<td>0.5</td>
</tr>
</tbody>
</table>

Fig. 6.4 - The ratio $r = xd_v/xu_v$ versus $x$[15].

The antiquark distributions, which refer respective-to the combinations $x\bar{q}^p = x\bar{u} + \frac{3}{4}x\bar{s}$ and $x\bar{q}^n = x\bar{d} + \frac{3}{4}x\bar{s}$, are also shown in Fig. 6.3; the slopes are almost the same and the fitted values for $\gamma$ are given in table 6.2. The total momentum fractions in the whole $x$-range, excluding elastic events, are respectively $\bar{Q}^p = 0.030\pm0.005$ and $\bar{Q}^n = 0.030\pm0.008$. The inclusion of the elastic contribution increase $\bar{Q}^n$ by one standard deviation. It must be remembered that, apart from a small difference in the strange quark content of the antiquark combinations, charm threshold effects are expected to play different roles in neutrino and antineutrino reactions and may affect the analysis.
Fig. 6.5 - Comparison of $(F_2^{\nu n} - F_2^{\nu p})/6$ measured in a $\nu D$ experiment with $F_2^{\mu n} - F_2^{\mu p}$ from a muon experiment\cite{19}.

Callan-Gros relation. The effect of a violation of the Callan-Gros relation may be investigated by taking $R = 0.1$ (instead of $R = 0$): the total antiquark momentum fractions become: $Q^p = 0.024 \pm 0.005$, $Q^n = 0.019 \pm 0.008$, which represent a negligible decrease.

Comparison with muon data. $(F_2^{\nu n} - F_2^{\nu p})$ has a quark description very close to that of $F_2^{\mu n} - F_2^{\mu p}$:

\begin{equation}
F_2^{\nu n} - F_2^{\nu p} = 2x(u_v - d_v), \tag{6.50}
\end{equation}

\begin{equation}
6(F_2^{\mu p} - F_2^{\mu n}) = 2x(u_v - d_v) + 4x(\bar{d} - \bar{u}). \tag{6.51}
\end{equation}

In Fig. 6.5 the neutrino results are compared with data from the EMC collaboration obtained from $H_2$ and $D_2$ targets\cite{20}. The agreement is good (the average $Q^2$ in the EMC data is much higher than in the $\nu(\bar{\nu})$ data).

Adler sum-rule. In the QPM, the Adler sum rule provides the difference between the number of valence quarks and antiquarks in the nucleon. The sum rule can be written separately for $\bar{\nu}$ and $\nu$.

\begin{equation}
\lim_{z_{\text{min}} \to 0} \frac{3\pi}{2G^2m_NE_{\nu}} \left( \int_{z_{\text{min}}}^{1} \frac{d\sigma^{\nu p}}{dz} \frac{dz}{x} - \int_{z_{\text{min}}}^{1} \frac{d\sigma^{\nu s}}{dz} \frac{dz}{x} \right) = \frac{1}{2}(2 + 3R)(U_v - D_v) + 2(\bar{D} - \bar{U}), \tag{6.52}
\end{equation}

\begin{equation}
\lim_{z_{\text{min}} \to 0} \frac{\pi}{2G^2m_NE_{\nu}} \left( \int_{z_{\text{min}}}^{1} \frac{d\sigma^{\nu n}}{dz} \frac{dz}{x} - \int_{z_{\text{min}}}^{1} \frac{d\sigma^{\nu s}}{dz} \frac{dz}{x} \right) = \frac{1}{2}(2 + R)(U_v - D_v) - \frac{2}{3}(\bar{D} - \bar{U}), \tag{6.53}
\end{equation}
where $U_\nu$, $D_\nu$, etc., represent the number of quarks. The left hand sides of (6.52) and (6.53) have been evaluated directly, computing the average values of $1/x$ in the $\bar{\nu}p$, $\bar{\nu}n$, $\nu n$ and $\nu p$ samples and using the known cross sections for normalization. Combining the $\nu$ and $\bar{\nu}$ data and assuming $R=0$ one obtains[19]

$$U_\nu - D_\nu = 1.01 \pm 0.08 \pm 0.18, \quad \overline{D} - \overline{U} = 0.05 \pm 0.05 \pm 0.11$$

(6.54)

where the first error is mainly statistical and the second one systematical. If $R=0.1$ both $U_\nu - D_\nu$ and $\overline{D} - \overline{U}$ decrease by one third of the total error. One may conclude that the Adler sum-rule is in good agreement with the QPM prediction.

Flavor composition of the nucleon. Fig. 6.6 shows the $\nu$-dependence of the flavor composition of the nucleon as determined by the CDHSW collaboration[11].

Where $0.6 < \epsilon < 0.9$ depending on $Q^2$. The behaviour of $x\bar{u}$ and $x\bar{d}$ is remarkably similar in the whole $Q^2$ range covered by the experiment; there is no sizeable $Q^2$ dependence. The fractional momentum carried by both $\bar{u}$ and $\bar{d}$ is practically zero for $x > 0.4$.

6.5 $Q^2$ dependence of the structure functions

The $Q^2$ dependences at fixed values of $x$ of the structure functions $F_2$, $xF_3$, $u_\nu$, $d_\nu$, $\bar{q}$ are shown in Figures 6.7-6.8[21,24]. With the exception of $\bar{q}$, all show a common pattern of scaling violation, that is a characteristic $Q^2$-dependence: the structure functions rise with $Q^2$ at small values of $x$, are almost $Q^2$-independent at $x \sim 0.2$ and decrease with increasing $Q^2$ at larger values of $x$. This is more evident in Fig 6.9, which shows the logarithmic derivative of $\bar{F}_2$. While there are relatively large experimental errors on the absolute values of the structure functions (the new
CDHSW values are \( \sim 40\% \) larger than the old values\(^{[24]}\). The slopes are better determined and there is good agreement between the various experiments.

The evolution in \( Q^2 \) of the structure functions to leading order in perturbative QCD is given by the Altarelli-Parisi equations\(^{[25]}\). The analysis is simpler for the flavor non-singlet cases, like for \( x F_3 \); the \( Q^2 \)-evolution of the flavor singlet and of the antiquark structure functions are coupled to the gluon distribution, which is difficult to determine.

**Analysis of \( x F_3(x, Q^2) \)** - One has to remember that \( x F_3 \) is obtained from differences of cross sections and is therefore subject to relatively large experimental uncertainties. The Altarelli-Parisi evolution equation of \( x F_3 \) may be written as

\[
\frac{\partial x F_3(x, Q^2)}{\partial \ln Q^2} = \frac{\alpha_s(Q^2)}{2\pi} \int_0^1 P_{q\bar{q}}(x/y) x F_3(y, Q^2) dy/y
\]  

where \( P_{q\bar{q}}(x/y) \) is one of the splitting functions given by the theory; the strong (running) coupling constant, in leading order (LO), is equal to

\[
\alpha_s(Q^2) = \frac{12\pi}{[33 - 2N_f] \ln(Q^2/\Lambda^2)}
\]
Fig. 6.8 - $Q^2$-dependence of (a) the sea distributions $x\bar{q}$, $x\bar{u}$ and (b) the $F_2$ structure functions from $\nu N$ and $\mu N$ interactions\[17].

Fig. 6.9 - Comparison between the logarithmic derivatives of different experiments.
<table>
<thead>
<tr>
<th>Collab.</th>
<th>ref.</th>
<th>year</th>
<th>reaction</th>
<th>type of fit</th>
<th>0</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>$\Lambda$/MeV</th>
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<td>[102]</td>
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<tr>
<td>BCDMS</td>
<td>[106]</td>
<td>1981</td>
<td>$\mu N(C)$</td>
<td>NS</td>
<td>X</td>
<td></td>
<td>X</td>
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<td></td>
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<tr>
<td>EMC</td>
<td>[102]</td>
<td>1982</td>
<td>$\mu N(Fe)$</td>
<td>NS</td>
<td>X</td>
<td>X</td>
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<td></td>
<td>X</td>
<td></td>
</tr>
<tr>
<td>BFP</td>
<td>[107]</td>
<td>1983</td>
<td>$\mu N(Fe)$</td>
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<td>X</td>
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<td>NS</td>
<td>X</td>
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<td>$\nu N(CaC_{6})$</td>
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<td>X</td>
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<td>1983</td>
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<td>CUSB</td>
<td>[108]</td>
<td>1983</td>
<td>$\Gamma(T\rightarrow s s)$</td>
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<td>$\Gamma(T\rightarrow g g)$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

**Fig. 6.10 - Values of the parameter $\Lambda$ from different experiments[2].**

where $\Lambda$ is a free parameter of the theory and $N_f$ is the number of quark flavor, which we shall fix at three. There is a further complication; one should avoid the region where a $Q^2$-dependence could be due to higher order (twist) effects: one has to apply the following cuts $Q^2 > 2\, GeV^2$ and $W^2 > 10\, GeV^2$.

Different procedures were suggested in the last few years to determine $\Lambda$. In leading order, the programs written by Odorico[26] and Abbott-Barnett[27] require a parametrization of $x F_2$ (at fixed values of $Q^2$), which was chosen to be (at $Q^2 = 1\, GeV^2$)

$$x F_2(x, Q^2) = A x^\alpha (1-x)^\beta (1 + \gamma x)$$

(6.57)

$\alpha$, $\beta$, $\gamma$ are free parameters, whereas $A$ is given by the QPM sum rules, $\int_0^1 x F_2 \, dx = 3(1 - \alpha_s/\pi)$.

Alternatively a program written by Furmansky-Petronzio[22] does not require an explicit phenomenological parametrization of $x F_2$, since it is expressed as a series of Laguerre polynomials.

All analyses yielded values of $\Lambda_{LO}$ in the range $100 < \Lambda_{LO} < 250\, MeV$, with large errors (see Fig. 6.10). One may conclude that leading order QCD fits are able to represent the experimental data.

**Analysis of $F_2(x, Q^2)$ and $z\bar{q}$** - The structure function $F_2$ is obtained from the sum of $\nu N$ and $\bar{\nu} N$ data. As already stated the QCD analysis is more difficult, since it depends on the QCD parameter $\Lambda$ and on the gluon distribution functions. For $x > 0.4$ the contribution of the sea quarks is negligible and one may analyze in a correlated way $F_2$ alone.
The change of $F_2$ with respect to $\ln Q^2$ arises from two contributions, one due to the process $q \to q + g$ and the other from $g \to q + \bar{q}$. This means that the gluon distribution function is a necessary input. In order to get a handle on the shape of the gluon distribution functions, the CDHSW collaboration considered a suitably chosen combination of the differential $\nu N$ and $\bar{\nu} N$ cross sections:[29]

$$\bar{q}^T \equiv x(\bar{u} + \bar{d} + 2\bar{s}) = \frac{1}{1 - (1 - y)\Lambda^2} \frac{1}{1 - (1 - y)^4} \frac{d^2}{dxdy} \left( \sigma(\nu N) - (1 - y)^2 \sigma(\nu N) \right) + \delta$$

(6.58)

where $\delta$ depends upon the strange (and charmed) sea $x s (-x \bar{s})$ and the longitudinal structure function $F_L = F_2 - 2xF_s$ with the property that $\delta \to 0$ as $y \to 1$. The term $(1 - y)^2 \sigma(\nu N)$ substracts the amount of scattering off quarks. Since at $y=0.5$ this represents about 50% the specific sea quark combination $x(\bar{u} + \bar{d} + 2\bar{s})$ is directly measurable. At large $x$, $x > 0.4$, the sea $\bar{q}^T$ is negligible over the measured $Q^2$-range. This implies a constraint on the shape of the gluon distribution function. Qualitatively speaking its shape cannot be too broad, otherwise the $Q^2$-evolution of $\bar{q}^T$ would generate a nonnegligible contribution at large $x$ due to $g \to q + \bar{q}$.

The simultaneous evaluation of $F_2$ and $\bar{q}^T$ yields $\Lambda_{\overline{MS}}$ and the gluon distribution function $G(x)$. The measured $\bar{q}^T$ distribution can be used to obtain for $x > 0.3$

$$F_2^{NS}(x, Q^2) = F_2(x, Q^2) - 2(\bar{q}^T(x, Q^2) - x s(x, Q^2)),$$

(6.59)

which is now independent of the sea, like $xF_2$. Thus a nonsinglet analysis is possible and has been performed in both $\nu$ and $\mu$ experiments. The EMC collaboration[20] used the following parametrization ($Q_0^2 = 5 GeV^2$)

$$F_2(x, Q^2) = \frac{5}{18} F_2^S(x, Q^2) + \frac{1}{6} F_2^{NS}(x, Q^2)$$

$$F_2^{NS}(x, Q^2) = A x^\alpha (1 - z)^6 + B (1 - y)^7$$

$$F_2^{NS}(x, Q_0^2) = 0.29 x^{0.52} (1 - z)^{5.26} (1 + 8.9z)$$

$$x G(z, Q_0^2) = D (1 - z)^7$$

$$R(z, Q^2) = R^{QCD}(z, Q^2)$$

(6.60)

This next-to-leading order nonsinglet QCD fit to $F_2$ yielded $\Lambda_{\overline{MS}} = 100$ MeV with errors of 50-80 MeV. Furthermore they found that the fraction of the momentum of the nucleon carried by gluons is $\sim 56\%$ at $Q^2 = 22.5 GeV^2$ and that the gluon distribution function at $Q^2 = 5 GeV^2$ is consistent with $x G(z, Q^2) = 3.8(1 - x)^{6.7}$.

In conclusion, there is good agreement between various experiments; all agree in the observation of substantial scaling violation yielding values of $\Lambda$ in the range $100 \sim 200$ MeV. If interpreted within QCD the strong interaction coupling constants is small in the $Q^2$-range from 5 to 100 GeV$^2$. However, the systematic uncertainties are for the time being too large to conclude about the running of $\alpha_S(Q^2)$. It should be mentioned that there is a second type of QCD analyses based upon the moments of the structure functions.
6.7 The EMC effect

We shall not discuss this effect in detail, since it was already discussed in other lectures at this school. We shall only recall that there are data from electron, muon and neutrino (CC) interactions that indicate a significant difference between the inelastic structure functions of nucleons in heavy nuclei (Al, Ne, Fe) and in deuterium nuclei[30]. Within the quark-parton model, the deviation of the ratio $F_2^{\text{heavy nucleons}}/F_2^D$ from unity suggests a distortion of the quark distributions for nucleons bound in a nucleus.

We also recall that there are indications[11] in high-energy, high-$p_t$, proton-proton interactions that there may be di-quark substructures in the proton, similar to $\alpha$-particle structures in nuclei.

6.8 Polarization of the $\mu^+$ from $\bar{\nu}_\mu + Fe \rightarrow \mu^+ + X$

In the standard model, the space-time structure of the charged current interactions is assumed be of the V-A type (vector-axial vector). The y-distribution in CC interactions (see Fig. 6.2) is in agreement with this hypothesis. However that distribution could be obtained with a mixture of scalar (S), pseudoscalar (P) and tensor (T) terms. The measurement of the $\mu^+$ helicity in the inclusive reaction

$$\bar{\nu} + Fe \rightarrow \mu^+ + X$$  \hspace{1cm} (6.61)

resolves this ambiguity, since vector and axialvector interactions conserve this helicity, while S, P and T interactions do not (Fig. 6.11).

An experiment was performed at the CERN-SPS using the CDHS detector as an instrumented target and the CHARM detector as a polarimeter of the stopped positive muon. The interaction took place in the CDHS detector; the $\mu^+$ was stopped in the CHARM detector, where the decay $\mu^+ \rightarrow e^+$ was also observed (Fig.6.12). The result of the experiment is in agreement with the V-A hypothesis and places an upper-limit to the maximum of S and P interactions, $\sigma_{SP}/\sigma_{alt} < 0.07$ (95% confidence level)[31].

Fig. 6.11 - Helicity of the incoming neutrino and of the outgoing muon in reaction (6.61) for V, A and S, P, T currents.
Fig. 6.12 - Layout of the $\mu^+$ polarization experiment at the CERN-SPS wide band beam[51].

7. NEUTRAL CURRENT. LEPTON – NUCLEON INTERACTIONS

7.1 Introduction

Neutral currents were introduced theoretically in order to remove divergences. For instance the diagram shown in Fig. 7.1a, corresponding to the process $\nu + \bar{\nu} \rightarrow W^+ + W^-$, gives a badly diverging amplitude. The divergence may be cancelled with the neutral current, $Z^0$ exchange diagram of Fig. 7.1b (a similar result is obtained with the exchange of a heavy electron, $E^+$).

Neutral current effects have been observed in: i) neutrino-electron elastic scattering; ii) electron-positron annihilation into muons (Fig. 7.2); iii) neutrino-hadron scattering; iv) deep

![Diagrams](image)

Fig. 7.1 - $\nu + \bar{\nu} \rightarrow W^+ + W^-$: (a) CC diverging diagram and (b) neutral current diagram which cancels the divergence of (a).

![Graph](image)

Fig. 7.2 - Muon asymmetry as measured at PETRA at 34.5 GeV centre-of-mass energy for $e^+e^- \rightarrow \mu^+\mu^-$: — QED+weak, - - - QED; JADE, MARK J, TASSO, PLUTO[11].
inelastic scattering of polarized electrons on deuterium; v) parity violating effects in atoms; vi) deep inelastic scattering of muons on nuclei\[1\].

In the following we shall consider only iii).

7.2 Formulae for neutrino-quark neutral current scattering

The neutrino-quark interaction via neutral current, \(\nu + N \rightarrow \nu + X\), is usually described by a phenomenological Lagrangian of the type

\[
L_{\nu q}^{\text{eff}} = -\frac{G}{\sqrt{2}} [\bar{\nu} \gamma_\alpha (1 + \gamma_5) \nu] \cdot \{\bar{u} \gamma_\alpha [u_L (1 + \gamma_5) + u_R (1 - \gamma_5)] \bar{u} + \bar{d} \gamma_\alpha [d_L (1 + \gamma_5) + d_R (1 - \gamma_5)] d + \ldots \}
\]  

(7.1)

where the dots represent analogous terms for other quarks. The coupling constants \(u_R, u_L, d_R, d_L, \ldots\) were introduced by Sehgal[57] and are usually called chiral coupling constants. 7.1 was written under the following assumptions: i) the neutral current is diagonal in all flavours, ii) the momentum transfer in the reaction is negligible in comparison with the \(Z_\alpha\) mass \((Q^2 << \langle M^2 \rangle)\), iii) the space-time structure of the interaction is only of the type V and A. If one neglects the contribution of strange and heavier quarks in the sea of the nucleon and forgets scaling violation effects one obtains the following cross-sections for neutrino-nucleon neutral current scattering:

\[
\frac{d\sigma^{NC}_{\nu N}}{dxdy}(\nu N) = \frac{G^2 m_\nu E_\nu}{\pi} \left\{ (u_L^2 + d_L^2)q + \bar{q} + q(1 - y)^2 \right\} \quad \text{(7.2)}
\]

\[
\frac{d\sigma^{NC}_{\bar{\nu} N}}{dxdy}(\bar{\nu} N) = \frac{G^2 m_\nu E_\nu}{\pi} \left\{ (u_L^2 + d_L^2)\bar{q} + q + \bar{q}(1 - y)^2 \right\} \quad \text{(7.3)}
\]

The neutral- to charged-current cross-sections ratios are given by

\[
R_\nu = \frac{\sigma_\nu^{NC}}{\sigma_\nu^{CC}} = (u_L^2 + d_L^2) + r(u_R^2 + d_R^2) + \text{corrections}
\]

(7.4)

\[
R_{\bar{\nu}} = \frac{\sigma_{\bar{\nu}}^{NC}}{\sigma_{\bar{\nu}}^{CC}} = (u_L^2 + d_L^2) + \frac{1}{r}(u_R^2 + d_R^2) + \text{corrections}
\]

(7.5)

where

\[
r = \frac{\sigma_{\bar{\nu}}^{CC}}{\sigma_\nu^{CC}} = \frac{\bar{q} + q/3}{q + \bar{q}/3}
\]

(7.6)

Other combinations have been considered

\[
R_+ = \frac{\sigma^{NC}(\nu N) + \sigma^{NC}(\bar{\nu} N)}{\sigma^{CC}(\nu N) + \sigma^{CC}(\bar{\nu} N)} = u_L^2 + u_R^2 + d_L^2 + \text{corrections}
\]

(7.7)

In the GWS theory, the chiral coupling constants are simple functions of the mixing angle \(\theta_\omega\)

\[
u_L^2 = \left(\frac{1}{2} - \frac{2}{3}\sin^2 \theta_\omega \right)^2, \quad u_R^2 = \left(\frac{2}{3}\sin^2 \theta_\omega \right)^2
\]

(7.8a)

\[
d_L^2 = \left(-\frac{1}{2} + \frac{1}{3}\sin^2 \theta_\omega \right)^2, \quad d_R^2 = \left(\frac{1}{3}\sin^2 \theta_\omega \right)^2
\]

(7.8b)
Thus one has

\[ R_\nu = \left[ \frac{1}{2} - \sin^2 \theta_w + \frac{5}{9} \sin^4 \theta_w \right] (1 + r) \quad (7.9) \]

\[ R_\tau = \frac{F_\tau - R_{\bar{\nu}}}{1 - r} = \frac{1}{2} (1 - 2 \sin^2 \theta_w) + \text{corrections} \quad (7.10) \]

7.3 Experimental results

Experiments on isoscalar targets measured $u_L^2 + d_L^2$ and $u_R^2 + d_R^2$. Values of $u_L^2$ and $d_L^2$ have been obtained by combining the results from an isoscalar target with a measurement of neutrino interactions in hydrogen, and from neutrino interactions with proton and neutron separately as observed in deuterium. Only in one experiment were measured simultaneously the neutral current deep inelastic scattering by neutrinos and antineutrinos on proton and neutron. The resulting values for the neutral to charged current cross section ratios are (for $E_h > 5 \text{ GeV}$, $p_h > 1.5 \text{ GeV/c}$)\(^{[58]}\):

\[ R^{\nu p} = 0.49 \pm 0.05, \quad R^{\bar{\nu}p} = 0.26 \pm 0.04, \quad R^{\nu n} = 0.25 \pm 0.02, \quad R^{\bar{\nu}n} = 0.57 \pm 0.09, \quad (7.11) \]

and $R^{\nu N} = 0.33 \pm 0.02, \quad R^{\bar{\nu}N} = 0.35 \pm 0.04$. The quoted errors are statistical only. Notice the cuts on the total hadron energy, $E_h$, and in the transverse momentum of the total hadronic system, $p_t$. This last cut is needed to reduce the background from neutral hadron interactions which yield low $p_t$ values and which would simulate a neutrino muon-less interaction. From the measured values (7.11) one obtains for the individual couplings the following values:

\[ u_L^2 = 0.133 \pm 0.026 \pm 0.015, \quad u_R^2 = 0.020 \pm 0.019 \pm 0.004, \]

\[ d_L^2 = 0.192 \pm 0.026 \pm 0.015, \quad d_R^2 = 0.002 \pm 0.019 \pm 0.004. \quad (7.12) \]

where the first error is statistical and the second systematic. The values (7.12) yield, if $\rho$ is considered a free parameter, the values $\rho = 1.01 \pm 0.06$ and $\sin^2 \theta_w = 0.202 \pm 0.054$, consistent with the predictions of the standard model ($\rho = 1$).

The best values for $R^{\nu N}$ and $\sin^2 \theta_w$ were obtained recently by electronic experiments. In particular the CDHSW collaboration obtained from their run with the NBB at 160 GeV/c\(^{[8]}\):

\[ R^{\nu N} = 0.3059 \pm 0.0022 \pm 0.005 \quad \text{and} \quad \sin^2 \theta_w = 0.217 \pm 0.007 \pm 0.006 \]

A global fit of all the neutrino-nucleon data yields for the chiral coupling costants of the light quarks\(^{[8,34,36]}\):

\[ u_L = 0.344 \pm 0.026, \quad d_L = -0.419 \pm 0.022, \]

\[ u_R = -0.153 \pm 0.022, \quad d_R = 0.076 \pm 0.041; \quad (7.13) \]

Sciulli at the recent 1985 Kyoto conference\(^{[8]}\) quoted the following values for $\sin^2 \theta_W$, corrected in order to be valid at the W mass:

\[ \text{from } \nu N : \quad \sin^2 \theta_w (m_W) = 0.227 \pm 0.005 \pm 0.006 \]

\[ \text{from } \bar{\nu}p : \quad \sin^2 \theta_w (m_W) = 0.221 \pm 0.001 \quad (7.14) \]
He also commented on the recent results on the differential x-distributions for neutral current events, which seem to be smooth functions of x, with at most only a minor x dependence\textsuperscript{[26,28]}.

Fig. 7.3 shows a compilation of values of $\sin^2 \theta_w$ obtained from a variety of experiments at low energies and at the $\bar{p}p$ collider. It is remarkable that a single value explains many different experiments in such a wide range of energies.

In the standard model of electroweak interactions, the nucleon analyzed in neutral current (NC) inelastic neutrino-nucleon scattering is expected to be nearly equal to that measured in CC scattering. But there is comparatively little data on NC neutrino-nucleon interactions.

The neutral current structure functions have quark contributions, different from the case of CC scattering:

$$F_2(x) = \left(u_2^2 + d_2^2 + u_1^2 + d_1^2\right)[xq(x) + z\bar{q}(x)] - \left(u_2^2 - d_2^2 + u_1^2 - d_1^2\right)2[xs(x) - xc(x)] \quad (7.15)$$

$$xF_3(x) = \left(u_2^2 + d_2^2 - u_1^2 - d_1^2\right)[xq(x) - z\bar{q}(x)] \quad (7.16)$$

The fine-grained, 340 ton detector at Fermilab was recently exposed to the Fermilab narrow band beam\textsuperscript{[37,38]}. The fine grain structure and the knowledge of the neutrino energy allowed to reconstruct the kinematical variables of NC and CC events using only the observed energies and angles of the recoil hadronic showers. The NC/CC ratios obtained as function of x are shown in Fig 7.4. The use of ratios eliminates a number of systematic errors. The ratios are 1 within

Fig. 7.4 - The ratios of neutral to charged current-cross sections, $R_\nu$ and $R_\tau$, plotted versus x (data from the FMM collaboration at Fermilab)\textsuperscript{[37]}. 
experimental errors, indicating that the x-distributions of NC and CC interactions are the same. An evaluation of the structure functions yielded the conclusions that they are equal to the CC case, thus confirming the expectations of the standard model.

8 PROPERTIES OF THE HADRON SYSTEM IN CC INTERACTIONS

8.1 Introduction

The properties of the hadronic system produced in neutrino-nucleon, charged current interactions, have been studied extensively in several bubble chamber experiments. In fact it is only the bubble chamber technique which offers at present the possibility of studying the details of the hadronic system.

Most properties of the hadronic system produced in $\nu N$, CC collisions turn out to be very similar to those of the hadronic system produced in hadron-hadron collisions. The most important variable seems to be the total energy of the hadronic system; it is the value of this variable which determines the properties of the system, irrespective of the specific reaction.

At the relatively low values of $W^2$ covered in the present $\nu N$ experiments there are a variety of small effects, which may be called of a kinematical nature. They arise from the baryon mass, (not negligible in comparison to the total hadronic energy W), to the 2-body and 3-body final states, which are different or are not allowed in all $\nu N$ reactions, different charge states of target remnants, differences in the y-distributions between $\nu N$ and $\bar{\nu} N$ reactions, etc.

8.2 Charged hadron multiplicities.

Fig. 8.1 shows as function of $W^2$ the normalized topological cross sections $P_n = \sigma_n/\sigma_{in}$ for the reaction $\bar{\nu}_\mu p \rightarrow \mu^+ + X$. The typical behaviour of every $P(n)$ versus $W^2$ is the following: $P_n$ rises with $W^2$, reaches a maximum and then decreases.

![Fig. 8.1 - Normalized topological cross sections for charged hadron multiplicities as a function of $W^2$ for $\bar{\nu} p$ interactions][30]
Fig. 8.2 - Average charged hadron multiplicity \(< n_{ch} >\) as a function of \(W^2\). The lines through the experimental points represent \(A + B \ln W^2\) fits to the data.

Fig. 8.3 - Mean charged multiplicities in the forward and backward centre of mass directions as a function of \(W^2\). The lines represent the Lund Monte Carlo predictions.

The average charged hadron multiplicity, \(< n_{ch} >\), is plotted in Fig. 8.2 versus \(W^2\) for \(\bar{v}n\), \(\bar{v}p\) and \(\nu p\) interactions. \(< n_{ch} >\) increases with \(W^2\); the increase is consistent with a linear dependence on \(\ln W^2\) and fits to the expression \(< n_{ch} > = A + B \ln W^2\) yield the following values for \(\bar{v}p\): \(< n_{ch} > = (0.02 \pm 0.20) + (1.28 \pm 0.08) \ln W^2\). This is quite consistent with what one finds for, say, \(\pi^- p\) interactions. It is probable that for a large \(W^2\)-range one has to add a term of the type \(C \ln^2 W^2\).

The forward and backward mean hadron multiplicities are shown in Fig. 8.3 for \(\nu p\), \(\nu n\), \(\bar{v}p\) and \(\bar{v}n\) interactions. Forward and backward refer to the hadronic c.m. system and correspond to
$z_F > 0$, $z_F < 0$ respectively. An increase of all multiplicities with increasing $W^2$ is observed. The increase is of the $A + B \ln W^2$ type and is larger for the forward than for the backward direction. The difference in slope between the forward and backward multiplicities is interpreted as an effect of the mass of the baryon in the backward hemisphere. This effect arises when the baryon rest energy is a considerable fraction of the total energy. At higher energies the differences should disappear.

8.3 $z_F$-distributions.

The invariant $z_F$-distributions are defined as

$$F(z_F) = \frac{1}{N} \frac{E^*}{\pi p_{l\text{max}}^*} \frac{dN}{dz_F}$$

(8.1)

where $N$ is the total number of events, $E^*$ is the c.m. energy of each particle and $p_{l\text{max}}$ is the maximum longitudinal c.m. momentum. The analysis of the $z_F$-distributions of Fig. 8.4 indicates that pions are produced preferentially forward and that there are great similarities in the production of pions with "leading" charge (positive particles for neutrino interactions, negative for antineutrinos). Forward pion production is qualitatively interpreted as due mainly to the following diagram (which leads to a leading $\pi^+$ in the forward hemisphere):

![Diagram of $z_F$-distributions](image)

The invariant distributions may be fitted to functions of the type

$$F(z_F) = a(1 - |z_F|)^n$$

(8.2)

In the forward hemisphere one obtains values of $n \approx 1.6$ for leading charges and $n \approx 3.2$ for non-leading charges.

8.4 $p_T^2$-distributions.

The $p_T^2$-distributions of charged hadrons are approximately exponential, Fig. 8.5. The average transverse momentum, $<p_T^2>$, increases with $W^2$, reaching probably a plateau (see Fig. 8.6; Note the good agreement between various measurements).

The analysis of the transverse momentum balance is sensitive to the influence of soft gluons and to the quark primordial transverse momentum inside the struck nucleon. The various analyses point to a primordial $KT \approx 400 \text{MeV}$. 
Fig. 8.4 - Invariant distributions for positive and negative pions produced in $\nu p$, $\nu n$, $\bar{\nu}p$ and $\bar{\nu}n$ interactions\cite{40}. a-d refer to the production of particles with leading charge, e-h to nonleading charges. Applied cuts are: $W > 3$ GeV, $Q^2 > 1$ (GeV/c)$^2$ and $z > 0.15$. The curves are the predictions of the Lund model.

