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ABSTRACT

The CERN School of Physics is intended to give young experimental physicists an introduction to the theoretical aspects of recent advances in elementary particle physics. These Proceedings contain reports of lecture series on the following topics: introduction to gauge fields, perturbative QCD, proton-antiproton collider physics, lattice quantum field theories, experiments on weak decays of leptons and quarks, lepton-hadron interactions, supersymmetry, grand unified theories and cosmology. They also include reports of special lectures on sum rules and hadron properties in QCD, on quark distribution in nuclei, and on the scientific programme of JINR.
Presented to the School by the town of Cagli
PREFACE

The 1985 CERN-JINR School of Physics was held in Urbino, Italy, from 1 to 14 September. The School was the ninth in the series of joint schools organized every two years by CERN and JINR (Dubna). It was attended by 99 young experimental physicists from 65 laboratories and universities; 33 students came from JINR and its Member States.

The basic aim of the School was to teach the fundamental aspects of present-day particle theories, and to review the recent experimental and theoretical developments in high-energy physics. Leading scientists from various countries gave eight series of lectures and seven special lectures.

The School was based in the modern colleges of the University of Urbino. Some of the afternoon lectures were given in the old buildings in the centre of the Renaissance town.

The School was a good example of scientific cooperation and of the promotion of mutual understanding between participants from different countries.

Many people and many organizations contributed to the success of the School; however, it is not possible to mention all of them here. But I would especially like to acknowledge the University of Urbino and its staff; in particular I thank Professor F. Grianti, Miss S. Urbinati, Mrs. M. L. Cangini, and Mr. G. Bernardini. The tireless efforts of Miss D. A. Caton, Miss. P. Nicoli and of Dr. W. O. Lock were of fundamental importance for the smooth running of the School. I also wish to acknowledge the contributions from the Istituto Nazionale di Fisica Nucleare, the University of Bologna, the Pesaro Rotary Club, the Pesaro Round Table, the Banca Popolare Pesarese, and the local authorities in Urbino, Pesaro, and Cagli. I thank all lecturers, discussion leaders, students, and members of the international and local Organizing Committees for their active participation. The CERN Composition and Printing Group contributed with their usual welcome efficiency.

G. Giacomelli,
Director of the School
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Dear colleagues, ladies and gentlemen,

I have the honour of greeting the traditional joint CERN-JINR School of Physics. Conceived as a means of widening the scientific horizons of our younger specialists working in various fields of particle physics, our School has fully justified the expectations of its founders. The scientific programme of the School acquaints young scientists who for the most part are experimenters, with important theoretical achievements and experimental results obtained at the major accelerators of the world.

The lecturers and discussion leaders of the School are prominent scientists of the CERN and JINR Member States and other countries. As you know, our first joint School was held in Finland in 1970. Since then quite a number of such Schools have been held in various European countries. Now we may already speak of certain traditions that have crystallized in both the scientific and organizing aspects of the School. As a rule, about 100 persons participate in it. The number of the countries represented is about 20. I think that these figures are good. The educational methods used at the School have proved to be most efficient. They are: review lectures on most important topics of modern physics of the microworld; seminars on most promising trends of experimental and theoretical research; informal discussions of the participants.

Here I should also mention the beautiful surroundings of the School venues and the interesting social and cultural programme that enable the students to learn more about the history and the traditions of the host country.

We hope that the School will acquaint you with most important work carried out at CERN and JINR as well as in other scientific centres. We also hope that the School will help us to get to know one another, and will generally benefit mutual understanding between scientists of various countries.

I wish a very good success to the IXth CERN-JINR School of Physics.

N N Bogolubov
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INTRODUCTION TO GAUGE FIELDS

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ABSTRACT

The aim of these lectures is to give an introduction to quantum field theory in the framework of the functional integration method. We outline a functional integral scheme of field quantization and its modification for systems with constraints. A general quantization scheme is developed. This scheme is applied to quantum electrodynamics, Yang-Mills fields, etc. The role of anomalies in gauge theories is discussed briefly, as well as the problem of extended objects.

LECTURE 1.

Systems with constraints and their quantization

Field theory can be looked upon as an infinite-dimensional analog of a mechanical system. In such an approach the theory of gauge fields is an analog of mechanical systems with constraints [1, 2].

A classical action of the finite-dimensional system with constraints is equal to

$$S = \int \left( \sum_{i=x}^{n} P_i \dot{q}_i - H(p, q) - \sum_{a=x}^{m} \lambda_a \varphi^a(p, q) \right) dt$$

(1.1)

It contains besides coordinates $q$ and momenta $p$ the variables $\lambda_a$, which come in linearly and play the role of Lagrange multipliers. The coefficients $\varphi^a(p, q)$ have the meaning of constraints. The variables $p, q$ generate the phase space of dimension $2n$. The number of constraints shall be denoted as $m$. We suppose that $m < n$ and that the constraints $\varphi^a$ and Hamiltonian $H$ are in involution, i.e. that they fulfill the conditions

$$\{ H, \varphi^a \} = \sum_{\xi} C_{\xi}^a \varphi^\xi, \ \{ \varphi^a, \varphi^\xi \} = \sum_{d} C_{d}^a \varphi^d$$

(1.2)

Here $C_{\xi}^a, C_{d}^a$ are functions of $p, q$ and $\{ f, g \}$ is the Poisson bracket

$$\{ f, g \} = \sum_{i=x}^{n} \left( \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} - \frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} \right)$$

(1.3)
The system of equations of motion for action functional (1.1) contains besides canonical equations
\[ \dot{q}^i = \frac{\partial H}{\partial p_i} + \sum_{a \neq i}^{m} \lambda_a \frac{\partial \varphi^a}{\partial q^i}, \quad \dot{p}_i = -\frac{\partial H}{\partial q^i} - \sum_{a \neq i}^{m} \lambda_a \frac{\partial \varphi^a}{\partial q^i} \quad (1.4) \]
also the constraint equations
\[ \varphi^a(p, q) = 0, \quad a = 1, \ldots, m \quad (1.5) \]

In principle, we can exclude some variables \( p, q \) using constraint equations. But in practice the solution of constraint equations often turns out to be rather difficult. So it is desirable to have a formalism where explicit solutions of constraint equations are not required.

Constraint equations (1.5) define the surface \( M \) of the \( 2n-m \) dimensions in phase space \( \Gamma \). The involution conditions (1.2) guarantee, for arbitrary functions \( \lambda_a(t) \) the fulfillment of constraint equations (1.5), provided those equations are satisfied for initial conditions. In other words, a trajectory which starts on the manifold \( M \) does not leave it.

We shall regard as observables on the manifold \( M \) the variables which are not influenced by arbitrariness in the choice of \( \lambda_a(t) \). This requirement is fulfilled by the functions \( f(p, q) \), which obey the conditions
\[ \{f, \varphi^a\} = \sum_{\ell} d^a_{\ell} \varphi^\ell \quad (1.6) \]
Indeed, in the equations of motion for those functions
\[ \ddot{f} = \{H, f\} + \sum_a \lambda_a \{\varphi^a, f\} \quad (1.7) \]
the \( \lambda_a \)-depending terms vanish on \( M \).

The function \( f(p, q) \), defined on \( M \) and satisfying conditions (1.6) does in fact depend on all variables. Conditions (1.6) can be looked upon as a system of \( m \) differential equations of the first order on \( M \) for which equations (1.2) are conditions of integrability. The function \( f \) is therefore unambiguously defined by its values on a submanifold of the systems initial conditions which has the dimension \( (2n-m)-m = 2(n-m) \). It is convenient to take as such a manifold a surface \( \Gamma^{*} \),
defined by constraint equations (1.5) and \( m \) additional conditions

\[ \mathcal{X}_a(p,q) = 0, \quad a = 1, \ldots, m. \]  

(1.8)

The function \( \mathcal{X}_a \) must satisfy the condition

\[ \det \| \{ \mathcal{X}_a, \mathcal{X}_b \} \| \neq 0 \]

(1.9)

because only in that case \( \Gamma^* \) can play the role of an initial surface for the equation (1.6). It is convenient to suppose that \( \mathcal{X}_a \) mutually commute

\[ \{ \mathcal{X}_a, \mathcal{X}_b \} = 0 \]

(1.10)

We mean two functions \( f, g \) commute, if their Poisson bracket is equal to zero ( \( \{ f, g \} = 0 \) ). In such a case it is possible to introduce canonical variables onto the manifold \( \Gamma^* \). Indeed, if condition (1.9) is satisfied, then using canonical transformation in \( \Gamma \), we can introduce a new set of variables where \( \mathcal{X}_a \) take a simple form:

\[ \mathcal{X}_a(p,q) = p_a, \quad a = 1, \ldots, m, \]

(1.11)

where \( p_a (a = 1, \ldots, m) \) is a subset of canonical momenta of the new system of variables. Condition (1.9) can be written in terms of the new variables as

\[ \det \| \frac{\partial q^a}{\partial q^b} \| \neq 0 \]

(1.12)

and the constraint equation (1.6) can therefore be solved with respect to \( q^a \). Finally, the surface \( \Gamma^* \) is given by the equations

\[ p_a = 0, \quad q^a = q^a(p^*, q^*) \]

(1.13)

on \( \Gamma \), so that \( p^* \) and \( q^* \) are independent variables on \( \Gamma^* \).

Now let us discuss the quantization procedure for finite-dimensional systems in the functional integral framework. We begin with the one-dimensional dynamical system with Hamiltonian \( \mathcal{H}(p,q) \). The principle of canonical quantization for such a system consists of replacing the coordinate \( q \) and momenta \( p \) by operators \( \hat{q} \) and \( \hat{p} \) according to the rule

\[ q \rightarrow \hat{q} = q, \quad p \rightarrow \hat{p} = -i\hbar \frac{\partial}{\partial q} \]

(1.14)

where \( \hbar \) is the Planck constant. In the following we shall use the system of units with \( \hbar = 1 \). The operators act on Hilbert space of complex
functions \( \psi(q) \).

The time evolution of a state is determined by the Heisenberg equation

\[
\frac{i}{\hbar} \frac{\partial \psi}{\partial t} = \hat{H} \psi,
\]

where \( \hat{H} \) is the energy operator, obtained from the classical Hamiltonian function \( H(p,q) \) \( p \) and \( q \) with operators \( \hat{p} \) and \( \hat{q} \) according to (1.14).

We can write down the formal solution of (1.15) as

\[
\psi(t) = \hat{U}(t,t_0) \psi(t_0),
\]

where the evolution operator

\[
\hat{U}(t,t_0) = \exp\left(i(t - t_0) \hat{H}\right)
\]

is the exponential of the energy operator \( \hat{H} \).

The method of functional integration allows us to express the matrix element of the evolution operator as a mean value of the expression

\[
\exp i \int [t_0, t]
\]

over trajectories in the phase space where

\[
\int [t_0, t] = \int_{t_0}^{t} (p(\tau) \dot{q}(\tau) - H(p(\tau), q(\tau))) d\tau
\]

is a classical action, corresponding to the trajectories \( p(\tau), q(\tau) \)

\( t_0 \leq \tau \leq t, \quad \dot{q}(\tau) = dq/d\tau \).

The mean value over trajectories is called Feynman functional integral. Usually this is defined as a limit of finite dimensional integrals.

We shall present here one of the possible definitions.

We divide the interval \([t_0, t]\) with \( \tau_1, ..., \tau_N \) points into \( N \) equal parts. Let us consider the functions \( p(\tau) \) defined on the interval which are constant on the intervals

\[
[t_0, \tau_1), (\tau_1, \tau_2), ..., (\tau_{N-1}, t]
\]

and the continuous functions \( q(\tau) \) linear on the intervals (1.20). We fix the values of the function \( q(\tau) \) at the end points of the interval \([t_0, t]\), putting

\[
q(t_0) = q_0, \quad q(t) = q
\]

The trajectory \((p(\tau), q(\tau))\) is determined by values of the
piecewise linear function $q(\tau)$ in the points $\tau_1, \ldots, \tau_{N-1}$ (we denote them $q_1, \ldots, q_{N-1}$) and by values of the piecewise constant function $p(\tau)$ on intervals $(\tau_{k}, \tau_{k+1})$. We denote those by $p_1, \ldots, p_N$.

Let us consider the finite dimensional integral

$$(2\pi)^N \int dp_1 dq_1 \cdots dp_N dq_N \exp \left(i \int_{t_o}^{t} S[q, q', t'] \right) \equiv \mathcal{J}_N(q_o, q^*, t_o, t), \quad (1.22)$$

where $S[t_o, t]$ is the action (1.13) for the described trajectory $(p(\tau), q(\tau))$ defined by the parameters $p_1, \ldots, p_N, q_1, \ldots, q_{N-1}$. The basis assertion says that the limit of the integral (1.22) for $N \to \infty$ is equal to the matrix element of the evolution operator

$$\ell_{\text{lm}} \mathcal{J}_N(q_o, q^*, t_o, t) = \langle q^* | \exp \left(i(t_o - t) \hat{H} \right) | q_o \rangle \quad (1.23)$$

Here we do not dwell on the proof of this statement. It is not hard to check it in the case, when the Hamiltonian $\hat{H}$ is a function of the coordinate or the momentum only. For the Schrödinger equation the proof is known only if the energy function $\hat{H}$ is a sum of a function of momentum and a function of coordinate

$$\hat{H} = H_1(p) + H_2(q) \quad (1.24)$$

Namely the Hamiltonians of the (1.24) type are used in nonrelativistic quantum mechanics.

We denote the functional integral, defined as the $N \to \infty$ limit of the expression (1.24) by the symbol

$$\mathcal{J}_N^\infty \left(\int_e \exp \left(i \int_{t_o}^{t} S[q, q', t'] \right) \prod_{\tau} \frac{dp(\tau)}{2\pi} \frac{dq(\tau)}{2\pi} \right) \quad (1.25)$$

This form is convenient but it does not reflect the fact that in the pre-limit expression (1.22) the number of integrations over momenta is higher by one order than that over the coordinates.

The generalization of the functional integral formalism to a system with an arbitrary finite number of degrees of freedom is straightforward.

The action of a mechanical system with $n$ degrees of freedom has the form

$$S[t_o, t] = \int_{t_o}^{t} \left(\sum_{i=1}^{n} p_i \dot{q}_i - H(p, q)\right) dt \quad (1.26)$$
Here \( q^i \) is the \( i \)-th canonical coordinate, \( p_i \) is the canonically conjugated momentum, \( H(p, q) = H(p_1, \ldots, p_n, q^1, \ldots, q^n) \) is the Hamiltonian.

By definition the functional integral for the evolution operator matrix element is a limit of the finite-dimensional integral obtained from (1.22) by the replacement

\[
(2\pi)^{-N} \to (2\pi)^{-N-n}, \quad dq_k \to \prod_{i=1}^{n} dq_k^i, \quad dp_k \to \prod_{i=1}^{n} dp_k^i, \quad (1.27)
\]

where \( q_k^i \) are the values of the \( i \)-th coordinate at the point \( \tau_k \) \((k = 1, \ldots, N\), and \( p_k^i \) are the values of the \( i \)-th momentum on \( (\tau_{k-1}, \tau_k) \) interval. It is necessary to keep all the coordinates \( q^1, \ldots, q^N \) simultaneously fixed at both ends of the time interval \( [t_0, t] \).

We will denote the functional integral defined in such a way by the symbol

\[
\int_{q(t') = q'}^{q(t''')} \exp \left\{ i \int_{t_0}^{t} \sum_{i=1}^{n} \left[ \frac{d}{dt} p_i^i(t) - H(p(t), q(t)) \right] dt \right\} \prod_{i=1}^{n} d\mu(p(t), q(t)), \quad (1.28)
\]

Let us study now what the functional integral for the finite dimensional mechanical system with constraints looks like.

We shall introduce additional conditions \( \mathcal{X}_a(p, q) \) so that relations (1.9) and (1.10) are satisfied.

The basis assertion is that the evolution operator matrix element is given by the functional integral

\[
\int \exp \left\{ i \int_{t_0}^{t} \left( \sum_{i=1}^{n} p_i^i(t) - H(p(t), q(t)) \right) dt \right\} \prod_{i=1}^{n} d\mu(p(t), q(t)), \quad (1.29)
\]

where the integration measure is given by the formula

\[
d\mu(t) = (2\pi)^{m-n} \det \parallel \mathcal{X}_a \parallel \prod_{a} \delta(\mathcal{X}_a(p)) \prod_{i=1}^{n} \delta(p_i, q_i), \quad (1.30)
\]

To prove the assertion we transform integral (1.29) with measure (1.30) to the integral (1.28), where integration is taken along the trajectories in the physical phase space \( \Gamma^* \). Using the abovementioned coordinates \( q^a, q^*, p_a, p^* \) we may transform (1.29) into the integral with the measure

\[
d\mu = (2\pi)^{m-n} \det \parallel \frac{\partial \mathcal{X}}{\partial q^a} \parallel \prod_{a} \delta(p_a) \delta(q^a) \prod_{i=1}^{n} dp_i dq_i, \quad (1.31)
\]
which can be rewritten as
\[ \prod_{\alpha} \delta(p_{\alpha}) \delta(q^{a} - q^{a}(p^{a}, q^{a})) \prod_{j=1}^{\kappa} \int \frac{dp_{j}^{a} dq_{j}^{a}}{2\pi} \]  

One need not integrate over \( p_{\alpha} \) and \( q^{a} \) thanks to the \( \delta \) -functions. As a result the integral (1.29) takes the following form
\[ \int \exp \left[ \sum_{t_{0}}^{t} \left( \sum_{i=1}^{n} p_{i}^{a} \dot{q}_{i}^{a} \right) - H^{*}(p^{a}, q^{a}) \right] \prod_{\tau=1}^{\kappa} \int \frac{dp_{\tau}^{a} dq_{\tau}^{a}}{2\pi} \]  

which coincides with (1.28).

Let us note, that integral (1.29) can be rewritten as
\[ \int \exp \left[ \sum_{t_{0}}^{t} \left( \sum_{i=1}^{n} p_{i}^{a} \dot{q}_{i}^{a} - H - \sum_{\alpha} \lambda_{\alpha} \varphi^{a} \right) d\tau \right] \times \]
\[ \times \prod_{\tau} \det \left\{ \chi_{\alpha}, \varphi^{a} \right\} (2\pi)^{m-n} \prod_{\alpha} \delta(\chi_{\alpha}) \prod_{i=1}^{n} \int \frac{dp_{i}^{a} dq_{i}^{a}}{2\pi} \prod_{\alpha} \Delta\tau \frac{d\lambda_{\alpha}}{2\pi} \]  

The symbol \( \prod_{\alpha} \delta(\tau) \left( d\lambda_{\alpha} / 2\pi \right) \) shows that in the prelimit expression there are integrals over \( \lambda_{\alpha}(\tau_{i}) \) (\( \tau_{i} \) are dividing points of the intervals \( [t_{0}, t] \)) of the type
\[ \int \exp \left( -i \sum_{i, \alpha} \lambda_{\alpha}(\tau_{i}) \varphi^{a}(p(\tau_{i}), q(\tau_{i})) \Delta\tau \right) \prod_{i, \alpha} \Delta\tau \frac{d\lambda_{\alpha}}{2\pi} \]  

Expression (1.35) is equal to the product of \( \delta \) -functions
\[ \prod_{i, \alpha} \delta(\varphi^{a}(p(\tau_{i}), q(\tau_{i}))) \]  

It means that in integral (1.36) we can carry out the integration over \( \lambda_{\alpha} \) and return again to integral (1.29).

It is not difficult to prove, that functional integral (1.29) does not depend on the choice of additional conditions (1.11).

LECTURE 2

Gauge Field Quantization

Field theory can be regarded as the theory of a mechanical system with an infinite number of degrees of freedom. The functional integral in
field theory can be constructed using various methods. First, it is possible to start with field action written in the Hamiltonian form and construct the functional integral over the phase space of a system with an infinite number of degrees of freedom. Second, it is possible to start with the action not written explicitly in Hamiltonian form and study the functional integral over all fields. This approach enables us to construct explicitly relativistic theory. In the Hamiltonian approach relativistic invariance is often not explicit and requires special proof.

Let us consider an example the theory of scalar field with action

$$\mathcal{S} = \int d^4x \left( \frac{1}{2} g^{\mu \nu} \frac{\partial \varphi}{\partial x^\mu} \frac{\partial \varphi}{\partial x^\nu} - \frac{m^2}{2} \varphi^2 - \frac{g}{3!} \varphi^3 \right)$$  \hspace{1cm} (2.1)

Here $\varphi(x)$ are field functions depending on a point $x = (x^0, x^1, x^2, x^3)$ of pseudo-Euclidean space $V_4$ and $g^{\mu \nu}$ is the diagonal Minkowski tensor $(1, -1, -1, -1)$. The action is the sum of the functional $\mathcal{S}_c$ — the action of the free field theory, quadratic in $\varphi$ and the integral over $(-g/3!) \varphi^3$ that describes the selfinteraction with the coupling constant $g$.

To define the functional integral over all fields the finite-dimensional approximation is frequently used.

In the space $V_4$ we take a big cubic volume $V$, divided into $N^d$ equal small cubes $V_i$ ($i = 1, \ldots, N^d$). We approximate function $\varphi(x)$ in the volume $V$ by a function constant in the volumes $V_i$ and the first derivatives $\partial \varphi/\partial x^\mu$ by the finite differences

$$\frac{1}{\Delta \ell} \left[ \varphi \left( x^\nu + \delta^\nu_\mu \Delta \ell \right) - \varphi \left( x^\nu \right) \right]$$  \hspace{1cm} (2.2)

where $\Delta \ell$ is the length of the edge of the cube $V_i$. Approximating piecewise constant function $\varphi(x)$ is defined through its values in volumes $V_i$.

Let us consider the finite-dimensional integral

$$\int \exp \mathcal{S} \prod d\varphi(x)$$  \hspace{1cm} (2.3)

over the values of function $\varphi(x)$ is volumes $V_i$. Here $\mathcal{S}$ is the action integral for the approximating function $\varphi(x)$ (with (2.2) as an app-
roximation for its first derivatives).

Finite-dimensional integrals of type (1.3) are present in the preli-
mit expressions used for the definition of functional integrals encount-
red in field theory. We shall define the Green function as

$$G(x, y) = -i \langle \varphi(x) \varphi(y) \rangle =$$

$$= -i \lim_{V \to \infty, V_i \to 0} \frac{\int (\exp \{i S \}) \varphi(x) \varphi(y) \prod x d\varphi(x)}{\int (\exp \{i S \}) \prod x d\varphi(x)}$$

(2.4)

We denote the limit on the r.h.s. of (2.4) by the symbol

$$\frac{\int (\exp \{i S \}) \varphi(x) \varphi(y) \prod x d\varphi(x)}{\int (\exp \{i S \}) \prod x d\varphi(x)}$$

(2.5)

The method of functional integration over all fields can be explained
and justified if it is possible to transform the functional integral ob-
tained here into integrals of a Hamiltonian form which represent a field
theory generalization of the integrals obtained above, in the quantiz-
ton of the finite-dimensional mechanical systems.

Continuing the examination of the scalar field example, we shall wri-
te down in Hamiltonian form the functional integral

$$\int \exp \{i S \} \prod x d\varphi(x)$$

(2.6)

To proceed we consider the integral

$$\int \exp \{i S[\varphi, \pi] \} \prod x d\varphi(x) d\pi(x)$$

(2.7)

where the expression

$$S[\varphi, \pi] = \int (\pi \partial_x \varphi - \frac{\pi^2}{2} - \frac{i}{2} (\bigtriangledown \varphi)^2 - m^2 \varphi^2 - \frac{\alpha}{3} \varphi^3)$$

(2.8)

coincides with action (2.1) provided \( \partial_x \varphi(x) \) is substituted for \( \pi(x) \).
Action (2.8) is of Hamiltonian form and the corresponding Hamiltonian function is

$$H = \int d^3x \left( \frac{1}{2} \pi^2 + \frac{1}{2} \nabla \varphi^2 + \frac{m^2}{2} \varphi^2 + \frac{g}{3!} \varphi^3 \right)$$  \hspace{1cm} (2.9)$$

where the functions $\varphi(x)$, $\pi(x)$ have the meaning of coordinate and conjugated momentum densities respectively. We show that integral (2.7) over variables $\varphi$ and $\pi$ results in integral (2.6) over all fields. To achieve this, it is sufficient to notice that the integral over $\pi$ in formula (2.5) can be expressed explicitly if the shift

$$\pi(x) \rightarrow \pi(x) + \partial_0 \varphi(x)$$  \hspace{1cm} (2.10)$$

is performed which causes the integral to transform into the product of integral (2.6) over $\varphi$ and the integral over $\pi$

$$\exp \left( -\frac{i}{2} \int d^3x \nabla \pi \right) \prod_x d\pi(x)$$  \hspace{1cm} (2.11)$$

leading to the product of normalization factors.

In such a way we have succeeded in expression the functional integral of scalar field theory through the Hamiltonian form, artificially introducing an integral over a new variable - the canonical momentum. Such an approach turns out to be useful for the proof of the Hamiltonicity of given systems of quantum field theory and statistical physics.

The scheme of functional integration over all fields produces a method of quantization of Bose fields.

Quantization of Fermi fields can be performed using the functional integral over anticommuting variables. The following basic facts are necessary here.

The integral over Fermi fields (over an infinite Grassmann algebra) is defined as a limit of the integral on an algebra with a unit element and a finite number of generators $X_i, X_i^*$ $(i = 1, \ldots, n)$ obeying commutation relations

$$X_i X_j + X_j X_i = 0 \hspace{0.5cm} X_i^* X_j^* + X_j^* X_i^* = 0 \hspace{0.5cm} X_i X_j^* + X_j X_i^* = 0$$  \hspace{1cm} (2.12)$$

Any of the elements of the algebra $\{X, X^*\}$ is a polynomial of the form
\[
\int (x, x^*) = \sum_{q_i \neq 0, \ell_i \neq 0} c_{q_i} c_{\ell_i} a_{q_i} b_{\ell_i} x_i^{q_i} x_i^{\ell_i} x_i^{q_i} x_i^{\ell_i} \cdots (x_i^*)^{q_i} (x_i^*)^{\ell_i} (x_i^*)^{q_i} (x_i^*)^{\ell_i} \tag{2.13}
\]

On the algebra we can introduce the integral
\[
\int \int (x, x^*) dx^* dx = \int \int (x_i, x_n, x_i^*, x_n^*) dx_i^* dx_i \cdots dx_n^* dx_n \tag{2.14}
\]

This integral is defined through the relations
\[
\int dx_i = 0, \quad \int dx_i^* = 0, \quad \int x_i dx_i = 1, \quad \int x_i^* dx_i^* = 1 \tag{2.15}
\]

We demand also that the symbols $dx_i, dx_i^*$ anticommute with each other and with the generators and we impose the natural condition of linearity
\[
\int (c_1 s_1 + c_2 s_2) dx^* dx = c_1 \int s_1 dx^* dx + c_2 \int s_2 dx^* dx \tag{2.16}
\]

When we integrate the sum (2.13), only the contribution of the term with $a_i = b_i = 1$ for all $i = 1, \ldots, n$ is different from zero.

The following two formulae will be used later on
\[
\int \exp (-x^* A x) dx^* dx = \det A \tag{2.17}
\]
\[
\frac{\exp (-x^* A x + \eta^* x + x^* \eta)}{\exp (-x^* A x)} dx^* dx = \exp (\eta^* A^{-1} \eta) \tag{2.18}
\]

where
\[
x^* A x = \sum_{i, \kappa} a_{q_i} x_i^{q_i} x_k \tag{2.19}
\]
is a quadratic form of the generators $x_i, x_i^*$, corresponding to the matrix $A$.

The expressions
\[
\eta^* x = \sum_i \eta_i^* x_i, \quad x^* \eta = \sum_i x_i^* \eta_i \tag{2.20}
\]
are linear of the generators $x_i, x_i^*$, whose coefficients $\eta_i, \eta_i^*$...
anticommute with each other and with the generators. The elements \( \eta^*, \eta \) together with the generators \( \chi, \chi^* \) can be regarded as generators of a larger algebra. The expression \( \eta^* A^{-1} \eta \) in (2.20) is a quadratic form of the matrix \( A^{-1} \) inverse to \( A \).

Now we are ready to discuss the gauge field quantization. Gauge fields can be regarded as connections on a fibre bundle, its base being the space-time \( V_\eta \) and its fiber a finite-dimensional space carrying a representation of a group.

The geometrical nature of gauge fields also demonstrates itself when constructing the corresponding quantum theory. The most suitable way of gauge field quantization is to use the functional integration approach.

The method of quantization is based on the following idea. The fields obtained from other fields through gauge transformation (e.g. \( A_\mu \) and \( A_\mu + \partial_\mu \lambda \) in electrodynamics) describe the same physical (geometrical) situation and are therefore physically indistinguishable. This leads to an idea that the classes of those fields which can be obtained from other fields through gauge transformations should be the basic objects of the theory. In such a way all fields of the type \( A_\mu + \partial_\mu \lambda \) are unified into one class.

The action in the gauge field theory is the same for all fields obtained through gauge transformations. In order words the action is a functional defined on classes.

In the functional integral formalism it is possible to obtain a theory whose basic objects are classes if we can write down the functional integral as an integral over all classes. It can be accomplished, e.g. if the integration is taken over the surface in the manifold of all fields whose elements intersect each of the classes once. Then each class will have exactly one representative on that surface. The integration measure arising on such surfaces changes with variation of the surface, but all physical results must be independent of the choice of the surface.