Fig. 8.5 - $p_T$ Distributions of charged hadrons in two $W^2$ intervals. The data come from $\nu D$ and $\bar{\nu} D$ experiments\cite{51}. 
8.5 Fragmentation functions.

The fragmentation function of a quark $q$ into a pion $\pi$ is defined as

$$D_q^\pi (z) = \frac{1}{N} \frac{dN}{dz}$$  \hspace{1cm} (8.3)

where $z = E/\nu = E/E_H$ (=Fraction of the total energy transfer carried by the final pion (hadron)) has a role similar to that of $x$ for the structure functions.

The experimental data (see Fig. 8.7) indicate specific forms for the fragmentation functions and also that there are differences between leading and non leading charges as for the $x_F$-distributions\textsuperscript{[40, 42]}.

The $Q^2$-dependence of the fragmentation functions may be interpreted in terms of QCD, for instance in terms of higher twists.

Fig. 8.7 - Fragmentation functions for positive and negative hadrons in $\nu D$ and $\bar{\nu} D$ interactions \textsuperscript{[10]}. Applied cuts: $W^2 > 5 (GeV)^2$, $Q^2 > 1 (GeV/c)^2$. The lines represent the Lund model predictions.
In many of the graphs are shown some curves labelled "Lund". They refer to the predictions of the Lund model, which is applied in a Montecarlo form that generates complete events. The model considers meson and baryon production in the current and target fragmentation regions. The Montecarlo takes into account the neutrino energy spectra and experimental resolutions. It leads to predictions which are in general agreement with the experimental data. But in order to obtain agreement one may have to change some internal parameters from their default values (for instance the probability to create $q - \bar{q}$ pairs).

8.6 $\rho^0$ production.

Experimental studies of high energy interactions have shown that a large fraction of the produced pions comes from the decays of meson and baryon resonances. Thus the study of resonance production may yield direct information on the quark fragmentation processes.

Fig. 8.8 shows the $\pi^+\pi^-$ effective mass distribution for the following inclusive charged current reactions

$$\bar{\nu}_\mu + D \rightarrow \mu^+ + (\pi^+ + \pi^-) + X$$  \hspace{1cm} (8.4a)

$$\nu_\mu + D \rightarrow \mu^- + (\pi^+ + \pi^-) + X$$  \hspace{1cm} (8.4b)

Fig. 8.8 - The invariant $(\pi^+\pi^-)$ mass distribution for the events from the inclusive processes (8.4)\(^{[44]}\). The solid line is a fit of the distribution to a background term plus a Breit-Wigner formula, see text. The dashed line shows the background term in the $\rho^0$ mass region.
The $\pi^+\pi^-$ mass distributions were fitted to an expression which represents a background plus a Breit-Wigner form times a phase space distribution (assumed to be equal to the background)$^{[43,44]}$:

$$\frac{dN}{dm_{\pi\pi}} = (1 + \alpha_1 BW')BG$$

(8.5)

The background, $BG$, was assumed to have the following form

$$BG = \alpha_1(m_{\pi\pi} - 0.28)^2exp(\alpha_2 m_{\pi\pi})$$

(8.6)

which takes into account the threshold effect (at $m_{\pi\pi} = 2m_\pi = 0.28$ GeV) and the exponential fall-off of the distribution at high values of $m_{\pi\pi}$. The Breit-Wigner form for the $\rho^0$ was written as

$$BW = \frac{\Gamma_\rho^2}{(m_{\pi\pi}^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2}$$

(8.7)

with

$$\Gamma_\rho = \Gamma_{\rho\rho} \left(\frac{p}{p_{\rho\rho}}\right)^{2l+1} \rho(m_{\pi\pi}) \rho(m_\rho) = \frac{1}{p^2 + p_{\rho\rho}^2}$$

(8.8)

where $p$ is the pion momentum in the $\pi\pi$ centre of mass system; $p_{\rho\rho}$ is the pion momentum at $m_{\pi\pi} = m_\rho$. The following values for the $\rho^0$ mass, width and $p_{\rho\rho}$ were used: $m_\rho = 0.769$ GeV, $\Gamma_{\rho\rho} = 0.154$ GeV, $p_{\rho\rho} = 0.358$ GeV/c. The four parameters, $\alpha_1$, $\alpha_2$, $\alpha_3$, $\alpha_4$, have to be determined from fits. The best results were obtained when the $0.5 < m_{\pi\pi} < 1.6$ GeV mass interval was used. The average number of $\rho^0$ per CC event was found to be $0.222 \pm 0.014$.

The fragmentation function for $\rho^0$ production is shown in Fig. 8.9. For $z = 0.7$ the muon data on $\rho^0$ production yield values which are similar to those from neutrino interactions$^{[43]}$. The

![Fig. 8.9 - The differential distribution for $\rho^0$ production in CC events plotted versus the $z$ variable$^{[43]}$. The lines are as in Fig. 8.10.](image)
ratio $< n_{\rho^+} > / < n_{\pi^-} >$ is very small for $z = 0$, it increases with $z$ and reaches values around 0.6 - 1 for $z = 0.7$.

There is a considerable difference in the $x_F -, z -, \text{ and } p_T^2 -$-distributions of $\rho^0$ production compared to those for negative particles ($\pi^-$). The three distributions for $\pi^-$ are steeper than those for $\rho^0$: $\pi^-$ production is concentrated at smaller $x_F$, $z$ and $p_T^2$ values than for $\rho^0$. This is a strong indication that the $\rho^0$'s are produced directly, while the majority of $\pi^-$ come from the decays of resonances, such as the $\rho^0$. A rough estimates indicates that more than 60% of the charged pions come from resonance decays.

The main features of $\rho^0$ production are similar in hadron-hadron, lepton-hadron and positron-electron reactions. In particular the dependence on $W^2$ is of the $A + B \ln W^2$ type, with $B \approx 0.10$ in all cases, but $\bar{p}p$.

### 8.7 Production of neutral strange particles in $\nu N$, CC interactions.

A large bubble chamber, like BEBC or the 15 ft, is particularly suited for the study of neutral strange particles ($K^0$, $\Lambda^0$, which appear as $\nu \overline{\nu}$)[40, 47].

The $x_F$ differential distributions for $\rho^0$ production, given in Fig. 8.10, show that $\rho^0$ mesons are produced preferentially in the forward direction, that is in the direction of the current. The $x_F$-dependence for $\rho^0$ production is weaker than for charged pion production, Fig. 8.4.

![Fig. 8.10. The differential distribution for $\rho^0$ production in CC events as a function of the Feynman $x_F$-variable[44]. The lines represent smooth curves of the $x_F$ distributions for negative particle production in charged current $\nu D$ (dashed curve) and $\bar{\nu} D$ interactions (full line).](image-url)

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[40, 47]: References to the original sources.
Fig. 8.11 - Fractions (relative to the CC interactions) of inclusive $CC + K^0$ and $CC + \Lambda$ events [$F(K^0)$ and $F(\Lambda)$] as a function of incident neutrino (antineutrino) energy, $E_\nu$ [47]. Production of: (a) $\Lambda$'s in $\nu p$ and $\nu n$, (b) of $\Lambda$'s in $\bar{\nu} p$ and $\bar{\nu} n$, (c) of $K^0$'s in $\nu p$ and $\nu n$, (d) of $K^0$'s in $\bar{\nu} p$ and $\bar{\nu} n$ interactions. The dashed and solid lines represent the prediction of the Lund fragmentation model for interactions on neutrons and protons respectively.

The fractions of charged current events containing a $V^0$ (either $K^0$ or $\Lambda^0$) relative to all charged current events are shown in Fig. 8.11 as function of neutrino energy for $\nu p$, $\nu n$, $\bar{\nu} p$ and $\bar{\nu} n$ interactions. $\Lambda^0$ production is essentially independent of neutrino energy (as is independent of $Q^2$ and $W^2$) and it occurs on the average at the level of 7% of the CC events. $K^0$ production is slowly rising with neutrino energy (it rises also with $Q^2$, $W^2$,) and it occurs at the level of 25% of the CC events. $\bar{\Lambda}$ is at the level of 0.4%. The inclusive production of two $V^0$'s is at the level of 7%, of which 4% $K^0\bar{K}^0$ and 3% $K^0\Lambda^0$.

In the hadronic c.m. frame the $K^0$ are produced mostly forward and the $\Lambda$'s mostly backward (see Fig. 8.12); the asymmetry parameters (defined as $A = (N_F - N_B)/(N_F + N_B)$ are $A(K^0) \approx 0.43$ and $A(\Lambda) \approx -0.60$. One also finds abundant production of $K^+(898)$ and $\Sigma^\pm(1385)$, as shown in Fig. 8.13.

Some of the qualitative features of the experimental data may be explained in the framework of the quark-parton model, where the exchanged intermediate charged boson interacts mainly with a $d(u)$ quark. Two processes on valence quarks should be considered. One corresponds to associated strange particle production

$$\nu + d_{\nu} \rightarrow \mu^- + u + s + \bar{s}, \quad (8.9a)$$

$$\bar{\nu} + u_{\bar{\nu}} \rightarrow \mu^+ + d + s + \bar{s}; \quad (8.9b)$$

the other corresponds to single strange particle production

$$\bar{\nu} + u_{\bar{\nu}} \rightarrow \mu^+ s. \quad (8.10)$$
Fig. 8.12 - Inclusive distributions normalized to the total number of CC events for $K^0$ and $\Lambda$ production as a function of the Feynman variable $x_F$ (dashed and solid lines as in Fig. 8.8)[47].

Fig. 8.13 - Invariant mass distributions for (a) the $\Delta\pi^+$ system in the $\nu p$ reaction and the $\Delta\pi^-$ system in the $\bar{\nu}n$ reaction; (b) the $\Delta\pi^+$ system in $\bar{\nu}p$, $\bar{\nu}n$, $\nu n$ interactions and $\Delta\pi^-$ system in $\nu p$, $\nu p$, $\nu n$ interactions[47]. The smooth lines represent the estimated background.
No conclusion has been drawn about the relative importance of these reactions. The emerging quarks fragment into forward going hadrons (current fragmentation) and the remaining spectator diquark system fragments into backward going hadrons (target fragmentation). This picture, as illustrated below for reaction 8.9a, leads to the prediction of a predominant backward production of Λ's and to both backward and forward K^0 production, as is experimentally observed.

More quantitative predictions may be obtained from various models, like the Field and Feynman model and the Lund fragmentation model. The predictions of these models, given via Montecarlo computer programs, reproduce well the dominant features of neutral strange particle production.

8.8 Bose-Einstein correlations

Among the many other features of the hadronic system we shall briefly mention the Bose-Einstein correlations. These allow to determine the size of the region from where pions originate in high energy collisions. The interference originates from the ambiguity in the paths of the two pions. The amplitude arising from the pion produced by source α(β) hitting detector A(B) interferes with the amplitude arising from the pion produced by source β(α) hitting detector A(B). (One does not know which source gave which π). The interference depends on the dimensions of the source and on the detectors positions. It is constructive for identical pions and is zero for non-identical pions.

\[
A \approx e^{ik\alpha_1} e^{ik\beta_1} + e^{ik\alpha_2} e^{ik\beta_2}, \quad I \approx \langle A \rangle^2
\]  

(8.11)

For a source of radius R, which emits pions with an intensity decreasing with the distance from the centre as \( \exp \left( -\frac{r}{R} \right) \), one expects

\[
I = \frac{N_L}{N_U} = \gamma \left( 1 + \alpha e^{-\beta q^2} \right)
\]  

(8.12)

where \( N_L \) and \( N_U \) are the distributions for like-pairs and unlike-pairs (considered equal to the uncorrelated distribution); \( q = |p_{\pi_1} - p_{\pi_2}| \) in the plane perpendicular to \( p_{\pi_1} \cdot p_{\pi_2} \), \( q_0 = E_1 - E_2 \). \( \gamma \) is a normalization constant, \( \alpha \) may be called the chaoticity parameter (\( \alpha = 1 \) for maximum chaoticity, \( \alpha = 0 \) for no chaoticity) and \( \beta \) is connected with the radius of the emitting region, \( R = 0.197\sqrt{\beta} \).
Fig. 8.14 - The $N_L(q_t)$ distribution for like-pairs divided by the $N_U(q_t)$ for unlike-pairs for $\nu_\mu D$ and $\bar{\nu}_\mu D$ interactions (WA25, very preliminary). The line represents a fit to Eq. 8.12.

In fig. 8.14 the ratio of the $q_t$ distribution for like-pairs relative to unlike-pairs is plotted for $\nu D$ and $\bar{\nu}D$ interactions. The data show, within their large statistical errors, the expected behaviour, with a Gaussian-like maximum at small values of $q_t$, superimposed on a background, essentially flat in $q_t$. A fit of the data to Eq. 8.12 gave $R \approx 0.8$ fm.

8.9 Search for $\mu^+\pi^\pm$ mass enhancements

This subject is not strictly connected with the study of the hadronic system. It is discussed here as an example of the many (unsuccessful) searches made in neutrino-nucleon interactions.

A structure (a peak) in the $\mu\pi$ mass spectrum could be due to the decay of a short-lived massive neutral lepton. In order to be observed in a bubble chamber experiment, the neutral heavy lepton should be produced in neutrino-nucleon interactions and should subsequently decay in the semileptonic channel $\mu\pi$. If the neutral-heavy-lepton mean life is smaller than about $10^{-12}$ such events do not differ topologically from usual charged-current events.

Indications for an enhancement near 430 MeV in $\mu^\pm\pi^\mp$ mass spectra were reported by a number of authors$^{[50]}$. More recent, higher statistics data do not show, any indication for such state, see for instance Fig. 8.15.

Fig. 8.15 - Invariant-mass distributions for $\mu^+\pi^-$ and $\mu^-\pi^+$ in a) 10-MeV and b) 50 MeV bins for $\nu D + \bar{\nu}D$.$^{[51]}$
The data may yield upper limits; for instance, at the level of three standard deviations they may be computed as \(3\sqrt{2N_k/(0.8N_{CC}F)}\), where \(N_k\) is the number of mass combinations in bin K; F takes account of losses in the number of mass combinations due to experimental cuts; \(N_{CC}\) is the number of CC events, and the factor 0.8 arises from the tails of the assumed mass peaks. The mass resolution must be comparable or smaller than the bin size (it is found to be 9 MeV around \(m_{\nu\pi} \approx 500\) MeV). The upper limits vary from \(8 \cdot 10^{-8}\) to \(1.5 \cdot 10^{-8}\) for both \(\mu^+\pi^-\) and \(\mu^-\pi^+\) masses.

9. CONCLUSIONS

Let us recall the main qualitative features that we have learned from lepton-hadron scattering. The measurements of the charge distribution in nuclei (via elastic scattering) revealed details of the nuclear structure. The observation of quasi-elastic peaks in electron-nucleus scattering verified the presence of nucleons in nuclei. Deeper scattering of electrons, of neutrinos and of muons from nucleons demonstrated that the neutron and the proton are composite of quarks, and that the neutron is not all neutral. The energy-conservation sum rules yielded evidence that there are also neutral constituents in the nucleons (the gluons).

The comparison of charged lepton-nucleon and neutrino-nucleon scattering gave a measure of the mean charge of the constituents which interact weakly and electromagnetically. The picture is consistent with the existence of point-like, fractional-charge quarks.

The features of strange and charm particle production gave evidence for the existence of a "sea" of quark-antiquark pairs in the nucleon.

Finally: none of these experiments has provided evidence for free quarks; this led to our conviction that quarks are permanently confined inside hadrons, or at least that they are extremely difficult to liberate.

The building blocks of "elementary" particles are quarks and leptons; the electroweak interactions are well understood and also QCD seems to be in good shape. But there is some arbitrariness and things unexplained. For instance the GWS "standard" theory does not predict the value of the weak mixing angle, \(\theta_w\), which has to be determined by experiments (it is actually remarkable that experiments at so many different energies give the same result). Details of the neutral current have still to be explored thoroughly. There is no explanation of the lepton spectrum. Quarks and leptons look so similar that they eventually should be put in a single extended family. Some of the larger unification schemes, GUT and/or SUSY may be the answer to these problems.

As already stated, at the present level of experimentation, the quarks and leptons appear to be the basic constituents of matter, at the level of approximately \(10^{-\text{16}}\) cm. But this does not mean that structures could not be expected at a lower level. If, for example, the quarks are composite, the nucleon structure functions may start changing when the appropriate threshold or degree of resolution is reached. A possible evolution is shown in Fig. 9.1, where are shown the cross sections \(d\sigma/dx_i\) for quarks and for nucleons. The \(d\sigma/dx_i\) for an individual quark at
Fig. 9.1 - A possible scenario for quark substructures and the consequences for the proton structure functions. The size of the quarks is denoted by $r$. The top three figures indicate the $x$-dependence for each individual quarks at "low" energies (top left), immediately above a threshold for exciting the quarks (top center) and at "high" energies (top right). The three bottom graphs are the corresponding $x$-dependences for the nucleon structure functions.

Present values of $Q^2$ is a $\delta$-function at $x_i = 1$, because quarks are point-like and can only scatter elastically. The corresponding $d\sigma/dx$ or hadron structure function is of the kind we have observed in present experiments (see for example Fig. 6.3).

If at higher $Q^2$ the quarks can be excited into resonant states, or shaken apart, the cross section $d\sigma/dx_i$ would extend to lower values of $x_i$ and would contain some ripples, as in electron-proton cross sections in the resonance region. The corresponding proton structure function would start narrowing in $x$. Far above threshold, the quark $d\sigma/dx_i$, or better its structure function, should resemble the proton structure functions of today. The corresponding proton structure functions will move to much smaller values of $x$. This would signal a new level of structure, which would have rather profound implications.

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SUPERSYMMETRY

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1. INTRODUCTION
1.1 Why Higgs?

It has been said that those who do not know history are condemned to repeat it. One might add that historical knowledge is no guarantee that repetition can be avoided. In the case of particle physics, there has been a recurrent theme during recent years, whose prospective reappearance is the main topic of these lectures. The central idea is that whenever you encounter a divergence, you introduce a new particle (or particles) to cancel it.

A classic example is the $W^\pm$. The four-fermion effective interaction, seen in Fig. 1a, gave weak interaction cross-sections which grew with energy:

$$\sigma \left( \bar{f}f \to \bar{f}f \right) \sim \xi^2 \frac{S}{16\pi \alpha} \frac{S}{m^2}, \quad S = E^2_{cm}$$

(1.1)

When one tried to calculate higher-order diagrams such as that in Fig. 1b, they were uncontrollable, since the integral over the internal loop momentum $k$ was quadratically divergent:

$$\frac{S^2}{16\pi} \propto \int \frac{d^4 k}{k^2}$$

(1.2)

The remedy was to introduce $[1]$ into Fig. 1a a vector gauge boson as in Fig. 1c, whose exchange modified the high-energy behaviour (1.1):

$$\sigma \left( \bar{f}f \to \bar{f}f \right) \propto \frac{\alpha^2}{S} \quad : \quad \alpha = \frac{\alpha}{4\pi}$$

(1.3)

The gauge-coupling $g$ is now dimensionless, and is related to the original dimensional coupling $G_F$ by

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\pi \alpha}$$

(1.4)

which provides the estimate $m_W = 80(100)$ GeV for $\alpha^2 \sim 1/30$. The modification of (1.1) to the form (1.3) meant that the loop integral of Fig. 1b was also modified (see Fig. 1d) and became finite. Whilst other higher-order diagrams were still infinite, these were controllable in the sense that the theory could be made renormalizable by introducing $[2]$ the Higgs particle, which we will meet shortly.

Before doing so, let us briefly recall another particle which was postulated in order to arrange cancellations in higher-order diagrams, and whose existence has also been confirmed by experiment, namely charm $[3]$. Diagrams such as that in Fig. 1d gave flavour-changing neutral interactions of the form

$$\mathcal{L}_{\text{eff}} \sim \mathcal{O}(\xi_F \alpha) \bar{c} \bar{d} (\bar{c} \bar{d})$$

(1.5)

that were much larger than experiment, which could only accommodate $[4]$
\[ \mathcal{L}_{\text{eff}} \propto \mathcal{O} \left[ g_F \left( \frac{5 \text{GeV}^3}{m_W^2} \right) \alpha \right] (\bar{s}d)(\bar{d}d) \] \hfill (1.6)

The answer to this apparent contradiction was to postulate \([3]\) the charm quark which could also circulate in the loops of Fig. 1d and reduce formula (1.5) to

\[ \mathcal{L}_{\text{eff}} \sim \mathcal{O} \left[ g_F \left( \frac{m_c^3 - m_u^3}{m_W^2} \right) \alpha \right] (\bar{s}d)(\bar{d}d) \] \hfill (1.7)

As you know, the charm quark with mass \(m_c \sim 1.5 \text{ GeV}\) was duly found.

This completes our brief review of past history. How about future history? Once one has introduced the \(W^\pm\), one must also worry about \(O(\bar{f}f + \bar{W}^+ W^-)\). Unfortunately, the cross-section for producing longitudinally polarized \(W^\pm\) still blows up at high energies:

\[ \sigma(\bar{f}f \to W^+_L W^-_L) \sim \left( \frac{\alpha^2}{s} \right) \left( \frac{s}{m_W^2} \right)^2 \sim \frac{s}{m_W^4} \] \hfill (1.8)

because of the uncooperative high-energy behaviour of the longitudinal polarization vector \(E = |p|/m_W\). However, the high-energy behaviour (1.8) is easily tamed by including diagrams like that in Fig. 1e, which involve the non-Abelian coupling of three gauge bosons [5]. They give the acceptable behaviour

\[ \sigma(\bar{f}f \to W^+_L W^-_L) \sim \frac{\alpha^2}{s} \] \hfill (1.9)

if \(m_f = 0\), but still have an unpleasant high-energy behaviour

\[ \sigma(\bar{f}f \to W^+_L W^-_L) \sim \frac{\alpha^2}{s} \left( \frac{s}{m_W^2} \right) \left( \frac{m_F}{m_W^2} \right) \] \hfill (1.10)

if \(m_f \neq 0\), which could give uncontrollable infinities when higher-order diagrams were calculated. The solution [5] is to introduce a Higgs boson [2], as in Fig. 1f, with couplings

\[ g_{\bar{f}f} \times g_{HWW} \sim g_{W^+_L W^-_L} \] \hfill (1.11)

which cancels the undesirable behaviour (1.10), and leaves us with the desirable behaviour (1.9). One can go further, analysing \(O(WW + WW)\) and the loop diagrams it involves. Then one can [5] fix independently

\[ g_{\bar{f}f} \sim g_{m_F/m_W}, \quad g_{HWW} \sim g_{m_W} \] \hfill (1.12)

but the mass of the Higgs boson is not tightly specified by these arguments, unlike the previous cases of the \(W^\pm\) and charm quark whose masses were predicted in advance quite precisely.

This approach to the introduction of the Higgs boson has been resolutely phenomenological. For a more conventional discussion of the Higgs mechanism for giving masses to vector bosons, see Ref. [6]. A massless vector boson has only two helicity states (+1), whereas a massive vector boson also needs a third helicity-0 state. This is provided by a
massless spin-0 boson, called a Goldstone boson, which is 'eaten' by the massless vector, giving it a mass. This conventional Higgs mechanism serves as a model for the super-Higgs mechanism to be discussed in Lecture 2.

There are some subsidiary arguments why one should expect

$$m_H = m_W \times O(\alpha^{-1})$$

(1.13)

and hence $m_H = 0(10$ to $1000)$ GeV. The upper bound $m_H < m_W/\alpha$ comes from the demand that the Higgs boson does not have strong self-couplings [7]. In the minimal Weinberg-Salam model [2], the Higgs potential

$$V(H) = -\mu^2 |H|^2 + \lambda(|H|^2)^2$$

(1.14)

is minimized when

$$\nu \equiv \langle 0 | H | 0 \rangle = \frac{\mu}{\sqrt{2} \lambda}$$

(1.15)

which yields

$$m_W = \frac{g}{\sqrt{2}} \langle 0 | H | 0 \rangle = \frac{g}{\sqrt{2}} \mu$$

(1.16)

whereas

$$m_H = \sqrt{2} \mu$$

(1.17)
Taking the ratio between the $W$ mass (1.16) and the Higgs mass (1.17), we find

$$\frac{m_H}{m_W} = 2\sqrt{2} \left( \frac{\lambda}{\alpha} \right)$$  \hspace{1cm} (1.18)

If we want to avoid strong coupling for the Higgs, so that $\lambda \lesssim O(1)$, Eq. (1.18) tells us that $m_H \approx m_W (\alpha/\lambda)^{-1} \approx 1$ TeV [7]. Equation (1.18) can also be used to derive a lower bound on $m_H$ [8]. Loop diagrams such as that in Fig. 1g give an unavoidable contribution to $\lambda$ which is

$$\delta\lambda \sim O(\alpha^2) \sim O(\alpha^2) \propto \lambda$$  \hspace{1cm} (1.19)

Combining Eqs. (1.19) and (1.18) we find that $m_H \gtrsim m_W (\alpha/\lambda) \sim 10$ GeV [8].

Thus the same arguments that brought you charm and the $W^\pm$ should also bring you the Higgs, with a mass within an order of magnitude of $m_W$.

1.2 Why supersymmetry?

Many different motivations for supersymmetry (SUSY) [9] have been mooted by different people at different times. It is beautiful. It is the only symmetry not yet used. It unifies matter and force. It reduces the divergences of quantum gravity. However, our motivation for postulating SUSY will be the hierarchy problem [10], namely that of reconciling 'small' mass scales such as $m_W$ with 'large' ones such as the grand unification scale $m_X = 10^{15}$ GeV or the Planck scale $m_p$ associated with gravitation: $G_N = 1/m_p^2$, $m_p = 10^{19}$ GeV. The hierarchy problem occurs at two levels: one is that of creating the hierarchy, i.e. the origin of $m_W$, and the other is that of maintaining it once it has been created, i.e. the natural stability of $m_W$. We will actually be concerned with this second problem of naturalness.

The value of a physical parameter is said to be natural [11] if quantum corrections to it are no larger than its physical value. A classic example is a fermion mass $m_f$, which gets corrections from the loop of Fig. 2:

$$\delta m_f = O(\alpha/\pi) m_f \ln(m/m_f)$$  \hspace{1cm} (1.20)

where $\Lambda$ is the cut-off in the loop integral. As long as $\Lambda \lesssim m_f \exp O(\alpha/\pi)$, $\delta m_f \lesssim m_f$ and the fermion mass parameter can be naturally small. This natural protection is actually a relic of the chiral symmetry [12] which is present as $m_f \approx 0$.

In our case, the smallness of $m_W$ is linked [see Eq. (1.13)] to the smallness of $m_H$, which is where the problem arises, since $m_H$ is notoriously unstable [10, 11] in the

Fig. 2 Loop diagrams contributing quantum corrections to $m_f$. 

Fig. 3  a) Quadratically divergent loop diagrams contributing to $\delta m^2_H$: b) GUT Higgs contributions to $\delta m^2_H$, and c) loop diagram renormalizing coupling to GUT Higgs.

Standard Model. Quantum corrections $\delta m^2_H$ to $m^2_H$ due to loops like those in Fig. 3a are quadratically divergent:

$$\delta m^2_H = g^2 \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} = O\left(\frac{\alpha}{\pi}\right)\Lambda^2$$  \hspace{1cm} (1.21)

where $\Lambda$ is a cut-off representing the threshold for some new physics beyond the Standard Model. In order for $\delta m^2_H$ [Eq. (1.21)] to be less than $m^2_H$, and hence technically natural [11], we need

$$\Lambda \approx O(1) \text{ TeV}$$  \hspace{1cm} (1.22)

Some physicists say: Why worry? after all, the divergences (1.21) are renormalizable, and hence can be absorbed into the specification of the bare parameters. However, one is entitled to be uneasy about corrections that are much larger than the observed value of $m^2_H$, and there is evidence that the large corrections (1.21) are symptoms of a serious disease which could be fatal in models which are more unified. For example, in Grand Unified Theories (GUTs), the Weinberg-Salam Higgses must couple [13] to 'heavy' Higgses $\varphi$ with vacuum expectation values $\langle 0 | \varphi | 0 \rangle = O(m_X)$, as illustrated in Fig. 3b. These $H-\varphi$ couplings therefore give

$$\delta m^2_H = \lambda_{H\varphi} \langle 0 | \varphi | 0 \rangle^2 = \frac{m^2_H}{m_X} \gg O\left(10^{15} \text{ GeV}\right)^2$$  \hspace{1cm} (1.23)

which is much larger than the allowable range (1.13). Of course, one could postulate that Eq. (1.23) is cancelled by the bare Higgs mass through some bizarre symmetry mechanism, but none has yet been found. In any case, radiative corrections such as those in Fig. 3c would change the effective coupling $\lambda_{H\varphi}$ and regenerate

$$\delta m^2_H = O\left(\frac{\alpha}{\pi}\right) m_X^2 \gg O\left(10^{15} \text{ GeV}\right)^2$$  \hspace{1cm} (1.24)
Whatever symmetry mechanism is postulated must also deal with radiative corrections to \(O(a^{12})\). One may not like GUTs and hence reject the above argument, but then one must still reckon with the effects of quantum gravity, which have been argued [14] to give corrections

\[
\delta m_H^2 = O(m_F^2) = O(10^{19} \text{ GeV})^2
\]  

(1.25)

to elementary scalar masses.

What strategies have been proposed for taming the large unnatural corrections to \(m_H^2\) [Eqs. (1.21), (1.23), (1.24), and (1.25)]? One strategy [15] is to dissolve the Higgs, making it composite on a distance scale

\[
\Lambda_T = O(\text{TeV}) \quad \Lambda_T \sim 1 \text{ TeV}
\]  

(1.26)

This would introduce a cut-off \(\Lambda - \Lambda_T\) in the loops of Fig. 3a. The scale of compositeness is where new technicolour interactions become strong. If the constituents of the Higgs are massless fermions, there is a chiral symmetry, analogous to that responsible for small \(m_F\) being natural [Eq. (1.20)], which preserves some scalars as massless Goldstone bosons to be eaten by the \(W^\pm\) and \(Z^0\) to give then their masses through the traditional Higgs mechanism [2, 6]. This scenario is very elegant, but it encounters difficulties when one tries to get non-zero quark and lepton masses [16]. Solutions to this problem involve extensions of the technicolour model which lead to problems with unseen flavour-changing neutral-current interactions [17] and light, charged, composite scalar particles [18]. For these reasons, such technicolour models [19] have largely been abandoned by theorists, although there is no no-go theorem excluding technicolour models.

The other strategy [10] is to cancel the loops of Fig. 3a among themselves. The boson and fermion loops have opposite signs,

\[
\delta m_H^2 = O\left(\frac{\alpha}{\pi}\right) \left[ \left( \Lambda^2 + O(m_B^2) \right)_{\text{Bosons}} - \left( \Lambda^2 + O(m_F^2) \right)_{\text{Fermions}} \right]
\]  

(1.27)

traceable to the difference between Bose-Einstein and Fermi-Dirac statistics. The idea is to postulate boson/fermion pairs with the same quantum numbers and hence identical couplings, and similar masses. Then the diagrams of Fig. 3a give a residual

\[
\delta m_H^2 = O\left(\frac{\alpha}{\pi}\right) \left| m_B^2 - m_F^2 \right|
\]  

(1.28)

which is 'naturally' small: \(\delta m_H^2 \lesssim m_H^2\) if the effective cut-off

\[
\Lambda^2 \sim \left| m_B^2 - m_F^2 \right| \lesssim O(1 \text{ TeV})^2
\]  

(1.29)

Identical couplings and similar masses mean approximate SUSY, which would also remove the problems (1.23), (1.24), and (1.25) associated with GUTs and quantum gravity. The naturalness condition (1.29) means that supersymmetric partners of the known particles can weigh no more than \(O(1)\) TeV.
1.3 What is SUSY?

Supersymmetry is the last possible symmetry of the S-matrix. All previous internal symmetries such as SU(3), SU(2), U(1), etc., had scalar charges interrelating different particles of the same spin:

$$\left| Spin \right> \rightarrow \left| Spin \right>$$

(1.30)

Indeed, a no-go theorem was proven [20] which demonstrated the impossibility of mixing internal symmetries and Lorentz space-time symmetries in a non-trivial way. The basic ingredient in this 'proof' was the demonstration that the only possible conserved tensor charges are the four-momentum $p_\mu$ and Lorentz scalars. There is an example which illustrates the principle underlying this demonstration. Consider 2-to-2 scattering: $1 + 2 + 3 + 4$, and suppose there is a conserved tensor charge $I_{\mu \nu}$. Lorentz invariance allows only the following form for its diagonal matrix elements:

$$\langle a | S_{\mu \nu} | a \rangle = A p_\mu^a p_\nu^a + B q_\mu q_\nu$$

(1.31)

Conservation of $I_{\mu \nu}$ in 2-to-2 scattering would require

$$p_\mu^1 p_\mu^1 + p_\mu^2 p_\mu^2 = p_\mu^3 p_\mu^3 + p_\mu^4 p_\mu^4$$

(1.32)

which is only possible if $p_\mu^1 = p_\mu^3$ or $p_\mu^2 = p_\mu^4$, corresponding to forward scattering only. An S-matrix with only forward scattering would not be analytic, and thus conflict with basic results of analyticity and unitarity [20]. Clearly the above argument can be extended to conserved charges with more tensor indices. However, it does not apply to spinor charges $Q_{\alpha}$ which have no diagonal matrix elements in a state containing a particle with a definite spin: $\langle a | Q_{\alpha} | a \rangle = 0$.

What is the possible form of a SUSY algebra containing several such spinorial charges $Q_{\alpha}$? To be a symmetry, we must have

$$[Q^i, H] = 0$$

(1.33)

and hence

$$\left\{ Q^i, Q^j \right\}, H = 0$$

(1.34)

etc. Then by the previous no-go theorem [20],

$$\left\{ Q^i, Q^j \right\} \propto p_\mu$$

(1.35)

where $p_\mu$ is the momentum operator. [There could in principle also [21] be additional Lorentz scalar 'central' charges $Z$ on the right-hand side of formula (1.35), but these seem to play no phenomenological role. They are in any case absent for simple $N = 1$ SUSY, and so will be neglected hereafter.] In fact, it has been shown [21] that the essentially unique form of formula (1.35) is
\[ \left\{ Q^i_\alpha, Q^j_\beta \right\} = 2i \hat{\gamma}^\mu \left( \mathcal{P} C \right)^{\alpha\beta} \]  

where \( \gamma^\mu \) and \( C \) is the charge-conjugation matrix.

In the Weyl representation, \( C \) is defined by

\[ \gamma^c = C \gamma_0 \gamma^* \quad C \gamma_\mu C^{-1} = -\gamma_\mu^T \]

and has the following form in the Bjorken-Drell [22] notation:

\[ C = -C^\dagger = -C^T = -C^T = i \gamma_2 \gamma_0 \]

In the Majorana representation, one has

\[ \gamma^c = \gamma^* \quad \gamma_\mu = -\gamma_\mu^* \quad C = \gamma_0 \]

with

\[ \gamma_0 = \begin{pmatrix} 0 & \sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} i \sigma_3 & 0 \\ 0 & i \sigma_3 \end{pmatrix}, \quad \gamma_2 = \begin{pmatrix} 0 & -\sigma_2 \\ \sigma_2 & 0 \end{pmatrix}, \]

\[ \gamma_3 = \begin{pmatrix} -i \sigma_3 & 0 \\ 0 & i \sigma_3 \end{pmatrix}, \quad \gamma_5 = i \sigma_0 \gamma_1 \sigma_2 \gamma_3 = \begin{pmatrix} \sigma_0 & 0 \\ 0 & -\sigma_0 \end{pmatrix} \]

Either of these representations may be useful in different circumstances.

1.4 A simple SUSY field theory

The simplest SUSY model [23, 24] contains just a free fermion and a free boson:

\[ L_0 = i \bar{\psi} \chi \psi + |\mathcal{F}|^2 \]

The spinorial transformation for a single SUSY charge Q corresponds to a single infinitesimal spinor E. The simplest possible transformation law for the scalar is:

\[ S \rightarrow S + \delta S \quad : \quad \delta S = E \psi \]

where \( \psi \) is taken to be a left-handed spinor, and \( E \) a right-handed one. The SUSY transformation law for the fermion can take the general form:

\[ \psi \rightarrow \psi + \delta \psi \quad : \quad \delta \psi = -a i (\bar{\delta} S) E - \mathcal{F} E^c \]

where \( F \) is a so-called 'auxiliary field' to be determined later. Combining Eqs. (1.42) and (1.43) we find the following transformation of the full Lagrangian (1.41):

\[ \delta L_0 = \mathcal{F} \left[ \bar{\psi} E \sigma^\mu S + E \sigma^\mu \sigma^* \chi \psi \right] \]

if

\[ a = 1, \quad F = 0 \]
(Later we will find a more complicated form for \( F \) when interactions are included.) The transformation (1.44) gives no net change in the action:

\[
\delta A = \int d^4x \, \delta L_0 = 0
\]  

(1.46)

Thus the simple model (1.41) exhibits SUSY in a trivial form.

Let us now go to a more complicated model [23, 24] with more scalar/fermion pairs \((S_i, \phi_i)\) and general interactions of the form

\[
L = L_0 + V(S_i, S_i^*) - \frac{1}{2} M_{jk}(S_i, S_i^*) \bar{\psi}^i \gamma^k \psi^k
\]  

(1.47)

We can determine what forms of \( V \) and \( M_{jk} \) are compatible with SUSY by considering [24] the different variations of the various terms in Eq. (1.47). For example, using (1.42) we see that the \( M \) term in Eq. (1.47) yields a piece,

\[
\delta (M \bar{\psi}^c) \equiv \frac{\partial M}{\partial S^*} \bar{\psi}^c \psi
\]  

(1.48)

which cannot be cancelled by the variation of any other term. We conclude that \( M_{jk} \) is a function of \( S \) alone, not \( S^* \). This leaves us with

\[
\delta (M \bar{\psi}^c) \equiv \frac{\partial M}{\partial S^*} \bar{E} \psi^i \bar{\psi}^c \psi^k
\]  

(1.49)

which vanishes if and only if \( \delta^2 M_{jk} / \partial S_i \partial S_i^* \) is symmetric in \( i, j, \) and \( k \). This is only possible if

\[
M_{jk} = \frac{\partial P}{\partial S^*} \partial S^k
\]  

(1.50)

for some function \( P(S_i) \) called the superpotential. There are other pieces in the variation of \( \Phi^c \) including

\[
\delta (M \bar{\psi}^c) \equiv M_{jk} \bar{\psi}^c \psi^k \bar{E} + M_{jk} \bar{\psi}^c (i \gamma^5 \gamma^k) \bar{E}
\]  

(1.51)

The second term in formula (1.51) can be cancelled by a term in the variation of \( \bar{\psi} \Phi_i \):

\[
\delta (\bar{\psi} \Phi_i) \equiv -i \bar{\psi} \gamma^i \bar{E} + (\text{herm. cond.)}
\]  

(1.52)

This cancellation occurs if

\[
\frac{\partial F^x_j}{\partial S^k} = M_{jk}
\]  

(1.53)

which, when combined with the previous result (1.50), tells us that

\[
F^x_j = \frac{\partial P}{\partial S^j}
\]  

(1.54)
Notice that although $F \neq 0$, it has no kinetic term in the Lagrangian and hence does not correspond to a physical particle. The first term in the variation (1.51) can be cancelled by the variation in the potential:

$$\delta V \equiv \frac{\partial V}{\partial \bar{S}^i} \bar{E} \psi^i + (\text{herm. conj.)}$$

(1.55)

if

$$\frac{\partial V}{\partial \bar{S}^i} = M_{ij} F^j$$

(1.56)

Combining (1.54) and (1.56) we see that

$$V = \left| \frac{\partial \bar{S}^i}{\partial \bar{S}^j} \right|^2 = |F|^2$$

(1.57)

and we have finally determined the form of the SUSY Lagrangian for many supermultiplets $(S^i, \Psi^i)$ with interactions. It is

$$L = i \bar{\psi} \gamma^\mu \psi_i + \bar{\psi} \gamma^\mu \psi_S^i - \left| \frac{\partial \bar{S}^i}{\partial \bar{S}^j} \right|^2 - \frac{i}{2} \frac{\partial \bar{S}^i}{\partial \bar{S}^j} \bar{\psi} \gamma^\mu \psi_S^k + (\text{herm. conj.)}$$

(1.58)

which is invariant under

$$\delta \bar{S}^i = \bar{E} \psi^i$$

$$\delta \psi^i = -i \bar{\psi} \gamma^\mu S^i E - F^i E^c$$

$$F^i = \left( \frac{\partial \bar{S}^i}{\partial \bar{S}^j} \right)^*$$

(1.59)

The simplest case [23] is that of a single supermultiplet $(S, \Phi)$ with the superpotential

$$W = \frac{\lambda}{3} S^3 + \frac{m}{2} S^2$$

(1.60)

Combining this simple case with the general form (1.58) we find

$$L = i \bar{\psi} \gamma^\mu \psi + \left| \frac{\partial \bar{S}^i}{\partial \bar{S}^j} \right|^2 - m S + \lambda S^2 \bar{\psi}^i \psi^i - \lambda S \bar{\psi}^i \psi^i$$

(1.61)

which has some interesting features worth noting.

Clearly the scalar and fermion masses are equal, as we might expect in view of the conservation (1.33) of the SUSY charge: $[Q, H] = 0$. Note also that the quartic scalar couplings are directly related to the Yukawa couplings of the scalar to the fermions:

$$\lambda S \bar{\psi}^i \psi^i \iff \left| S \right|^2 \left( S^2 \right)$$

(1.62)

In the real world, there are no pairs of fermions and bosons with the same mass, so we must break SUSY. However, we must be careful to preserve relations such as (1.62), which is a prototype for the relations between couplings of particles with different spins which are essential to the stabilisation of the gauge hierarchy [10].
2. STRUCTURE OF SUPERSYMMETRIC THEORIES

2.1 SUSY REPRESENTATIONS

We saw in the last subsection of the previous lecture the simplest example of a SUSY field theory containing spin-0 and spin-1/2 fields. We will now explore the general form of representations [24, 25] of the SUSY algebra, which we recall takes the form

\[ \{ \overline{Q}^i, Q^j_\beta \} = 2 \delta^{ij} (\theta_\alpha)_{\beta\gamma} \]  

(2.1)

It is convenient to look at representations for massless particles: although most particles are not massless, all the known ones have masses \( \ll m_P \), and seem to acquire their masses by spontaneous gauge symmetry breakdown, so zero mass seems a good place to start. For massless particles, the algebra (2.1) tells us that

\[ \{ (\overline{Q}^i)^\alpha, Q^j_\beta \} = 0 \]

(2.2)

because \( \theta_\alpha = P^2 = 0 \). We will try to make a harmonic oscillator-like raising and lowering algebra out of Eqs. (2.1) and (2.2). Consider first the case of a single SUSY charge \( Q \). The anticommutator (2.2) means that we can regard \( \theta \), which takes the form

\[ \theta = E \begin{pmatrix} Q_1 + Q_4 \\ Q_2 + Q_3 \\ Q_1 - Q_2 \\ -Q_1 + Q_4 \end{pmatrix} \]

(2.3)

in the Majorana representation, as a constant. Therefore we only need consider operators made out of \( Q_2 - Q_4 \) and \( Q_2 + Q_4 \), which we take to be

\[ S = \frac{1}{\sqrt{2}} \left[ (Q_2 + Q_4) + i (Q_2 - Q_4) \right] \]

(2.4)

and its complex conjugate. The basic starting-point (2.1) tells us that these obey the simple algebra

\[ \{ S, S^\dagger \} = 0, \quad \{ S^\dagger, S \} = 0, \quad \{ S, S^\dagger S \} = 1 \]

(2.5)

which has the familiar form of a raising and lowering algebra. Starting from any given helicity \( \lambda; S^\dagger |\lambda\rangle = 0 \), we can construct a state of lower helicity \( |\lambda - 1/2\rangle = S |\lambda\rangle \), but no others if there is a single SUSY charge \( Q \). If there are several charges \( Q^i \), the corresponding \( S^i \) obey the obvious extension of the simple algebra (2.5), with

\[ \{ S^i, S^j_\dagger \} = S^i_\dagger \]

(2.6)

as the only non-zero anticommutators. Then, starting from a helicity state \( |\lambda\rangle, S^i_\dagger |\lambda\rangle = 0 \), we get states

\[ |\lambda\rangle \rightarrow S^i |\lambda\rangle \rightarrow |\lambda - 1/2\rangle \rightarrow S^i S^j_\dagger |\lambda\rangle \rightarrow |\lambda - 1/2\rangle S^i S^j_\dagger |\lambda\rangle \rightarrow ... \]

(2.7)

where the internal indices \( i, j, ... \) are antisymmetrized.
In the case of \( N = 1 \) simple SUSY, the possible supermultiplets of interest are the following:

\[
\text{chiral (matter)} : \begin{pmatrix} \frac{1}{2} \\ 0 \end{pmatrix}, \quad \text{gauge (vector)} : \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}, \quad \text{graviton} : \begin{pmatrix} 2 \\ \frac{1}{2} \end{pmatrix}
\] (2.8)

In the rest of these lectures we will concentrate on the case \( N = 1 \): the following simple argument [25] indicates why \( N > 1 \) is not very useful for phenomenology. In the case of \( N = 2 \) SUSY's, starting from an initial state with helicity +1/2, one gets two states of helicity 0 and one of helicity -1/2. The states of helicity \( \pm 1/2 \), i.e. right- and left-handed fermions, must sit in identical representations of the gauge group. Thus the \( \tilde{W}^\pm \), for example, must couple in the same way to \( f_L \) and \( f_R \), and their couplings cannot violate parity. This argument can be extended to the \( N = 2 \) supermultiplet starting with helicity +1, which has two states of helicity +1/2 and one of helicity 0. Since vector particles can only sit in the adjoint representation of the gauge group, so also must the particles of helicity +1/2. To this \( N = 2 \) supermultiplet must be added the conjugate supermultiplet which has one state of helicity -1, two of helicity -1/2, and one of helicity 0. Thus the helicity -1 and hence the helicity -1/2 states must again be in the adjoint representation of the gauge group. Once again, fermions of helicity \( \pm 1/2 \) have identical couplings, and parity cannot be violated in theories with \( N = 2 \) SUSY. One can try to make models in which \( N = 2 \) SUSY is broken in such a way that the known particles are accompanied by 'mirror' particles in which parity is violated in the opposite way [26], but such models would be very cumbersome, and it is not clear that \( N = 2 \) SUSY can be broken in the desired way.

2.2 SUSY gauge theories

Now that we have found the gauge supermultiplet (2.7), we should also find the pure SUSY gauge Lagrangian [27]. It contains vectors \( A^a_\mu \) and Majorana fermions \( \chi^a \) in the adjoint representation of the gauge group. There is a unique Lagrangian for this pair of particles, which can be written as:

\[
L = \frac{i}{2} \bar{\chi}^a \gamma^\mu D_\mu \chi^a - \frac{i}{2} F^a_{\mu
u} F^{a\mu
u} - \frac{i}{2} (D^a)^2
\] (2.9)

where the covariant derivative

\[
D^a_\mu = \delta^a_{\mu} \partial_\mu - g f^{abc} A^b_\mu A^c_\mu
\] (2.10)

and the gauge field strength

\[
F^a_{\mu
u} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu
\] (2.11)

[You may wonder what \( D^a \) is doing in Eq. (2.9): actually the equations of motion for (2.9) tell you that \( D^a = 0 \). In general, when matter fields are coupled to the gauge supermultiplet, \( D^a \) is a non-zero auxiliary field which does not have a kinetic term and hence is
not associated with any physical particle.] It is easy to check that there is a unique set of candidate SUSY transformations which leave (2.9) invariant:

$$\delta \chi^a = - \frac{i}{2} F^a_{\mu \nu} \chi^a \gamma^\mu \gamma^\nu \epsilon + D^a \gamma^\mu \epsilon$$  \hspace{1cm} (2.12b)

$$\delta D^a = - i \bar{\chi} \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\lambda \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\lambda \epsilon$$  \hspace{1cm} (2.12c)

It is not difficult to add matter fields to the pure gauge Lagrangian (2.9). You add in the conventional gauge covariant kinetic terms for the scalars $s^i$ and for their fermionic partners $\psi^i$, as well as

$$S_L = - i \int \bar{\psi} i \gamma^\mu \partial_\mu \psi + \text{(higher-order terms for } \psi^i \text{)}$$

$$+ \int \bar{s} \gamma_\mu \partial_\mu s + \text{(higher-order terms for } s^i \text{)}$$

Using Eq. (2.13), the equation of motion for $D^a$ now tells us that

$$D^a = \sum_i s^+_i (T^a)_j^i s_j$$  \hspace{1cm} (2.14)

and hence the full scalar potential for the model is

$$V = \sum_i f_i \left| F_i \right|^2 + \sum_a \left| D^a \right|^2$$  \hspace{1cm} (2.15)

where $F_i = (\partial F_i / \partial \phi_1)^i$ as seen in subsection 1.4, and $D^a$ is now given by Eq. (2.14).

This is all the formal machinery that we need to construct field theories with SUSY, and we shall now go back to the original motivations for SUSY discussed in subsection 1.2, to see to what extent our hopes for SUSY are realized.

2.3 Why SUSY is useful

As previously advertized, there are no quadratic divergences in supersymmetric theories [28]:

$$\sum_{m_B} = \mathcal{O}\left(\frac{s^2}{16\pi^2}\right) \left[ (\Lambda^2 + \mathcal{O}(m_B^2)) \right]_{\text{Bosons}} - (\Lambda^2 + \mathcal{O}(m_\phi^2)) \right]_{\text{Fermions}}$$

$$= \mathcal{O} \left[ \frac{\alpha}{4\pi} \right]$$

This cancellation is a result of having equal numbers of bosons and fermions in the supermultiplets (2.8) with identical couplings in the Lagrangians (1.62), (2.9), (2.12), and (2.14). The residual contribution (2.16) to the Higgs mass squared is

$$\sum_{m_H} = \mathcal{O}\left(\frac{\alpha}{4\pi} \right) \left[ m_B^2 - m_\phi^2 \right]$$  \hspace{1cm} (2.17)

which is $\mathcal{O}(m_B^2)$ if $|m_B^2 - m_\phi^2| \leq 1 \text{ TeV}^2$. Note, however, that so far SUSY is unbroken, so the fermion and boson masses are equal, as can be seen directly in Eq. (1.61).
In addition to the quadratic divergences being absent, many logarithmic divergences also vanish \([29]\). In particular, there is no renormalization of the parameters \((m, \lambda)\) in the superpotential \(P\), apart from that induced by wave-function renormalizations of the matter fields. This means, in particular, that if we set some particle mass to zero, such as the Higgs mass \(m_h\),

\[
P \ni m_h H \quad ; \quad m_h \to 0
\]

then it will not be regenerated by loops \([10]\). This is a major difference from conventional gauge theories in general, and GUTs in particular \([13]\), and removes the problems we had in subsection 1.2 when arranging the gauge hierarchy in GUTs. SUSY also prevents elementary scalars from getting large mass corrections from quantum gravity.

2.4 Breaking SUSY

We have already seen that SUSY cannot be an exact symmetry: if one has fermions \(|F\rangle\) and bosons \(|B\rangle\) related by the SUSY charge \(Q, Q|B\rangle = |F\rangle\) and \(Q|F\rangle = |B\rangle\), and if SUSY is a good symmetry, \([Q, H] = 0\), then \(m_B = m_F\). However, no experiment has ever found a sparticle, so \(m_e^e \gg m_e^e, m_q^q \gg m_q^q\) etc. We could break SUSY either explicitly, by terms in the Lagrangian, or spontaneously, by having a non-invariant vacuum \(Q|\Omega\rangle \neq 0\). Explicit SUSY-breaking is not only ugly, but it also gives problems with unitarity for the gravitino when gravity is introduced \([30]\). Spontaneous SUSY breaking is the obvious analogue of what we believe happens in gauge theories \([2, 5, 6]\), so we pursue this possibility.

It is easy to show that SUSY can be broken spontaneously if and only if the field energy is strictly positive. The basic algebra \((2.1)\) tells us that

\[
\{Q, Q^\dagger\} = 2E
\]

Taking the vacuum expectation value of this equation, we see that \(\langle 0|E|0\rangle > 0\) if and only if there is a fermion \(\lambda\) coupled to the vacuum by the SUSY charge \(Q\):

\[
\langle 0|Q|\lambda\rangle \equiv \int^E \neq 0
\]

The fermion \(\lambda\) is known as a Goldstone fermion, by analogy with the well-known massless Goldstone boson which appears when a conventional global symmetry is spontaneously broken \([6]\). The 'decay constant' \(f_\lambda\) in Eq. \((2.20)\) is defined by analogy with the familiar \([12]\) decay constant \(f_\pi\) of the pion,

\[
\langle 0|Q|\pi\rangle = \int^E
\]

where \(Q_5\) is the axial charge in QCD.

Now then can we arrange to have positive vacuum energy:

\[
V = \sum F_i^2 + \sum A_\alpha (\partial \phi_\alpha)^2 > 0
\]

This can be done by arranging to have either the first term positive \([31]\), called F-breaking, or the second term positive \([32]\), called D-breaking. A consistent model with
D-breaking requires an additional $U(1)$ gauge group \([33]\), and many new matter fields in order to cancel out all anomalies \([34]\). Then there is a severe danger that the gauge couplings will rise so rapidly with increasing energy that they will blow up at energies less than the Planck mass $m_P$ or even the conjectured grand unification scale $m_X$ \([34]\). Even if this problem can be avoided, it is difficult to be sure of avoiding a SUSY minimum in some corner of field space, which would have zero vacuum energy and hence be energetically favoured over any state with spontaneously broken SUSY. Arranging F-breaking \([31]\) needs additional chiral fields with artificial-seeming couplings which appear ugly. The simplest example involves three gauge singlet chiral fields $A$, $B$, and $C$:

$$ P = \alpha A^2 + \beta C(B^2 - \mu^2) $$

(2.23)

Using formula (1.57) for the $F_i$, we deduce

$$ F_A = \alpha B^2 , \quad F_B = 2B(\alpha A + \beta C) , \quad F_C = \beta (B^2 - \mu^2) $$

(2.24)

so that

$$ V = \sum_{i=1}^n |F_i|^2 = |2B(\alpha A + \beta C)|^2 + |\alpha B|^2 + |\beta (B^2 - \mu^2)|^2 $$

(2.25)

It is clear that the last two terms in the potential (2.25) cannot vanish simultaneously, so we deduce that necessarily $V > 0$ and hence SUSY is spontaneously broken.

2.5 Local supersymmetry: supergravity

So far, all our SUSY transformations have been global, meaning that the infinitesimal SUSY spinor $\epsilon$ has not been allowed to depend on the space-time coordinates $x_\mu$. We will now try to make SUSY invariance local, $E \rightarrow E(x)$ in analogy with local gauge transformations, and we will immediately see that this requires the introduction of general coordinate invariance and hence gravity. If we make two independent SUSY transformations (here we introduce spinors $\psi$: $E = 1/2$ $(1 + \gamma_5)$ $\epsilon = i/2\gamma^\mu E$):

$$ S_{\tilde{1},2} \gamma^\mu \epsilon_{1,2} \psi + ... , \quad S_{\tilde{1},2} \psi = -i \gamma^\mu (i \gamma_5 \epsilon) + ... $$

(2.26)

then it is easy to check that

$$ \left[ S_{\tilde{1}}, S_{\tilde{2}} \right] (S \circ \psi) = -2 \left[ \tilde{\epsilon}_{\tilde{2}} \gamma_\mu \epsilon_{\tilde{1}} \right] \gamma^\mu (S \circ \psi) $$

(2.27)

Thus if $\epsilon_{1,2}$ are independent of $x$, $[S_{\tilde{1}}, S_{\tilde{2}}]$ corresponds to making global translations $P_\mu$. However, if the $\epsilon_{1,2}$ depend on $x$, then the translations (2.26) are local, by amounts $\epsilon_\mu(x) \gamma^\mu \epsilon_{1,2}(x)$. The appearance of these local coordinate transformations suggests that we need a gauge field, and a connection with gravity \([30]\). To see how this comes about, let us consider \([24]\) the simplest SUSY Lagrangian,

$$ L = i \bar{\psi} \gamma^\mu \partial_\mu \psi + i \bar{\psi} \gamma^\mu \partial_\mu \psi $$

(2.28)
and try to make it invariant under local SUSY transformations

$$\delta S(x) = \bar{\psi}(x) \gamma(x) \chi(x) , \delta \chi(x) = -i \gamma S(x) \psi(x)$$ (2.29)

We find that

$$\delta L = \partial_{\mu} \left[ \bar{\psi} \gamma_{\mu} \chi + \bar{\chi} \gamma_{\mu} \psi \right] + 2 \gamma_{\mu} \gamma_{5} \chi \partial_{\mu} \psi + \text{(higher order terms)}$$ (2.30)

The first term is just what we had before in the case of global SUSY. It is a total derivative and hence gives an invariant action. However, the second term in Eq. (2.30) must be cancelled by something else. At this point we note an analogy with conventional local gauge theories [6]. In that case, when we make a local phase transformation $\phi(x) \to e^{i \epsilon(x)} \phi(x)$ we find a term

$$\left( \gamma_{\mu} \gamma_{5} \psi \right) \times \partial_{\mu} \epsilon(x)$$ (2.31a)

Here we find

$$\left( \gamma_{\mu} \gamma_{5} \chi \right) \times \partial_{\mu} \psi(x)$$ (2.31b)

In the gauge case we added to the Lagrangian term

$$\Delta L = g \gamma_{\mu} \gamma_{5} \chi \psi \chi_{\mu} : \delta \chi_{\mu}(x) = \frac{\partial_{\mu} \epsilon(x)}{g}$$ (2.32a)

where $\lambda^{\mu}$ is a gauge boson which compensates (2.31a), and in the SUSY case we can add [24]

$$\Delta L = \kappa \gamma_{\mu} \gamma_{5} \chi \psi \chi_{\mu} : \delta \chi_{\mu}(x) = -2 \gamma_{5} \epsilon(x)$$ (2.32b)

where $\psi^{\mu}$ is a gauge fermion which compensates (2.31b). Just as the gauge boson has spin one unit more than the corresponding infinitesimal transformation $\epsilon(x)$, so the gauge fermion must have spin one unit more than $E(x)$, namely spin $3/2$. Thus $\psi^{\mu}$ is a gravitino, and the simplest Lagrangian involving it and the graviton is [30]

$$L = -\frac{1}{2} \kappa^{2} \sqrt{-g} R - \frac{i}{2} \varepsilon^{\mu \nu \rho \sigma} \gamma_{\mu} \gamma_{5} \partial_{\nu} \psi_{\rho} \psi_{\sigma}$$ (2.33)

where $g = \det g_{\mu \nu}$, $R$ is the curvature tensor, and $D_{\mu}$ is the generally covariant derivative,

$$D_{\mu} \equiv \partial_{\mu} + \frac{i}{4} \omega_{\mu}^{\rho \sigma} \left[ \gamma_{\rho}, \gamma_{\sigma} \right]$$ (2.34)

Here $m$ and $n$ are local Lorentz indices related to the general coordinates by

$$g_{\mu \nu} = \varepsilon_{m}^{\mu} \varepsilon_{n}^{\nu} \eta_{mn}$$ (2.35)
where $\epsilon^m_\mu$ is the vierbein and $\eta_{mn}$ the Minkowsky metric, and $\omega^m_\mu$ is the spin connection. Our luck is in again, because the Lagrangian (2.33) is invariant under the local SUSY transformations:

$$
\delta \epsilon^m_\mu = \frac{i}{2} \epsilon(x) \omega^m_\mu (x) \tag{2.36a}
$$

$$
\delta \omega^m_\mu = 0 \tag{2.36b}
$$

$$
\delta \gamma_\mu = \frac{i}{2} \epsilon(x) \gamma_\mu \epsilon(x) \tag{2.36c}
$$

The Lagrangian (2.33) is the prototype of a supergravity theory. In the next subsection we will see that such a supergravity theory with local SUSY is not only of academic interest, but is also useful for breaking SUSY spontaneously.

2.6 The super-Higgs mechanism

We continue to follow the analogy with conventional gauge theories [6]. There are massless gauge bosons, e.g. the $W_\mu$, with two helicity states $\pm 1$, and now we have a massless gravitino $\psi_\mu$ with two helicity states $\pm 3/2$. The gauge boson can acquire a mass through the Higgs mechanism [2, 5, 6], whereby it eats a massless Goldstone boson with a single state of helicity 0, and then has the three helicity states (0, $\pm 1$) needed by a massive vector boson. In our case, the gravitino can [35] eat a massless Goldstone fermion with two helicity states $\pm 1/2$, and then has the four helicity states needed by a massive spin-3/2 particle. The mass of the vector boson is

$$
m_W = \frac{g_5}{F_{\text{Goldstone}}} \tag{2.37a}
$$

whereas in our case the gravitino mass is

$$
m_{\psi_\mu} = \frac{g_5}{F_{\lambda}} \tag{2.37b}
$$

Thus the gravitino mass is no longer equal to the mass of its supersymmetric partner, the graviton, and hence local SUSY is spontaneously broken.