We now formulate the quantization scheme of gauge fields in the formalism of functional integration over all fields.

We shall denote the gauge fields by \( A \), its components \( A_\mu^a \), where \( \mu = 0, 1, 2, 3 \) is a space-time index, and \( a \) is an isotopic index.
The gauge group is a direct product of the groups $G_c$, operating at every point $x$ of space-time

$$G = \bigotimes_x G_c(x)$$  \hfill (2.21)

Let $\Omega$ be an element of the gauge group which is a function on $V_\gamma$ with functional values in $G_c$. We denote $A_\Omega$ the result of action of the element $\Omega$ on the field $A$. The set of fields $A_\Omega$ with $A$ fixed, $\Omega$ running through gauge group $G$ is called the gauge group orbit.

We have seen that quantization of a field with action $S$ leads to averaging $\exp iS$ over all fields. In the theory of gauge fields the action $S[A]$ is gauge invariant, i.e.

$$S[A_\Omega] = S[A]$$  \hfill (2.22)

The measure in the functional integral exhibit the property of gauge invariance

$$d\mu[A_\Omega] = d\mu[A]$$  \hfill (2.23)

as well as the action $S[A]$. The invariance of action $S[A]$ and the measure $d\mu[A]$ with respect to gauge transformations $A \rightarrow A_\Omega$ implies that the corresponding functional integral

$$\int \exp i S[A] \, d\mu[A]$$  \hfill (2.24)

becomes proportional to the "orbit volume" i.e. to the functional integral

$$\int \bigotimes_x d\Omega(x)$$  \hfill (2.25)

over gauge group $G$. Here $\bigotimes_x d\Omega(x)$ is an invariant measure on group $G$ which is equal to the product of measures on $G_c$, operating at every point of space-time $V_\gamma$.

The approach to the integration over classes consists of the explicit factorization of that factor from the functional integral. Such a factorization can be realized by several methods.

One of them consists of the transition from integral (2.24) over all fields to the integral over the surface in the manifold of all fields, whose elements intersect with each of the gauge-group orbits just once.
Let the equation of the surface be
\[ \int (A) = 0 \] (2.26)
The equation \( \int (A^\Omega) = 0 \) should have a unique solution with respect to \( \Omega (x) \) for any \( A (x) \).

We introduce the functional \( \Delta_f [A] \) defined by the condition
\[ \Delta_f [A] \int \prod_x \delta \left( \int (A^\Omega (x)) \right) d\Omega (x) = 1 \] (2.27)
Here the integration is carried out over gauge group \( G \) of the infinite dimensional \( \delta \)-function \( \prod_x \delta \left( \int (A^\Omega (x)) \right) \). Such a \( \delta \)-function is a functional defined by the specification of the rules for its integration with other functionals. In the following we shall demonstrate several specific examples of the evaluation of integrals of type (2.27). Let us notice that the functional \( \Delta_f [A] \) is gauge invariant, i.e.
\[ \Delta_f [A^\Omega] = \Delta_f [A] \] (2.28)
To factorize factor (2.25) from functional integral (2.20) we insert the left hand side of (2.27) (which is equal to one) into the integral and make the substitution \( A^\Omega \rightarrow A \). The measure \( d\mu [A] \) and functionals \( S[A], \Delta_f [A] \) are invariant under such a substitution. Integral (2.24) leads to the multiplication of the group's volume \( \int \prod_x d\Omega (x) \) by the integral
\[ \int \exp \left( i S[A] \right) \Delta_f [A] \prod_x \delta \left( \int (A) \right) d\mu [A] \] (2.29)
Just this integral provides a starting point for the quantum theory of gauge fields.

It is easy to demonstrate that integral (2.29) formally depending on the choice of the surface \( \int (A) = 0 \), is in fact invariant with respect to the choice of the surface. To prove it we insert into the integrand (2.29) "another unit element"
\[ 1 = \Delta_g [A] \int \prod_x \delta \left( g \left( A^\Omega (x) \right) \right) d\Omega (x) \] (2.30)
where \( g (A) = 0 \) is an equation of another surface which, like the surface \( \int (A) = 0 \), intersects each of the orbits of group \( G \) only once.

Interchanging the integration over \( A \) and \( \Omega \), performing then the
shift $\hat{\mathcal{A}} \to \mathcal{A}$ and finally interchanging again the integration over $\mathcal{A}$ and $\mathcal{O}$, we can express integral (2.29) as

$$\int \exp(i S[\mathcal{A}]) \Delta_{\mathcal{O}} [\mathcal{A}] \prod_{x} \delta(g(A)) \, d\mu [\mathcal{A}]$$

(2.31)

The method described allows one to pass in the functional integral from one surface to another or, we can say, from one gauge to another. Especially, such a method is suitable for the transition from the Hamiltonian form of the functional integral to the integral in the relativistic gauge. There exists a method for the factorization of the volume of a gauge group from the functional integral which is more general than the method just described. Let us take the functional $F[\mathcal{A}]$ which is not gauge invariant. We define a gauge invariant functional $\phi[\mathcal{A}]$ by the equation

$$\phi[\mathcal{A}] = \int F[\mathcal{A}] \prod_{x} d\mathcal{O}(x) = 1$$

(2.32)

It is necessary, however, to require that the functional on the l.h.s. of (2.32) really exists. Inserting the l.h.s. of (2.32) into integral (2.34) and then performing the shift $\hat{\mathcal{O}} \to \mathcal{O}$ we obtain the product of the group volume (2.35) with the integral

$$\int \exp(i S[\mathcal{A}]) \phi[\mathcal{A}] \, F[\mathcal{A}] \, d\mu [\mathcal{A}]$$

(2.33)

Integral (2.29) is a special case of (2.33). The independence of integral (2.33) on the choice of functional $F[\mathcal{A}]$ can be proved in the same way as the independence of integral (2.29) on the choice of the surface $\mathcal{O}(A) = 0$.

In the theory of gauge fields Green's function is defined as an expectation value of the product of field functions at different points of space-time $V_{\eta}$. The generating functional of Green's functions has the form

$$Z[\eta] = \frac{\int \exp\left\{ i S[\mathcal{A}] + i \int A d^4 x \right\} F[\mathcal{A}] \phi[\mathcal{A}] \, d\mu [\mathcal{A}]}{\int \exp(i S[\mathcal{A}]) \, F[\mathcal{A}] \phi[\mathcal{A}] \, d\mu [\mathcal{A}]}$$

(2.34)

where $S[\mathcal{A}]$ is the action of the field $\mathcal{A}$, $d\mu [\mathcal{A}]$ is the local gauge invariant measure, the functionals $F$ and $\phi$ are defined above. The linear functional
\[ \int \left( \sum_{\mu, \lambda} \mathcal{N}_\lambda \left( A^\lambda_{\mu} (x) \right) A_{\mu}^\lambda (x) \right) d^4x \quad (2.35) \]

is denoted as \( \int \eta A \, d^4x \), where \( \mathcal{N}_\lambda \) are arbitrary test functions.

Green's functions - functional derivatives of function (2.34) - depend on the choice of gauge i.e., on the choice of the functional \( F [A] \). Physical results, obtained by averaging gauge-invariant functionals, however, do not depend on the choice of gauge.

LECTURE 3

Examples of gauge theories. Electrodynamics, Yang-Mills field.

The simplest example of gauge field is electromagnetic field. The action of a free electromagnetic field

\[ S = -\frac{1}{4} \int \left( \partial_\mu A_\nu - \partial_\nu A_\mu \right)^2 d^4x \quad (3.1) \]

is invariant under the Abelian group of gauge transformations

\[ A_\mu (x) \rightarrow A_\mu (x) + \partial_\mu \lambda (x). \quad (3.2) \]

We have seen that the quantization of gauge fields is realized using the functional integral of the functional \( \phi \int \exp \left( i S \right) \), where \( S \) is the action, \( F \) is an arbitrary nongauge invariant functional, and \( \phi^{-i} \) is the expectation value of \( F \) averaged over the gauge group. The local integration measure

\[ d^3 [A] = \prod_x \prod_{\mu=0}^3 dA_\mu (x) \quad (3.3) \]

is evidently gauge invariant. Functionals of the type

\[ F_1 [A] = \prod_x \mathcal{S} \left( \partial_\mu A_\mu (x) \right), \]

\[ F_2 [A] = \prod_x \mathcal{S} \left( \partial_\mu \phi \partial_\nu \phi (x) \right), \]

\[ F_3 [A] = \exp \left( -\frac{i}{2} \int (\partial_\mu A_\mu)^2 d^4x \right) \]

turn out to be most convenient for the construction of the perturbation theory. The functionals \( F_1 \) and \( F_3 \) lead to an explicitly relativistic
quantization and the use of $F_2$ is convenient when passing to the Hamilton theory. The corresponding gauge invariant functionals are given by the formulae

$$
\Phi_1^{-1}[A] = \int \prod_x \delta (\partial_\mu (A_\mu + \partial_\mu \lambda)) d\lambda(x),
$$

$$
\Phi_2^{-1}[A] = \int \prod_x \delta (i \nu (A_\mu + \partial_\mu \lambda)) d\lambda(x), \quad (3.5)
$$

$$
\Phi_3^{-1}[A] = \int \exp \left( -\frac{i}{2\alpha} \int \left( \partial_\mu (A_\mu + \partial_\mu \lambda) \right)^2 d\lambda(x) \right) \prod_x d\lambda(x).
$$

All these functionals do not, in fact, depend on the field $A_\mu(x)$, as can be seen if we perform the shift $\lambda \rightarrow \lambda - \Delta^{-1} \partial_\mu A_\mu$ in the first and third functionals and $\lambda \rightarrow \lambda - \Delta^{-1} i \nu A_\mu$ in the second one. Thus, with precision up to an (infinite) constant factor we can take

$$
\Phi_1 = \Phi_2 = \Phi_3 = 1 \quad (3.6)
$$

Now the form of functional integral is defined in all 3 cases. The use of the functional $F_2$ means the integration over the fields satisfying the equation

$$
div A = 0 \quad (3.7)
$$

This is a well-known Coulomb-gauge condition. We shall show how the integral with the functional $F_2$ can be transformed into the integral of an explicitly Hamilton type. Such a transformation is possible if the integral over auxiliary fields is introduced. In our case we are led to the functional integral of the form

$$
\int \exp \left( i \left[ S[A_\mu, F_{\mu\nu}] \prod_x \delta (i \nu \vec{A}(x)) \prod_{\mu} dA_\mu(x) \prod_{\mu \neq \nu} dF_{\mu\nu}(x) \right] \right) 
$$

with the action

$$
S[A_\mu, F_{\mu\nu}] = \int \left( \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{i}{2} F_{\mu\nu} (\partial_\mu A_\nu - \partial_\nu A_\mu) \right) d^4x \quad (3.8)
$$
depending not only on the vector $A_\mu(x)$, but also on the antisymmetric tensor $F_{\mu\nu}(x)$. In the classical theory $F_{\mu\nu}$ are the electromagnetic field strengths:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

(3.10)

Here we regard $A_\mu$, $F_{\mu\nu}$ as independent variables and integrate over them as over independent variables. The integral over $F_{\mu\nu}$ in (3.9) can be evaluated exactly. To accomplish this it is sufficient to perform a shift

$$F_{\mu\nu} \rightarrow F_{\mu\nu} + \partial_\mu A_\nu - \partial_\nu A_\mu$$

(3.11)

which transforms integral over $A_\mu$ into the product of the integral over $A_\mu$ and the integral over $F_{\mu\nu}$

$$\int \exp \left( \frac{i}{\hbar} \int F_{\mu\nu} F_{\mu\nu} d^4x \right) \prod_x \prod_{\mu\nu} dF_{\mu\nu}$$

(3.12)

which is just a normalization constant.

We rewrite action (3.9) using 3-dimensional notations

$$\int \left( \vec{E} \cdot \partial_t \vec{A} - \frac{1}{2} \vec{E}^2 + \frac{1}{2} \vec{H}^2 - (\vec{H}, \text{rot} \vec{A}) + A_0 \text{div} \vec{E} \right) d^4x,$$

(3.13)

where

$$E_i = F_{0i}, \quad H_i = F_{23}, \quad H_2 = F_{31}, \quad H_3 = F_{12}$$

(3.14)

We shall integrate over $\vec{H}$ in (3.8). This leads to the substitution $\vec{H} \rightarrow \text{rot} \vec{A}$ in action (3.13). Then we integrate over $A_0$, which yields the functional

$$\prod_x \delta(\text{div} \vec{E}(x))$$

(3.15)

We obtain the integral

$$\int \exp \left( i \int [\vec{A}, \vec{E}] \right) \prod_x \delta(\text{div} \vec{A}(x)) \delta(\text{div} \vec{E}(x)) \prod_i dA_i(x) dE_i(x)$$

(3.16)

with a Hamiltonian-type-action

$$\int \left( \vec{E} \cdot \partial_t \vec{A} - \frac{1}{2} \vec{E}^2 - \frac{1}{2} (\text{rot} \vec{A})^2 \right) d^4x$$

(3.17)
Integral (3.16) is an analog of integrals considered in Lecture 1 when we quantized finite-dimensional systems with constraints. Here the role of the constraint is played by divE, the role of the additional condition by the Coulomb gauge equation (3.7). It is possible to use the transverse (in the 3-dimensional sense) components of vectors \( \vec{A} \) and \( \vec{E} \) as independent variables.

The Lagrangian of spinorial quantum electrodynamics

\[
\overline{\psi}(\chi) \left( i \gamma^\mu \left( \frac{\partial}{\partial x^\mu} - i e A^\mu(x) \right) - m \right) \psi(x) - \frac{1}{4} \left( \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) \right)^2 \quad (3.16)
\]

where \( \gamma^\mu \) are Dirac matrices, also contains besides the electromagnetic potential \( A^\mu(x) \), the four-component spinors \( \psi(x) \), \( \overline{\psi}(x) \), describing the Fermi electron-positron field. Lagrangian (3.16) is invariant under the Abelian group of gauge transformations

\[
A^\mu(x) \rightarrow A^\mu(x) + \partial^\mu \lambda(x), \quad \psi(x) \rightarrow e^{i \lambda(x)} \psi(x), \quad \overline{\psi}(x) \rightarrow \overline{\psi}(x) e^{-i \lambda(x)} \quad (3.19)
\]

In the functional integration scheme we shall consider the components of spinors \( \psi^\alpha(x) \), \( \overline{\psi}^\alpha(x) \) to be anticommuting elements of Grassmann algebra and we shall integrate \( \exp i S \) with the measure

\[
F[A] = \int \prod_x dA^\mu(x) \prod_A d\overline{\psi}^\alpha(x) d\psi^\alpha(x) \quad (3.20)
\]

Here \( F \) is equal to \( F_1 \) or \( F_2 \) or \( F_3 \).