To apply this breaking of SUSY, we must couple ordinary matter to the graviton supermultiplet (2.8) via an extension of the Lagrangian (2.33). Here the necessary result will just be stated without proof. The kinetic terms for the matter fields and the superpotential $P$ are generalized [35] by introducing a real function $G(\psi, \psi^*)$ of the matter fields $\psi$ and their complex conjugates $\psi^*$. This provides derivative terms which generalize the kinetic terms (1.58) to

$$
\frac{\partial^2}{\partial \psi^*_\mu \partial \psi^*_\nu} \psi^*_\mu \psi^*_\nu \tag{2.38}
$$

for the spin-0 components of the supermultiplets, and analogously for their fermionic partners. We see from formula (2.38) that if $G \gg |\psi|^2$, then the kinetic terms (2.38) just reduce to the old canonical ones (1.58). The scalar potential is [35]
\[
V = e^{\frac{\alpha^i}{\tilde{g}_i}} \left[ \frac{\partial \phi_i}{\partial \phi_i} \left( \frac{\partial^2 \phi_i}{\partial \tilde{g}_i \partial \phi_i} \right) - \frac{\partial \phi_i}{\partial \phi_i} - \frac{3}{2} \right] + \frac{i}{2} (\Omega^i)^2
\]

(2.39)

If the function \( G \left| \ln P \right|^2 \) as well as canonical kinetic terms, then formula (2.39) gives

\[
V = \left[ \frac{\alpha^i}{\tilde{g}_i} \left| \frac{\partial \phi_i}{\partial \phi_i} \right|^2 + \frac{3}{2} \left| \phi_i \right|^2 \right] + \frac{3}{2} \left| \phi_i \right|^2
\]

and we see the glimmerings of the \( F \)-terms of global SUSY. In fact, if we assume that \( V(\phi_i^0) = 0 \), and look at fields \( |\phi_i|^2 \ll 1 \), then

\[
V \propto \frac{\alpha^i}{\tilde{g}_i} \left| \frac{\partial \phi_i}{\partial \phi_i} \right|^2 + \left| \phi_i \right|^2 \left( \frac{\partial^2 \phi_i}{\partial \tilde{g}_i \partial \phi_i} + \text{hem. cong.} \right) + \frac{i}{2} (\Omega^i)^2
\]

(2.40)

(2.41)

The first term in Eq. (2.41) has exactly the form of the \( F \)-terms of global SUSY, whereas the other terms break SUSY. In the model with minimal kinetic terms \[35\]

\[
m_{3/2} = e^{\alpha/2} \propto |P|
\]

(2.42)

but this is not necessarily true. A more general form is \[36\]

\[
V = \frac{\alpha^i}{\tilde{g}_i} \left| \frac{\partial \phi_i}{\partial \phi_i} \right|^2 + m_0 \left| \phi_i \right|^2 + m_0 \left[ A \phi_3(\theta) + B \phi_2(\theta) \right] + \text{(hem. cong.)}
\]

(2.43)

where \( m_0 \) is a SUSY-breaking scalar mass, which is \( m_{3/2} \) but not necessarily equal to it, and \( A \) and \( B \) are model-dependent parameters of \( O(1) \). The effective low-energy theory is also characterized by a non-minimal kinetic term for the vector supermultiplet \[35\], which is determined by a chiral function \( f(\omega) \) of the matter superfields. This can also give a SUSY-breaking mass \( m_{1/2} \) for the gaugino fermions, which is also \( m_{3/2} \) but not necessarily equal to it. The parameters \( m_0 \) and \( m_{1/2} \) appearing in the tree-level Lagrangian (2.43) are significantly renormalized by radiative corrections \[37\]. These have the general effect of making physical squark masses larger than physical slepton masses, and the gluino heavier than other gauginos \[38\].

When we discuss SUSY phenomenology in the next lecture, we will treat \( m_0 \) and \( m_{1/2} \) as free SUSY-breaking parameters to be constrained, or better determined, by experiment.

3. **Searches for Sparticles**

3.1 **The Spectrum**

No known particle can be the supersymmetric partner of any other. Consider, for example, the quark: we know of no elementary spin-0 particles, but the supersymmetric partner of the quark cannot have spin 1, because in a renormalizable field theory all vector bosons must be gauge bosons. Or consider the gluon: its supersymmetric partner would have
spin 1/2, like the quark, but be an $\mathbf{f}$ of colour. Thus we must invent a complete set of
supersymmetric partners for the known particles, along with all their sames shown in
Table 1.

The fact [39] that no charged sparticle has been seen at PEP or PETRA tells us that

$$m_q, m_{\tilde{c}^\pm}, m_{\tilde{\nu}^\pm}, m_{\tilde{\chi}^0} \geq 20 \text{ GeV} \quad (3.1)$$

and the absence [39] of any new strongly interacting sparticle in hadron-hadron collisions
or quarkonium decays tells us that

$$m_{\tilde{\rho}} \geq 3 \text{ GeV} \quad (3.2)$$

Later in this lecture we will review these limits in more detail. Conventional laboratory
experiments do not provide direct model-independent limits on neutral, weakly interacting
sparticles, but there are important constraints from cosmology, as we will see shortly.

Most of these bounds are based on a general phenomenological property of supersym-
matic theories, namely that all sparticles carry a new, multiplicatively conserved quantum
number called R-parity [33]:

$$R = \begin{cases} +1 & \text{for particles} \\ -1 & \text{for sparticles} \end{cases} \quad (3.3)$$

This quantum number is clearly conserved by the SUSY interactions we have met:

$$\tilde{f} f V \rightarrow \tilde{f} f \tilde{V}, \quad \tilde{f} f H \rightarrow \tilde{f} f \tilde{H}, \quad |\tilde{f} f|^2, \quad |\tilde{f} H|^2, \ldots \quad (3.4)$$
and its conservation can be related to that of other quantum numbers, namely spin $S$, baryon number $B$, and lepton number $L$:

$$ R = \left( \frac{1}{2} \right)^{2S} \left( \frac{1}{3} \right)^{3(B-L)} $$

(3.5)

Conservation of $R$-parity has three important phenomenological consequences:

1) Sparticles are always produced in pairs, e.g.

$$ e^+ e^- \rightarrow \tilde{e}^+ \tilde{e}^- , \quad \tilde{\nu} \tilde{\nu} \rightarrow \tilde{\chi} \tilde{\chi}^* + X $$

(3.6)

2) Heavier sparticles decay into lighter sparticles, e.g.

$$ \tilde{e}^+ \rightarrow e^+ \tilde{\chi} , \quad \tilde{\nu} \rightarrow \tilde{\nu} \tilde{\nu} \text{ or } q \tilde{\chi} , \quad \tilde{\chi} \rightarrow q \tilde{\nu} \tilde{\chi} $$

(3.7)

3) The lightest supersymmetric particle (LSP) is absolutely stable, because it has no legal decay modes.

It is this last property which means that the Universe should contain many supersymmetric relics from the Big Bang, and hence provides important cosmological constraints on the LSP. It must be electrically neutral and have no strong interactions [40]. If not, after being produced in the early Universe it would have condensed along with ordinary matter, and thus show up as anomalous heavy isotopes. These would have an abundance [41]

$$ \frac{n(\text{heavy})}{n(\nu)} = O(10^{-6} \text{ to } 10^{-13}) $$

(3.8)

to be compared with experimental limits [42]

$$ \frac{n(\text{heavy})}{n(\nu)} < O(10^{-20} \text{ to } 10^{-30}) $$

(3.9)

for heavy isotope masses up to about 1 TeV, which covers the range expected for the LSP.

The following are the neutral, weakly interacting supersymmetric particles in the minimal supersymmetric Standard Model, listed in Table 1, which are candidates for the LSP:

$$ \begin{array}{cccc}
\text{Spin} & 0 & \frac{1}{2} & 1 & \frac{3}{2} \\
\text{Sparticle} & \tilde{\nu} & \tilde{\tau} & \tilde{\chi} & \text{gravitino}
\end{array} $$

(3.10)

Of these, the sneutrino $\tilde{\nu}$ generally weighs $O(m_{\nu})$ in most models, whilst the gravitino usually weighs at least as much as the $e$, and perhaps much more. Thus these are not favoured candidates for the LSP [43]. The spin-1/2 candidates $\tilde{\tau}$ and $\tilde{\chi}^0$ mix in general [40], and one or the other, if not both, can be much lighter than the $e$. The couplings of the $\tilde{\chi}^0$ to ordinary matter are $O(m_e/m_{\tilde{\chi}^0})$ and hence quite small. This means that the $\tilde{\chi}^0$ cannot annihilate very efficiently, and hence would have too high a relic density if it was the LSP [40]. Thus the best candidate for the LSP is the photino [44], although this is not necessarily the unique choice even in the minimal SUSY Standard Model.
3.2 Experimental signatures and past searches

The facts that all sparticles decay into the LSP, and that this must be neutral and
weakly interacting, and thus able to escape undetected from any collision, mean that the
general signature for SUSY is missing energy-momentum $\not{p}_T$. The LSP, e.g. the $\tilde{\gamma}$, escapes from
any apparatus without any secondary interaction, much like a neutrino. Thus in $e^+e^-$ annih-
ation to a pair of sleptons, about 50% of the centre-of-mass energy would be invisible
as a result of

$$e^+e^- \rightarrow (\gamma^\ast \rightarrow \gamma^\ast \tilde{\gamma}^\ast)(\gamma^- \rightarrow \gamma^- \tilde{\gamma}^-) \text{ or } e^+e^- \rightarrow (\mu^+ \rightarrow \mu^+ \tilde{\mu}^+)(\mu^- \rightarrow \mu^- \tilde{\mu}^-).$$

In hadron-hadron collisions, it is difficult to measure the longitudinal momentum balance
very precisely, so the experimental signature is missing transverse-momentum $p_T$, which
would in general be of the order of the transverse mass $m_T^*$. From

$$p_T \rightarrow (\gamma \rightarrow \tilde{\gamma} \tilde{\gamma})(\gamma \rightarrow \tilde{\gamma} \tilde{\gamma}) + \chi \text{ or } p_T \rightarrow (\tilde{\gamma} \rightarrow \tilde{\gamma} \tilde{\gamma})(\tilde{\gamma} \rightarrow \tilde{\gamma} \tilde{\gamma}) + \chi.$$  

Later we will discuss in detail the search for such a signature at the CERN $pp$ Collider.

The cross-section for $e^+e^- \rightarrow \tilde{\nu}_R \tilde{\nu}_L$ or $\tilde{\tau}^+\tilde{\tau}^-$ is given by

$$\sigma(e^+e^- \rightarrow \gamma^\ast \rightarrow \tilde{\tau}^+\tilde{\tau}^-) \approx \left( \frac{1}{4} \frac{m_T^*}{\sqrt{s}} \left( 1 \text{ or } 2 \right) \right),$$

whilst $\sigma(e^+e^- \rightarrow e^+e^-)$, has an additional contribution from $\tilde{\gamma}$, etc. exchange [45]. The
factor of 1/4 in Eq. (3.13) is due to spin, whilst $\beta = v/c = \sqrt{1 - (m_T^*/s)}$ is a threshold
factor, and the factor of 1 (or 2) depends on whether the supersymmetric partner of only
one (or both) helicity state(s) of the charged lepton ($\tilde{\gamma}_L$ and/or $\tilde{\gamma}_R$) is being pair-produced.
There is in general no reason why the $\tilde{\gamma}_L$ and $\tilde{\gamma}_R$ should have equal masses, although sleptons
with the same gauge quantum numbers, e.g. $\tilde{e}_R$, $\tilde{\nu}_R$, and $\tilde{\tau}_R$, may have similar masses. The signature
for $e^+e^-$ or $\mu^+\mu^-$ production is an acoplanar, acoplanar pair of leptons of the same
flavour as seen in Fig. 4a, a clear distinction from the pair-production of a new heavy
lepton which can produce lepton pairs of different flavours. The absence of this signature
means [39] that

$$m_{\tilde{e}} > 22 \text{ GeV}, \quad m_{\tilde{\nu}} > 20 \text{ GeV} \quad (m_{\tilde{\nu}} = m_{\tilde{\tau}}),$$

whilst the slightly more difficult search for $\tilde{\tau}^+\tilde{\tau}^-$ production yields [39]

$$m_{\tilde{\tau}} > 17 \text{ GeV} \quad \text{(for } m_{\tilde{\nu}} = m_{\tilde{\tau}} \text{)}.$$

Fig. 4 Missing energy signatures:
a) $\tilde{\gamma}^+\tilde{\gamma}^-$ + $\not{p}_T$ for $\tilde{\gamma}^+\tilde{\gamma}^-$, and
b) jet-jet + $\not{p}_T$ for qq.
The cross-section for $e^+e^- \rightarrow \tilde{q}\tilde{q}$ production is given by

$$\frac{\sigma(e^+e^- \rightarrow \tilde{q}\tilde{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \frac{1}{4} \beta^3 \frac{3}{2} \epsilon_q^2 (1 + \beta)$$ \hspace{1cm} (3.16)

where the factor of 3 is due to colour, and $\epsilon_q$ is the charge of the squark in units of the electron charge. Once again, the $\tilde{q}_L$ and the $\tilde{q}_R$ may have different masses, although the masses of squarks in at least the first two generations with the same gauge quantum numbers may be nearly equal. If $\tilde{q} \rightarrow q\tilde{\chi}$ are the dominant decays, final states will mainly contain two acoplanar, acollinear hadronic jets with $|p_T| < 1/2 \sqrt{s} (E_{\text{cm}} = \sqrt{s})$ as seen in Fig. 4b. It has been easy to exclude such events at PEP and PETRA, and one deduces [31] that

$$m_{\tilde{q}} > 21 \text{ GeV} \left( m_{\tilde{q}_L} = m_{\tilde{q}_R} \right)$$ \hspace{1cm} (3.17)

if $\tilde{q} \rightarrow q\tilde{\chi}$. If $\tilde{q} \rightarrow q\tilde{\chi}$, followed by $\tilde{q} \rightarrow q\tilde{\chi}$, final states have more jets, which are less easy to distinguish, and $|p_T|$ is smaller. Nevertheless, one has the limits [39]

$$m_{\tilde{q}} > 17 \text{ GeV} \left( m_{\tilde{q}_L} = m_{\tilde{q}_R} \right)$$ \hspace{1cm} (3.18)

We will soon discuss the improvements obtained from the CERN $p\bar{p}$ Collider.

Limits on the $\tilde{q}$ mass come from hadron-hadron beam-dump experiments [46]. In one class of limits, one assumes that $\tilde{q} \rightarrow q\tilde{\chi}$ decay occurs before the $\tilde{q}$ (or more accurately, the hadron containing the confined $\tilde{g}$) has a chance to interact. Then the $\tilde{\chi}$ decay product escapes from the dump and may interact in a neutrino detector downstream, as in Fig. 5. In this case, the $\tilde{g}$ lifetime and $\sigma(\tilde{g}N)$ depend on $m_{\tilde{q}}$, and hence the bound on $m_{\tilde{q}}$ also depends on $m_{\tilde{q}}$ [46], as seen in Fig. 6. One can also look for longer-lived $\tilde{g}$ hadrons which travel an

Fig. 5 Schematic layout of a beam dump experiment.

![Fig. 5 Schematic layout of a beam dump experiment.](image)

Fig. 6 Domains of $(m_{\tilde{g}}, m_{\tilde{q}})$ excluded by beam dump experiments.
observable distance before decaying [47], and such limits are also shown in Fig. 6. Such longer-lived $\tilde{g}$ hadrons have also been sought [48] in quarkonium decays: $\Upsilon' \rightarrow \Upsilon + \tilde{p}_t (b\bar{b}) + \Upsilon, \quad \tilde{p}_t (b\bar{b}) \rightarrow \tilde{q} \tilde{q} X$ [49]. Combining all these limits, one obtains
\begin{equation}
m_{\tilde{g}} \gtrsim 3 \text{ GeV}
\end{equation}
and we will now see how this can be improved at the CERN $p\bar{p}$ Collider.

3.3 Hadronic missing-momentum events at the CERN $p\bar{p}$ Collider

These would include monojet, dijet, and trijet events with missing transverse momentum $p_T$. The search for such events was at first greeted with optimism. Just a handful of such events with $p_T > 4\sigma$ (where, in GeV units, $\sigma = 0.7 / E_T$ is the standard error in measuring the transverse energy $E_T$) were seen by the UA1 Collaboration among its 1983 data [50]. These were mainly monojet events, and could be interpreted as upper limits on $\sigma(\tilde{q}, \tilde{q})$ and hence as lower limits on $m_{\tilde{q}, \tilde{g}}$ [51] (see Fig. 7):
\begin{equation}
m_{\tilde{q}, \tilde{g}} \gtrsim 40 \text{ GeV}.
\end{equation}
Super-optimists even hoped [52, 53] that the few missing-momentum events observed could be due to the pair-production of either $\tilde{q}$ or $\tilde{g}$ with masses $\sim 40$ GeV. In either of these cases, one could expect $m^2_{\tilde{q}} - m^2_{\tilde{q}}$ in the former case, the mass relations [38] of the minimal SUSY Standard Model would lead one to expect [53, 54] $m^2_{\tilde{q}} - m^2_{\tilde{q}}$, in which case the lower bound on the common sparticle mass would be [55]
\begin{equation}
m_{\tilde{q}} \sim m_{\tilde{q}} \gtrsim 60 \text{ GeV}
\end{equation}
and some dijet + $p_T$ events should be seen if this inequality were saturated.

![Fig. 7 Calculated $p_T$ cross-sections a) for $p\bar{p} \rightarrow \tilde{g} \tilde{g} + X$, $\tilde{q} \rightarrow q\tilde{q}$, and b) for $p\bar{p} \rightarrow \tilde{q} \tilde{q} + X$, $q \rightarrow q\tilde{q}$.


Table 2
Backgrounds to monojets
(per 100 nb$^{-1}$ at $\sqrt{s} = 540$ GeV)

<table>
<thead>
<tr>
<th>Event Description</th>
<th>Ref. 56</th>
</tr>
</thead>
<tbody>
<tr>
<td>jet + (Z$^0 + \nu\bar{\nu}$)</td>
<td>0.36</td>
</tr>
<tr>
<td>W + e lost</td>
<td>0.12</td>
</tr>
<tr>
<td>W + \mu lost</td>
<td>$&lt; 0.01$</td>
</tr>
<tr>
<td>W + \tau lost</td>
<td>0.25</td>
</tr>
<tr>
<td>W + \tau seen</td>
<td>0.50</td>
</tr>
</tbody>
</table>

However, the backgrounds to monojet events were not small, and the UA1 Collaboration said from the beginning [50] that their events with $p_T^2 < 1000$ GeV$^2$ were all explicable as backgrounds due to measurement fluctuations in ordinary jet-jet events, production of heavy flavours such as b\bar{b} or c\bar{c} followed by semileptonic decays producing neutrinos, and W + rv decays. The backgrounds to events with $p_T^2 > 1600$ GeV$^2$ were not negligible either, as seen from the results of a representative calculation [56] shown in Table 2. The number of events shown could well be an underestimate by a factor of 2. These are to be compared with the five events seen in the 1983 data [50]. More data were clearly needed before one could tell whether the observed monojets represented a true signal, or were just due to the backgrounds.

Before discussing the 1984 data [57] of the UA1 Collaboration, another possible interpretation of the 1983 data should be discussed. Even before they were available, it had been pointed out [58] that if the \tilde{q} were relatively light, say 0(5) GeV, and the \tilde{\bar{q}} were relatively heavy, say 0(100) GeV, then the p and \bar{p} would have a significant probability of containing an intrinsic virtual \tilde{g}, which could collide with a q or \bar{q} from the p or \bar{p}, fusing to form a \tilde{q} or \tilde{\bar{q}}. This could then decay into q or \tilde{q} + \tilde{\bar{q}}, giving gold-plated monojets [59]. A possible difficulty [60] with this interpretation of the observed monojets was that one would expect a very large cross-section for the production of light gluino pairs, which could also give events with \pt. Most of these would be lost because of the way the UA1 detector was triggered and the data were analyzed [50]. Nevertheless, sufficient \pt events would have remained [60] that such light \tilde{g} could presumably be excluded (3.20). Or could they? It was argued [59] that the uncertainties in the cross-section for light \tilde{g} production and decay, particularly in the way an energetic light \tilde{q} jet fragments to give a \tilde{q} hadron, were such that a \tilde{g} in the range

$$5 \text{ GeV} \leq m_{\tilde{g}} \leq 20 \text{ GeV}$$

(3.22)

could not be excluded. The existence of such a 'window of opportunity' seemed to us unlikely [60] (see Fig. 8), particularly since the reduction in the observable cross-section owing to \tilde{q} jet fragmentation would be largely compensated by an increase due to additional events with \tilde{g} + \tilde{g} final states [61]. Moreover, having such a light \tilde{g} with such a heavy \tilde{\bar{g}} would give severe cosmological problems [62] in the minimal SUSY Standard Model: the relic
density of photinos would exceed observational limits. These suggestive but inconclusive arguments against the light $\tilde{g}$ scenario became moot after the 1984 data of the UA1 Collaboration.

These data [57] revealed no new monojets with $p_T \gtrsim 50$ GeV, and about the same number of events with $40$ GeV $< p_T < 50$ GeV as had been seen in the 1983 data. This corresponded to a reduction in the apparent rate of monojet events by a factor $0(3)$, since the integrated luminosity in 1984 was higher than it had been in 1983, and the centre-of-mass energy was higher: 630 GeV instead of 540 GeV. In view of the background estimates in Table 2, this meant that the gross number of monojet events observed was compatible with the backgrounds ($\tau$, $b\bar{b} + c\bar{c}$, $W$, $Z + q$, $t\bar{t}$, ...) not only for $p_T \lesssim 40$ GeV as before, but now also for $p_T \gtrsim 40$ GeV. It remains possible that there may be some more detailed difference between the events seen and the backgrounds, for example in the narrowness of the jets [57], but for the moment the best one can do is to quote improved lower limits [63] on the squark and gluino masses:

$$m_{\tilde{q}} > 504 \text{ GeV}, \quad m_{\tilde{g}} > 454 \text{ GeV}, \quad m_{\tilde{g}} = m_{\tilde{q}} > 664 \text{ GeV}. \quad (3.23)$$

### 3.4 Other $\bar{p}p$ Collider Limits

The decays $W^\pm \rightarrow (Z^\pm + e^\mp) + \bar{\nu}$ [64] would be an additional source of $l^\pm + p_T$ events beyond $W^\pm \rightarrow l^\pm + \nu$. They would give events with $|p_T| \lesssim 30$ GeV and $\cos \theta_{lep} < 0$, unlike conventional $W^\pm \rightarrow l^\pm + \nu$ events. After including UA1 cuts and resolutions, we estimate [65]

$$\frac{\sigma(W \rightarrow \ell \rightarrow e) / \sigma(W \rightarrow e)}{\sigma(W \rightarrow \mu / \sigma(W \rightarrow \mu)} \gtrsim 0.1 \quad (3.24)$$
for $m_T \lesssim 30$ GeV. Note that the $W$ decays into $e^+\nu$ only, not into $e^-\bar{\nu}$. The UA1 Collaboration has looked unsuccessfully for such an excess of these $l^+ + p_T$ events, and quotes [57] the lower limit

$$m_{e^-_L} \gtrsim 26 \text{ GeV}$$

(3.25)

if $m_{p_T} = m_{e^-}$. The decay $z^0 \rightarrow (\bar{1}^+ + 1^+\bar{\gamma})(\bar{1}^- + 1^-\gamma)$ [66] would give $1^+1^- + p_T$ events with $\Delta p_T(1^+\gamma') | - | p_T | < 20$ GeV and $m(1^+1^-) - 40$ GeV. The presence of $p_T$ would distinguish such events from any Drell-Yan background. Implementing UA1 cuts and resolutions, we estimate [65]

$$\sigma(z^0 \rightarrow e^+e^- p_T) / \sigma(z^0 \rightarrow e^+e^-) \gtrsim 0.1$$

(3.26)

for $m^{\pm \pm} \lesssim 30$ GeV. Note that in this case one is sensitive to both $\bar{1}^-_L$ and $1^+_R$. No limit on $m_{p_T}$ has yet been quoted based on the non-observation of this process, although it should give a clearer signature than $W \rightarrow 1^-\bar{1}^+$ and have a sensitivity similar to (3.25).

One can also look for winos and zinos in $W^\pm$ and $z^0$ decay. If $m_{\tilde{W}^\pm}, m_{\tilde{z}^0} \gtrsim m_{p_T}$, the decays $\tilde{W}^\pm \rightarrow \bar{1}^+ \bar{\nu} + 1^+\nu \rightarrow 1^+\gamma \nu$ and $z^0 \rightarrow \bar{1}^- \bar{\nu} + 1^-\nu \rightarrow 1^-\gamma \nu$ should dominate. Thus the decay $W^\pm + z^0$ could give $1^+1^- + p_T$ events with $\sigma(\mu^+\mu^- + p_T) \gtrsim 20$ pb if $m_{\mu} < 30$ GeV and $m_{\tilde{W}^\pm} \gtrsim 40$ GeV, and $p_T \sim 10$ GeV [67]. Analogously, $z^0 \rightarrow W^- W^+$ decay followed by $\tilde{W}^\pm + z^0$ decay could give $1^+1^- + p_T$ events with $\sigma(\nu^+\nu^- + p_T) \gtrsim 40$ pb if $m_{\nu} \lesssim 30$ GeV and $m_{\tilde{W}^\pm} \lesssim 40$ GeV, and $p_T \sim 15$ GeV [67]. Such events have not been seen, so it seems fair to conclude [67] that

$$m_{\tilde{W}^\pm} \gtrsim 40 \text{ GeV} \quad \Rightarrow \quad m_{\tilde{W}^\pm} > m_{\tilde{T}^\pm}$$

(3.27)

This limit is much more severe than that from $e^+e^-$ collisions [39]:

$$m_{\tilde{W}^\pm} \gtrsim 22 \text{ GeV}$$

(3.28)

albeit more model-dependent.

### 3.5 Searches for $e^+e^- \rightarrow \gamma +$ nothing

The idea here is to look for $e^+e^- \rightarrow \gamma + p_T$, where the missing energy-momentum is carried off by unseen neutrals such as $\nu\bar{\nu}$, $\nu\gamma$, or $\gamma\gamma$. The otherwise unobservable final state is tagged by a bremsstrahlung photon as in Fig. 9, which has a characteristic soft

![Fig. 9 Basic diagram for e^+e^- \rightarrow γ + Nothing.](image-url)
spectrum peaked at small angles relative to the $e^+ e^-$ beams. In the prototype case of "nothing" = $\nu \bar{\nu}$ [68],

$$\frac{d^2 \sigma}{d \chi d \Theta} \sim \frac{E_{\text{cm}}^2}{\alpha_s \sin^2 \Theta} \left(\left(1 - \chi \frac{q^2}{4} \left(1 - \sin^2 \Theta \right)\right)^2 + \frac{\pi^2}{4} \left(1 - \chi \frac{q^2}{4} \left(1 - \sin^2 \Theta \right)\right)\right) \times \left\{ \frac{1}{2 \sqrt{N_\nu}} \left(1 + N_\nu \right) \right\} ,$$

(3.29)

where $\chi = 2E_e / E_{\text{cm}}$ and $N_\nu$ is the total number of neutrino species. The cross-section (3.29) is not strictly linear in the number of neutrinos because $\sigma(e^+ e^- \rightarrow \nu \bar{\nu})$ has a crossed-channel $Z^0$ exchange diagram contributing to it, which is absent for the other neutrinos that only have a direct-channel $Z^0$ diagram (see Fig. 10a). The best upper limit on $\sigma(e^+ e^- \rightarrow \gamma + \text{nothing})$ comes from the ASP Collaboration [39, 69]. Its result tells us that

$$N_\nu < 14 ,$$

(3.30)

which is not as restrictive as the $p\bar{p}$ Collider limit from $Z^0$ production and decay [70],

$$N_\nu < (5.4 \pm 1.0) ,$$

(3.31)

but has fewer systematic uncertainties. In the case of "nothing" = $\gamma \gamma$, the squiggly bracket in Eq. (3.29) is replaced [71] by

$$\left\{ \left(\frac{1}{2} \right)^3 \alpha^2 \sqrt{\frac{1}{m_{\ell R}^2} + \frac{1}{m_{\ell L}^2}} \right\} x \left(\text{phase space} \right) ,$$

(3.32)

where the dominant diagram is crossed-channel $\bar{e} e$ exchange as in Fig. 10b; the factor exhibited in square brackets is appropriate in the double limit $m_{\gamma} \ll E_{\text{cm}} \ll m_{\ell}$, and is modified by a phase-space factor away from this limit [71]. From its upper limit on $\sigma(e^+ e^- \rightarrow \gamma + \text{nothing})$ the ASP Collaboration [39, 69] deduces that

$$m_{\ell} \geq 51 \text{ GeV} \quad \text{if} \quad m_{\ell} = 0 \geq 45 \text{ GeV} \quad \text{if} \quad m_{\ell} = 54 \text{ GeV} \quad \left(m_{\ell L} = m_{\ell R}\right)$$

(3.33)

if $m_{\ell} = m_{\ell}$ is assumed. This lower limit on $m_{\ell}$ is the most stringent available if $m_{\ell}$ is small, but there is no significant limit at all on $m_{\ell}$ if $m_{\ell} \geq 12 \text{ GeV}$. "Nothing" could also

---

**Fig. 10** a) Diagrams contributing to $e^+ e^- \rightarrow \nu \bar{\nu}$; b) diagram contributing to $e^+ e^- \rightarrow \gamma \gamma$; and c) diagram for $e^+ e^- + Z^0 \rightarrow H \gamma + (\gamma \gamma)?$. 

---
be $\tilde{\nu}\tilde{\nu}$, but since we expect $m_\tilde{\nu} \approx m_\tilde{\nu} > 20$ GeV, there is no significant constraint from $\sigma(e^+e^- \rightarrow \gamma + \text{nothing})$ at present centre-of-mass energies. However, this reaction could be interesting at energies just above the $Z^0$ pole, where $\sigma(e^+e^- \rightarrow \gamma + \text{nothing})$ could be measured sufficiently accurately to determine $N_\nu$ with a precision of less than unity [72]. Then, since

$$
\frac{\sigma(e^+e^- \rightarrow Z^0 \rightarrow \tilde{\nu}\tilde{\nu})}{\sigma(e^+e^- \rightarrow Z^0 \rightarrow \nu\bar{\nu})} = \frac{1}{2} \left(1 - \frac{4m_e^2}{m_e^2}\right)^{3/2},
$$

(3.34)

where $m = m(\tilde{\nu}\tilde{\nu}$ or $\nu\bar{\nu}$) one could also count the number of $\tilde{\nu}$ species if the $\tilde{\nu}$ are either stable, long-lived, or decay invisibly, e.g. into $\gamma + \nu$.

A final example of a process contributing to $\sigma(e^+e^- \rightarrow \gamma + \text{nothing})$ is $e^+e^- \rightarrow Z^0 \rightarrow \tilde{H}^0 + \tilde{\gamma}$ [73], where we exploit the fact that the neutral supersymmetric fermion mass eigenstates are not pure $\tilde{H}^0$ and $\gamma$, but mixtures [40]. If $m_{\tilde{H}^0}$ and $m_{\gamma}$ are both $\lesssim 30$ GeV, this process could have a large cross-section at the $Z^0$ peak. The heavier mass eigenstate, which is probably the $\tilde{H}^0$, for the reasons discussed earlier [40], would then decay into $\gamma + \gamma$ + nothing events at the $Z^0$ peak [73], as in Fig. 10c.

3.6 Prospects

In these lectures we have seen why SUSY is such an appealing idea for theorists, and have looked at the way in which phenomenologically viable supersymmetric models can be constructed. Unfortunately, as we have seen in this last lecture, there is as yet no experimental evidence for SUSY. However, we should not be discouraged. According to theory, supersymmetric particles could have any mass up to about 1 TeV, and there are good prospects for exploring all this mass range with new accelerators in construction or under discussion. Moreover, the phenomenological signatures for SUSY discussed in this lecture are also applicable at higher energies. Perhaps you will be luckier than the previous unsuccessful shunters?
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GRAND UNIFIED THEORIES AND COSMOLOGY

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ABSTRACT

This is an introduction to grand unified theories and their cosmological implications. The SU(5) model as the simplest example of a GUT is considered in some details. Two major low energy applications are considered, just proton decay and mu-oscillations. The SO(10) model is briefly reviewed. Among cosmological implications of GUTs there are considered the generation of the baryon asymmetry of the Universe as well as a variety of high temperature transitions in the very early Universe in the framework of the SU(5) model.

1. INTRODUCTION

A great deal of attention has been paid in the last several years by all the particle physics community to the Grand Unified Theories (GUTs) describing in the common framework and by means of the unique coupling constant the strong, weak and electromagnetic interactions. These theories have not only predicted a number of new physical phenomena (proton instability, neutron-antineutron oscillations etc.) but enriched essentially our cosmological knowledge having explained for example, the (possible) origin of the baryon asymmetry of the Universe.

There were a number of lectures and talks devoted to the subject delivered at various schools and training courses, from which I have benefited so much while preparing my lectures. I would apologize that I have mentioned not all beautiful lectures I would like but only some of them.

In these lectures I shall briefly describe the main properties of GUTs using basically the SU(5) model as an example and omitting details of minor importance, which could be found in a number of reviews and original papers.

The starting point in the construction of a GUT in most cases is the requirement that at low energies the theory must reduce to the electro-weak SU(2) x U(1) model of Glashow-Salam-Weinberg and the quantum chromodynamic (QCD) based on the color group SU(3)c and describing the strong interactions. The QCD is nothing but the gauge theory of interactions of colored object. The color itself was introduced by Bogolubov, Struminsky and Tavkhelidze and by Han and Nambu to provide a solution to the hadron classification problems. The combination of these two theories, i.e. the gauge model on the group SU(2) x U(1) x SU(3)c is called usually the Standard Model.

The validity of the SM at low energies is justified by numerous experiments, which we have no time to discuss. Instead we recall in a somewhat
detailed way those shortcoming of the SM which allow one to appreciate the attractive features of GUTs.

Let us first of all recall the main features of the SU(2) x U(1) model. It contains three gauge fields $A_i$ (i = 1, 2, 3) transforming as a triplet under SU(2) and interacting with the coupling constant $g$, and the single field $B$ corresponding to the group U(1) and interacting with the coupling constant $g'$. The following field combinations correspond to the neutral weak boson $Z^0$ and to photon

$$Z = A_3 \cos \theta + B \sin \theta$$  
$$A = B \cos \theta - A_3 \sin \theta$$  

where the Weinberg angle $\theta$ is given by

$$\sin^2 \theta = \frac{g'^2}{g^2 + g'^2} .$$  

The angle $\theta$ is the very important parameter since it determines the (relative) strength of neutral currents. However, in the framework of SM it is by means fixed, so it has to be extracted from experiment. This is due to the fact that the group SU(2) x U(1) is not simple but it is the product of two simple groups, therefore there are two independent coupling constants $g$ and $g'$ in the theory. If, however, one embeds SU(2) x U(1) into the more wide simple group with the gauge coupling constant $g_{GU}$ (say, into SU(5) group) then both coupling constants $g$ and $g'$ will prove to be proportional to $g_{GU}$, so that Eq.(2) may allow to predict the value of $\theta$.

In the SM quarks and leptons are assigned to SU(2) multiplets as follows:

1-st generation:

$$\left( \begin{array}{c} u_i \\ d_i \end{array} \right)_L , \left( \begin{array}{c} \nu_e \\ e \end{array} \right)_L , \nu^-, u_{iR} , d_{iR}$$

2-nd generation:

$$\left( \begin{array}{c} c_i \\ s_i \end{array} \right)_L , \left( \begin{array}{c} \nu_\mu \\ \mu \end{array} \right)_L , \nu^\mu , c_{iR} , s_{iR}$$

3-rd generation:

$$\left( \begin{array}{c} t_i \\ b_i \end{array} \right)_L , \left( \begin{array}{c} \nu_\tau \\ \tau \end{array} \right)_L , \nu^\tau , t_{iR} , b_{iR} ,$$

here $i = 1, 2, 3$ is the SU(3)$^C$ color index.

Neither the structure of multiplets in the generations nor the fermion mass spectrum as well as the number of generations are explained by the SM.

What could be said about the electric charges of quarks and leptons in the SM? Here the relation takes place

$$Q = T_3 + Y/2 ,$$
Q being the (electric) charge of a fermion, $T_3$ being the 3-rd component of weak isospin $Y$ and being the fermion hypercharge with respect to U(1), which is by as means fixed by the model and can be chosen arbitrary. Thus, the model restricts only the difference between charges of the pair of fermions in a SU(2) doublet:

$$Q_u - Q_d = 1, \quad Q_\nu - Q_e = 1, \quad \text{etc.} \quad (5)$$

The absolute values of quark charges remain arbitrary. In other words, the SU(2) x SU(1) model does not explain the quantization of electric charge.

So, let us list once more the shortcomings of the SM:
1. The Weinberg angle is a free parameter of the theory.
2. It does not follow from the theory that the electric charges are quantized.
3. The reason of the baryon, lepton, fermion numbers conservation is unclear, as well as the origin of the baryon-antibaryon asymmetry of the Universe.
4. There is no answer on the question, why there exist quarks and leptons, i.e. triplets and singlets under SU(3)$^C$.
5. No relations between quark and lepton masses.
6. No reason for existence of fermion generations.
7. No explanation at all of why the (dimensionless) coupling constants of EM and strong interactions differ (so strongly).
8. The SM contains at least 27 free parameters (gauge, Yukawa, Higgs coupling constants; angles characterizing the vacuum state).

In the framework of GUTs one may hope to find solutions at least to some of these (and other) Grand Problems.

Let me say a few words about the phylosophy of grand unification. When speaking on the GU we have in mind that gluons, weak bosons and photon are components of the unique (vector) field $W^\mu$, which at high energies interacts with the (nearly) same strength with quarks and leptons and can transfer one into another. This sets one thinking of the construction of the unified theory using a simple gauge group implying the unique coupling constant. Then the Weinberg angle could be determined by the geometry of the group while the charge quantization would be due to the fact that all charges are proportional to the unique coupling constant.

The unified description of known interactions is not only attractive aesthetically and allows to explain well known facts. Remarkably enough that unifying EM, weak and strong interactions one inevitably put into consideration the essentially new type of interactions, just the superweak ones violating the baryon, lepton, fermion number conservation. It is just this new interaction which is responsible for the most exciting physical consequences of GUTs: proton instability, $\nu$-$\bar{\nu}$-oscillations, the explanation of the origin of the cosmological baryon excess. (Remind, that the similar situation occured at the electroweak unification in the frame-
work of the SU(2) x U(1) model which resulted in the prediction of weak NC).

The idea of gauge unification of interactions was first proposed by Pati and Salam in 1973 \(^4\). Georgi and Glashow in 1974 proposed the unified model on the gauge group SU(5) \(^5\). However, the rapid growth of the number of publications on GUTs began only since about 1978. This was due to a number of reasons: 1) a raising number of experiments had given some evidences of validity of both SU(2) x U(1) model and QCD; 2) the more precise value of the Weinberg angle proved to be \(\sin^2 \theta_W \approx 0.23\) in 1978 instead of \(\sin^2 \theta_W \approx 0.4\) in 1974; 3) it had become clear that GUTs can not only explain the very existence of the baryon asymmetry of the Universe (BAU) but its magnitude too; 4) the \(b\)-quark mass was predicted.

2. **GRAND UNIFIED THEORIES**

2.1. **SU(5) model**

After these introductory remarks let me proceed to the consideration of the most popular and simultaneously the simplest GUT, namely, the minimal model on the gauge group SU(5) \(^5\). Why just SU(5)? I would say simply that the SU(5) model is the most economic in the number of new (yet unobserved) particles which it contains. The known 15 low energy fermionic states of SM transform with respect to SU(3)xSU(2) group as follows

\[
[(u_4 d_4)_L + (u_1 u_2 + d_1 d_2) (v_e e^-)_L + e^+_L] = \\
= (3,2) + 2(\overline{3},1) + (1,2) + (1,1).
\]

They could be placed into two lowest representations of SU(5), antiquintet and decuplet:

\[15 = \overline{5} + 10\]

in the following way:

(7)

\[
\begin{pmatrix}
\begin{pmatrix}
\underline{d_0^c} \\
\underline{d_1^c} \\
\underline{d_2^c} \\
\underline{d_3^c} \\
\underline{e^+}
\end{pmatrix} \\
-\nu_e
\end{pmatrix}
= (\overline{3},1) \oplus (1,2) \quad \text{in the following way:}
\]

(8)
Recall that 10 is the antisymmetric part of the product \( 5 \times 5 \):

\[
5 \times 5 = (10)_{\text{antisym}} + (15)_{\text{sym}}.
\]

Comparing (6), (7) and (8), we see that the sum of the multiplets \( \overline{5} \) and 10 include just all the 15 fermionic states. No one extra (new, unknown) state.

The gauge interactions in the SU(5) model proceed via exchange of 24-plet of vector bosons \( A_\mu \) of the adjoint representation of the group SU(5). With respect to the group SU(3)^c \times SU(2) they transform as follows:

\[
24 = (8,1) + (3,2) + (1,1) + (3,2) + (1,3). \quad (10)
\]

8 gluons \( W^+, Z^0 \) photon 12 leptoquarks \( X_\parallel, Y_\parallel \)

The leptoquarks, whose appearance is characteristic for any GUT, violate conservation of baryon, lepton and fermion numbers; it is just these particles (and scalar leptoquarks too) which are responsible for most interesting consequences of GUTs. It is seen that the leptoquarks \( X_\parallel \) and \( Y_\parallel \) are triplets under SU(3)^c and doublets under weak group SU(2). Their interactions with fermions are shown in Fig.1. The electric charges of \( X_\parallel \) and \( Y_\parallel \) are respectively:

\[
Q_X = 4/3, \quad Q_Y = 1/3. \quad (11)
\]

The part of the Lagrangian, describing X and Y interactions with fermions, is

\[
L = 2^{-1/2} g_5 \overline{X}^\mu \left( \overline{d}_R \gamma_\mu e^+_L + \overline{u}_L \gamma_\mu u_L + \overline{d}_L \gamma_\mu e^+_L \right) + \\
+ 2^{-1/2} g_5 \overline{Y}^\mu \left( \overline{d}_R \gamma_\mu \nu_R^c + \overline{u}_L \gamma_\mu d_L + \overline{u}_L \gamma_\mu \nu_R^c \right) + \text{h.c.} \quad (12)
\]

Now we can easily see what are the diagrams describing proton decay in the SU(5) model (see Fig.2). Similar diagrams can be drawn for the decays

\[
n \rightarrow e^+ + \pi^-, \quad n \rightarrow \overline{\nu} + \pi^0, \quad \text{etc.}
\]

Fig.1. Typical lepto-quark and diquark vertices for the leptoquarks X and Y.

It is convenient to separate the further description of the properties and the predictions of the SU(5) model into two parts: first we consider the properties which are independent on the details of the mechanism of spontaneous symmetry breaking (i.e. independent on structure of the Higgs sector of the model); then we shall proceed to the effects which, on the contrary, are related with the SU(5) symmetry breaking.
Fig. 2. Diagrams for $p \to e^+ u \bar{u}$ or $e^+ d \bar{d}$ and for $p \to \nu_d u$.

2.2. "Higgs-independent" features of the SU(5) model

1. Here takes place the charge quantization. Since the operator $Q$ is the generator of SU(5), its trace equals zero for each representation

$$\sum Q_i = 0.$$  \hspace{1cm} (13)

Then, in particular, for $\bar{5}$ we have

$$Q_d = \frac{1}{3} Q_3^-,$$  \hspace{1cm} (14)

and consequently, since $Q_W = Q_e$,

$$Q_u = \frac{2}{3} Q_e^+.$$  \hspace{1cm} (15)

It was assumed here that the quark charge does not depend upon color, i.e. quarks are fractionally charged. In the case of spontaneously broken color symmetry quarks could be integer charged (for details see Ref. 6), so instead of Eqs. (14) and (15) we have

$$\bar{Q}_d = \frac{1}{3} Q_3^-, \quad \bar{Q}_u = \frac{2}{3} Q_e^+,$$  \hspace{1cm} (16)

$\bar{Q}$ being quark charge averaged over color. The fact that the (averaged over color) charges of quarks proved to be multiple to $1/3$ $Q_3^-$ follows directly from the choice of color group (SU(3)$^c$). For example, would there be color group SU(n), one would have

$$Q_d = \frac{1}{n} Q_3^-, \quad Q_u = \frac{n-1}{n} Q_e^+.$$  \hspace{1cm} (17)

2. The Weinberg angle is exactly fixed in the model rather than the free parameter. Indeed, by definition

$$\sin^2 \theta_W = \frac{e}{g^2},$$  \hspace{1cm} (18)

e being the electric charge of electron, and $g$ being the weak SU(2) charge.
The covariant derivative is

$$D_\mu = \partial_\mu + \sum_k \frac{g_k}{c_k} T_k \gamma_\mu,$$

(19)

$g_k$, $T_k$ being coupling constants and generators of the chosen gauge group. Physics should not obviously change if one takes "other" coupling constants and generators

$$\frac{g_k}{c_k} T_k = \frac{g_k}{c_k} T_k \equiv g'_k T'_k, \quad c_k \equiv \text{numbers}.$$

Now we use the convention that we shall call the coupling constant the factor in front of the generators normalized in the following way

$$Sp T_i T_j = k \delta_{ij},$$

(20)

$k = 1/2$ for the fundamental representation. The SU(2) and SU(3) coupling constants has been already constructed just in this way. Until we do not embed QED in higher groups, we can not proceed in the same way with respect to electric charge. However, the situation is now different in GUTs. For example, in SU(5) model the electric charge being the generator of the group in the fundamental representation takes the form

$$Q = c \begin{pmatrix} 1/3 \\ \cdot \\ 1/3 \\ \cdot \\ 1/3 \\ \cdot \\ -1 \\ \cdot \\ 0 \end{pmatrix}.$$  

(21)

The normalization of this generator according to (20) gives


Thus, the electric charge of electron is

$$e = -c g_5 = -\sqrt{\frac{3}{5}} g_5,$$

(22)

$g_5$ being the unique SU(5) coupling constant. Hence,

$$\sin^2 \theta_W = c^2 = 3/8.$$  

(23)

It is convenient to use the general formula (for arbitrary representation):

$$\sin^2 \theta_W = \frac{Sp T_3^2}{Sp Q^2}.$$  

(24)

Recall, that the experimental value is $^7$

$$\sin^2 \theta_W = \begin{cases} 0.242 \pm 0.011 \pm 0.005 & \text{CCFR} \\
0.226 \pm 0.008 \pm 0.014 & \text{UA1} \\
0.216 \pm 0.010 \pm 0.007 & \text{UA2} \end{cases}.$$  

(25)

At first sight (23) and (25) contradict one to another. One has, however,
not to forget that the prediction (23) is valid only at superhigh energies, when strong, weak and electromagnetic interactions become equal in strength. The renormalization of the Weinberg angle from high energies to low energies will be yet discussed a little bit later, however, I would like to give already now the result

\[
\sin^2 \theta_W \left( Q^2 \sim \text{low} \sim \text{GeV}^2 \right) \sim \frac{g^2(g^2)}{g^2(Q^2)} \approx 0.20
\]

which has to be compared with the experimental value (25).

3. The presence in the theory of B-violating leptoquarks results in proton instability. In order to find the proton lifetime one has to know leptoquark masses or, in other words, the scale of SU.

4. To find the scale of grand unification \( M \) one has to take into account that the effective coupling constants \( \alpha_1, \alpha_2, \alpha_3 (\alpha_1 \equiv g_1^2/4\pi) \) corresponding to the groups U(1), SU(2) and SU(3) respectively, are in fact not constants at all, but depend upon the \( Q^2 \) transferred, and their energy dependence is governed by the equations of the renormalization group. The variation of coupling constants is shown schematically in Fig.3: the constants \( \alpha_2 \) and \( \alpha_3 \) are diminishing with energy (this is known as the "asymptotic freedom" behaviour) while the constant \( \alpha_1 \), on the contrary, is slowly raising. At some energy \( M \) (which is called the point of unification) all three curves coincide. Since the evolution of coupling constants goes very slowly (logarithmically), the unification scale \( M \) proves to be enormously large

\[
M \approx 10^{14} \times 10^{15} \text{ GeV.}
\]

The origin of such a great magnitude of \( M \) could be understood from the rough formula

\[
M \sim \Lambda \exp(k/\alpha),
\]

\( \Lambda \) being the scale parameter of strong interactions (\( \Lambda \approx 0.1 - 0.3 \text{ GeV} \)), \( \alpha \) being the fine structure constant, and \( k \) - being some numerical group factor.

![Fig.3. Rough evolution of the coupling constants of SM](image)

![Fig.4a. Naive evolution of coupling constants beyond \( M_X \)](image)

![Fig.4b. Evolution of the coupling constants in a unified theory](image)
If one were to keep following the evolution of the $\alpha'_{i}$s beyond $M$, one would obtain the rather disturbing picture of Fig.4a. In GUTs (and here in the SU(5)) would at $M$ new physics come in which forces $\alpha_{1}$, $\alpha_{2}$, and $\alpha_{3}$ to evolve in the same way, as shown in Fig.4b. This is a true unification. It is due to non (massless) bosons contribution.

Let us describe now in a little bit more details the procedure of finding of $M$ \[1/8\]. The renormalization group equations describing the evolution of coupling constants $g_{1}$, $g_{2}$, $g_{3}$ are

$$
\mu \frac{\partial g_{1}(\mu)}{\partial \mu} = - \frac{g_{1}^{3}}{16\pi^{2}} \left(-\frac{4}{3}N_{f} + \ldots \right)
$$

$$
\mu \frac{\partial g_{2}(\mu)}{\partial \mu} = - \frac{g_{2}^{3}}{16\pi^{2}} \left(+\frac{22}{3} - \frac{4}{3}N_{f} + \ldots \right)
$$

$$
\mu \frac{\partial g_{3}(\mu)}{\partial \mu} = - \frac{g_{3}^{3}}{16\pi^{2}} \left(11 - \frac{4}{3}N_{f} + \ldots \right)
$$

(29)

where $N_{f}$ is the number of fermion generations and dots denote omitted terms due to higher loops contributions as well as scalar ones. The following boundary conditions have to be satisfied:

$$
g_{1} = g_{2} = g_{3} \bigg|_{\mu = M}
$$

$$
\frac{g_{3}^{2}}{4\pi} \bigg|_{\mu = m} = \alpha_{s}(m)
$$

$$
\frac{25}{9} g_{1}^{-2} + \frac{15}{9} g_{2}^{-2} = e^{-2} \bigg|_{\mu = m}
$$

(30)

$m$ being an ordinary mass of order GeV. The solution to the system (29) with these boundary conditions gives

1) $\ln \frac{M}{m} = \frac{\pi}{11} \left(\frac{1}{\alpha} - \frac{8}{3} \frac{1}{\alpha_{s}(m)}\right)$

2) $\sin^{2} \theta(m) = \frac{3g_{1}^{2}(m)}{5g_{2}^{2}(m) + 3g_{1}^{2}(m)} = \frac{1}{6} + \frac{5\alpha}{9\alpha_{s}(m)} \approx 0.21$

(31)

3) $\alpha'_{\text{U}} = \alpha'_{i}(M) \approx 1/40$.

2.3. "Higgs-dependent" features of the SU(5) model

1. Scheme of symmetry breaking. The symmetry breaking occurs in two stages: first the SU(5) is broken down to the group $G = SU(3)^{c} \times SU(2) \times U(1)$ and then $G$ is broken down to SU(3)$^{c} \times U(1)_{em}$. The appropriate representa-
tions of scalar fields providing the first stage of symmetry breaking should contain a singlet under $G$ (neutral under $U(1)$). The minimal dimension representation with such properties is 24-plet of scalars $\Phi$ of the adjoint representation with the vacuum expectation value of the form

$$\langle \Phi \rangle = V_{\text{diag}}(1, 1, 1, -3/2, -3/2).$$ (32)

The leptoquarks $X$ and $Y$ get the masses

$$M_X^2 = M_Y^2 = \frac{25}{6} g^2 v^2 \approx M^2,$$ (33)

all other gauge bosons remain massless. Since the values of $g_1$, $g_2$, $g_3$ at low energies and the evolution curves of these constants are known, one can find the unification scale where the field $\Phi$ has developed the vacuum expectation value (VEV):

$$\langle \Phi \rangle \approx M \sim 10^{14} \text{ GeV}.$$ (34)

The subsequent symmetry breaking $SU(3)^c \times SU(2) \times U(1) \rightarrow SU(3)^0 \times U(1)_{\text{em}}$ is provided by the scalar field transforming as $(1, 2)$ with respect to $SU(3)^c \times SU(2)$. The minimal dimension representation with such properties is 5, with the VEV of the form

$$\langle H \rangle = (0, 0, 0, 0, v/\sqrt{2}).$$ (35)

The VEV of 5 we know, since it gives masses to $W^\pm, Z^0$:

$$\langle H \rangle \sim M \sim 10^2 \text{ GeV}.$$ (36)

The SU(5) model with the scalar sector consisting of only the representations 5 and 24 is called for simplicity the minimal SU(5).

We see that

$$\langle H \rangle / \langle \Phi \rangle \sim 10^{-12}$$ (37)

which is a very small number. What is the origin of this great difference in VEVs? This is what is referred to as a "hierarchy problem".

2. Fermion masses. The fermion masses are coming from VEVs of Higgs fields $\Phi$ interacting with fermions by means of Yukawa coupling

$$f \cdot \Psi_L^C \Psi_L \cdot \Phi + \text{h.c.},$$ (38)

where $C$ is the charge conjugation matrix, $\Psi_L$ are fermionic fields and $f$ is the Yukawa coupling constant. Since in the SU(5)

$$\begin{align*}
5 \times 5 &= 10 + 15 \\
\overline{5} \times 5 &= 5 + 45 \\
10 \times 10 &= \overline{5} + 45 + 50
\end{align*}$$ (39)

only the Higgs fields from the following multiplets are interacting directly with fermions

$$5, 10, 15, 45, 50.$$ (40)
Note that 24 does not interact with fermions that is very good, otherwise fermions would be superheavy. If SU(3)x U(1) is unbroken then a non-zero VEV may have (and, consequently, give masses to fermions) only the neutral color singlet. Such a field is presented only in
\[ 5, 15, 45 \quad . \] (41)

In the minimal SU(5) only 5 gives masses to fermions. Then, since the VEV of 5 possesses is SU(4) symmetric, one would have, without accounting for renormalization effects,
\[ m_d = m_e, \quad m_s = m, \quad m_b = m \tau \quad . \] (42)

After the renormalization to low energies \( 9 \)
\[ m_b/m = \left[ \frac{\alpha_3}{\alpha} \right]^{33} \frac{4}{11-\frac{4}{3}N} \approx 2.8 \] (43)

which is in excellent agreement with experiment:
\[ m_b/m \tau \sim 2.5 \quad . \] (44)

However, the predicted ratio
\[ m_s/m_d = m/m_e \approx 206 \] (45)
is in a very bad agreement with the experiment
\[ m_s/m_d \sim 20 \quad . \] (46)

This could be considered as an indication on the necessity of some modification of the scalar sector of the model. Using 45 in addition to 5 one can obtain more satisfactory mass relations.

2.4. Proton decay

To find the proton lifetime one has to:
1) evaluate hadronic matrix elements of the form \( <p | qq | p^0> \) etc., and
2) find the effective 4-fermion coupling constant \( G \):
\[ q \rightarrow e^+ \rightarrow q, G = g \frac{X, Y}{X, Y} ; \] \[ G \sim \frac{g^2(m_p)}{M_X^2} \quad . \]

The dimensional estimate gives
\[ \tau_p \sim \frac{M_X^4}{\frac{\alpha_3}{\alpha} \frac{M_X^2}{N}} \sim 10^{32} \quad . \] (47)

More accurate calculations taking into account corrections to the renormalization group equations, overlapping of quark wave functions inside nucleon etc, give \( 10 \)
\[ \tau_p = (0.2 + 4) \times 10^{28}(M_X/2.4 \times 10^{14})^4 \] (48)
\[ M_X = 2.4 \cdot 10^{14}(1.5)^{\pm 1} \left( \frac{\Lambda_{\overline{MS}}}{160 \text{ MeV}} \right) \text{ GeV}. \] (49)

The main uncertainty in \( \tau_p \) comes from the incorrect knowledge of the strong interaction scale parameter \( \Lambda \), \( \tau_p \sim \Lambda^4 \). The expected branching ratios for proton decay into various channels are (3):

i) inclusive modes
- \( p \to e^+ + \text{non-strange} \) 83% 
- \( p \to \bar{\nu} + \text{all} \) 15% 
- \( p \to \mu^+ + \text{non-strange} \) 1% 
- \( p \to \mu^+ + \text{strange} \) 1% 
- \( p \to e^+ + \text{strange} \) < 1% 

(50)

ii) exclusive modes
- \( p \to \pi^0 e^+ \) 40% 
- \( p \to \rho^0 e^+ \) 20% 
- \( p \to \eta e^+ \) 14% 
- \( p \to \omega e^+ \) 26% 

(51)

Note that muonic mode is suppressed. This is mainly due to the Cabibbo angle.

With the "extremum" values of all parameters (10)

\[ \Lambda_{\overline{MS}} = 200 \text{ MeV}, \quad M_X = 3.2 \cdot 10^{14} \text{ GeV} \] (52)

one obtains

\[ \tau_p = 1.4 \cdot 10^{30} (M_X/2.4 \cdot 10^{14})^4 = 4.3 \cdot 10^{30} \text{ y} \] (53)

\[ \tau(p_{\text{max}} \to e^+ \pi^0) = 10 \tau_p \leq 4.3 \cdot 10^{31} \text{ y} \]

The recent experimental bounds are

\[ \tau(p \to e^+ \pi^0) > 2 \cdot 10^{32} \text{ y} \] (11)

\[ \tau(p \to e^+ \pi^0) > 1.2 \cdot 10^{32} \text{ y} \] (12)

(54)

Thus, the minimal SU(5) model seems to be in trouble.

How to lengthen the proton lifetime? There are two ways:

A. to increase the unification scale \( M_X \), or
B. to change the effective Lagrangian describing the proton decay, or
do both A and B.

Consider these two possibilities consequently. Since \( M_X \) is determined by the renormalization group equations, one can increase \( M_X \) changing the equations themselves (or changing the boundary conditions). The equations will change if one introduces new additional fields in the theory. One may extend particle content either in the fermionic sector, or in the scalar sector, or in the gauge boson sector as well. The first two points can be realized yet in the framework of the SU(5) model. For example, additional fermionic multiplets \( 5 + \bar{5} \) or scalar multiplet(s) 45 could be added.

On the other hand, if one would like to extent the gauge boson sector, that
would mean that the gauge symmetry group should be wider than the SU(5).
The "minimal" embeddings are

\[ \text{SU}(5) \subset \text{SO}(10) \subset \text{E}(6) \]  

(55)

Going in this way, one would extent of course the variety of fermionic as well as scalar fields too.

2.5. The SO(10) model \(^{14}\)

In this model fermions of each generation are placed in one irreducible spinor 16-component representation

\[ \Psi = (u_1^u u_2^u u_3^d d_1^d d_2^d d_3^d \nu_e e^- u_1^c u_2^c u_3^c c_1^c c_2^c c_3^c \nu_e e^+ )_L \]  

(56)

which under SU(5) in reduced as

\[ 16 = 10 + \bar{5} + 1 \]  

(57)

Here is only one additional fermion (a right handed neutrino?) The fermions are Yukawa interacting with the following scalar multiplets

\[ 16 \times 16 = 10 + 120 + 126 \]  

(58)

All these scalar multiplets contain neutral color singlets and so they may give masses to fermions. Notice that under SU(5)

\[ 120 = 5 + \bar{5} + 10 + \bar{10} + 45 + 45 \]  

\[ 126 = 1 + \bar{5} + 10 + \bar{15} + 45 + 50 \]  

(59)

This means that by proceeding to consideration of the SO(10) model, one introduces in the scalar sector a number SU(5) multiplets.

The group SU(5) is a subgroup of SO(10). Another important subgroup is SU(4) \( \times \) SU(2) \( \times \) SU(2). If the SO(10) is first broken down to SU(5) then one will have a "low energy" SU(5) model and theoretical predictions differ little from the SU(5) model predictions with additional scalar multiplets. In the case of another SO(10) symmetry breaking

\[ \text{SO}(10) \rightarrow \text{SU}(4) \times \text{SU}(2) \times \text{SU}(2) \]  

(60)

one may consider leptons as the carriers of the 4-th color:

\[ 16 = (4, 1, 2) + (\bar{4}, 2, 1) \]  

(61)

After the further symmetry breaking

\[ \text{SU}(4) \times \text{SU}(2) \times \text{SU}(2) \rightarrow \text{SU}(3) \times \text{SU}(1) \times \text{SU}(2) \times \text{SU}(2) \]  

(62)

\[ 16 = (3, 2, 1) + (1, 2, 1) + (\bar{3}, 1, 2) + (1, 1, 2) \]  

(63)

quarks  leptons  \( \bar{q}, l \)

one may identify SU(2) \( \times \) SU(2)\( \times \)SU(1) group as a left-right symmetric electro-weak group

\[ \text{SU}(2) \times \text{SU}(2) \equiv \text{SU}(2)_L \times \text{SU}(2)_R \]  

(64)

and quark and lepton multiplets as shown above.
Therefore, in the SO(10) model before spontaneous symmetry breaking there is conserved parity in the gauge interactions (in contrast with the SU(5) model).

The group SO(10) has a number of subgroups and, consequently, there could be many different ways of spontaneous symmetry breaking. We show two of them, which are the most interesting ones:

\[
\begin{align*}
SO(10) & \xrightarrow{M_1} SU(5) \xrightarrow{M_2} SU(3)^C \times SU(2) \times U(1) \xrightarrow{M_W} SU(3)^C \times U(1)_{\text{em}} \\
SO(10) & \xrightarrow{M_X} SU(4) \times SU(2)_L \times SU(2)_R \xrightarrow{M_C} SU(3)^C \times SU(2)_L \times SU(2)_R \xrightarrow{M_R} SU(3)^C \times U(1) \xrightarrow{M_W} SU(3)^C \times U(4)_{\text{em}}
\end{align*}
\]

Here \(M_i\) are characteristic energies connected with different stages of symmetry breaking. (We imply \(M_1 \gg M_2 \gg M_W; M_X \gg M_C \gg M_R \gg M_W\). We emphasize that here, contrary to the case of the minimal SU(5), one has 1 or 2 additional intermediate mass scales, i.e., partial population of the Grand Desert. This reduced the predictive power of the theory, since the value of the unification mass depends essentially upon the intermediate mass scales. On the other hand however, the proton lifetime could be varied in higher degree that provides a possibility to avoid contradiction with the experiment.