The form of Green's functions depends on the choice of functional \( F[A] \), but all the physical results do not depend on the gauge condition.

The theory of Yang-Mills fields is the simplest example of the theory with a nonabelian gauge group \( G \).

The vector Yang-Mills field, connected with simple compact Lie group \( G \) can be described by matrices \( B^\mu_\alpha(x) \) acquiring values in the Lie algebra of that group

\[
B^\mu_\alpha(x) = \sum_{a=1}^n \theta^a_\mu(x) \tau_a. \quad (3.21)
\]

Here \( \tau_a \) are linear independent matrices in the adjoint representation of
the Lie algebra, normalized by the conditions

$$
tr \tau_a \tau_b = -2 \delta_{a,b},
$$

(3.22)

$n_\lambda$ is the number of group parameters, $\theta^a_\mu(x)$ is a $C$-number function with the vector index $\mu$ and the "isotopic" index $a$. As is well known, in the adjoint representation it is possible to use the latter index for the enumeration of matrix elements, so that

$$
(B_\mu)_{a b} = (\tau^c_a)_{a b} \theta^c_\mu = \tau_{a b c} \theta^c_\mu,
$$

(3.23)

where $\tau_{a b c}$ are group-structure constants, antisymmetric in all three indices.

The Lagrangian of the Yang-Mills field

$$
\frac{i}{8} tr F_{\mu\nu} F^{\mu\nu},
$$

(3.24)

where

$$
F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + \epsilon [B_\mu, B_\nu],
$$

(3.25)

is invariant under the gauge transformations

$$
B_\mu \rightarrow \Omega B_\mu \Omega^{-1} + \epsilon^{-1} \partial_\mu \epsilon \Omega \epsilon^{-1}
$$

(3.26)

with the matrix $\Omega$ acting in the adjoint representation of the group.

The analogues of the functionals $F[A]$, used above for quantum electrodynamics, are suitable also for quantization of Yang-Mills field. Now they have the form

$$
F_1[\mathcal{B}] = \prod_x \delta(\partial_\mu \mathcal{B}_\mu(x)) = \prod_x \delta(\partial_\mu \theta^a_\mu(x)),
$$

$$
F_2[\mathcal{B}] = \prod_x \delta(d_{\mu\nu} \mathcal{B}(x)) = \prod_x \delta(\partial_\mu \theta^a_\mu(x)),
$$

(3.27)

$$
F_3[\mathcal{B}] = \exp \left( \frac{i}{4 \omega} \int tr (\partial_\mu \mathcal{B}_\mu)^2 d^4x \right) = \exp \left( -\frac{i}{2 \omega} \int \sum_a (\partial_\mu \theta^a_\mu)^2 d^4x \right)
$$
Here, the functionals $F_1$ and $F_2$ select among all fields, those satisfying the conditions

$$\int L[B] = \partial^\mu B_\mu = 0 \quad \text{for} \quad F_1 \quad \int R[B] = d_i \nu \hat{B} = 0 \quad \text{for} \quad F_2$$ (3.28)

Each of these equations is a matrix equation and in fact it represents additional conditions (according to the number of parameters of the group $G$).

The factor $\phi_i$ corresponding to the functional $F_i$, will be denoted by $\Delta_L$. In the functional integral, this factor stands before the $S$-function of $\partial^\mu B_\mu$ and it is therefore sufficient to know its values only for the transverse fields ($\partial^\mu B_\mu = 0$). In such a case, the whole contribution to the integral

$$\Delta_L^{-1}[B] = \int \prod_x S(\partial^\mu B_\mu(x)) d\Omega(x)$$ (3.29)

comes from a neighbourhood at the unit element. Here the substitution

$$\Omega(x) = 1 + \varepsilon \nu(x)$$ (3.30)

can be performed ($\nu(x)$ is an element of the Lie algebra) and only the terms linear in $\nu$ remain in

$$\partial^\mu B_\mu = \partial^\mu (B_\mu + \varepsilon [\nu, B_\mu] + \partial^\mu \nu) =$$

$$= \Box \nu + \varepsilon [B_\mu, \partial^\mu \nu]$$ (3.31)

where $\Box = \hat{A}_\alpha$ is the d’Alambert operator. Instead of the matrices $\nu(x)$ we introduce the column

$$\nu(x) = \sum_{\alpha} \tau_\alpha \nu_\alpha(x)$$ (3.32)

which the operator $\hat{A}$ acts on according to the rule

$$(\hat{A} \nu)_\alpha = (\Box \nu - \varepsilon [B_\mu, \partial^\mu \nu])_\alpha = (\Box \delta_\alpha^\epsilon - \varepsilon \partial^\mu \nu_\alpha) \nu_\epsilon =$$

$$= \Box \nu_\alpha - \varepsilon \delta_\alpha^\epsilon \delta_\mu^\theta \partial^\mu \nu_\epsilon$$ (3.33)
Integral (3.29) can be written as

$$\Delta_L^{-1} [B] = \int \prod_{x, x'} \delta((\hat{A} \cdot u)_a) \, du_a(x) \tag{3.34}$$

Formally $\Delta_L [B]$ is the determinant of the operator $\hat{A}$. Putting out the trivial (infinite) factor $\det \Box$ it is possible to expand the logarithm $\Delta_L$ into series in $\varepsilon$:

$$l_n \Delta_L [B] = l_n \det \hat{A} / \hat{A}_0 = \text{Tr} \, l_n \left( 1 - \varepsilon \Box^{-1} B_{\mu} \partial_{\mu} \right) =$$

$$= - \sum_{n=2}^{\infty} \frac{\varepsilon^n}{n} \int d^n x_1 \ldots d^n x_n \, \text{tr}(B_{\mu_1}(x_1) \ldots B_{\mu_n}(x_n)) \times$$

$$\times \partial_{\mu_1} \tilde{D}(x_1 - x_2) \ldots \partial_{\mu_n} \tilde{D}(x_n - x_1) \tag{3.35}$$

$\tilde{D}(x)$ is the Green's function of the d'Alambert operator $\Box$. $\text{Tr}$ in (3.35) means the trace in an operator sense in contradiction to $\text{tr}$ - the trace of a matrix.

The corresponding factor in the Coulomb gauge is denoted by $\Delta_R$.

Analogous evaluation leads to the formula

$$l_n \Delta_R [B] = \text{Tr} \, l_n \left( 1 - \varepsilon \Delta^{-1} B_{\mu} \partial_{\mu} \right) =$$

$$= - \sum_{n=2}^{\infty} \frac{\varepsilon^n}{n} \int d^n x_1 \ldots d^n x_n \, \text{tr}(B_{i_1}(x_1) \ldots B_{i_n}(x_n)) \times$$

$$\times \partial_{i_1} \tilde{D}(x_1 - x_2) \ldots \partial_{i_n} \tilde{D}(x_n - x_1) \tag{3.36}$$

where

$$\tilde{D}(x) = - \frac{i}{(2\pi)^n} \int \frac{d^4 k}{k^2} \, e^{i k \cdot x} = - \delta(x_0) \left( 4 \pi \left| x^1 \right| \right)^{-1} \tag{3.37}$$

The indices $i_1 \ldots i_n$ in (3.36) acquire the values 1, 2, 3.

It is not difficult to construct the perturbation theory in the Lorentz gauge $\partial_{\mu} B_{\mu} = 0$. It arises as a result of the expansion of the
functional
\[ \Delta_L[B] \exp(iS[B]) = \exp(iS[B] + \mathcal{L}_n \Delta_L[B]) \] (3.38)

into series in \( \varepsilon \). The expression \( \mathcal{L}_n \Delta_L \) may be interpreted as an addition to the action \( S \). The term of \( n \)-th order in the expansion of \( \mathcal{L}_n \Delta_L \) into series in \( \varepsilon \) leads to the diagram vertex with \( n \) outgoing lines. The explicit expression for this term, which follows from (3.35), suggests its interpretation as a circle with \( n \) outgoing lines along which the ghost scalar particle propagates. This statement can be interpreted exactly if we write down the determinant as an integral over the anticommuting variables
\[ \det \left( \Box - \varepsilon B_\mu \partial_\mu \right) = \]
\[ = \int \exp \left( i \int L(B_\mu, \tilde{\eta}, \eta) d^4x \right) \prod_{x, \alpha} d\tilde{\eta}^{\alpha}(x) d\eta^{\alpha}(x) , \] (3.40)

where
\[ L[B_\mu, \tilde{\eta}, \eta] = \frac{1}{2} tr \tilde{\eta} \left( \Box - \varepsilon B_\mu \partial_\mu \right) \eta = \]
\[ = \tilde{\eta}^\alpha \Box \eta - \varepsilon t_a t_c \delta_{\alpha c} \partial_\mu \tilde{\eta}^\alpha \partial_\mu \eta^c \] (3.41)

Formula (3.40) is an infinite-dimensional integral of the (247) type.

So, our system can be looked upon as a system of Bose fields \( \vartheta_\alpha^a(x) \) interacting with each other and with scalar Fermi fields \( \eta^a(x) \), \( \tilde{\eta}^a(x) \).

The elements of the diagram technique in the Yang-Mills theory are lines of two types, corresponding to the transverse vector and ghost scalar particles, and also the vertices describing the interaction of vector particles with the scalar ones and with each other.

We shall represent the vector particles with solid lines and the ghost particles with dashed lines. The elements of the diagrams are the lines and vertices of the form
\[ \begin{align*}
\eta^a & \, \rho \, \vartheta^a \\
\mathcal{G}_{\eta^a} \left( \rho \right) & \quad \mathcal{G}^{\eta^a} \left( \rho \right)
\end{align*} \]
The expressions of the diagram elements (3.42) are

\[ G_{\mu \nu}^{a \ell} (p) = - \delta_{a \ell} \left( p^2 \delta_{\mu \nu} - p_{\mu} p_{\nu} \right) \left( p^2 + i \alpha \right)^{-2}, \]

\[ G_{\alpha}^{a} (p) = - \delta_{a \ell} \left( p^2 + i \alpha \right)^{-1}, \]

\[ \nabla_{\mu, \nu \rho}^{a \ell c} = i \epsilon \left( t_{a \ell c} \left( p_{\mu} \delta_{\nu \rho} - p_{\nu} \delta_{\mu \rho} \right) \right), \quad (3.43) \]

\[ \nabla_{\mu, \nu \rho \delta}^{a \ell c} = \epsilon^2 \left( t_{a \ell c} t_{c \delta \epsilon} \left( \delta_{\mu \rho} \delta_{\nu \delta} - \delta_{\mu \delta} \epsilon_{\nu \rho} \right) \right), \]

\[ \nabla_{\mu}^{a \ell c} = \frac{i \epsilon}{2} \left( t_{a \ell c} \left( p_{\mu} - p_{\ell} \right) \right). \]

To find the contribution of a given diagram it is necessary to integrate over independent 4-momenta the product of expressions which correspond to all its elements, sum over all independent discrete indices and multiply the result by

\[ Z^{-1} \left( \frac{i}{2 \pi} \right)^{\ell - \nu - 1} \left( -2 \right)^S \quad (3.44) \]

where \( \nu \) is the number of diagram vertices, \( \ell \) is the number of its internal lines, \( S \) is the number of closed loops of ghost scalar particles and \( Z \) is the order of the symmetry group of the diagram. Let us remark
that \( l - \nu - \tau = \zeta \) is the number of independent contours of the diagram.

This perturbation theory is not the only possible one. Another form of perturbation theory and diagram technique emerges in the so-called first order formalism. This formalism can be obtained if Lagrangian (3.24) is written as

\[
- \frac{1}{8} \tau \zeta F_{\mu \nu} F_{\mu \nu} + \frac{1}{4} \tau \zeta F_{\mu \nu} \left( \partial_\nu B_\mu - \partial_\mu B_\nu + \varepsilon \left[ B_\mu, B_\nu \right] \right) \tag{3.45}
\]

and the integration over \( B_\mu, F_{\mu \nu} \) as over independent variables is performed. The expression "first order formalism" means that the symbol of the derivative enters Lagrangian (3.45) in an order not higher than the first.

Using the Lorentz gauge we obtain the functional integral of the form

\[
\int \exp \left( iS \left[ B, F \right] \right) \Delta_L \left[ B \right] \prod_x \delta (\partial_\nu B_\mu) d^4 B d^4 F, \tag{3.46}
\]

where the expression

\[
d \left( B(x) \right) d \left( F(x) \right) = \prod \prod d \varphi^a (x) \prod d f^a (x) \tag{3.47}
\]

as well as \( d (B(x)), \) is gauge invariant.

The formalism of the first order is convenient for the passage to the canonical quantization. We shall examine such a transition starting from the integral over \( B_\mu, F_{\mu \nu} \) in the Coulomb gauge

\[
\int \exp \left( iS \left[ B, F \right] \right) \Delta_R \left[ B \right] \prod_x \delta (d_\nu B) d^4 B d^4 F \tag{3.48}
\]

In 3-dimensional notation Lagrangian (3.45) acquires the form

\[
- \frac{1}{8} \tau \zeta F_{\iota \zeta} F_{\iota \zeta} + \frac{1}{4} \tau \zeta F_{\iota \zeta} F_{\iota \zeta} + \frac{1}{4} \tau \zeta F_{\iota \zeta} \left( \partial_\zeta B_\iota - \partial_\iota B_\zeta + \varepsilon \left[ B_\iota, B_\zeta \right] \right) - \frac{1}{2} \tau \zeta F_{\iota \zeta} \partial_\zeta B_\iota - \frac{1}{2} \tau \zeta B_\iota \left( \partial_\zeta F_{\iota \zeta} - \varepsilon \left[ B_\iota, F_{\iota \zeta} \right] \right) \tag{3.49}
\]

We can integrate over \( B_\zeta, F_{\iota \zeta} \). Integration over \( B_\zeta \) is equivalent to the appearance of \( \delta \) -functional

\[
\prod_x \delta (\partial_\nu F_{\iota \zeta} - \varepsilon \left[ B_\iota, F_{\iota \zeta} \right]) \tag{3.50}
\]
The integration over $\int_{\epsilon_k}^\kappa$ reduces to the replacement $\int_{\epsilon_k}^\kappa$ by
\begin{equation}
H_{i\kappa} = \partial_k B_i - \partial_i B_k + \varepsilon \left[ B_i, B_k \right]
\end{equation}
in the integral over the remaining variables $B_i, F_{0i}$.