2.6. \(n\bar{n}\)-oscillations

Introducing (even in the framework of the SU(5) model) of additional scalar multiplets gives rise to new physical phenomena. In particular, the effect of \(n\bar{n}\)-oscillations could take place. For this phenomenon to occur, one needs diquark vertices of the form

\[
\begin{align*}
q & \rightarrow \Phi \\
\bar{q} & \rightarrow \Phi
\end{align*}
\]

Then the process \(n \leftrightarrow \bar{n}\) could proceed as

\[
\begin{align*}
n \left\{ q \rightarrow \Phi, \overline{\bar{q}} \rightarrow \overline{\Phi} \right\} \rightarrow \bar{n}
\end{align*}
\]

Since the amplitude of this process is \(\sim M_\Phi^6\), where \(M_\Phi\) is the mass of \(\Phi\), the \(M_\Phi\) should be small (in comparison with \(M_X\)) to provide experimental observability of the process. Experimentally \(^{16}\) the \(n\bar{n}\)-oscillation period is bounded from below by \(T > 10^{-8}\) s. The effective strength of the transition \(\varepsilon\) and the oscillation period are simply related:

\[
\varepsilon \frac{n}{\bar{n}} = \frac{2\pi}{\varepsilon} \\
T = \frac{2\pi}{\varepsilon}
\]

We have roughly

\[
\varepsilon \sim \frac{1}{M_\Phi^6} \cdot M_\Phi \cdot m_n^6 < 2\pi \cdot 10^{-8}\text{s}^{-1} \approx 10^{-31}\text{ GeV}.
\]
Hence
\[ M_\varphi > 10^6 \text{ GeV} \quad . \] (71)

Leptoquarks have also the diquark vertices like (67). However, they can not provide observable \( n\bar{n} \) transitions since \( M_{\text{leptoquark}} \approx 10^{14} \) to \( 10^{15} \text{GeV} \. We have \( 3 \times 3 = 6 + \bar{3} \) in color space,
\[
\begin{array}{c}
q \rightarrow 5 \quad 6 \text{ or } 3 \\
q \rightarrow 5 \\
\end{array}
\] (72)

However, \( \{ \varphi \} = \bar{3} \) may have the coupling with \( (1\bar{3}) \) current too
\[
\begin{array}{c}
\bar{q} \rightarrow 3 \\
\bar{q} \rightarrow 3 \\
l \rightarrow \varphi \\
\end{array}
\] (73)

and is of no interest for the \( n\bar{n} \)-transition business. Therefore, one has to have \( \{ \varphi \} = \bar{6} \) under \( SU(2)_W^0 \). Further, we have more explicitly (do not pay attention, for a moment, to scalar field assignments):
\[
\begin{array}{c}
\begin{array}{c}
\bar{d} \\
\bar{d} \\
\bar{d} \\
\end{array}
\end{array} \rightarrow \frac{4}{5} - \frac{15}{5} - \frac{15}{5} \rightarrow \begin{array}{c}
\bar{d} \\
\bar{d} \\
\bar{d} \\
\end{array} \rightarrow \bar{n} \\
\] (74)

So, we need both vertices
\[
\begin{array}{c}
d \\
Q = -2/3 \\
\end{array}
\] (75)

and vertices
\[
\begin{array}{c}
d \\
Q = -2/3 \\
\end{array}
\] (76)

Taking into account the vertex
\[
\begin{array}{c}
u \\
Q = -4/3 \\
\end{array}
\] (77)

too, we obtain that the field \( \varphi \) has to be a triplet under \( SU(2)_W \), \( \{ \varphi \} = 3_W \). It is easy to find the corresponding hypercharge of \( \varphi : Y_\varphi = -1/3 \). Therefore, to realize the \( n\bar{n} \) transition phenomenon, one has to have the scalar field with the following quantum numbers under \( SU(3)_c \times SU(2)_W \times U(1)_E \):
\[
\{ \varphi \} = (\bar{6}, 3, -1/3) \quad . \] (78)

Remarkable that even in the \( SU(5) \) model there exists such a field:
\[
50 = (\bar{6}, 3, -1/3) + ... \] (79)

which, fortunately, does interact with fermions (see (40)).

Moreover, since under \( SU(2)_L \), \( 2 \times 2 = 3 + 1 \),
\[
\begin{array}{c}
q \rightarrow 2 \quad 3 + 1 \\
q \rightarrow 2 \\
\end{array}
\] (80)
the field \( \varphi \) could be both \( 3 \) and \( 1 \). Hence, one has to have two fields, which are \( \bar{6} \) under \( SU(3)_c \) and singlets under \( SU(2)_W \) with different hypercharges. Fortunately, again, one of the necessary fields could be found in \( 15 \) while the other in \( 45 \).
\[ T \sim \frac{2}{\Lambda x^3} \]
\[ u \rightarrow 45 \quad d \rightarrow 15 \]
\[ d \rightarrow d \quad \tau \rightarrow \gamma \quad u \rightarrow t \]

\( \lambda \) and \( f \) being coupling constants of triple scalar and Yukawa interactions, respectively.

So, here also is a room for \( n \bar{n} \) transitions in the framework of the SU(5) model.

The \( n \bar{n} \) oscillation phenomenon was considered in a number of papers.

I would like to mention finally that first experimental searches for \( n \bar{n} \) oscillations in free neutron beam have been already carried out and a number of experiments are under preparation\(^{17}\).

Now I would like to mention briefly some approaches to proton stabilization by changing of the effective Lagrangian for proton decay\(^{10}\).

The introduction of the Higgs multiplet 45 leads, in general, to the appearance of (new) quark mixing angles which make proton more long-living. While in the Kobayashi-Maskawa mixing take part only left-handed components of quarks in the leptoquarks currents take part both left-handed and right-handed components. Usually one has \( p = \{ d u u \} \rightarrow e^+ u u^c \) (see Fig.2).

It is important that in the final state here stand \( u^c \) or \( d^c \) quarks.

Now, however, with the introduction of 45, there could arise new mixings (and even interchanging of the right-handed fermions from different generations), for example

\[ u^c \rightarrow u^c \cos \theta^c + t^c \sin \theta^c \quad (82) \]

At \( \phi^c = \pi /2 \) one would have

\[ p \left\{ \begin{array}{c}
- \frac{d}{x} e^+ \\
\frac{u}{x} t^c \\
\frac{u}{x} u^c \\
\end{array} \right. \]

so that proton cannot decay due to energy conservation reasons. The 10-plets will now have the form

\[ \Psi_u = \left( \begin{array}{c}
- \frac{e^+}{x} \\
\frac{u}{x} \gamma \\
\frac{d^*}{x} \gamma \\
\frac{t^c}{x} \gamma \\
\end{array} \right) L \]

\[ \Psi_t = \left( \begin{array}{c}
- \frac{\tau^+}{x} \\
\frac{u^c}{x} \gamma \\
\frac{b^*}{x} \gamma \\
\frac{u^c}{x} \gamma \\
\end{array} \right) L \quad (83) \]

the primed quantities denote usual Kobayashi-Maskawa mixed states.

It may be shown\(^{10}\) that by means of these new mixing and replacements the proton lifetime could not be made very long, though it...
could not be make absolutely stable (for example, in the model with the
replacement \( u^0 \leftrightarrow t^0 \) proton will decay because of \( d \)-quark admixture in
\( b' \)).

In the SU(5) model with 45 which is under SU(3) x SU(2) x U(1)

\[
45 = (1, 2, -1) + (3, 1, 2/3) + (3, 3, 2/3) + \\
\uparrow \downarrow \downarrow \\
m_1 m_2 m_3 \\
+ (3, 1, -8/3) + (3, 2, 7/3) + (6, 1, 2/3) + (8, 2, -1) \\
\uparrow \uparrow \uparrow \\
m_4 m_5 m_6 \\
\text{with appropriate splitting between mass scales } m_4, \text{ one obtains as strong}
\text{prolongation of proton lifetime as } 10^4 \text{.}
\]

\[
\tau_p^{(45)}/\tau_p \sim 3 \cdot 10^4 \text{. (85)}
\]

(Here care should be taken about the value of \( \sin^2 \theta_W \) because it is also
changed; for example, if the model mentioned, \( \Delta (\sin^2 \theta_W) = -0.017 \).)

Proton could be made absolutely stable \(^{48}\) though there is still
baryon nonconservation in the theory if some of usual fermions in genera-
tions are replaced by new (heavy) fermions. This is equivalent to the
introduction of a new, absolutely conserved quantum number (either additive
or multiplicative) which forbids proton decay into light particles. The
new conservation law forbids, for example, the decay \( p \longrightarrow e^+ \pi^0 \) while
allows \( p \longrightarrow \pi^0 e^+ \), \( E^+ \) being new heavy lepton, so this decay mode is
forbidden by energy conservation).

Let me say a few words about an alternative approach to the construc-
tion of a GUT, originally developed by Pati and Salam \(^4\). One could
imagine basically two approaches to unification. The first is the "uni-
fication of interactions" and the examples of such kind theories are the
SU(5) and the SO(10) models we have already considered. In the second
approach the starting point is the maximal symmetry between leptons and
quarks (say, lepton are considered as the fourth color state), so that it
could be called the "unification of particles". First of all here the
fermion multiplets are constructed in a desired way and then one takes the
appropriate symmetry group, which is semisimple \((G = G_1 \times G_2 \times \ldots, \ G_i
\text{ being simple groups})\). The minimal model constructed in such a way is the
SU(4)\(^4\) model \(^{49}\). Here are several interesting features of the model:
\( i) \) all fermions belong to fundamental representation of the gauge group,
\( \text{ii) the "early unification" is possible here, } \mathcal{M}_X \sim 10^{5+7} \text{GeV, but proton is}
\text{nevertheless fairly long-lived, since here } \tau_p \sim \mathcal{M}_X^{12} \text{ due to fermion num-
ber conservation, } \text{iii) proton decay modes are completely different from those predicted, say, by the SU(5) model, etc. I would say finally that} \)
only experiment may provide arguments in favour of one or another (if any) scheme of GU.

Here I would like to finish the first part of my lecture. I have not even touched a number of problems, such as neutrino masses, monopoles, catalysis of proton decay etc. I have not even outlined also the next step of unification, just supersymmetric unification - this is perfectly done in a number of review papers and talks, in particular this will be covered by the lecture course by J. Ellis at this school. I proceed now to the second part - cosmological implications of GUTs.

3. GUTs AND COSMOLOGY

The early Universe seems to be the only 'laboratory' where existed superhigh energies like $10^{15}$ GeV at which GU interactions could reveal themselves with the full strength. The application of GUTs to various cosmological problems proved to be rather successful. The baryon asymmetry of the Universe seems to be explained. Further, there exist approaches to the solution to horizon, global homogeneity and flatness problems as well as to the problem of inhomogeneities necessary for galaxy formation, etc.

We start with the description of the standard scenario of the Universe evolution.

3.1. Standard hot Universe

What are the observational data which convince us that our Universe at the early stages of evolution was hot?

1. There was discovered by Penzias and Wilson in 1965 the 3 K black-body radiation (proclaimed by G. Gamow in 1948), which proved to have a thermal spectrum and isotropic distribution.

2. The Hubble expansion of the Universe is the well established fact. It follows then that the earlier Universe was more dense and more hot. Now one needs to extrapolate a little bit (not more than, say, 28 orders of magnitude) to arrive at the temperature of about $T \sim 10^{15}$ GeV.

3. Calculations of relative abundances of light nuclei (mainly, $^4\text{He}/p$) in the model are in reasonable agreement with observations. So, one may conclude that we know rather well the rate of the Universe expansion at temperature $T \sim 1$ MeV.

We shall be interested mainly in the Universe evolution at temperatures of order of GU mass scales, $T \sim 10^{15} - 10^{10}$ GeV. Since $T \ll T_{Pl} \approx 10^{19}$ GeV, one may neglect quantum gravity effects and use the classical Einstein equations:

$$\frac{\dot{R}}{R} = - \frac{4\pi}{M_{Pl}^2} \left( p + \frac{\xi}{3} \right)$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi}{3M_{Pl}^2} - \frac{k}{R^2}$$

(86)  (87)
where $R$ is the scale factor, \( s \) and $p$ are energy and pressure densities, $k = 1, 0, -1$ for a closed, spatially flat and open world, respectively.

(The variation of temperature is given by the Eq. (87) with the term $(R/R)^2$ replaced by $(t/T)^2$.) At the very early stages of expansion one may neglect the term $k/R^2$. The solutions of these equations in two limiting cases take the following forms:

i) Radiation-dominated case. In this case the main contribution to the right-hand side of Eq. (87) is due to ultra-relativistic particles with the equation of state

\[ p = \frac{s}{3} \quad . \]

Then

\[ s(t) = \frac{3 M_{pl}^2}{32 \pi} \cdot \frac{1}{t^2} \quad . \]

Substituting into Eq. (89) the usual thermodynamical expression for radiation energy density $s = (N_{\text{eff}} \cdot \pi^2/30) T^4$, where $N_{\text{eff}}$ is the effective number of massless degrees of freedom, $N_{\text{eff}} = 1$ or 1/8 for each boson and fermion degree of freedom, respectively, one obtains the time dependence of temperature

\[ t = \frac{M_0}{\pi^2} \quad , \quad M_0 = M_{pl}/1.66 \cdot N_{\text{eff}}^{1/2} \quad . \]

The scale factor in the RD stage is varying as

\[ R \sim t^{1/2} \quad . \]

If the equation of state does not change, the RD stage is taking place till the first term in Eq. (87) is larger than the second one. When these two terms become equal, the expansion of the closed world turns to the collapse, while the open world continues to expand, curvature being dominating. The critical moment is characterized by the temperature

\[ T^* \sim M_{pl}/N^{1/3} \quad , \]

where $N$ is the coordinate entropy density,

\[ N = s(t) R^3(t) \quad , \]

\[ s(t) = \frac{4}{3} \frac{\pi^2}{30} N_{\text{eff}} \cdot T^3 \quad . \]

Thus, the closed Universe at $T = T^*$ turns to the collapse. With $N = 1$ it lives $t \sim t_{pl} \approx 10^{-43}$ s. (In the open Universe with small $N$ all the matter is scattered at $T < T^*$ and the formation of stars, say, is impossible in such a Universe). For the Universe in RD stage to survive till the temperature $3^\circ K$, there should be

\[ RT \sim N^{1/3} \gg M_{pl}/3^\circ K \sim 10^{28} \quad . \]
ii) Vacuum-energy domination. Now the equation of state is $p = -\varepsilon_{\text{vac}}$. This takes place in phase transitions at strong supercooling when thermal energy of particles decreases as a result of expansion $\varepsilon_{\text{part}} \ll \varepsilon_{\text{vac}}$. (This is just the case in the 'inflating' Universe). The solution of Eq. (87) at $t \gg \sqrt{3M_{\text{Pl}}^2/18\pi\varepsilon}$ has the form

$$\varepsilon = \text{const}$$

$$R \sim \exp\left\{\frac{8\pi\varepsilon}{3M_{\text{Pl}}^2} \cdot t\right\}$$

(95)

3.2. Baryon asymmetry of the Universe

At present there are no indications that somewhere in the Universe there is antimatter in amount comparable with matter. This is definitely so in our local cluster of galaxies, otherwise one would detect a $\gamma$-flux from matter-antimatter annihilation. Extrapolating these conclusions to the whole Universe one may suppose that there exists the global BAU by definition is nothing but

$$\Delta(t) = \frac{n(t) - \overline{n}(t)}{n(t) + \overline{n}(t)}$$

(96)

where $n(t)$ and $\overline{n}(t)$ are number densities of baryons and antibaryons at the time $t$, respectively. At present, the asymmetry is complete, i.e. $\Delta(\text{now}) = 1$.

At temperatures $T > n_N$ (or $t < 10^{-6}$s) the number densities of baryon-antibaryon (or quark-antiquark) pairs coincide with that of photons. Hence,

$$\Delta(t < 10^{-6}s) \sim \frac{n - \overline{n}}{n} = \delta$$

(97)

The value of $\delta$ depends little upon time since at $T \lesssim 1$ GeV the rate of B-nonconserving processes is negligible. Thus

$$\delta = \frac{n}{n_N} \bigg|_{t = 3 \cdot 10^{17}s} = 10^{-8} - 10^{-10}$$

(98)

The photon concentration is known very well: $T \approx 2.7^\circ \text{K}$, $n_\gamma \approx 500 \text{ cm}^{-3}$. The density of baryon number is known much worse: the bounds on deceleration parameter $2q_0 = \frac{8\pi}{3H^2}M_{\text{Pl}}^2 = \rho/\rho_c$ (where $H = \dot{R}/R$ is the Hubble constant, $\rho_c$ is the critical density; the cases $\rho > \rho_c$, $\rho = \rho_c$, $\rho < \rho_c$ correspond to closed, flat and open Universe, respectively) give $n < 3 \cdot 10^{-6}\text{cm}^{-3}$. On other hand, from the mass of visible matter of galaxies, $n > 3 \cdot 10^{-8}\text{cm}^{-3}$. Thus, $21, 22$

$$\delta = 10^{-8} - 10^{-10}$$

(99)

i.e. at $t < 10^{-6}$s there was one extra baryon per $10^8 - 10^{10}$ BB-pairs. The
quantity δ is a fundamental characteristic of the Universe which origin has to be understood.

The most attractive seems to be the explanation of the origin of baryon excess, in which the Universe was first symmetric in B and then, at some stage of expansion, there appears a matter asymmetry (in the visible part) of the Universe. If B is strictly conserved then one needs either matter-antimatter separation at macroscopic scales (this seems to be a rather difficult job) or the B absorption by, say, black holes which could, in principle, with account of CP-violation in microprocesses, separate matter from antimatter. Of course, one could consider the Universe as being globally asymmetric from the very beginning, however, this seems not to be an explanation at all.

The most natural therefore seems to be the approach first suggested in Refs. [25, 26] where the (global) BAU was just generated starting with a B-symmetric initial state. It is clear that one needs [25-27]:

i) B-nonconservation, in order to obtain a B ≠ 0 state from B = 0 one,

ii) the B-nonconserving interactions have to violate C-parity, since C could be the exact symmetry only if B = 0 (C transfer a quark into an antiquark); the same is true with respect to CP-parity,

iii) finally, these B-nonconserving interactions at the moment of the B-excess generation have not to be in thermal equilibrium otherwise the GPT-conservation provides neutrality of the system with respect to all nonconserved charges, here with respect to B.

The BAU generation scenario proposed in Refs. [25, 26] has not in fact changed so much except for the invention of GUMs. In Ref. [26] there were formulated the main thermodynamical conditions of the BAU generation and the (B-violating and CP, C-nonconserving) decay of heavy particles was suggested as an origin of the macroscopic BAU. The BAU generation problem in the framework of gauge theories with spontaneous symmetry breaking was first studied in Refs. [28]

The synthesis of GUTs and the theory of hot Universe provides that all three mentioned conditions for the BAU production are naturally fulfilled. Indeed, GUTs predict B- and CP-violating interactions which are becoming non-equilibrium at some stage of expansion.

Let us first describe in a few words the standard baryon production scenario in the framework of GUTs [29-29]. At temperatures $10^{16}$ GeV $\gg T \gg 10^{13}$ GeV GU interactions are in equilibrium in cosmological plasma. Then at $T \ll 10^{13}$ GeV they come out of equilibrium due to their weakness and the fast enough expansion of the Universe. The equilibrium with respect to $\Delta B \neq 0$ stage washes out any initial value of B, i.e. at the equilibrium stage B vanishes irrespective to any initial value. At the non-equilibrium stage due to asymmetric decays of leptoquarks the Universe acquires a non-zero macroscopic value of B.

The criterion of equilibrium (with respect to the given process) in the expanding Universe may be taken as follows
\[ t_{\text{process}} < t_{\text{expansion}} \] (100)

\[ t_{\text{process}} \] being the characteristic time for the given process to occur, and \[ t_{\text{expansion}} \] is given by Eq. (90). For the two particle collision process one has

\[ t_{\text{process}} = (\sigma v)^{-1} \] (101)

where \( \sigma \) is the cross-section, \( v \) is the relative velocity, and \( n \) is the number density of particles under consideration. For a decay process and an inverse decay process one has, respectively,

\[ t \sim \Gamma^{-1} \quad \text{and} \quad t \sim (\Gamma n_{eq})^{-1} \] (102)

where \( \Gamma \) is the decay width and \( n_{eq} \) is the equilibrium number density of decaying particles. Among the B-nonconserving processes there are:

i) light fermion scattering, ii) decays inverse decays of vector and Higgs leptoquarks (we now denote them by \( \chi \)), and iii) boson scattering on fermions. It has been understood that light fermions scattering could not contribute considerably to the BAU. We shall discuss only \( \chi \) decays.

For the \( \chi \) -decays to lead to the BAU production, it is necessary that:

a) the baryon number generated at \( \chi \) -decays was not cancelled by the that from \( \chi \) -decays.

b) the decays have to take place at non-equilibrium conditions, i.e. \( \chi \) -concentration should differ from the equilibrium one.

c) at the stage of \( \chi \) -decays all the B-nonconserving interactions should be non-equilibrium too, otherwise they would wash out the B-excess arising as a result of \( \chi \) -decays.

Consider these requirements consequently

a) The relative quark excess over antiquarks arising from decays of \( \chi \) and \( \overline{\chi} \) in a vacuum, is given by

\[ \delta_{\text{(micro)}} = (\Gamma_{\chi \to i})^{-1} \left[ \sum (\Gamma(\chi \to i)B_i + \Gamma(\overline{\chi} \to \overline{i})B_{\overline{i}}) \right] \] (103)

where \( \Gamma (\chi \to i) \) is the partial decay width for the decay mode \( \chi \to i \), \( B_i \) is the baryon number of \( i \)-th channel. Obviously, it is necessary that \( \chi \) -boson to have at least two decay channels with different baryon numbers. Otherwise,

\[ \delta = B(\Gamma_{\chi \to i})^{-1} (\Gamma_{\chi \to i} - \Gamma_{\overline{\chi} \to \overline{i}}) = 0 \] (104)

due to CPT-invariance. Let us consider now, for example, the case, when \( \chi(\overline{\chi}) \) has just two decay modes with the partial widths \( \Gamma_1 \) and \( \Gamma_2 \) (\( \overline{\Gamma}_1 \) and \( \overline{\Gamma}_2 \), respectively) and baryon numbers \( B_1 \) and \( B_2 \) (correspondingly, \( -B_1 \) and \( -B_2 \)). Using Eq. (103) one gets
\[ \delta = (\Gamma_{\text{tot}}^X)^{-1}(B_1 - B_2)(\Gamma_1 - \Gamma_{\bar{1}}) \]  
(105)

Note that as we have already seen (Fig.1), say, in the SU(5) model the \( X \) and \( Y \) leptoquarks have several decay modes with \( B_1 = 2/3 \) (in quark-quark channel and \( B_2 = 1/3 \) (in the antiquark-antilepton channel).

The microscopic mechanism giving rise to the nonvanishing difference of partial decay widths in Eq. (105) is provided by CP-violtion effects. How can one calculate \( \Gamma_1 - \Gamma_{\bar{1}} \)? Let us consider diagrams contributing to the decays widths taking into account one-loop corrections, i.e. accounting for virtual exchanges (see Fig.5). The amplitudes of charge-conjugated processes are given by

\[ A (\chi^i \rightarrow \chi) = g^0 + \sum_k g^k A^k \]
\[ A (\chi \rightarrow \bar{\chi}) = g^0 + \sum_k g^* A^k \]  
(106)

where \( g^0 \) and \( g^k \) are the corresponding coupling constants and \( A^k \) is the radiation correction taken at the unit values of coupling constants. It follows then that

\[ \Gamma_1 - \Gamma_{\bar{1}} \sim \sum_k \text{Im} \, g^0 \, g_k^* \, \text{Im} \, A^k \]  
(107)

Fig.5. Radiation corrections to scalar leptoquark decays contributing to baryon excess

Thus, we see that both coupling constants and the corresponding diagrams are to have imaginary parts in order to provide \( \Gamma_1 - \Gamma_{\bar{1}} \neq 0 \). (The complexity of coupling constants means CP-noninvariance of the interaction). Is there the CP-violation present in GUTs? The answer is yes, there is new source of CP-violation in GUTs, which is absent in low energy standard model. For example, even in the minimal SU(5) model the Yukawa coupling of 5-plet of scalars \( H \) with fermions has the form

\[ f_{\alpha\beta} H_i^\alpha \bar{E}_{\alpha}^{ij} \Psi_{\beta}^j + g_{\alpha\beta} H_i^i \bar{E}_{\alpha}^{ijkl} \Psi_{\beta}^{jk(0)} \bar{E}_{\beta}^{lm} + h.c. \]  
(108)

where \( \Psi, \bar{\epsilon} \) are fermion multiplets 5 and 10, and \( f_{\alpha\beta} \) and \( g_{\alpha\beta} \) are (complex) coupling constants, latin indices correspond to SU(5) while Greek ones numerate fermion generations. It may be shown that by no redefinition of fermion phases one could make constants \( f_{\alpha\beta} \) and \( g_{\alpha\beta} \) real.

Let us proceed now to the thermodynamical conditions.

b) The \( \chi \) -bosons decay at \( t_{\text{decay}} \approx t_{\text{expansion}} \), and
where the factor $T/m_\chi$ accounts for the Lorentz transformation of time. The corresponding temperature is

$$T_{\text{decay}} = \begin{cases} \Gamma^{-1} & ; T \leq m_\chi \\ T(m_\chi \Gamma)^{-1} & ; T > m_\chi \end{cases} \quad (109)$$

Till the moment $T = T_{\text{decay}}$ the $\chi$ number density behaves as the conserved particle number density, i.e. $n_\chi \sim R^{-3}$, and it is equal to (at $t = t_{\text{decay}}$):

$$n_\chi = g \cdot \pi^{-2} T_{\text{decay}}^3 \quad (111)$$

g being the spin factor. However, the equilibrium concentration corresponding to $T_{\text{decay}}$, is

$$n = \begin{cases} g(m_\chi T_{\text{decay}}/2\pi)^{3/2} \exp(-m_\chi/T_{\text{decay}}) & ; T_{\text{decay}} > m_\chi \\ g\pi^{-2} T_{\text{decay}}^3 & ; T_{\text{decay}} \leq m_\chi \end{cases} \quad (112)$$

Comparing now Eqs. (112) and (111) we see that in the case $T_{\text{decay}} \gg m_\chi$ the $\chi$ concentration coincides with the $n_{eq}$. In addition, the rate of $\chi$ decay after $T_{\text{decay}}$ exceeds the expansion rate. Therefore, after $T_{\text{decay}}$ the $\chi$-disappearance from cosmological plasma will go in equilibrium way, i.e. concentration will be equilibrium and will be determined by the Universe temperature only. In this case, obviously, there is impossible the BAU generation.

At $T_{\text{decay}} \leq m_\chi$, on the contrary, the $\chi$-concentration at the moment $T = T_{\text{decay}}$ exceeds considerably the equilibrium value. This excess decay in non-equilibrium way, and the BAU production is possible in principle. The condition $T_{\text{decay}} \leq m_\chi$ leads to the following bound on the mass of particle responsible for the BAU production

$$m_\chi \geq \alpha_{\chi} m_0 \quad , \quad \alpha_{\chi} = \frac{\Gamma}{m_\chi} \quad (113)$$

Let us give, for example, the values of $\alpha_{\chi} m_0$ for gauge ($X$) and scalar ($H$) leptoquarks of the SU(5) model. Since

$$\Gamma_X = \alpha_{\text{GUT}} m_X \quad ; \quad \Gamma_H \simeq \frac{7}{4} \alpha_{\text{GUT}} (m_t/m_W)^2 m_H \quad , \quad (114)$$

$m_t$ and $m_W$ being $t$-quark and $W$-boson masses, respectively, one obtains

$$m_\chi \geq 2 \cdot 10^{16} \text{ GeV} \quad , \quad m_H \geq (10^{13} \text{ to } 10^{14}) \text{ GeV} \quad (115)$$

(we have taken $m_t \sim 50$ GeV).
c) For the baryon excess due to $\chi$-decays not to vanish, it is necessary the rates not to vanish, it is necessary that the rates of baryon-nonconserving processes were less than the expansion rate. For four-fermion scattering processes $qq \rightarrow \chi \rightarrow \overline{q}l$ one has:

$$
\frac{1}{\tau_{\text{scat}}} = 6\pi \sim \frac{\alpha^2 \chi T^2}{m^4 \chi} T^3 = \frac{\alpha^2 \chi M_0 T^3}{m^4 \chi} \tau^{-1}_{\text{expansion}}
$$

i.e. scattering processes are not effective if (113) is valid. The inverse decay rate $(qq \rightarrow \chi, \overline{q}l \rightarrow \chi)$ is given by

$$
\tau^{-1}_{\text{inv}} = \left(\frac{m \chi}{T}\right)^{3/2} \exp\left(-\frac{m \chi}{T}\right)
$$

and it is also smaller than the expansion rate at the decay moment. The exponential factor here is due to the fact that the number density of fermions which could create $\chi$-boson at the collision is determined by the equilibrium distribution function of quarks and leptons, $n \sim \exp\left(-\frac{E}{T}\right)$ $\sim \exp\left(-\frac{m \chi}{T}\right)$, $E$ being the fermion energy.

Thus, at $m \chi > \alpha \chi M_0$ the quark excess over antiquarks arising in $\chi$-decays is preserved till now. So, at $T \sim T_{\text{decay}}$ the baryon asymmetry coincides with $\delta$,

$$
\frac{N - \overline{N}}{N + \overline{N}} \bigg|_{T \sim T_{\text{decay}}} = \delta
$$

The present value of $\Delta N/n_{\chi}$ is somewhat smaller due to heating of background radiation by heavy particle annihilation at later stages and is given by $31$

$$
\frac{\Delta N}{n_{\chi}} = \frac{\zeta(3)}{4 \pi^4} \sum \frac{E_{\chi} S_{\chi}}{N_{\text{eff}}}
$$

Here the sum is over all particles-parents of the BAU. Therefore, at large enough leptoquark masses we have the direct expression of the fundamental quantity $\delta$ in terms of a GUT. If a grand unified scale is smaller than (115), one has to solve explicitly the kinetic equations for leptoquarks in plasma, in order to obtain the macroscopic suppression factors (i.e. to account for washing out of baryon excess). The expression for $\Delta N/n_{\chi}$ then reads

$$
\frac{\Delta N}{n_{\chi}} = \frac{\zeta(3)}{4 \pi^4} \sum \frac{E_{\chi} S_{\chi}}{N_{\text{eff}}}
$$

$S_{\chi}$ being macroscopic suppression factors. They are $S_{\chi} \lesssim 10^{-2} - 10^{-6}$.

With the natural choice of a GUT parameters one obtains

$$
\delta \sim 10^{-6} - 10^{-12}
$$
to be compared with observational value $8 \approx 10^{-8} - 10^{-6}$. Despite all uncertainties in calculations of $\phi$, the agreement with observational data seems to be quite satisfactory, so one may consider it as an indication that we are on the right way.

3.3. Cosmological phase transitions

At the present stage of the Universe evolution we are living in the $SU(3)^0 \times U(1)^{em}$ symmetric phase, which in the lowest minimum of the effective potential (i.e., energy) of scalar fields. The symmetry of the vacuum state at early stages of evolution could be a different one. It is determined by minimization of the free energy.

High temperature phase transitions in the early Universe are inevitable consequence of the Grand Unified Theories. The study of the phase transitions (besides the academic interest) is of great physical importance. Anyway, the account of the phase transitions has proved to be quite essential in considerations of almost all cosmological problems.

As we now believe there occurred at least the following phase transitions in the history of the Universe:

1. At $T \approx 10^2$ MeV the hadronization of hot quark plasma took place. Free quarks and gluons disappeared while forming hadrons.

2. At $T \approx 10^2$ GeV the electroweak symmetry $SU(2)_L \times U(1)$ was broken and $W^\pm, Z^0$ become massive.

3. At $T \approx 10^{15}$ GeV the symmetry of GU was broken. Leptoquarks become supermassive.

In this lecture we shall be interested mainly in the latter phase transition.