We shall insert into the integral (3.48) the factor
\begin{equation}
\int \prod_x \delta \left( \Delta C + \partial_i F_{0i} \right) dC(x)
\end{equation}
which in fact does not depend on $F_{0i}$, and then perform the shift $F_{0i} \rightarrow F_{0i} - \partial_i C$. The functional $\prod_x \delta \delta (\Delta C + \partial_i F_{0i})$ is transformed into $\prod_x \delta (\partial_i F_{0i})$ and the functional $\prod_x \delta \left( \partial_i F_{0i} - \varepsilon \left[ B_i, F_{0i} \right] \right)$ into the expression $\prod_x \delta \left( \Delta C - \partial_i F_{0i} - \varepsilon \left[ B_i, \partial_i C \right] + \varepsilon \left[ B_i, F_{0i} \right] \right)$, which is equal to $\prod_x \delta \left( \Delta C - \varepsilon \left[ B_i, \partial_i C \right] + \varepsilon \left[ B_i, F_{0i} \right] \right)$ due to $\partial_i F_{0i} = 0$.

Let $C_0(x)$ be a solution of the equation
\begin{equation}
\Delta C - \varepsilon \left[ B_i, \partial_i C \right] = - \varepsilon \left[ B_i, F_{0i} \right]
\end{equation}
which can be expressed in terms of the Green's function depending on $B$:
\begin{equation}
C_0(x) = - \varepsilon \int D(x, \gamma, B) \left[ B_i(\gamma), F_{0i}(\gamma) \right] d^3 \gamma
\end{equation}
After the shift $C \rightarrow C + C_0$ the functional $\prod_x \delta \left( \Delta C - \varepsilon \left[ B_i, \partial_i C \right] \right)$ originates and the function $C(x)$ can be put equal to zero everywhere except in the argument of the $\delta$-functional. The integral
\begin{equation}
\int \prod_x \delta \left( \Delta C - \varepsilon \left[ B_i, \partial_i C \right] \right)
\end{equation}
cancels with the factor $\Delta_{\mathcal{R}}[B]$. As a result, functional (3.45) takes the form
\begin{equation}
\int \exp \left( i \int \left[ B_i, F_{0i} \right] \right) \prod_x \delta (\partial_i B_i) \delta (\partial_i F_{0i}) \prod_i d F_{0i},
\end{equation}
where
\begin{equation}
\int \left[ B_i, F_{0i} \right] = \int d x_0 \left( \int f_{o_i}^2 \rho_i \rho_0^2 d^3 x - H \right)
\end{equation}
\[ H = \int d^3x \left( \frac{1}{4} g \varepsilon_{\alpha}^\kappa \varepsilon^\alpha_{\kappa} + \frac{i}{2} \varepsilon^\alpha_{\kappa} f^\alpha_{\kappa} + \frac{i}{2} \varepsilon_{\alpha}^\kappa \partial^\alpha \varepsilon^\kappa_{\alpha} - \tilde{J} \varepsilon^\alpha_{\alpha} \right) \]  \tag{3.58}

In those formulae \( \mathcal{S}[B, F] \) is the action corresponding to the Hamiltonian \( H \), where the transverse fields \( \varepsilon^\alpha_{\kappa}, f^\alpha_{\kappa} \) have the meaning of canonically conjugated coordinates and momenta.

As we have shown above, the functional integration over canonically conjugated coordinates and momenta is equivalent to the canonical quantization. When the canonical quantization is applied to the system described by Hamiltonian (3.58), it results in the replacement of functions \( \varepsilon_{\alpha}^\kappa, f^\alpha_{\kappa} \) through which \( h_{\alpha\kappa}, c_{\alpha}^\kappa \) are expressed, by operators \( \hat{\varepsilon}_{\alpha}^\kappa(x), \hat{f}_{\alpha}^\kappa(x) \) satisfying commutation relations

\[ \left[ \hat{\varepsilon}_{\alpha}^\kappa(x), \hat{f}_{\beta}^\delta(y) \right] = \delta_{\alpha\beta} \delta^{\kappa\delta} \left( x - y \right) \]

\[ = \frac{i}{(2\pi)^3} \int d^3x \varepsilon^{\alpha\kappa}(x) \left( \delta_{\kappa\delta} - \frac{\kappa \delta}{|x|^2} \right) \]  \tag{3.59}

Hamiltonian (3.58) becomes a self-conjugated and positive definite energy operator. Such a quantization of the Yang-Mills field has been suggested by Schwinger [9]. It has been shown how the functional integral formalism leads to Schwinger's canonical quantization. Let us emphasise that the existence of the factor \( \Delta_B \left[ \mathcal{B} \right] \) in the original integral (3.48) is important in bringing the integral to an explicitly Hamiltonian form.

LECTURE 4

**Electroweak Interactions, QCD, Extended Objects, Anomalies.**

Yang-Mills field theory discussed above is a fundament for construction more realistic nonabelian gauge theories which can be applied to the real high energy physics. Historically the first such theory was the Weinberg-Salam (W.S.) model of electroweak interactions.

The main idea of W.S. model is the spontaneous breaking of the original invariance with respect to the gauge transformation of massless vector
fields of the Yang-Mills type. The gauge group of the model is group $\mathcal{U}(2)$. This group is isomorphic to the group of $2 \times 2$ unitary matrices and equal to the product of the $\mathcal{U}(1)$ group of phase transformations and the group of $2 \times 2$ unitary matrices with unit determinant.

The connection generated by group $\mathcal{U}(2)$ consists of two types of vector fields - the Yang-Mills multiplet $A_\nu^a$ $(a = 1, 2, 3)$ and the field $B_\nu$. Besides these fields that induce the spontaneous symmetry breaking of the $\mathcal{U}(2)$ gauge invariance. The lepton fields involved in the W.S. model are the electron-type fields

$$L = \frac{i}{2} (1 + \gamma_5) \left( \begin{array}{c} \nu_e \\ \psi_e \end{array} \right), \quad R = \frac{i}{2} (1 - \gamma_5) \psi_e,$$

where $\psi_e$ is the electron field and $\nu_e$ is the electron neutrino field. The scalar fields form the doublet

$$\psi = \left( \begin{array}{c} \varphi_+ \\ \varphi_- \end{array} \right)$$

The Lagrangian of the model is

$$L = -\frac{i}{g} (\partial_\mu A_\nu - g [A_\mu, A_\nu])^2 - \frac{1}{4} (\partial_\mu A_\mu - \partial_\nu A_\nu)^2 - \bar{R} \gamma^\mu (\partial_\mu - ig' B_\mu) R - \bar{L} \gamma^\mu (\partial_\mu + ig \frac{\tau_3}{2} A_\mu + ig' B_\mu) L -$$

$$- \frac{i}{2} (\partial_\mu \varphi - ig \frac{\tau_3}{2} A_\mu \varphi - ig' B_\mu \varphi)^2 -$$

$$- g_e (\bar{L} \varphi R + \bar{R} \varphi L) - m^2 \varphi^T \varphi + h (\varphi^T \varphi)^2$$

where $g, g'$ are the coupling constants of the multiplet $A_\mu$ and singlet $B_\mu$ respectively.

The mechanism of spontaneous symmetry breaking and mass generation, which was first proposed by Higgs, is based on the appearance of the anomalous average value

$$\lambda = \langle \varphi^\nu \rangle$$
of the zeroth component of the $\varphi$ field. Such a mechanism is a well-known in the superfluidity theory. Let us proceed from the original fields to the "physical" ones subtracting from the $\varphi$ fields their anomalous average values. For the physical fields we take the $\varphi$ field and

$$
\varphi_1 = \left( \varphi^0 + \bar{\varphi}^0 - 2\lambda \right)/\sqrt{2} \quad \varphi_2 = \left( \varphi^0 - \bar{\varphi}^0 \right)/\sqrt{2}
$$

(4.5)

In the first order of the perturbation theory the quantity $\lambda$ is determined by the minimum of the expression $-M^2_\lambda (\varphi^0 \varphi^0 + 1/2 (\varphi^0 \varphi^0)^2)$ supposing that $\varphi^0 = \lambda$, $\varphi^0 = 0$. This leads to

$$\lambda^2 = M^2_\lambda / 2h
$$

(4.6)

After these operations the $\varphi_1$ fields acquire the mass $M_\lambda$ and fields $\varphi_2$, $\varphi_3$ remain massless. The appearance of massless excitations in models with spontaneous symmetry breaking was discovered by Goldstone. In this case, however, the excitations have no immediate physical meaning and can be removed by a gauge $U(2)$ transformation.

The mass of the $\varphi_1$ meson appears to be too large (compared to the electron mass me) and that is why the coupling of $\varphi$ to other fields may be neglected.

Eventually it became obvious that the effect of the appearance of anomalous value (4.4) can be reduced in the first order to the substitution of the $\varphi$ field by its vacuum expectation value

$$
\langle \varphi \rangle = \lambda \left( \begin{array}{c} 1 \\ 0 \end{array} \right)
$$

(4.7)

By this substitution Lagrangian (4.3) transforms to

$$
- \frac{1}{4} \left( \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} + i g [A_{\mu}, A_{\nu}] \right)^2 - \frac{1}{4} \left( \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu} \right)^2 - \bar{R} \gamma^\nu \left( \partial^\mu - ig \gamma^\alpha \gamma^\mu \right) \gamma^\nu - \bar{L} \gamma^\nu \left( \partial^\mu + ig \gamma^\alpha \gamma^\mu \right) \gamma^\nu - \frac{1}{8} \lambda^2 \bar{\varphi}^0 (\varphi^0)^2 + \frac{1}{8} \bar{\varphi}^0 \left( \varphi^0 \right)^2 - \frac{1}{8} \lambda \left( \varphi^0 + \bar{\varphi}^0 \right)^2 + \frac{1}{8} \left( \varphi^0 + \bar{\varphi}^0 \right)^2
$$

(4.8)
The electron acquires the mass

$$m_e = \frac{\alpha}{\sqrt{2}} G_e$$  \hspace{1cm} (4.9)

The charged vector field

$$\mathcal{W}_\mu = 2^{-1/2} \left( A^3_\mu + i A^2_\mu \right)$$  \hspace{1cm} (4.10)

describes an intermediate boson with the mass

$$M_\omega = \frac{\lambda}{2} g$$  \hspace{1cm} (4.11)

From the neutral fields $A^3_\mu$, $B_\mu$ the combinations

$$Z_\mu = (g^2 + g'^2)^{1/2} \left( g A^3_\mu + g' B_\mu \right),$$  \hspace{1cm} (4.12)

$$A_\mu = (g^2 + g'^2)^{-1/2} \left( -g' A^3_\mu + g B_\mu \right)$$

with the masses

$$M_Z = \frac{1}{2} \lambda (g^2 + g'^2)^{1/2}, \hspace{1cm} M_A = \omega$$  \hspace{1cm} (4.13)

can be formed. In this way, one component of the vector field multiplet has zero mass and it is therefore considered to be the photon field.

The interaction term of lepton and vector fields can be written as

$$\frac{i g}{2\sqrt{2}} \bar{\psi}_e \left( 1 + \gamma_5 \right) \gamma^\mu \mathcal{W}_\mu + \frac{i g g'}{(g^2 + g'^2)^{1/2}} \bar{\psi}_e \gamma^\mu \gamma^\nu A_\mu \gamma^\nu +$$

$$+ \frac{i (g^2 + g'^2)^{1/2}}{4} \left[ \frac{3(g^2 - g'^2)}{g^2 + g'^2} \bar{\psi}_e \gamma_\mu \gamma_\nu \gamma^\mu \psi_e - \bar{\psi}_e \gamma_\mu \gamma_\nu \gamma^\mu \psi_e + \bar{\gamma}_\mu (1 + \gamma_5) \gamma^\mu \psi_e \right]$$  \hspace{1cm} (4.14)

The second term in (4.14) implies that the electron charge $e$ is

$$e = g g' (g^2 + g'^2)^{-1/2}$$  \hspace{1cm} (4.15)

and is therefore smaller that any of the two original charges $q, q'$. Supposing that $\mathcal{W}_\mu$ is, as usual, coupled to hadrons and to a muon we obtain
\[ G_\omega \sqrt{2} = \frac{g^2}{8 M_\omega} = \frac{1}{\lambda^2} \] (4.16)

It follows from (4.12) and (4.16) that masses of intermediate bosons are very large

\[ M_\omega > 80 \text{ GeV}, \quad M_\omega > 40 \text{ GeV} \] (4.17)

compared not only to the electron mass but also to the hadron masses.

Nevertheless, such intermediate bosons were observed experimentally on the GERN collider, as well as the neutral currents, representing the other prediction of the W.S. model.

Strong interactions also can be included into the scheme of gauge theories. The best candidate for the theory of strong interactions is quantum chromodynamics (QCD). The main idea is that any strong interacting fermion (hadron) consists of 3 other fermions (quarks) which a described by the fundamental representation of \( SU(3)_c \) "colour" group, and also under the "flever" group \( G_f \), so the full gauge group is the product \( SU(3)_c \times SU(3)_c \times G_f \). The QCD Lagrangian

\[ -\frac{i}{8} \text{tr} F_{\mu \nu} F^{\mu \nu} - \bar{\psi} \left( i \gamma_\mu \partial^\mu - \xi B_\mu \right) m \psi \] (4.18)

is invariant under the gauge transformations of the type of (3.26), where \( \psi \) is a quark spinor field, which has spinorial indices \( \alpha \) and also isotopic indices.

Such a Lagrangian describes satisfactorily high energy strong interaction processes. The main problem is how to deal with the low energy interactions, for example, how to evaluate masses of strong interacting particles and low energy scattering amplitudes. The hypothesis of quark confinement explains why quarks can be observed only in hadrons, but not separately. This hypothesis can be justified in 3-dimensional field theories, but not in the real 4-dimensional case.

Another hypothesis in QCD is that of quark and gluon condensates. It states, that there exist the "anomalous averages".

\[ \langle \bar{\psi} \psi \rangle \quad \text{quark condensate} \]
\[ \langle F^a_{\mu \nu} F^a_{\mu \nu} \rangle \quad \text{gluon condensate} \] (4.19)
This supposition allows to move to the quantitative description of strong interactions. Here I am not going to dwell on this subject.

At the conclusion let me touch two more aspects of gauge theories. The first one is the presence of extended objects in gauge theories. These objects are connected with nontrivial classical solutions of the field equations. In the functional integral formalism we have to integrate over fields, which are "close" to some classical solutions. May be the most famous of such solutions is the Polyakov - t'Hooft monopole, which exists in the theory of the Yang-Mills field $\theta^a_\mu$, interacting scalar isotopic field $\varphi_a$. The action of such a system is

$$
-\frac{1}{2} \int \left[ \sum_{\alpha, \xi} \left( \partial_\mu \varphi_\alpha + \xi \epsilon_{\alpha \beta \xi} \theta^\beta_\rho \varphi_\xi \right)^2 - \lambda \sum_\alpha \varphi_\alpha^2 + \frac{g^2}{2} \left( \sum_\alpha \varphi_\alpha^2 \right)^2 \right] d^4x
$$

$$
-\frac{1}{2} \int \sum_{\alpha, \xi} \left( \partial_\mu \theta^a_\nu - \partial_\nu \theta^a_\mu + \epsilon_{\alpha \beta \xi} \theta^\beta_\rho \theta^a_\xi \right)^2 d^4x
$$

(4.20) We can look for a solution of the form

$$
\varphi_\alpha(x) = \chi_\alpha \mathcal{U}(\tau) \tau^{-1}, \quad \theta^a_\mu(x) = \epsilon_{\alpha \beta \xi} \chi_\alpha \left( \mathcal{Q}(\tau) - (\mathcal{E} \tau) \tau^{-1} \right)
$$

(4.21) It can be shown that this solution has a finite energy functional and consequently can be treated as a new particle[5,6]

The other interesting solution is the so called *instanton*, which is the solution of the Yang-Mills field equations in the euclidean space with a finite action functional. Such field configuration gives a contribution when performing functional integration over Yang-Mills fields.