At certain temperature the mean value of a scalar field $\phi$, which had corresponded to the ground state of the system, under consideration (the hot Universe filled by particles and field quanta) cease to be equilibrium. The main object of investigations of corresponding phase transitions is the effective potential $V$ which coincides with the free energy $f$ of a system (at vanishing chemical potential) and equals to pressure taken with the opposite sign:

$$V(\phi) = f(\phi) = -p(\phi) = -\varepsilon(\phi) - Ts(\phi),$$

(122)

where $s = -\partial f/\partial T$ is the entropy density. In a metastable state $V$ is a complex number, its imaginary part gives the probability of decay in unit volume per unit time. From Eq. (122) we that at $T = 0$ the equation of state is

$$p = -\varepsilon,$$

(123)

i.e., the state is a pure vacuum. Thus the value of $V$ at the point of extremum at $T = 0$ gives the energy density of the corresponding vacuum state.

In flat space-time the effective potential in one-loop approximation is given by $32-37)$
\[ V = V_{\text{tree}} + V_{\text{eff}}^{(1)} + V_T^{(1)}. \]  

(124)

Here \( V_{\text{tree}} \) is the tree potential of interactions of scalar fields \( \varphi_i \), which is a polynomial of \( \varphi \), which is invariant under closed symmetry group. The degree of the polynomial is not higher than 4; \( V_{\text{eff}}^{(1)} \) represents one-loop corrections which do not depend on temperature, and \( V_T^{(1)} \) is the temperature dependent part of the one-loop contribution to \( V \). \( V_T^{(1)} \rightarrow 0 \) at \( T \rightarrow 0 \). This corrections are given by \(32-36\)

\[ V_T^{(1)} = \frac{n_i^2}{2\pi^2} \left( \sum_{\text{bosons}} I_b \left( \frac{m_i^2}{T^2} \right) + \sum_{\text{fermions}} I_f \left( \frac{m_i^2}{T^2} \right) \right), \]  

(125)

where

\[ I_b(y) = \int_0^\infty x^2 \ln(1 - \exp(-\sqrt{x^2 + y^2})) \, dx, \]  

(126)

\[ y_i = \frac{m_i^2}{T^2}. \]

At low temperatures \( T^2 \ll m_i^2 \) all temperature dependent contributions vanish exponentially \( \sim \exp(-y) \). At high temperatures the main contributions come from the terms of zeroth and second degree in \( y \). Namely, at \( y \rightarrow 0 \) one has

\[ V_T^{(1)} = -\frac{\pi^2}{96} T^4 \left( \sum_b \frac{1}{8} \sum_f \right) + \frac{1}{24} T^2 \left( \sum_b m_i^2(\varphi) \right) + \frac{1}{2} \sum_f m_i^2(\varphi) = \]

\[ = -\frac{\pi^2}{96} T^4 N_{\text{eff}} + \sigma T^2 \sum_i \varphi_i^2 \]  

(127)

where \( \sigma \) is some function of coupling constants.

Since the effective potential is the function of \( T \), its global minimum at \( T = 0 \) may differ (which in fact is often the case) from the global minimum at high temperatures. That is why there take place phase transitions. At a given temperature the field \( \varphi \) could concentrate in a local minimum of \( V \). Such a state is metastable and decays. Similarly to the case of usual macroscopic physics this transition may be, in principle, of either first or second order. In the 2-nd order phase transition the order parameter \( \varphi \) is changing continuously along with temperature. In the 1-st order transition there is a discontinuity of the order parameter. The variation of effective potential with temperature and the temperature dependence of scalar field VEV are shown in Fig.6a and Fig.6b for the 2-nd order and 1-st order phase transition, respectively; \( T_C \) is the critical temperature; the dotted line in Fig.6b corresponds to the metastable local minimum of the effective potential.

Usually, in GUTs there take place 1-st order phase transitions \(38-40\). They are going by means of new vacuum bubble nucleation \(41-44\) in interiors of the old metastable phase. A complete analysis of the process...
implies: i) determination of the probability of formation of a single bubble of the new phase and of its initial size, ii) investigation of the bubble growth, and, finally, iii) construction of a general picture of the filling-up of the space by the new phase bubbles.

Strictly speaking, an embryo of new phase may be called a bubble only approximately. The real configuration has no sharp-boundary (see Fig. 7).

However, the thin-wall approximation gives in a number of cases the correct qualitative and, with a reasonable accuracy, a quantitative picture. In this approximation one needs to have: the energy-momentum tensor both a) inside \( T_{\text{in}}^{\mu \nu} \) and b) outside \( T_{\text{out}}^{\mu \nu} \) the bubble, as well as c) the surface density of energy-momentum tensor on the phase separation boundary (the 3-tensor \( S_{ij} \)).

It appears that spherically symmetric bubbles are nucleated most often. The following two cases are best studied: the nucleation of pure vacuum bubbles and the bubble creation in a thermostat.

The structure of energy-momentum tensor of a vacuum follows from Poincare-invariance of the vacuum state

\[
T_{\mu \nu}^V = \varepsilon \delta_{\mu \nu} , \quad \varepsilon = \text{const} .
\]  

(128)

If there is vacuum both inside and outside the bubble, then one also has

\[
S_{ij}^1 = S \delta_{ij} , \quad S = \text{const} .
\]  

(129)
Fig. 7. Schematic pictures of:

a) the effective potential with two minima I and II, at $\Phi = \Phi_1$ and $\Phi = \Phi_2$, respectively;

b) scalar field $\Phi$ variation with $r$, and

c) energy density variation with $r$. Between two phase I and II there is shown the wall. The space occupied by the phase II is the bubble.

From the energy conservation one has

$$\frac{4\pi}{3} r^3 \left( \varepsilon_{\text{out}} - \varepsilon_{\text{in}} \right) = 4\pi r^2 S .$$

(130)

Hence, the size of the nucleated bubble is

$$r_0 = \frac{3S}{\varepsilon_{\text{out}} - \varepsilon_{\text{in}}} .$$

(131)

During the bubble expansion the energy released is being transferred to bubble walls kinetic energy. Let $\tau$ be the proper time on the wall. Then this fact is expressed as follows

$$\frac{4\pi}{3} r^2 \left( \varepsilon_{\text{out}} - \varepsilon_{\text{in}} \right) = 4\pi r^2 S \sqrt{1 + \left( \frac{dr}{d\tau} \right)^2} .$$

(132)

Integrating this equation and transforming to coordinates of an observer which is at rest in the centre of the bubble, one obtains the trajectory

$$r^2 - t^2 = r_0^2 .$$

(133)

In the imaginary time $t \rightarrow i\tau$ one obtains that the equation (133) describes a $3$-sphere:

$$r^2 + i t^2 = r_0^2 .$$

(134)

i.e. in imaginary time the solution is $O(4)$ symmetric. Let $S_4$ be the action on this solution. Then in quasiclassical approximation the probability of a single bubble creation in unit volume per unit time is $41 - 43$.
\( \gamma = M^4 \exp(-S_4) \) .

Note that if coupling constants \( \lambda \) in the theory under consideration are small, then is a very large quantity, \( S_4 > 1/\lambda \) (when thin-wall approximation ceases to be valid, \( S_4 \sim 1/\lambda \) ).

What are the generic values of arising dimensional parameters? Roughly speaking, they are

\[
\begin{align*}
\varepsilon_{\text{in}} &\approx 0 , \\
\varepsilon_{\text{out}} &\sim M_X^4 , \\
S &\sim M_X^2 , \\
M &\sim M_X .
\end{align*}
\] (136)

One may ask, is there a particle creation during bubble expansion. One may, in principle, imagine the shell with \( S_0^2 = 0 \). Such a shell being "massless" could transfer all energy release not to kinetic energy but to particle production. In this case \( S_0^2 \neq 0 \). This shell describes just the phenomenon of 'vacuum burning' (47, 48).

The components of energy-momentum tenzor (in isotropic case) have the following physical meaning: \( T_{\mu\nu} = (\varepsilon, p, p, p) \). This means that in the vacuum state \( p = -\varepsilon \). Further, since \( \varepsilon_{\text{out}} > \varepsilon_{\text{in}} \), one gets \( p_{\text{in}} > p_{\text{out}} \). This inequality is a very general one. Inside the new phase bubble the pressure is always larger than in the surrounding medium. That is why the old phase is metastable, and the transition takes place from the maximum of effective potential to the minimum. The condition of hydrostatic equilibrium of a bubble (the equality of all forces) is

\[
T_0 = \frac{2S_0^2}{P_{\text{in}} - P_{\text{out}}} .
\] (137)

It is seen that \( S_0^2 \) has the meaning of the coefficient of surface tension.

In a thermostat the energy conservation should not be taken into account at the bubble nucleation. If the bubble size is \( r < r_0 \), then the bubble collapses. If, on the contrary, \( r > r_0 \) this bubble will expand infinitely.

The phase transition starts at \( T = T_c \) and finishes when new vacuum bubbles fill up all the Universe (there are created many new bubbles which are expanding). The various stages of phase transitions and of filling-up the space by the new phase are shown schematically in Fig. 8 in particular case of the SU(5) model.

The probability of bubble creation in thermostat is given by the Gibbs formula

\[
\gamma \sim T^4 \exp(-\Delta f/T) ,
\] (138)
\( f \) being the free energy change due to the bubble nucleation. In quasi-classical approximation \( \gamma \) is determined by the minimum of 3-dimentional action \( S_3 \),

\[
S_3 = \int d^3 x \mathcal{L},
\]

\[
\gamma \asymp T^4 \exp\left(-S_3/T\right),
\]

(139) (140)

In thin-wall approximation \((1/m_\phi \ll 2S/\Delta \varepsilon = R\), \(R\) being the bubble radius),

\[
\gamma \sim T^4 \exp\left(-16\pi S^2/3T(\varepsilon_{\text{out}} - \varepsilon_{\text{in}})^2\right)
\]

(141)

where \( S \) is the surface energy density,

\[
S = \int_0^\mathcal{V} d \Phi \sqrt{V(\Phi)}
\]

(142)

The tunneling probability depends very strongly upon temperature: at \( T = T_c \) \( \gamma = 0 \) since \( \Delta \varepsilon = 0 \). When \( T \) diminishes, \( \Delta \varepsilon \) is raising while the wall energy density decreases. Therefore, in those GUTs where the absolute instability of the phase with \( \Phi = 0 \) (i.e. potential barrier disappearance) takes place at \( T_1 \sim M_\chi \sim 10^{14} \text{ GeV} \), the phase transition occurs almost instantly \( T \sim 38 - 40, 49 \). It is followed by enormous energy release since the effective potential in two phases with broken and unbroken symmetry differs essentially. The thermalization of this energy release leads to the rehea-
ting of the Universe up to the temperature, which is determined by the energy conservation:

$$V(0) + \frac{\pi^2}{30} T_1^4 N_1 = V(\Phi_0) + \frac{\pi^2}{30} N_2 T_1^4 ,$$

(143)

$N_1$ and $N_2$ being effective numbers of massless degrees of freedom in phases with $\Phi = 0$ and $\Phi \neq 0$, respectively. One finds

$$T^4 = \frac{30}{\pi^2 N_2} (V(0) - V(\Phi_0)) + T_1^4 \frac{N_1}{N_2} .$$

(144)

So the temperature after the phase transition appears to be of order $M_X$. This is very important for the production of the BAU.

3.4. Old phase remnants

At final stages of the phase transition the new phase forms an infinite cluster (there is a percolation through the new phase) with islands of the old phase (see Fig. 8). The dynamics of these remnants was studied in Refs. 46. The outer mass of a remnant is not equal to zero (in contrast with the new phase bubble), and $P_{old} < P_{new}$. Due to these reasons old phase remnants tend to collapse and may form black holes with masses

$$m < (M_{Pl}/M_X)^2 M_{Pl} .$$

(145)

The SU black holes ($M_X \sim 10^{15}$ GeV) have already evaporated. The Weinberg-Salam black holes ($M_W \sim 10^2$ GeV) have masses $m \sim 10^{-4} M_\odot$, $M_\odot$ being the mass of the sun, and could survive.

3.5. Phase portrait of SU(5) model

Here we shall be interested in the chain of symmetry breaking in the SU(5) model. We have seen that in the minimal SU(5) there are two scalar multiplets whose VEVs break symmetry: the field $\Phi$ (24) with the VEV of order $M_X$ and the field H(5) with the VEV of order $M_W$.

The tree potential of interactions of these fields is

$$V(\Phi, H) = -\frac{1}{2} m^2 \text{Tr} \Phi^2 + \frac{\lambda_1}{4} \text{Tr} \Phi^4 + \frac{\lambda_2}{4} (\text{Tr} \Phi^2)^2 -$$

$$-m_H^2 H^H + \frac{\lambda_3}{4} (H^H)^2 + \lambda_4 H^H \text{Tr} \Phi^2 + \lambda_5 H^H \Phi^2 \Phi^2 H .$$

(146)

At $T \neq 0$ the second variation of at $\Phi = 0$, $H = 0$ is diagonal and has two different components:

$$m^2 \Phi_{eff} = -m^2 \Phi + \sigma \Phi T^2 ,$$

(147)
\[ m_{\text{eff}}^2 = -m_H^2 + \sigma_H T^2, \]  

(148)

where

\[ \sigma_{\Phi} = \frac{1}{60} (75g^2 + 130 \lambda_2 + 47 \lambda_1 + 50 \lambda_4 + 10 \lambda_5), \]

\[ \sigma_H = \frac{1}{60} (72g^2 + 30 \lambda_3 + 240 \lambda_4 + 48 \lambda_5), \]

(see Eq.(127), \( g \) being the gauge coupling constant. Therefore, at high enough temperatures the second variation is positively defined, and SU(5) symmetry is restored. At smaller temperatures the SU(5) phase ceases to be the local minimum of the potential. The symmetry breaking occurs at the moment when one of the quantities \( m_{\Phi}^2 \) or \( m_H^2 \) vanishes. If

\[ \frac{m_{\Phi}^2}{\sigma_{\Phi}} > \frac{m_H^2}{\sigma_H}, \]  

(149)

then the SU(5) phase is unstable with respect to the field \( \Phi \) fluctuations and first of all the field I condensate appears. Otherwise, the field H condensate appears first.

If the H-condensate appears only at \( T \leq M_W \), then at high temperatures one may investigate the effective potential as a function of \( \Phi = 24 \) only. It appears that depending on the value of coupling constants \( \lambda_1 \) and \( \lambda_2 \) in the tree potential \( \langle W \rangle \), the temperature evolution of the symmetry could vary considerably.

We give the phase pictures of the SU(5) model at two temperatures: at \( T = 0 \) (see Fig.9) and the moment of SU(5) breaking (Fig.10). The plane of variables \( \lambda_1/g^4, \lambda_2/g^4 \) in Fig.10 is divided into three parts, moreover each on the plane represents a peculiar SU(5) model. If, for example, the coupling constants in the initial tree Lagrangian are chosen in such a way that the point \( p = (\lambda_1/g^4, \lambda_2/g^4) \) lies to the right from the curve \( \langle W \rangle \) then the system transits out of the SU(5) phase directly into the phase SU(3) x SU(2) x U(1), while if the point \( p \) lies to the left from the phase SU(4) x U(1). Finally, we may conclude that there may appear the domain structure of the Universe with different vacua is various domains SU(4) x x U(1) and SU(3) x SU(2) x U(1) symmetric vacua respectively if the point \( p \) lies in between the curves \( \langle W \rangle \) and \( \langle M \rangle \).

Thus, the evolution of the Universe in the framework of the minimal SU(5) model could proceed in the following ways (if the VEV of 24 appears first)

\[ \text{SU}(5) \rightarrow \text{SU}(3) \times \text{SU}(2) \times U(1), \]

or

\[ \text{SU}(5) \rightarrow \text{SU}(4) \times U(1) \rightarrow \text{SU}(3) \times \text{SU}(2) \times U(1), \]  

(150)
Fig. 9. Phase portrait of the SU(5) model at zero temperature in terms of the variables \((\lambda_1, \lambda_2)\). Unlabelled curves correspond to equipotential levels of the system. Regions of the phase plane in which the effective potential has the global minimum on the phases SU(5), SU(4)xU(1), and SU(3)xSU(2)xU(1) are shown just in the figure. The curves L and S represent the boundaries of the stability regions of the phases SU(4)xU(1) and SU(3)xU(1) respectively. The line is the boundary of the stability region of the SU(5) phase. We have not drawn the curves L and S below the line P because in this region the system remains in the SU(5) symmetric phase till zero temperature. The dashed lines represent the asymptotics of the corresponding curves.

or

\[
SU(5) \rightarrow \left\{ \begin{array}{l}
SU(4) \times U(1) \\
SU(3) \times SU(2) \times U(1)
\end{array} \right\} \rightarrow SU(3)xSU(2)xU(1) .
\]  
(151)

Fig. 10. The phase picture of the system at the moment of the SU(5) breaking in terms of the variables \((\lambda_1, \lambda_2)\). The crosshatched region is forbidden.

or

\[
SU(5) \rightarrow \left\{ \begin{array}{l}
SU(4) \\
SU(3) \times SU(2) \times U(1)
\end{array} \right\} \rightarrow SU(3)xSU(2)xU(1) .
\]  
(151)

If the parameters of the model one chosen in such a way that the H-condensate \((H = \frac{5}{2})\) appears first when temperature decreased from \(T \sim M_{\text{Pl}}\), then the temperature evolution of the symmetry would be

\[
SU(5) \xrightarrow{T_1} SU(4) \xrightarrow{T_2} SU(3)xU(1) \xrightarrow{T_3} SU(3)xSU(2)xU(1) \xrightarrow{T_{\text{GWS}}} SU(3)xU(1) .
\]  
(152)

The temperature variation of VEVs of \(\Phi\) and \(H\) is shown schematically in Fig. 11. At temperatures \(T_2 < T < T_3\) quarks (as well as \(W^\pm, Z^0\)) appear to be supermassive. This could affect decisively the process of the BAU production. In this case the baryon excess is produced in the high temperature SU(3)xU(1) phase, resulting in the quite acceptable value of the BAU.
We would mention finally that in the SU(5) model with three quintets of scalars, there could take place a spontaneous CP-violation at high temperatures \(52-54\). This could result in the formation of matter-anti-matter domains in the Universe \(52-54\).

To conclude, I would say that there are many other applications of GUTs in cosmology. One may find approaches to solution to such problems as horizon problem, isotropy and flatness of the Universe, generation of inhomogeneities necessary for galaxies formation \(55-58\). There is interesting phenomenon of electroweak baryon number nonconservation at high temperatures resulting, possibly, in the BAU generation \(59\) etc. In view of back of time I here not even outlined these items there we encountered both successes and failures, but in any case GUTs implications are very rich and promising.

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THE JINR EXPERIMENTAL PROGRAMME IN HIGH ENERGY PHYSICS

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ABSTRACT
A review is given of recent works of the Dubna experimentalists in the high energy physics.

1. INTRODUCTION

The topic of my lecture is the scientific programme of the Joint Institute for Nuclear Research, Dubna, in particle physics.

Let me say firstly few words about the structure and organization of the Institute.

The Joint Institute for Nuclear Research in Dubna was established almost 30 years ago, in March 1956, on the basis of two Soviet scientific laboratories. One of them disposed of the 680 MeV synchrocyclotron which operated since 1949. In the other laboratory the construction of the 10 GeV synchrophasotron was in the final stage. This accelerator was put into operation in 1957.

At present eleven member states - Bulgaria, Hungary, Vietnam, the German Democratic Republic, the Korean People's Democratic Republic, Cuba, Mongolia, Poland, Romania, USSR and Czechoslovakia contribute to the Institute. JINR employs about one thousand scientists, more than six thousand people work in its laboratories, one thousand and a half visiting scientists from the member states come annually to work in Dubna.

Table 1 shows the structure of the Joint Institute Administration.

The supreme body of the Institute is the Committee of Plenipotentiaries of the governments of Member States which is convened annually and considers the major problems of the Institute's activity.

The Finance Committee is concerned with the annual budget, contributions from the member states, etc.

The research policy of JINR is determined by the Scientific Council which is convened twice a year. The Council submits its recommendations on experimental planning and finances, the construction of new facilities and buildings to the Committee of Plenipotentiaries. There are also 3 sections of the Council on High Energy Physics, Low Energy Physics and Theoretical Physics. The Sections of the Council have specialized committees which are concerned with research techniques.

The JINR directorate is reelected each three years. Since 1965 the Director of the Institute is the distinguished Soviet physicist and mathematician N.N. Bogolubov. The present Vice-directors are Professor Elias Entralgo from Cuba who is responsible for research in High Energy Physics, and Professor Aureliu Sandulescu from Romania, who is responsible for research in Low Energy Physics. Professor Yuri Denisov is appointed as
Administrative Director.

JINR has 6 laboratories, each almost equivalent to a large institute, and associated departments. Table 2 presents the structure of the JINR's laboratories and the subjects of their research.

As you see, the scale of the scientific research at the JINR is quite large. The Joint Institute is one of the rare centers where almost all branches of the nuclear sciences are developed, namely, research in High Energy Physics and Low Energy Physics, in physics of condensed matter, in neutron physics, theoretical physics, development of the accelerators. This fact proves that there are large possibilities to enrich the research in the intermediate branches of sciences.

The Institute provides good facilities to accomplish the various experiments. Table 3 gives the image of the facilities at the Institute.

But one of the most important instrument to probe Nature and promote science is the international cooperation owing to which physicists have the possibility to use most sophisticated machines at other laboratories and centers.

Let me proceed to the main topic.

As I already said, one of the principal trends of the JINR scientific programme is research in high energy physics. The experiments are carried out mainly at the Serpukhov proton-synchrotron and CERN's accelerators.

But there is another branch of physics developed in Dubna by the initiative of academician A.M. Balin which is called relativistic nuclear physics. The programme of the investigations in this field started in 1970 at the Dubna synchrophasotron and now is considerably enlarged. By 1988 the construction of the Nuclotron - accelerator of relativistic nuclei on superconducting magnets - is planned.

A number of experiments on elementary particle physics are carried out at the Dubna low energy facilities, for example, at the Dubna neutron reactor.

2. EXPERIMENT CARRIED OUT AT THE SERPUKHOV PS.

Let me say in more details about the physics programme and experiments carried out at the Serpukhov proton-synchrotron.

During more than 17 years the Joint Institute collaborates with the Institute of High Energy Physics in Serpukhov. 25 large experimental set up of the Institute were used to carry out experiments at the Serpukhov PS. 30-50% of operational time of the PS was used for JINR.

Actually the Serpukhov machine started to operate in the Booster mode. Table 4 shows the principal installations of the Joint Institute used at the Serpukhov PS.

2.1. The BIS-2 - filmless spark chamber spectrometer is an "old" set-up located in the neutron beam of the PS. The energy of neutrons - 20-70 GeV/c. Experiments are designed for a search of charmed particles produced in the diffraction processes. During last two years charmed baryons \( \Lambda_c \) produced
in the interaction of the 40–70 GeV neutrons with various targets have been studied. The analysis of the two channel production of $\Lambda_c^+$ was made:

$$\Lambda_c^+ - \Lambda_c^0 \pi^+ \pi^-;$$  \hspace{1cm} (1)

$$\Lambda_c^+ - K^0 p \pi^+ \pi^-.$$ \hspace{1cm} (2)

It was found that the invariant cross section which is described by the relation

$$\frac{d^3\sigma}{dp^3} = e^{Bp} \cdot (1 - X)^n,$$

where $p$ is transverse momentum, $X$ is scale variable has the following parameters:

$$B = (2.5\pm0.6) \text{ (GeV/c)}^{-1}; \quad n = 1.5\pm0.5.$$

The asymmetry of outgoing baryons $\Lambda^0$ and $P$ in the $\Lambda_c^+$ decays into two channels 1 and 2 relative to the $\Lambda_c^+$ plane production has been studied for the first time. The value of the asymmetry have been found as follows:

$$A(\Lambda^0) = +0.34\pm0.22; \quad A(P) = -0.23\pm0.12.$$

It may be considered as indications of $\Lambda_c^+$ being produced polarized (see fig. 1).

2.2. The liquid hydrogen bubble chamber "Liudmila" was used over many years in the experiments studying processes of multiparticle production at high energies. The chamber has a track sensitive target filled with a liquid deuterium. It was exposed in the beams of the 22.4 GeV antiprotons and 11.5 antideuterons.

The last results are concerned with the study of the interaction of antideuterons with deuterons and protons at 12.2 GeV.

On the basis of $\bar{d}d$-interaction analysis the topological cross sections $\sigma_n$ have been determined, as well as average multiplicities of charged particles $\langle n \rangle$, the ratio $\langle n \rangle / D$ (where $D$ is the standard deviation), the correlation function $f^{-}$ and KNO-distribution for $\bar{d}d$-interactions at 12.2 GeV/c and $\bar{n}n$-annihilation at 6.1 GeV/c. For the latter process the values of $\langle n \rangle = 4.67\pm0.05$ and $\langle n \rangle / D = 2.20\pm0.04$ have been obtained. In the case of multinucleon interactions the values of $\langle n \rangle$ and $\langle n \rangle / D$ are higher by 30%. The estimation of the bottom limit of the cross section of the full annihilation of antideuteron with deuteron has been found as follows:

$$\sigma_{\bar{n}n}^{\bar{d}d} > 0.12 \text{ mb}.$$

With the help of statistics on $dp$-events the analysis of elastic $\bar{n}p$-
interactions at 6.1 GeV/c in the reaction $\bar{p}\bar{p}\rightarrow pn$ has been performed. The total cross section of this channel of the reactions $(10.4\pm0.7)\text{ mb}$ and the dependence of the differential cross section over the square of the momentum transferred agree with the calculations by the Glauber model. For the elastic cross section of the $\bar{p}n$-interactions the slope of the diffraction cone $b=12.7\pm1.3$ in the region $|t|<0.2\text{ GeV}/c^2$ agrees with the value of the $b$ parameter for the $\bar{p}p$-interactions at the close energies. This result does not confirm the difference of the value $b$ (which was observed earlier from the $\bar{p}p$-interactions data) determined for the $\bar{p}n$-interactions.(see fig.2).

2.3. The next JINR installation operated at the Serpukhov PS is the 5 meter magnetic spark chamber spectrometer MIS.

With the use of this spectrometer the joint CERN-JINR experiment is carried out. The physicists from Italy (INFN) participated actively in this experiment. The analysis of the resonances in the 3 pion system produced in the coherent processes on nuclei in now completed. The following processes

$$\pi^-+A\rightarrow\pi^++\pi^-+\pi^-+A$$

were studied at the energy of the incident $\pi^-$-meson of 40 GeV.

As the result of the investigation of the 3 pion system, the spectrum of $\pi^-$-meson excitation levels has been experimentally studied for the first time both by orbital and by radial quantum number. The discovery of the two new states of the $\pi^-$-meson which are interpreted as radial excitations of the quark-antiquark system has been made. This proves the composite structure of the lightest hadron. Recently these results were adequately confirmed in the experiments carried out at the FNAL's machine in Batavia.

Besides this result, some indications about a possible existence of the resonance states corresponding to the radial excitations of the $A1$, $A2$ and $A3$ mesons have been obtained. The widths and possible probabilities of the $A3$-meson decay into three channels have been determined: $A3\rightarrow\pi\pi\pi$ and $\pi\pi\pi$.

Table 5 demonstrates the parameters of the resonance states in the 3 pion system found in the experiment performed with the MIS spectrometer.

2.4. The SIGMA-AJAX experimental set-up consists of a broad-aperture magnet and a gamma-spectrometer designed to study the structure of light hadrons. It is being used in the joint JINR-IHEP experiment. The study of the processes of the pion pair production by pions near threshold in the Coulomb field of nuclei has been completed. The cross section of the reaction

$$\pi^-+(A,Z)\rightarrow\pi^-+\pi^0+(A,Z)$$

was determined for nuclei C, Fe, Cu, in the field of the Coulomb production $t<10^{-3}\text{ (GeV/c)}^2$ and invariant masses of the system $(\pi^-\pi^0)$ $<10\text{ MeV}$. The results of this experiment agree with the generally adopted assumption about the availability of three colour degrees of freedom of quarks (included in 1965 by Bogolubov, Struminsky, Tavkhelidze, and Han, Nambu) and, therefo-

re, are an independent confirmation of the fundamental postulate of the QCD
taking into account the validity of the important theorem on chirality anomalies which relates the amplitudes of the processes $\gamma \rightarrow 3\gamma$ and $\pi^0 \rightarrow 2\gamma$.
(see fig. 3a).

The analysis of data obtained in this study of Compton effect on the pion allowed to determine for the first time the value of the pion electromagnetic polarizability equal to $\beta_\pi = (-7.1 \pm 2.8) \times 10^{-43} \text{cm}^3$. The experimental estimation of the sum of electric ($a_\pi$) and magnetic ($\beta_\pi$) polarizability of the pion was also obtained. It is equal to $a_\pi + \beta_\pi = (1.4 \pm 3.1) \times 10^{-43} \text{cm}^3$ (see fig. 3b).

2.5. The physics program of the experiment carried out with the help of the magnetic spectrometer "POSITRONIUM" is now completed. The value of the relative probability of the decay of $\pi^0$-meson into the positronium and gamma-quantum is obtained: $W=(1 \pm 2) \times 10^{-9}$. To compare, the rarest from the earlier observed particle decays, the $K^0 \rightarrow \pi^0 \mu^+ \mu^-$ has the probability equal to $W=(9.1 \pm 1.9) \times 10^{-9}$.

2.6. The analysis of data on measurement of the polarization in the $\pi^- + p \rightarrow \pi^0 + n$ charge exchange reaction has been performed in the joint JINR-IHEP experiment with the help of the "POLARIMETER" set-up. The energy of the incident pion was 40 GeV/c. Data published previously did not take into account the neutron recoil. The actual analysis has included both the neutron recoil and pions. It was found that the dependence of the polarization from the momentum transferred had a complicated character (see Fig. 4a and 4b) and cannot be explained in the framework of the standard theoretical approaches.

The azimuthal asymmetry was changed in the reactions $\pi^- p \rightarrow \omega n$ and $\pi^- p \rightarrow K^0 \Lambda$ within the momenta transferred of 0.15 $\leq t \leq$ 1.2 (GeV/c)$^2$. The average value of the asymmetry in this momentum range was $(22 \pm 5)$% for the reaction $(\pi^- p \rightarrow \omega n)$.

The asymmetry was measured for the first time in the reaction $\pi^- p \rightarrow \pi^0 + X$ of the inclusive production of zero pions at 40 GeV/c at the angle of 90° in the cms. An interesting result of these preliminary measurements is the asymmetry different from the zero in the inclusive process. The average value of the asymmetry within the momenta transferred of $1.2 \leq p_\perp \leq 2.2$ GeV/c is $(10.0 \pm 3.4)$%.

A number of joint JINR-IHEP experiments will be soon carried out with the help of installations developed and built by mutual efforts. At the end of this year the neutrino detector will be put into operation at the Serpukhov PS. The program to study the so-called "tagged" neutrinos is being developed, as well as the project of Hadron Spectrometer of a new generation is under way.

3. JOINT EXPERIMENTS PERFORMED AT THE CERN ACCELERATORS

3.1. Dubna physicists together with Italian colleagues from universities of Torino, Padua and Pavia participate in the experiments performed at the LEAR (low energy antiproton ring). The low energy antiproton interaction
with \(^4\)He and \(^{20}\)Ne nuclei has been studied. The cross sections of different channels of the annihilation of antiprotons with these nuclei have been determined. The data on \(\bar{p}^4\)He interaction were used to obtain a very important cosmological limitation of the antimatter quantity in the early Universe (at the period of time \(10^3 < t < 10^{13}\) sec from the beginning of expansion). It was Ya.B.Zeldovich who for the first time paid attention to a possible existence of the connection between the \(\bar{p}^4\)He annihilation characteristics and astrophysical parameters. The analysis of the LEAR experimental data allowed to obtain the value of the limitation for an admissible tolerance of antimatter in the early Universe as follows: \(R \leq (0.7 + 1.1) \times 10^{-3}\), where \(R = n_\bar{p}/n_p\).

3.2. In the joint JINR-CERN muon experiment to study the deep inelastic muon scattering on nuclei of deuterium, nitrogen, and iron at 280 GeV it was shown that the ratio of nucleon structure functions obtained for data with the use of iron, deuterium, and nitrogen, deuterium target and at the fixed value of the scalar variable \(X\), was not dependent from the square of 4-momenta transferred \(Q^2\). The decrease of the structure function ratio with the increase of \(X\) within \(0.2 < X < 0.7\) and at the energies of \(50 < Q^2 < 200\) \((\text{GeV/c})^2\) is well described by the linear law \(R = a + bx\), with the parameters: \(a = 1.16 \pm 0.03\); \(b = 0.56 \pm 0.08\) for Fe/D\(_2\) and \(a = 1.10 \pm 0.04\); \(b = -0.39 \pm 0.09\) for N\(_2\)/D\(_2\) (see Figs. 5a, b).

To fit data on the ratio of nucleon structure functions obtained in the deep inelastic electron-nuclei and muon-nuclei reactions in the region of \(X < 0.4\), it was proposed to use in the analysis of the eA-scattering an assumption about the dependence of the ratio of the longitudinal and transversal virtual photon cross sections from the atomic weight of the nucleus \(A\) in the form:

\[ R(A) = \sigma_L/\sigma_T = aA^{1/3} \]

with the parameter \(a = 0.08\).

3.3. It is known that JINR is responsible for a production and testing of the DELPHI Hadron Calorimeter detectors in the framework of the Agreement between JINR and CERN.

Now the workshop and technological line to produce streamer detectors are ready to be put into operation.

4. EXPERIMENTS CARRIED OUT AT THE DUBNA SYNCHROPHASOTRON

Relativistic nuclear physics is a new region of research which, from the one hand, opens large perspectives to develop nuclear physics and, from the other hand, is closely related to the elementary particle physics at high energies.

Relativistic nuclear physics gives possibilities to investigate not only the behaviour of nuclear matter at small internucleon distances and extreme conditions, for example, high pressure and temperatures, but to study the states of hadron matter - the quark-gluon plasma. This enables to verify in
principle the theory of strong interactions—quantum chronodynamics, to study closely such problems as confinement and multiquarks interactions.

The scientific program on relativistic nuclear physics in Dubna is carried out by using the unique beams of relativistic nuclei of the synchrophasotron. These beams with the energy of 4 GeV/nucleon were obtained due to the development and construction of new types of multicharged ion and nuclei sources and also to the improvement of a whole system of the accelerator. Table 6 demonstrates experimental facilities which are used in these investigations.

Let me say about some principal results:

The criterion is formulated to define the region of kinematical variables in which hadrons are failed to proceed as quasiparticles of nuclear matter:

\[ b_{jk} = -\frac{(P_j - P_k)^2}{m_j m_k} \geq 5 \]

where \( P_j \) is the 4 momenta of particles with the mass \( m_j \) participating in the reaction. The condition \( b_{II} > 5 \) for primary particles indicates that the energy of interacting particles per nucleon has to be more than 4 GeV when the limiting fragmentation begins. Thus, two conditions are essential to discriminate quark degrees of freedom in the cumulative particle production:

\( b_{II} > 5, \ X > 1. \) \( X \) is the scale variable - the ratio of the energy of produced particle to the initial energy. The region of \( 10^{-2} < b_{jk} < 5 \) is transitional from the domination of the nucleon degrees of freedom to the quark–gluon degrees which are manifested in relativistic nuclear collisions.

Experimental data for the ratio of \( K^+ / K^- \) mesons yielding in cumulative proton-nuclei interactions (see Fig. 6) show that the sea quark distribution in \( X > 1 \) is different from the momentum distribution of valence quarks by the constant \( S(X) = 1/50U(X) \).

By using the DISK-2 experimental set-up, the data have been obtained to determine the \( A \) dependence of the cumulative pion cross sections at \( X=1.3 \) and \( X=2.1 \) (see Figs. 7a,b) in proton-nuclei interactions at \( P=9 \) GeV/c. Two series of measurements have been performed. The first one corresponded to the pion momentum of 500 MeV/c and the emission angle of \( 168^\circ \), the second serie - to the momentum of 800 MeV/c and the emission angle of \( 162^\circ \). As targets the following elements were used:

- the first serie: \( ^6Li, ^7Li, Be, C, Mg, Al, Si, ^{54}Fe, ^{56}Fe, ^{58}Fe, ^{58}Ni, ^{61}Ni, ^{64}Ni, Cu, ^{64}Zn, ^{112}Sn, ^{118}Sn, ^{144}Sm, ^{154}Sm, ^{182}W, ^{186}W, Pb; \)
- the second serie: \( ^6Li, Be, C, Al, Cu, ^{144}Sn, ^{124}Sn, Sn, W, Pb. \)

The dependence of the cross section ratio per nucleon from the atomic number of the nucleus at \( X=1.3 \), namely, the increase of cross section with the increase of a \( A \) at \( A \leq 30 \) differs from the ratio obtained at SLAC for \( X < 1 \), where the ratio of structure functions decreases with the increase of \( A \). This indicates to the change of the interaction mechanism at the transition to the cumulative region.
Meanwhile, the character of the A-dependence of the ratio $\sigma_A/\sigma_{PB}$ at $X=2,1$ remains the same as at $X=1,3$ regardless of the decrease of the cross section absolute values approximately by a factor of 500.

The relativistically invariant description of multiparticle processes is proposed for the case when hadrons and nuclei collide in the space of relative 4-velocities $b_{ik}$. The main goal of the transition to the $b_{ik}$ variables instead of the classic variables - momenta and energies - is to demonstrate that at $b_{ik} \gg 1$ the function of distribution

$$ F(b_{11}, b_{12}, \ldots, b_{12}, \ldots, b_{11}) $$

which corresponds to the invariant cross section of the n-particle production is monotonously and sufficiently fast decreased with the increase of $b_{ik}$. This property may be presented as the principle of correlation relaxation which was proposed by N.N. Bogolubov in statistical physics. On the base of this conception a new relativistically invariant determination of the secondary particle jets has been done instead of the traditional approach by using the variables "sphericity" "trust" etc. Fig. 8 shows the distribution of negative pions by $b_k$ in jets produced in the $\pi^\pm$C interactions at the momentum of 40 GeV/c. It is seen that the distributions by the $b_k$ variable for the both jets (in the region of the nucleus fragmentation and in the region of fragmentation of the $\pi^\pm$-meson) coincide within the experimental errors. In the region of $b_k > 4$ of the $\pi^\pm$-mesons distribution the ratio of $dN/db_k$ is described by exponential function

$$ F(b_k) = A \exp(-b_k/\langle b_k \rangle) $$

with average value $\langle b_k \rangle = 4$ in both jets.

The study of the characteristics of $K_0$-mesons and $\Lambda$-hyperons produced in the quarks and diquark fragmentation processes of $\pi^-p$ and $\pi^\pm$C cumulative interactions at $P=40$ GeV/c has denoted that the fragmentation of quarks into $K_0$-mesons and $\Lambda$-hyperons was similar to the fragmentation in the $e^+e^-$-annihilation (See Figs. 9a,b). The functions of the diquark fragmentation for $\pi^-p$ and cumulative $\pi^\pm$C-interactions are identical. This conclusion demonstrates the universal character of the quark and diquark fragmentation in soft and hard processes.

The study of the cumulative hadrons (protons and pions) properties in the $pC$-interactions produced in the 2 meter propane chamber at 10 GeV/c was extended. The analysis of the invariant inclusive cross sections of cumulative hadrons and corresponding spectra of pions and protons emitted in the backward semisphere in l.c.s. lead to the conclusion of the independent character of the processes of the cumulative proton and pion emission.

The comparison of the $pC$, $dC$, $aC$ and $CC$ inelastic interactions shows that their general characteristics are satisfactorily described in the framework of the cascade model. Experimental data on multiplicity and relations of the $\pi$-meson inclusive cross sections in $dC$, $aC$ and $CC$-interactions are correctly explained by the model of multiple scattering. The character of the dependence of angular and momentum characteristics of mesons and protons from the cumulative variable is adequate for various types of colliding nuclei and is similar in the case of pions and protons.
With the help of the Recoil Particle Spectrometer the fragmentation of the nuclei Be, C, Al, Cu, Ag, Au in the beam of \( \alpha \)-particle has been investigated at 3.33 GeV/c. The doubled differential cross sections of the helium and hydrogen isotope production were measured in the region of fragmentation of the nucleus-target at the angles of 45°, 90°, 135°.

To describe data the thermodynamical model with a source moving inside the nucleus has been developed. It was shown that in the region of the fragment energies (\( E_2 \)) less than 50 MeV the mechanism of evaporation contributed in the great extent. When considering the ratio of the \(^3\)H output to the \(^3\)H\(_2\) output it was found that this ratio is greater than the ratio of the quantity of neutrons to the quantity of protons in the fragment emitting system. The analysis of the function

\[
R = \frac{\sigma_{^3\text{H}_1}}{\sigma_{^3\text{He}_2}} = R(E_2)
\]

has shown that the \(^3\)H\(_1\) and \(^3\)He\(_2\) fragments were "sensible" to the full Coulomb field of the nucleus-target.

The analysis of the data obtained and from the references in the framework of the adhesion model has shown that the region of the fragment formation was equal to 3.2 and depended neither on the nucleus of the target, nor on the beam nucleus.

The obtained boundaries of the limiting fragmentation of nuclei (3.5 ± 4.0 GeV/nucleon) allowed the Dubna synchrophasotron to get an exceptional position of the machine which possesses beams of nuclei with energies above this limit. The universality of these results obtained at the Dubna synchrophasotron has been largely proved in the lepton-nuclei experiments at the energies up to 400 GeV at CERN, FNAL and SLAC.

5. INVESTIGATIONS ON PARTICLE PHYSICS PERFORMED AT THE JINR PULSED REACTOR AND OTHER FACILITIES

In 1983 an interesting experiment on measurement of the spirality of a neutrino from the \(^{152}\)Eu decay has been carried out by using the gamma quanta circular polarization. This work was stipulated by a situation with regard to new calculations of the gamma quanta circular polarization \( H \) which took into account the heat motion of atoms of the emitter and absorber and also a possible probability of the electron capture from the L- and M- atom shells. The following value of \( H \) was obtained: \( H = -0.93 \). The previous theoretical value has been obtained without taking into account the above-mentioned effects and was equal to \( H = -0.84 \) that was in agreement with the results of the experiments carried out in Sweden and USA (see Table 7).

In contrast to the classical Godhaber experiment, in the present experiment the Ge(li) detector, 100 cm\(^3\) in volume, was used with a high energy resolution. The measurements were performed in the multichannel amplitude mode which allowed to separate all the background effects.
The data of this experiment have given such a value of the circular polarization $H_p = -0.87 \pm 0.10$ that agreed with a new theoretical value for $H_p$ and confirmed the assumption on the full left longitudinal polarization of the neutrino.

The JINR pulsed reactors of fast neutrons, IBR-30 and IBR-2, are used to perform such important investigations as the parity nonconservation in neutron reactions, the $\alpha(\alpha)\alpha$ reactions on radioactive nuclei, spectroscopy of the p-wave neutrons and others.

The study of the enhancement of the CP-violation in the total cross sections of interactions of the resonance neutrons with nuclei which has been performed in 1981-1982 at JINR, has verified a theoretical model of the compound state mixing of different parities.

According to the model, one should expect to get the enhanced effects in the neutron reactions with charged particles when the corresponding s- and p-wave resonances are available. The experiment to verify this prediction for the $(n,p)$ reaction on $^{35}\text{Cl}$ has been carried out by the collaboration of the Leningrad Institute of Nuclear Physics and Laboratory of Neutron Physics of the JINR. It was performed in the beam of the transversely polarized hot neutrons of the reactor VVR-M. The 0.6 MeV protons were registered by a proportional chamber in which the BaCl$_2$ target, 60x1100 mm$^2$ in size, has been located. The coefficient of the asymmetry $a_p = (\bar{N}-N)/(N+\bar{N})$ was measured where $\bar{N}$ and $N$ are the number of the chamber countings for the opposite directions of the neutron polarization which changed every 2.8 seconds. The p-odd asymmetry of the proton emission has been found. After introducing the correction for a partial polarization and average cosine of the angle of the proton outlet relatively the neutron spin, the following result has been obtained: $a_p = (1.5 \pm 0.3) \cdot 10^{-4}$.

In the course of the experiment the parity conserving effect of the wright-left asymmetry of the proton outlet in the plane perpendicular to the neutron spin direction has been measured: $a = (2.6 \pm 0.4) \cdot 10^{-4}$.

By using these results and parameters of the s- and p-mixing resonances in $^{35}\text{Cl}$ (at the energies of $E_s = -180$ eV and $E_p = +398$ eV) the experimental value of the matrix element violating the parity of interactions $W_{sp} = 0.1$ eV has been determined.

The interpretation of experiments on the parity nonconservation in the interactions of resonance neutrons with nuclei depends significantly from the spectrometric data, for example, from the partial parameters $\Gamma_{nj}$ of the p-resonance widths (by the full neutron spin). To obtain such an information the study of correlations in the gamma quanta angular correlations at the neutron capture in p-resonances has begun at the JINR IBR-30 reactor.

Using the HaK detector, 200x200 mm$^2$ in size, located at the angle of 90° to the neutron beam, the exit of the 9.3 MeV gamma-rays has been measured in the reaction $^{117}\text{Sn}(\alpha, \gamma)$ in the neighbourhood of the p-wave resonance of 1.33 eV with regard to the sign of polarization of the neutron.
beam, oriented perpendicularly to the reaction. Fig. 10 shows the results presented in the form the relation

\[ N^1, N^+ \text{ and } \epsilon = \frac{N^1 - N^+}{N^1 + N^+} \]

depending from the neutron time of flight. They correspond to the appearance of the right-left asymmetry of the gamma ray exit which has the resonance character. This effect is due to the interference of the S- and p-waves. The value of the relative amplitude of the neutron widths \( \sqrt{\Gamma_{n1/2}} / \sqrt{\Gamma_n} \) has been determined by the channel \( j = 1/2 \) for the resonance of 1.33 eV.

In my review I has considered only few aspects of the scientific program of JINR in the particle physics. Future investigations are related mainly to the constructions of such machines as the UNX Accelerator Storage Complex in Serpukhov, LEP Collider at CERN, the superconducting relativistic accelerator NUCLOTRON in Dubna.
Table 1

JINR Administration (JINR governing bodies)

STRUCTURE AND MEMBERSHIP

<table>
<thead>
<tr>
<th>COMMITTEE OF PLENI-POTENTIALIES</th>
<th>FINANCE COMMITTEE</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCIENTIFIC COUNCIL</td>
<td></td>
</tr>
<tr>
<td>Chairman – academician N.N. Bogolubov</td>
<td>professor E.Entsalgo</td>
</tr>
<tr>
<td>professor A. Sandulescu</td>
<td></td>
</tr>
<tr>
<td>Chief scientific secretary – doctor A.N. Sissakian</td>
<td></td>
</tr>
<tr>
<td>Bulgaria, Hungary, Vietnam, Germany, Korea, Cuba, Mongolia, Poland, Romania, USSR, Czechoslovakia</td>
<td></td>
</tr>
</tbody>
</table>

SC section on High Energy Physics

Chamber Committee

Electronic Experimental Committee

Photoemulsion Experiments Committee

SC section on Theoretical Physics

SC section on Low Energy Physics

Neutron Physics Committee

Nuclear Structure Committee

Heavy Ion Physics Committee

Table 2

The structure of the Institute. Scientific program of the JINR Laboratories

FREE INSTITUTE FOR NUCLEAR RESEARCH

STRUCTURE AND RESEARCH ACTIVITIES

DIRECTORATE

Director – N.N. Bogolubov
Vice-director – E. Entsalgo
Vice-director – A. Sandulescu

Laboratory of Theoretical Physics
Director – N.N. Bogolubov

Research activities:
- Elementary particles;
- Atomic nucleus theory;
- Condensed matter theory

Laboratory of High Energy Physics
Director – A.M. Balbin

Research activities:
- Nuclear structure;
- Strong, weak, el/μ interactions;
- Search of new particles;
- Nuclear structure

Laboratory of Nuclear Problems
Director – V.P. Zheleuzov

Research activities:
- Nuclear reactions;
- Search of superheavy elements;
- Transuranium isotopes;
- Complex nuclei interactions

Laboratory of Neutron Physics
Director – I.M. Frank

Research activities:
- Neutron spectrometry;
- Neutron basic properties;
- Nuclear structure & dynamics of solid & liquid matter;
- Light nuclei reactions

Laboratory of Computing Technique
Director – M.G. Maksimov

Research activities:
- Computation;
- Auto-systems for chamber film processing;
- Computer equipment of experimental facilities;
- Software systems

Main Divisions

Division of New Methods of Acceleration
Serpukhov Scientific Experimental Division
Division of Chief Scientific Secretary
Scientific & Technical Information Service
Experimental Physics Facilities Division
Table 3
JINR search facilities

1. SYNCHROPHASOTRON
   1. Injector  - Lines
      energy  - 10 GeV
      intensity  - $4 \times 10^{12}$ proton/cycle
   2. Injector  - Polarized Duterom Source
      energy  - 4.2 GeV/nucleon
      intensity  - $1 \times 10^{8}$ deuteron/cycle
      degree of polarization  - 50%
   3. Injector  - Laser
      energy  - 4 GeV/nucleon
      beam intensity per cycle:
      
      \[
      \begin{array}{cc}
      P & 2 \times 10^{12} \\
      \text{Li} & 2 \times 10^{9} \\
      \text{d} & 1.2 \times 10^{12} \\
      \text{C} & 5 \times 10^{8} \\
      \text{He} & 3 \times 10^{10} \\
      \text{Ne} & 2 \times 10^{10} \\
      \text{Li} & 1.5 \times 10^{8} \end{array}
      \]

2. SYNCHROCYCLOTRON (under reconstruction)
   energy  - 680 MeV

3. CYCLOTRONS
   1. Heavy ion cyclotron U-300
      energy of accelerated particles  - 250 $Z^2/A$ MeV
      mass/charge A/Z  - 4.5+7
      intensity  - $10^{11}$ to $10^{14}$ particle/sec
   2. Heavy ion cyclotron U-900
      energy of accelerated particles  - 650 $Z^2/A$ MeV
      mass/charge A/Z  - 4+20
      intensity  - $10^{12}$ to $10^{14}$ particle/sec
   3. Microtron MT-22
      energy of electrons  - 22 MeV
      average current  - 20 mA
      pulsed current  - 20 mA
      pulse duration  - 2.3 microsec

IV. FAST NEUTRON PULSED REACTORS

1. IBR-20
   average heat output  - 30 kW
   output per pulse  - 150 mW
   pulse duration  - $\sim$ 50 microsec
   intensity  - $5 \times 10^{14}$ neutr/cm$^2$.sec

2. IBR-2
   average heat output  - 2 M.W.
   output per pulse  - 1350 M.W.
   pulse duration  - 230 microsec
   intensity  - $10^{16}$ neutr/cm$^2$.sec
Table 4

Experimental facilities used at the Serpukhov PS

**BIS-2 Diffraction Spectrometer**
- with proportional chambers;
- angle resolution: 0.2±0.4 mrad;
- mass resolution: 2±5 MeV;
- (put in operation in 1978)

**HYPERON Missing Mass Spectrometer (JINR-IHEP)**
- Spark and proportional chambers;
- Shower electron and X-quanta detectors;
- (put in operation in 1978)

**LUZILRA Hydrogen Chamber**
- Volume: 860 l; track sensitive internal target of 10 l filled with liquid deuterium.
- (put in operation in 1978)

**POLARIMETER (JINR-IHEP) Spectrometer with a “frozen” polarized proton target; proportional chambers**
- (put in operation in 1976)

**RICE 3 Film Streamer Chamber**
- in a magnetic field;
- (put in operation in 1978)

**POSITRONIUM Magnetic Spectrometer**
- with a drift chambers;
- (put in operation in 1982)

**MIS 2 Magnetic Spark Spectrometer**
- with film data taking; proportional chambers and scintillation counter trigger system
- (put in operation in 1980)

**SIGMA-AJAX Magnetic Spectrometer (JINR-IHEP)**
- spark and proportional chambers;
- gamma spectrometer on lead glass;
- (put in operation in 1980)

**NEUTRINO DETECTOR (JINR-IHEP)**
- target-calorimeter; muon spectrometer; vertex detector;
- shower detector of e- and X-quanta;
- (to be put in operation in 1985)

Search for charged and narrow baryon resonances:
- Charged baryon have been registered via the decay:
  \[ \Lambda_c^- \rightarrow \Xi_c^- p \]
  \[ \Lambda_c^0 \rightarrow \Xi_c^0 p \]

\[ \sigma \Xi_c^- (\Xi^-) \propto 60 \text{ pb} \] per nucleon.

- New baryon resonance found \( \Xi^- \rightarrow \Xi^- (1385) K^- \)
- \( \sqrt{s} = 1956\text{ GeV/c}^2 \)
- \( \Gamma_{\Xi^-} = (172\pm 15) \text{ MeV/c}^2 \)

- Study of multiparticle processes and binary \( \Xi^- p \rightarrow \Xi^- \Xi^- (1385) \)
- Hyper charge exchange reactions at 12 GeV;
- Decays of \( \Xi^- \) and \( K^- \) mesons

Investigation of the \( \bar{d} - d \) interactions

Investigation of the polarization effects in the hadron-hadron collisions in a broad range of momenta transferred at the energies up to 60 GeV

Study of the \( \Xi^- \), \( K^- p \) interactions with hydrogen and different nuclei at 40 GeV. Search for new charm. Investigation of the transversal momenta processes: \( p_T > 1.0 \text{ GeV/c} \)

Study of the reaction \( \Xi^- \rightarrow \gamma (e^- e^+) \)-atom relativistic positronium interaction with matter

Investigation of the pion resonances in the dissociation processes of meson on nuclei. The mass and the width of the radial excitations of the \( \Xi^- \)-meson has been determined:
- \( M_{\Xi^-} = (1240\pm 30) \text{ MeV} \)
- \( \Gamma_{\Xi^-} = (360\pm 120) \text{ MeV} \)
- \( M_{\Xi^-} = (1179\pm 30) \text{ MeV} \)
- \( \Gamma_{\Xi^-} = (310\pm 50) \text{ MeV} \)

The first time the Compton effect on \( \Xi^- \)-meson has been investigated. Polarizability of the pion in the external field found; fundamental constant \( \sigma_{\Xi^-} \pm 6.8_{\pm 1.4} \cdot 10^{-13} \text{ cm}^3 \) was determined as well. Coupling constant \( g_{\Xi^-} \) was measured for the first time, \( g_{\Xi^-} = 13.5_{-1.5} \), the result confirming the chiral anomaly hypothesis.

The cross section production, life time and decay modes of the short lived particles in the beam of muon neutrinos. Weak current structure; deep inelastic scattering of neutrinos on nucleons
Fig. 1. Polarization in the inclusive production of the $\Lambda_c^+$ (BIS-2 experimental set-up).

Fig. 2. Probe of KNO scaling: charged multiplicity distributions for $\bar{p}n$ and $nn$ interactions, and $\pi n$ annihilations at 6.1 GeV/c, in the Liudmila hydrogen chamber.
Table 5
Resonance states observed in the process $\pi^+A \rightarrow \pi^+A$ using the MIS-2 spectrometer

<table>
<thead>
<tr>
<th>Particle</th>
<th>$J^P$</th>
<th>Mass (MeV/$c^2$)</th>
<th>Width (MeV)</th>
<th>Decay channel</th>
<th>Branching ratio (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^-$</td>
<td>0$^-$</td>
<td>1240±30</td>
<td>360±120</td>
<td>$\pi\pi$</td>
<td>100</td>
</tr>
<tr>
<td>$\pi^-'$</td>
<td>0$^-$</td>
<td>1770±30</td>
<td>360±50</td>
<td>$\pi\pi$</td>
<td>100</td>
</tr>
<tr>
<td>A1</td>
<td>1$^+$</td>
<td>1255±23</td>
<td>292±40</td>
<td>$\rho\pi$</td>
<td>98</td>
</tr>
<tr>
<td>A1'</td>
<td>1$^+$</td>
<td>1670±90</td>
<td>300±100</td>
<td>$\rho\pi$</td>
<td>100</td>
</tr>
<tr>
<td>A3</td>
<td>2$^-$S</td>
<td>1624±21</td>
<td>304±22</td>
<td>$\pi\pi$</td>
<td>60</td>
</tr>
<tr>
<td>A3'</td>
<td>2$^-$</td>
<td>-1850</td>
<td>-300</td>
<td>$\pi\pi$</td>
<td>30</td>
</tr>
<tr>
<td>A2</td>
<td>2$^+$</td>
<td>1320</td>
<td>-100</td>
<td>$\rho\pi$</td>
<td>10</td>
</tr>
<tr>
<td>A2'</td>
<td>2$^+$</td>
<td>1750</td>
<td>-</td>
<td>$\pi\pi$</td>
<td></td>
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</table>

Fig. 3a) Cross section of the reaction of pion pair production in the nuclei Coulomb field as a function of the $Z$ nucleus charge. Theoretical calculations for various numbers $N_C$ of the quark colour degrees of freedom are shown by dashed lines. Dots show a systematic error for cross section of the pion pair production on $^{12}$C. In the study of pion pair production in the $\pi$Al and $\pi$Fe interactions the systematic error is less than the statistical one.

b) Polarization of the pion.
(Results of the SIGMA-AJAX experiment.)
Fig. 4. Polarization in the reaction versus the square of the four-momentum transfer at an incident pion energy of 40 GeV/c. a) Without n-recoil; b) new data. (PROSA-POLARIMETER.)

Fig. 5. Ratio of the nucleon functions obtained in the deep inelastic muon scattering experiments at 280 GeV: a) with iron and deuterium targets; b) with nitrogen and deuterium targets (NA4 experiment).
Table 6
Experimental facilities for research in relativistic nuclear physics located in the synchrophasotron area.

- 411 -

2 m propane bubble chamber
Volume: 210 m 65 x 43 cm³
magnetic field: \( H = 15.5 \text{ ke} \)
\( \Delta E / E = 5-12\% \)

ALPHA JC Proportional and
drift chamber magnetic spectrometer
+ time flight and ionisation
hodoscopes; \( \Delta E_0 / E_0 = 0.8 \text{ mrad} \)

Recal Particle Spectrometer
semiconductor telescope; gas identifier;
scintillation magnetic spectrometer in
the internal beam of the accelerator

DIN 3 Cerenkov and scintillation counters
\( \Delta P / P = 6\% \); Time flight resolution = 1 nsec

150 channel Cerenkov Mass Spectrometer

RESONANCE Streamer chamber
with a liquid hydrogen target
in the magnetic field

MARSPEAK Magnetic spectrometer
with wire chambers

Hybrid Streamer Chamber Magnetic Spectrometer
including vertex detector, system of the beam
testing, fast processors, proportional chambers
volume: \( 2 \times 1 \times 0.6 \text{ m}^3 \)
magnetic field of 1.5 tesla

Study of interactions of relativistic nuclei and neutrino
with light and heavy nuclei for a search of multiquark
states

Search of highly excited states of the few nucleon
systems in experiments on scattering and fragmentation of
relativistic nuclei. Deuteron wave function was measured
in the reaction

Study of the p, \(^{3}\text{He}, ^{4}\text{He}, ^{6}\text{He}, ^{6}\text{Li}, ^{7}\text{Li}\) in the 6.6 GeV
proton interaction with nuclei. Behaviour of structure
functions at 6-400 GeV. Proof of the hypothesis of nuclear
scalar invariance.

Investigation of the particle cumulative production in
the relativistic nuclear physics

Search and study of the resonance production and decay
into \( \pi^+\pi^- \) pairs and \( \gamma \)-quanta
Search for 6 quark strange dibaryon systems

Study of nuclear interactions at relativistic
energies. Search for multiquark states in nuclei

Study of the inelastic nuclei-nuclei collisions;
compression of nuclear matter;
search for abnormal superdense nuclei.

Fig. 6. The cumulative \( \pi^- \) and K-meson events
in the proton-nuclei interactions at the inci-
dent proton momentum of 9 GeV/c.
Figs. 7. The A-dependence of the cumulative pion cross sections in proton-nuclei interactions at $P=9$ GeV/c: a) at $X$ values $X=1.3$; b) $X=2.1$.

Fig. 8. The distribution of mesons by the value of $b_k$ in jets produced in $\pi$-C interactions at $p=40$ GeV/c: (•) - in the region of the target fragmentation; (○) - flying toward the primary $\pi$-meson motion; the solid line is the result of the exponential dependence approximation.

Fig. 9. Invariant cross sections of the strange particle production in different interactions: a) for $K^0$-mesons; b) for $\Lambda$-hyperons, versus the part of energy which is carried away.
Table 7

The measurement of the circular polarization of gamma-quanta

<table>
<thead>
<tr>
<th>No.</th>
<th>Authors</th>
<th>Year</th>
<th>Detector</th>
<th>(H_\gamma(%))</th>
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<tr>
<td>1</td>
<td>Goldhaber et al.</td>
<td>1958</td>
<td>NaI(Tl)</td>
<td>(-67 \pm 10)</td>
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<tr>
<td>2</td>
<td>Marklund, Page</td>
<td>1958</td>
<td>NaI(Tl)</td>
<td>(-80 \pm 30)</td>
</tr>
<tr>
<td>3</td>
<td>Palathingal</td>
<td>1970</td>
<td>NaI(Tl)</td>
<td>(-61 \pm 12)</td>
</tr>
<tr>
<td>4</td>
<td>Average</td>
<td>-</td>
<td>-</td>
<td>(-65 \pm 8)</td>
</tr>
<tr>
<td>5</td>
<td>Calculated</td>
<td>1958</td>
<td>-</td>
<td>(-84)</td>
</tr>
<tr>
<td>6</td>
<td>Calculated</td>
<td>1983</td>
<td>-</td>
<td>(-93)</td>
</tr>
<tr>
<td>7</td>
<td>Present work</td>
<td>1983</td>
<td>Ge(Li)</td>
<td>(-87 \pm 10)</td>
</tr>
</tbody>
</table>

Fig. 10. The right-left asymmetry of the gamma-quanta emission with \(E_\gamma=9.3\) MeV in the p-wave resonance of 1.33 eV in the nucleus of \(^{117}\)Sn: a) experimental spectra of the emitted rays at the angle of 90° to the beam for the opposite directions of the neutron spin; b) the right-left asymmetry reduced to the 100% polarization of neutrons.
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