Attempts to find out other more realistic field theoretical models with vortex-line solutions belong to very actual problems. It is not excluded that the key to the construction of a successive strong interaction theory lies just on that path.

The last thing I should like to touch in these lectures is the "anomaly". Anomalous were invented by Adler, Bardeen, Gross, and Jackiw. Recently a new approach to the anomalies was suggested by I.D. Paddeev and S.L. Shatashvili.[7-9]

Let us consider Lagrangian, describing left fermions interacting the Yang-Mills field $A$ ( $A$ are matrices in the Lie algebra).
\[ L = i \bar{\Psi} \left( \gamma_\mu \partial_\mu + \frac{1}{2} \left( 1 - \gamma_5 \right) A_\mu \right) \Psi \]  \hspace{1cm} (4.22)

This Lagrangian is invariant under gauge transformations

\[ A_\mu \rightarrow \Omega^{-1} A_\mu \Omega \quad + \quad \Omega^{-1} \partial_\mu \Omega \quad , \quad \Psi \rightarrow \Omega^{-1} \Psi \]  \hspace{1cm} (4.23)

But, as it is well-known, the functional integral

\[ Z[A] = \int \mathcal{D} \bar{\Psi} \mathcal{D} \Psi \exp \left( i \int L \, d^4 x \right) \]  \hspace{1cm} (4.24)

is not gauge invariant, i.e. \( Z[A^\alpha] \neq Z[A] \).

In order to characterize the behaviour of \( Z[A] \) more precisely, let us introduce the generators of infinitesimal gauge transformations

\[ T^a(x) = - \partial_\mu \frac{\delta}{\delta \alpha^a_\mu(x)} = - \left( \partial_\mu \frac{\delta}{\delta \alpha^a_\mu} + t^a_{\;\beta\gamma} \alpha^\beta_\mu \frac{\delta}{\delta \alpha^\gamma_\mu} \right) \]  \hspace{1cm} (4.25)

These operators obey the commutation relations

\[ \left[ T^a(x), T^b(y) \right] = t^a_{\;\beta\gamma} T^c(y) \delta(x-y) \]  \hspace{1cm} (4.26)

The calculations which were performed by Bardeen, Gross and Jackiw show, that

\[ T^a(x) Z[A] = U^a(x) Z[A] \]  \hspace{1cm} (4.27)

where \( U^a(x) \) are polynomials on \( A^a_\mu(x) \) and its derivatives.

It is not difficult to show the following relation

\[ T^a(x) U^b(y) - T^b(y) U^a(x) = t^a_{\;\beta\gamma} U^\beta(y) \delta(x-y) \]  \hspace{1cm} (4.28)

This is the so called Wess-Zumino consistency condition.

Let us introduce operators

\[ X^a(x) = T^a(x) + i \gamma^a U^a(x) \]  \hspace{1cm} (4.29)

where \( \gamma^a \) are real. We have

\[ \left[ X^a(x), X^b(y) \right] = t^a_{\;\beta\gamma} X^\beta(y) \delta(x-y) \]  \hspace{1cm} (4.30)
We can say that the operators $X^a(x)$ give us some new representation of gauge group. Formula (4.27) implies that $Z[A]$ is invariant under this new representation, if we put $\mathcal{M} = -1$.

These results imply the interpretation of the anomaly as the infinitesimal 1-cocycle. It is not surprising that it can be expressed via geometrical constructions such as the second Chern-Simons class.

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PERTURBATIVE QCD

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1. INTRODUCTION

QCD now is a rather old theory. This year, e.g., marks the 20th anniversary of colour (1-4), the concept that is a cornerstone of QCD. The last year was the 20th anniversary of the quark hypothesis (5,6), and two years ago the QCD world celebrated the 10th anniversary of asymptotic freedom (7-9) and the ultimate formulation of QCD itself (8,10,11). The literature on QCD is vast (see, e.g., the review papers (12-26) and books (27-29)), and it is impossible to discuss here all aspects of the QCD theory. Moreover, the lectures on QCD are usually assumed to contain also the discussion of the experimental status of QCD. At the present school, however, some of QCD-related experimental data will be discussed in lectures by G. Giacomelli and F. Kalmus. So, in these lectures I will try to present an outline of only the theoretical development in the QCD area during the last 10-15 years. I will also try to avoid discussing too technical details of the QCD theory that are probably of a limited interest to experimentalists. The only exception is the theory behind the fundamental QCD scale \( \Lambda \) that is discussed in more detail. There are many experiment groups measuring \( \Lambda \), so I think it is worth spending some time on clarifying what \( \Lambda_{QCD} \) really is.

2. BASIC PRINCIPLES OF QCD

2.1 QCD Lagrangian

Formally, QCD can be defined as a field-theoretical scheme describing coloured quarks \( \Psi_{iA} \) interacting with each other through exchange of the non-Abelian vector gauge fields - the gluons \( A^a_\mu \). Its Lagrangian has the form

\[
\mathcal{L}_{QCD} = -\frac{i}{4} G^a_{\mu\nu} G^{a\mu\nu} + \sum_{A,A' = 1}^{N_c^2-1} \sum_{i = 1}^{N_f} \overline{\Psi}_{iA}(i \gamma^\mu D^\mu + m_i)_{AA'} \Psi_{iA'}^+ + \text{(gauge-fixing terms)} \tag{1}
\]

The spinor fields \( \Psi_{iA} \) correspond to quarks that have in addition to the standard Dirac indices (not written explicitly) the colour \( A \) \( (A = 1, \ldots, N_c = 3) \) and flavour \( i \) \( (i = u, d, s, c, b, t, \ldots) \) ones. The gluonic fields \( A^a_\mu \) \( (a = 1, \ldots, N_c^2-1 = 8) \) form an octet. They enter into the Lagrangian (eq. (1)) through the field-strength

\[
G^{a}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \frac{\epsilon^{abc}}{2} A^b_\mu A^c_\nu \tag{2}
\]

and the covariant derivative

\[
D_\mu = \partial_\mu - i q A^a_\mu \tau^a \tag{3}
\]
where the matrices \( \tau^a \) are related with the \( 3 \times 3 \) Gell-Mann matrices \( \lambda^a \) by \( \tau^a = \lambda^a / 2 \), and \( f_{abc} \) are structure constants of the \( SU(3) \) group.

Thus, the QCD Lagrangian can formally be characterized by the quark-mass parameters \( m_u, m_d, \ldots, m_s, \ldots \) and the coupling constant \( q(\mu) \) for some (in principle, arbitrary) convention concerning the subtraction of the ultraviolet divergences related to a scale \( \mu \).

2.2 The rationale for the QCD Lagrangian choice

The choice of the QCD Lagrangian in the above form was, in a sense, dictated to the theoreticians by experimental data and phenomenological schemes that existed at the beginning of the 70s. These data can be treated now as experimental tests of QCD.

a) Quarks

A successful classification of the hadrons within the framework of the Gell-Mann-Zweig quark model \(^5^, ^6\) provided a strong evidence in favour of the hypothesis that all the observed hadrons should be considered as the bound states of more elementary constituents — quarks. Another evidence for quarks was yielded by the results of deep-inelastic-scattering experiments indicating the existence of pointlike constituents \(^30-31\) inside the hadrons — the partons \(^32\). According to the parton model \(^32\), the data require that the partons should have quantum numbers of the quarks.

b) Colour

However, the interpretation of the baryons as bound states of quarks faced with the problem of quark statistics. Being the particles with spin \( 1/2 \), the quarks should obey the Fermi-Dirac statistics and Pauli exclusion principle that forbids the presence of two or more quarks in the same quantum state. However, within the quark model the baryons like \( \Delta^{++} \) and \( \Omega^- \), say, correspond to the \( u^+u^+u^- \) and \( s^+s^+s^- \) states in which the quarks of the same flavour have the same spin direction and zero angular momentum. The quark model also gave no answer to the question why all the observed hadrons are either \( q\bar{q} \) (mesons) or \( qqq \) (baryons) bound states while there are no experimental indications for existence of other quark systems. The analysis of these problems resulted eventually in the coloured-quarks concept formulated 20 years ago \(^1-4, 33\). According to this concept, the quarks have additional quantum degree of freedom—the colour, corresponding to a new colour symmetry group \( SU(3)_c \), with the hadrons being singlets (i.e., "colourless" states) with respect to this group.

It should be noted that an earlier attempt to solve the statistics problem by assuming that quarks obey the so-called para-Fermi-statistics of rank \( 3 \) \(^34\) is less satisfactory since the corresponding gauge theory...
is based on $SO(3)$ symmetry group $^{35}$, in which, e.g., quarks have the same "colour charge" as antiquarks.

c) Number of colours

The quark–statistics problem mentioned above is solved if the number of colours $N_c$ is not less than three. In fact, due to the existence of the triangle anomaly of the axial current $^{36}$ one can exactly calculate within the quark model the width of the $\pi^0 \rightarrow 2\gamma$ decay which, according to that calculation, is proportional to $N_c^3$. This enables one to determine $N_c$ from the experimental value of $\Gamma_{\pi^0 \rightarrow 2\gamma}$ $^{37}$

$$N_c = 2.98 \pm 0.11$$ (4)

Another estimate of $N_c$ can be extracted from the analysis of the $e^+e^-\rightarrow$-annihilation into hadrons (its cross section is proportional to $N_c$). Available data are in agreement with $N_c = 3$ within 10 – 15% accuracy $^{37}$.

d) Gluons

The attempts to find a dynamical explanation of the rule that only colour-neutral objects may exist in a free state resulted in the hypothesis that quarks are bound inside the hadrons by tremendous forces responsible for the confinement of coloured particles "imprisoned" inside the hadronic "bags". The first bag model of hadrons was developed at Dubna 20 years ago $^{2}$. It was assumed in this model that quarks are very heavy objects and the necessary defect of quark masses as well as the quark confinement were explained by a large depth of the potential well. However, it was established that the quarks inside the well behave as quasi-independent $^{38}$, or effectively free. The concept of quasi-independent quarks developed by P.N.Bogoliubov $^{38}$ is a prototype of the modern concept of the asymptotic freedom. An important step in a further development was the introduction of the gluons $^{39,10}$, vector particles responsible for the quark interactions. It is worth emphasizing here that the one-gluon exchange potential is attractive for the colourless $\bar{q}q$ and $qqq$ systems only if the gluons are vector particles corresponding to the octet representation of the colour group. Note that the gluons themselves are then coloured particles and, hence, they should interact with each other as well. Self-interaction of that kind is a specific feature of the non-Abelian gauge field theories proposed by Yang and Mills in the 50's and practically forgotten in the 60's. However, at the beginning of the 70's they were a subject of a considerable interest due to the progress in constructing the unified electroweak theory based on the group $SU(2) \times U(1)$. An essential contribution to this progress came from the development of a general method of quantizing the non-Abelian gauge fields by Faddeev and Popov $^{40}$ (see also $^{41,42}$) and the proof $^{43}$ that the non-Abelian gauge fields are renormalizable. The ultimate formulation of QCD appeared after the asymptotic freedom was discovered $^{7,8}$ and
incorporated to explain the approximate scaling in deep inelastic scattering. A subsequent analysis \(44\) resulted in the observation that the asymptotic freedom, i.e., the situation when the scale-dependent effective coupling constant falls down to zero the momentum transferred grows (i.e., at short distances), may take place only in the non-Abelian gauge theories. The asymptotic freedom provides a natural explanation of successes of the naive parton model in which it is assumed that the partons do not interact at all.

1.3 Feynman rules for QCD

The smallness of the effective coupling constant \(\tilde{g}(Q)\) for the processes with high momentum transfer \(Q\) gives a possibility to calculate various characteristics of these processes using the well-developed machinery of perturbation theory \(45\). The corresponding branch of QCD is referred to as "perturbative QCD". The basic element of perturbative QCD calculations is the Feynman rules for the QCD Lagrangian \((1)\). In particular, the expressions for vertices, e.g., the quark-gluon, three-gluon and four-gluon ones, immediately follow from eq. \((1)\). The propagators can be extracted from the free-field Lagrangians using the standard rules. One should write first the free-field equations of motion, e.g., for quarks

\[
(i\gamma^\mu - m)_{\mu \nu} \psi_\nu = 0
\]

and find the relevant Green function determined by the inverse of the operator \((i\gamma^\mu - m)\). The latter has the following form (in the momentum representation):

\[
\mathcal{S}_{\mu \nu}^\mu = \frac{i}{\mathcal{S} - m} \mathcal{E}_{\mu \nu} = \frac{i}{p^2 - m^2} \mathcal{E}_{\mu \nu}
\]

For gluons the procedure is more complicated since one has in this case

\[
(q_{\mu \nu} \gamma^2 \partial_\nu A_\mu) = 0
\]

in place of eq. \((5)\), and the operator \((q_{\mu \nu} \gamma^2 \partial_\nu A_\mu)\) being a projector onto the transverse vector states has no inverse. To avoid this difficulty, one should add a "gauge-fixing term" \(\Delta L\) to the original Lagrangian. To get the so-called covariant gauges \(\partial_\mu A_\mu = 0\), one must take

\[
\Delta L = -\frac{1}{2\alpha} \sum_\alpha (\partial_\mu A^\mu_\alpha)^2
\]

Then the free-field equation of motion takes the form

\[
(q_{\mu \nu} \gamma^2 - (1 - \frac{1}{\alpha}) \partial_\mu \partial_\nu) A^\mu_\nu = 0
\]

and one may easily find the inverse operator

\[
\mathcal{D}^\alpha_{\mu \nu} = \frac{1}{p^2 + i\epsilon} \left[ q_{\mu \nu} + (1 - \alpha \frac{p_\mu p_\nu}{p^2 + i\epsilon}) \delta^\alpha_{\mu \nu} \right]
\]
The most popular are the Fermi–Feynman (\( \alpha = 1 \)) and Landau (\( \alpha = 0 \)) gauges.

For the Abelian fields adding the gauge-fixing terms is sufficient for a consistent quantization of the theory. For the non-Abelian gauge fields, however, using some gauges (including the covariant ones) one should add also the contributions of nonphysical auxiliary fields – Paddeev–Popov "ghosts" 40)

\[
\Delta Q^{ghost} = (a_q \varphi^a) (\varphi^b + \sum_b c^b \xi^b \xi^c)
\]

(11)

the role of which reduces to a complete subtraction of the contributions due to unphysical degrees of freedom of the gluonic field (further details may be found in the lectures by V.N.Popov). In principle, there exist also "ghost-free" gauges, e.g., axial gauges \( \eta^{\mu} A^{\mu} = 0 \) where \( \eta \) is some fixed vector (more precisely, axial gauges are those with \( \eta^2 < 0 \), for \( \eta^2 = 0 \) one has the lightlike gauge and for \( \eta^2 > 0 \) – the temporal or Coulomb-like gauge). The gauge-fixing term in this case is

\[
\Delta Q = - \frac{1}{2\alpha} \sum_q \left( \eta^{\mu} A^{\mu} \right)^2
\]

(12)

and in the limit \( \alpha \to 0 \) one gets the propagator

\[
D_{\mu,\nu}^{\alpha,\xi}(\rho) = \frac{i}{\rho^2 + \xi^2} \left[ - q_{\rho} + \eta_{\xi} \rho_{\rho} + \eta_{\rho} \rho_{\xi} - \rho^2 \frac{P_{\rho} P_{\xi}}{\eta^2} \right]
\]

(13)

3. ASYMPTOTIC FREEDOM AND THEORETICAL STATUS OF \( \Lambda_{QCD} \)

3.1 Effective coupling constant in QCD.

The strength of the quark-gluon (and also the gluon-gluon) interactions is characterized by the magnitude of the coupling constant \( \tilde{q} \), defined by some normalization condition characterized by a scale \( \mu \) : \( \tilde{q} = \tilde{q}(\mu) \). For instance, one may define \( \tilde{q}(\mu) \) as a \( q_\nu \)-term of the quark-gluon vertex function \( \Gamma_\nu(q_\alpha, p_\beta, p_\gamma) \) at the point \( p_\nu^2 = p_\alpha^2 = p_\gamma^2 = -\mu^2 \) (see, e.g., 45); this convention is an example of the so-called MOM (re)normalization schemes 46). Of course, physical ("observable") quantities should not depend on a particular convention used to specify \( \tilde{q} \).

The set of transformations \{ \( \mu \to \mu' \); \( \tilde{q}(\mu) \to \tilde{q}(\mu') \) \} leaving unchanged the physical quantities constitutes the renormalization group 45). The RG method enables one to recalculate the results from one convention to another and, hence, to compare the results obtained within different conventions. The basic element of all applications of the RG is the dependence of the coupling constant \( \tilde{q}(\mu) \) on the relevant scale \( \mu \). In QCD (and also in any other renormalizable theory) this dependence is governed by the equation
\[ \frac{dG}{dL} = \beta(G) = -\sum_{k=0}^{\infty} b_k G^k \]  

(14)

where \( G = \frac{1}{2}(\phi_m)^2 \), \( L = \ln \mu^2 \), \( d/dL = \mu^2 d/d\mu^2 \).

Integrating eq. (14) one obtains

\[ \ln \left( \frac{\mu^2}{\mu^{'2}} \right) = \int \frac{G(\mu)}{\beta(x)} \, dx \equiv \Phi(G(\mu)) - \Phi(G(\mu')) \]

(15)

where \( \Phi \) is an indefinite integral.

Note that the combination

\[ \ln \left( \frac{\mu^2}{\mu^{'2}} \right) + \Phi(G(\mu')) = \Phi(G(\mu)) \]

(16)

does not depend on \( \mu' \). This is possible only if

\[ \Phi(G(\mu')) = \ln \left( \frac{\mu^2}{\Lambda^2} \right) \]

(17a)

where \( \Lambda \) is some parameter with the dimension of mass. As a result,

\[ G(\mu) = \Phi^{-1} \left[ \ln \left( \frac{\mu^2}{\Lambda^2} \right) \right] \]

(17b)

where \( \Phi^{-1} \) is the inverse of the function \( \Phi \).

One should remember that the function \( \Phi \) is defined by eq. (15) only up to an arbitrary additive constant. However, changing

\[ \Phi \rightarrow \Phi + 2\varphi \]

(18)

one should also redefine \( \Lambda \), according to eq. (17):

\[ \Lambda \rightarrow \Lambda + \varphi \]

(19)

The value of \( G(\mu) \) remains unchanged under this transformation. One deals, in fact, with the one-parameter set of possible expressions of \( G(\mu) \) through \( \ln \mu^2 \)

\[ G(\mu) = \Phi^{-1} \left[ \ln \left( \frac{\mu^2}{\Lambda^2} \right) \right] \]

(20)

Fixing some \( \varphi \) one fixes a particular form of the functional dependence of \( G(\mu) \) on \( \ln \left( \frac{\mu^2}{\Lambda^2} \right) \). To illustrate this point, let us consider the simplest example. Assume that for some \( \varphi = \varphi_0 \) the function \( G(\mu) \) has the simple form

\[ G(\mu) = \frac{1}{b_0} \ln \left( \frac{\mu^2}{\Lambda^2} \right) \]

(21)

Then, if one chooses another \( \varphi = \varphi_0 + \delta \varphi \), \( G(\mu) \) will be given by the infinite series
\[ G(\mu) = \frac{1}{\mu_0^2} \ln \left( \frac{\mu^2}{\Lambda^2} \right) \left[ 1 + \sum_{n=1}^{\infty} \left( \frac{\delta_n}{\ln \left( \frac{\mu^2}{\Lambda^2} \right)} \right)^n \right] \] 

(22)

that evidently corresponds to another form of the functional dependence of \( G(\mu) \) on the relevant \( \ln (\mu^2/\Lambda^2) \), though the summation of the series (22), with account of eq. (19), gives the same result for \( G(\mu) \) as given by eq. (21).

The \( \beta \) -function (14) was calculated up to \( G^4 \)-terms \((7, 8, 47, 48)\)

\[ \beta_0 = 11 - \frac{2}{3} N_f \]

(23)

\[ \beta_1 = 402 - \frac{32}{3} N_f \]

(23a)

\[ \beta_2 \bigg|_{\text{MS}} = \frac{2854}{2} - \frac{5033}{18} N_f + \frac{325}{54} N_f^2 \]

(24)

where \( N_f \) is the number of quark flavours. Note that the third coefficient \( \beta_2 \) calculated at Dubna \((48)\) depends on the choice of the renormalization convention, and the result given by eq. (24) corresponds to the minimal subtraction scheme (or \( \text{MS} \) - type scheme) based on the dimensional regularization \((46)\). The value of \( \beta_2 \) in other renormalization schemes is still unknown.

In the lowest approximation one gets from eqs. (21) and (23)

\[ \alpha_s(Q^2) = \frac{g^2(Q^2)}{4\pi} = 4\pi G(Q) = \frac{4\pi}{(1 - \frac{2}{3} N_f) \ln \frac{Q^2}{\Lambda^2}} \]

(25)

Eq. (25) states that the effective coupling constant \( \frac{g}{\Lambda} \) vanishes as \( G^2 \to 0 \) (provided that \( N_f \leq 16 \)). This property of QCD-asymptotic freedom \((7, 8)\) is a cornerstone of all approaches relying on the quantum field theoretic perturbation expansion in QCD.

3.2 Form-dependence ambiguities

Eq. (25) preserves its form under the change \( \Lambda \to \Lambda' \) up to higher-order terms. Hence, to fix arbitrariness in the definition of \( \Lambda \) (and, as a consequence, also in the functional dependence of \( \alpha_s \) on \( \ln (\mu^2/\Lambda^2) \) ), one should also include the next terms of the expansion (14). In this case eq. (14) for small \( G \) acquires the form

\[ \ln \frac{\mu^2}{\Lambda^2} = \frac{4}{\beta_0 \lambda} + \frac{\beta_1}{\beta_0^2} \ln (\beta_0 G) + 2 \phi + \frac{\beta_0 \beta_2 - \beta_1^2}{\beta_0^3} G + O(G^2) \]

(26)

where \( \phi \) is just the parameter reflecting the arbitrariness in question (cf. eq. (16)). From this relation one can immediately derive the expression relating \( \Lambda \) to \( \alpha_s = 4\pi G \):
\[ \Lambda^2 = \mu^2 \exp \left\{ - \frac{4}{b_0} - \frac{b_2}{b_0^2} \ln(b_0 \Gamma) - \varphi + \frac{b_2 b_0 - b_2^2}{b_0^2} \Gamma + O(\xi^2) \right\} \]

(27)

To obtain the dependence of \( \alpha_s \) on \( \mu \) (and \( \Lambda \)), one should solve eq. (27); e.g., by iterations:

\[ G_T^{(0)} = \frac{4}{b_0} \Gamma \]

(28)

\[ G_T^{(1)} = \frac{4}{b_0} \left( \Gamma + L_\lambda \right) \]

(29)

\[ G_T^{(2)} = \frac{4}{b_0} \left( L + L_\lambda + \frac{b_2}{b_0^2} \ln \left( \frac{L + L_\lambda}{L} \right) - \frac{b_2 b_0 - b_2^2}{b_0^2 L^2} \frac{L + L_\lambda}{L} \right)^{-1} \]

(30)

and so on, where

\[ L_\lambda = \frac{b_2}{b_0^2} \ln L - 2 \varphi. \]

(31)

Very popular (especially, in applications) is the expansion of \( \alpha_s \) over \( \lambda/L \):

\[ \alpha_s(\mu^2, \varphi) = \frac{4\pi}{b_0 L} \left\{ 1 - \frac{L_\lambda}{L} + \frac{4}{L^2} \left( L_\lambda - \frac{b_2}{b_0^2} \ln L + \frac{b_2 b_0 - b_2^2}{b_0^2 L^2} \frac{L + L_\lambda}{L} \right) + O \left( \frac{1}{L^3} \right) \right\} \]

(32)

The usual choice \( \varphi = 0 \) is motivated by the absence of terms of \( \text{const}/L^2 \)-type in the expansion:

\[ \alpha_s(\mu^2, \varphi = 0) = \frac{4\pi}{b_0} \left( \frac{L_\lambda}{L} - \frac{b_2}{b_0^2} \ln \frac{L}{L_\lambda} + O \left( \frac{1}{L^3} \right) \right) \]

(33)

However, a more reasonable attitude is to make the convention-dependent corrections smaller rather than "shorter". The choice \( \varphi = 0 \) is not the best in this aspect: the \( O(1/L) \)-correction in eq. (33) is 30% compared to the \( O(4/L) \)-term for \( L = 3 \) (i.e., \( \alpha_s = 0.5 \)) and 15% for \( L = 10 \) (\( \alpha_s = 0.1 \)). As a result, the value of \( \Lambda \) extracted from the lowest-order formula

\[ \Lambda^{(0)} = Q \exp \left\{ - \frac{2\pi}{b_0 \alpha_s(Q^2)} \right\} \]

(34)

strongly differs from that obtained from the two-loop one.
\[ \Lambda^{(n)} = Q \exp \left\{ -\frac{2\pi}{\tilde{b}_0 \alpha_s(Q)} + \frac{\tilde{b}_s}{2\tilde{b}_0^2} \ln \left( \frac{\alpha_s^2}{\alpha_s} \right) \right\} = \Lambda^{(0)} \left( \frac{\alpha_s}{\alpha_s^0} \right)^{\tilde{b}_s/2\tilde{b}_0^2} \approx Q \]  

(35)

that results form eq. (27) if one defines \( \alpha_o \) by

\[ \varphi = \frac{\tilde{b}_s}{2\tilde{b}_0^2} \ln \left( \frac{\alpha_o^2}{\alpha_s} \right) \]

Large difference between \( \Lambda^{(n)} \) and \( \Lambda^{(0)} \) (the ratio \( \Lambda^{(n)}/\Lambda^{(0)} \)) is 1.83 for \( \alpha_s = 0.3 \); 2.15 for \( \alpha_s = 0.2 \), and 2.83 for \( \alpha_s = 0.1 \). for most "practical" values of \( \alpha_s \) is due to the fact, that the choice \( \varphi = 0 \) corresponds to \( \alpha_o = 1.4 \).

Thus, for practical purposes it is much wiser to use the arbitrariness in the choice of the functional form of \( \alpha_s(Q^2) \) to make the ratio \( \left( \alpha_s/\alpha_o \right) \) most close to unity for the above-mentioned values of \( \alpha_s \).

If one takes, e.g., \( \alpha_o = 0.2 \) (this corresponds to \( \varphi = \varphi_{opt} = \approx \frac{\tilde{b}_s}{2\tilde{b}_0^2} \ln \frac{\alpha_s}{\alpha_o} \));

\[ \alpha_s(Q^2, \varphi; \varphi_{opt}) = \frac{4\pi}{\tilde{b}_0} \ln \left[ 1 - \frac{\tilde{b}_s}{\tilde{b}_0^2} \ln \left( \frac{L}{\varphi} \right) + O\left( \frac{L}{\varphi^2} \right) \right] \]  

(36)

and the two-loop correction in eq. (36) is small because \( L_s = 7 \) is just the the average value of \( L \) in the region where \( \alpha_s = 0.1 - 0.3 \). The ratio \( \Lambda^{(n)}/\Lambda^{(0)} \) in this case is 0.85, 1.00, and 1.31 for \( \alpha_s \) equal to 0.3; 0.2 and 0.1, respectively. The stability of the \( \Lambda \)-value with respect to the approximation used is by an order of magnitude better than that for the standard choice. The relation between \( \Lambda_{\text{stand}} = \Lambda \left( \alpha_o = 1, \varphi \right) \) and \( \Lambda_{opt} = \Lambda \left( \alpha_o = 0.2 \right) \) can be obtained from eq. (19):

\[ \Lambda_{opt} = 0.46 \Lambda_{\text{stand}}. \]  

(37)

for \( N_f = 3 \).

3.3 Renormalization-scheme dependence of \( \Lambda \)

The choice of the renormalization prescription defining \( q(\mu) \) is far from being unique. Even if one decides to use the momentum subtraction (MOM) convention, i.e., to define \( q(\mu) \) by the magnitude of a vertex function \( \Gamma (p_1^2, p_2^2, q^2) \) at the symmetric point \( p_1^2 = p_2^2 = q^2 \), one is left with at least three possibilities for the vertex: quark-gluon, ghost-gluon or three-gluon ones. The MOM definitions of \( q(\mu) \) suggest that if one deals with some process involving momenta of an order of some scale \( Q \), the relevant expansion parameter is \( \alpha_s(Q) \). In principle, it is possible to define \( q(\mu) \) also by the value of, say, \( \Gamma (-q^2, -q^2, -q^2) \). It is evident that this \( q(\mu) \) (call it \( q_2(\mu) \)) is related to the original \( q(\mu) \) one by \( q_2(\mu) = q(2\mu) \). This means that \( \Lambda \)'s corresponding to these two definitions differ : \( \Lambda_2 = \frac{1}{2} \Lambda^{(1)} \).
(provided, of course, that both $Q_{A(\mu)}$ and $Q_{(\mu)}$ are given by the same function of $\Lambda/\Lambda_{\mu}$). The MOM-schemes, however, are not very convenient for higher-order calculations, and much more popular are MS-type schemes \(^{49}\) based on the dimensional regularization for the loop integrals

$$\frac{d^q k}{(2\pi)^q} \rightarrow \frac{d^{q-2\epsilon} k}{(2\pi)^{q-2\epsilon}} \frac{\Lambda^{2\epsilon}}{(\mu^2)^{2\epsilon}}$$  \( (38) \)

combined with "minimal" subtraction of the $\mathcal{UV} \mathcal{N}^c -$ poles. The coefficient $\mathcal{Q}$ in eq. (38) specifies a particular choice of the MS-type scheme: $\mathcal{Q} = 1$ corresponds to the original MS-scheme \(^{49}\), while $\mathcal{Q} = \exp(\chi/\Lambda P)$, ($\chi = 0.577$ being the Euler constant) to the more popular MS-convention \(^{50}\). The motivation behind the MS-choice is that L-loop integrals for $\mathcal{Q} = 1$ normally contain the $\epsilon$-dependent factor

$$\Gamma(1 + \Lambda \epsilon) \left( \frac{\Lambda}{\mu} \right)^{\Lambda \epsilon} = \left( e^{\chi/\Lambda P} \right) \left( \frac{\Lambda}{\mu} \right)^{\Lambda \epsilon}$$  \( (39) \)

resulting in the meaningless $\mathcal{B}(\epsilon)$ and $\chi$ contributions. The factor $\exp(\chi/\Lambda P)$ just cancels them for all orders.

The physical meaning of $Q(\mu)$ defined in the MS-type scheme is not as clear, of course, as that in a MOM-type scheme. It is not clear a priori, e.g., what is the relevant expansion parameter for, say, $\Gamma(-Q^2, -Q^2, -Q^2)$ in these schemes. However, the relevant $\Lambda$-parameters of MS-schemes may be related to those of MOM schemes just like $\Lambda_{(1)}$ and $\Lambda_{(2)}$ in the above simplified example.

To get such a relation, one should calculate the same renormgroup-invariant quantity in two schemes. Consider as an example the expansion for $R(Q^2) = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$:

$$R(Q^2) = \sum Q \left\{ \lambda + \frac{\chi}{\Lambda} + d_2 \left( \frac{\alpha_s}{\mu^2} \right) \left( \frac{\alpha_s}{\Lambda^2} \right) + \ldots \right\}$$  \( (40) \)

for which the scheme-dependent three-loop coefficient $d_2$ was calculated first in the MS-scheme \(^{51}\)

$$d_2 \left( \frac{\Lambda^2}{Q^2} \right) \bigg|_{MS} = \frac{1}{12} + \left[ \frac{11}{4} - 4\chi(3) + \left( \beta_0 + \beta_1 + \beta_2 \right) \frac{\Lambda}{\mu} \right] \frac{\Lambda}{\mu}$$  \( (41) \)

and then in the MOM-scheme \(^{52}\)

$$d_2 \left( \frac{\Lambda^2}{Q^2} \right) \bigg|_{MOM} = \frac{35}{36 \sqrt{3}} \sum_{n=1}^{\infty} \frac{\Lambda}{n^2} \left( \frac{\pi^2}{3} - \frac{43}{48} + \left( \frac{13}{24} - \chi(3) \right) \beta_0 + \frac{\beta_2}{\Lambda} \right) \left( \frac{\Lambda^2}{Q^2} \right)$$  \( (42) \)

resulting in

$$d_2 \left( \frac{\Lambda^2}{Q^2} \right) \bigg|_{MOM} = \frac{35}{36 \sqrt{3}} \sum_{n=1}^{\infty} \frac{\Lambda}{n^2} \left( \frac{\pi^2}{3} - \frac{43}{48} + \left( \frac{13}{24} - \chi(3) \right) \beta_0 + \frac{\beta_2}{\Lambda} \right) \left( \frac{\Lambda^2}{Q^2} \right)$$
Now, using the explicit $\Lambda$-dependence of $\alpha_s$:
\[
\frac{1}{\ln \frac{\mu^2}{\Lambda^2}} = \frac{\Lambda}{\ln \frac{\mu^2}{\Lambda^2} + \ln \frac{\Lambda^2}{\Lambda^2}} = \frac{\Lambda}{\ln \frac{\mu^2}{\Lambda^2}} \left\{ 1 - \frac{b_0}{4} \ln \left( \frac{\Lambda^2}{\mu^2} \right) \right\}.
\]
(43)
one obtains from eqs. (41), (42) that
\[
\Lambda_{\text{MOM}} \approx 2.10 \Lambda_{\overline{\text{MS}}}
\]
(44)
The relation between $\Lambda_{\text{MS}}$ and $\Lambda_{\overline{\text{MS}}}$ follows immediately from eq. (38)
\[
\frac{\Lambda_{\overline{\text{MS}}}^2}{\Lambda_{\text{MS}}} = 4\pi \exp(-\gamma_E) \Lambda_{\overline{\text{MS}}}^2 \approx (2.7 \Lambda_{\text{MS}})^2
\]
(45)
It is also instructive to write down $d_2$ in the $\overline{\text{MS}}$-scheme
\[
d_2 \bigg|_{\overline{\text{MS}}} = 1.986 - 0.114 N_f^3 + \frac{b_0}{4} \ln \left( \frac{\Lambda^2}{Q^2} \right)
\]
(46)
Note that for $\mu^2 = Q^2$ the $d_2$ coefficient is very large in the MS-scheme, but rather small in MS and MOM schemes. In other words, the $\alpha_s(Q^2)$ expansion in the MS and MOM schemes is better convergent than that in the MS-scheme. One may also hope that studying the relevant momenta flowing through the vertices for other processes one would be able to make educated guesses about the argument of the MOM-scheme-defined $\alpha_s$ yielding rapidly convergent expansions. That this procedure works was checked in many cases. Thus, $\Lambda_{\text{MOM}}$ apparently has all chances to be the most natural (and physically most sensible) definition of $\Lambda_{\text{QCD}}$. However, the most popular definition of $\Lambda$ is $\Lambda_{\overline{\text{MS}}}$. There would be no surprise if $\Lambda_{\text{MOM}}/\Lambda_{\overline{\text{MS}}}$ would differ from 1 by 20-30%, say. The popularity of $\Lambda_{\overline{\text{MS}}}$ would then be explained by the calculational convenience of the MS-type schemes. But $\Lambda_{\text{MOM}}/\Lambda_{\overline{\text{MS}}} = 2$ and there should be another explanation.

An important observation is that the total correction to the leading $O(\Lambda_{\text{QCD}})$ result is determined in eq. (40) not only by the magnitude of the $d_2$ coefficient but also by the magnitude of the two-loop correction to $\alpha_s$ itself. As discussed in the previous section, this correction is rather small for the optimized choice of the $\varphi$-parameter in eq. (26). However, it is rather large for the standard choice of $\varphi$
\[
\alpha_s^{(1)}(Q^2) \bigg|_{\varphi = \varphi_{\text{stand}}} = \alpha_s^{(0)} \left\{ 1 - \frac{b_1}{4\delta_0} \ln \left( \frac{Q^2}{\Lambda^2} \right) \frac{d_s}{\pi} + \ldots \right\} \approx
\]
\[
\approx \alpha_s^{(0)} \left\{ 1 - (3-4) \frac{d_s}{\pi} + \ldots \right\}
\]
(47)
Thus, to get the total $O(1/\Lambda^2)$-correction to $R(Q^2)$ for the standard choice of $\varphi$, one should subtract from $d_2$ something like 3 to 4.
As a result \( (d_2^{\text{MOM}})_{e_f} \approx -(5-\varepsilon) \) whereas \( (d_2^{\text{MS}})_{e_f} \approx -(1-2) \). In other words, the total correction for the standard choice of \( \varphi \) in the \( \text{MS} \)-scheme is close to that for the optimized choice of \( \varphi \) in the \( \text{MOM} \)-scheme. This can be easily understood also by combining eqs. (37) and (44):

\[
\Lambda_{\text{MOM}}(\varphi=\varphi_{\text{opt}}) = 0.97 \Lambda_{\text{MS}}(\varphi=\varphi_{\text{stand}})
\]

Eq. (48) means that the popularity of \( \Lambda_{\text{MS}}(\varphi=\varphi_{\text{stand}}) \) is due to that the "bad" choice of \( \varphi \) (\( \varphi=\varphi_{\text{stand}} \)) is precisely compensated by the equally "bad" choice of the renormalization scheme.

3.4 Higher orders and \( \Lambda \)

A natural question now is how the above discussion is modified by inclusion of higher-order effects. Fortunately, there are no additional higher-order ambiguities in \( \Lambda \); e.g., the relation (44) between \( \Lambda_{\text{MOM}} \) and \( \Lambda_{\text{MS}} \) is exact. This may be understood in the following way. Using the \( \Lambda_{\text{MS}} \)-expansion for \( d_3(Q^2) \) one can write for \( R(Q^2) \) the expansion

\[
R(Q^2) = \sum_{n=0}^{\infty} \sum_{k=\max(0,n-\eta)} \mathcal{A}_{nk}^{(S)} \frac{\ln^k(L/L_0)}{L^n}
\]

where "S" stands for the renormalization scheme. The only possible change in this expansion (for fixed \( L_0 \)) is the change of \( \Lambda \) compensated by an appropriate change of \( \mathcal{A}_{nk}^{(S)} \) s to render \( R(Q^2) \) unchanged. As a consequence, the first three coefficients (\( \mathcal{A}_{00}, \mathcal{A}_{10} \) and \( \mathcal{A}_{11} \)) in eq. (49) are scheme-independent. Furthermore, fixing the coefficient

\[ a_{20} = \left| \frac{d_2(\eta)}{\beta_0} \right| \]

one unambiguously fixes all other coefficients \( \mathcal{A}_{nk} \).

3.5 Threshold effects in \( d_3(Q^2) \)

The \( \beta \) -function coefficients (23)-(25) are explicitly dependent on the number \( N_f \) of quark flavours. As is well-known, all the "old" hadrons can be built up from quarks of 3 flavours u, d, s. To describe the new heavy particles discovered in the 70's, one should also use two heavy quarks - c, b. Just recently the quarks of the 5th-top flavour have been discovered. In principle, there can exist even heavier quarks. Thus, there arises a natural question: what \( N_f \) should be taken in eqs. (23)-(25)? The standard practice is to use \( N_f = k \) in the region \( Q^2 \gtrsim (2m_k)^2 \), i.e., above the "would-be" threshold for production of a quark-antiquark pair of \( k \)-th flavour. I wrote "would-be" because the parameter \( Q^2 > Q^2 \) refers to the spacelike region while the thresholds may appear "on the other side" - in the timelike region. Hence, the points \( Q_k^2 > 4m_k^2 \) have a meaning of the "mirror" rather than real thresholds.

Just from this observation it follows that one should not expect any sharp threshold phenomena in the behaviour of \( d_3(Q^2) \).

It should also be noted that in the class of the so-called "mass-independent" renormalization schemes (including all modifications
of the MS-scheme) the $\beta$ -function (and, consequently, $\alpha_s(Q^2)$) does not depend on masses. These schemes are most suited to describe phenomena in the region where $Q^2$ is much larger than all mirror thresholds and masses appear only as power corrections of the $(\mu^2/Q^2)^N$ type. Much more suitable for the analysis of the threshold effects is the momentum subtraction scheme. The $\beta$ -function for this scheme in the lowest order has the following dependence on the quark masses \(^{55}\)

$$\mu^2 \frac{d G}{d \mu^2} = - G^2 \left\{ 11 - \frac{2}{3} \sum_{i=4}^{N_f} \left[ 1 - 6 \frac{m_i^2}{\mu^2} + 12 \frac{m_i^2/\mu^4}{\sqrt{1 + 4m_i^2/\mu^2}} \right] \ln \left( \frac{1 + \sqrt{1 + 4m_i^2/\mu^2}}{1 + \sqrt{1 + 4m_i^2/\mu^2}} \right) \right\}^{-1}$$ \hspace{1cm} (50)

Very useful for explicit calculations is the approximate formula \(^{55}\)

$$\beta(G, m, \mu) \simeq - G^2 \left\{ 11 - \frac{2}{3} \sum_{i=4}^{N_f} \frac{1}{1 + 5m_i^2/\mu^2} \right\}$$ \hspace{1cm} (51)

that has a true asymptotic behaviour both for $m_i^2/\mu^2 \to \infty$ and $m_i^2/\mu^2 \to 0$ and with a good accuracy (within few per cent) approximating eq. (50).

Assuming that $m_i$ are constant one can integrate eq. (50) using the approximation (51). As a result, one obtains

$$\frac{4}{G(\mu)} - \frac{4}{G(\mu_0)} = 11 \ln \left( \frac{\mu^2}{\mu_0^2} \right) - \frac{2}{3} \sum_{i=4}^{N_f} \ln \left( \frac{\mu^2 + 5m_i^2}{\mu_0^2 + 5m_i^2} \right)$$ \hspace{1cm} (52)

It is clear from this formula that in the region of interest ($\mu^2 \gtrsim 1$ GeV\(^2\)) one may neglect the light quark masses ( $m_u, d \leq 10$ MeV, $m_s = 150$ MeV \(^{56}\)). Taking for definiteness $\mu_0 = 1$ GeV\(^2\), we rewrite eq. (52)

$$G(\mu) = \frac{\alpha_s(\mu_0^2)}{4\pi} = \left\{ 9 \ln \left( \frac{\mu^2}{\Lambda_3^2} \right) - \frac{2}{3} \sum_{i=4}^{N_f} \ln \left( \frac{5m_i^2 + \mu^2}{5m_i^2 + \Lambda_3^2} \right) \right\}^{-1}$$ \hspace{1cm} (53)

The constant $\Lambda_3$ accumulated, as usual, information about $\alpha_s(\mu_0)$; the sum in eq. (52) vanishes for $\mu = \mu_0$.

Now let us assume for the sake of simplicity, that $N_f = 4$ and investigate the effects due to the c-quark. From eq. (53) it is evident that in the asymptotic limit $\mu^2 \to \infty$ the $\ln \mu^2$ coefficient becomes $9 - \frac{2}{3} \equiv \beta_s$ ($N_f = 4$), and in this sense one can use the "massless" formula (25) with $N_f = 4$. It is not clear, however, whether one should use the same $\Lambda$ (i.e., $\Lambda_3$) or a different one. To answer this question, let us rewrite eq. (53) for $N_f = 4$ in the form

$$G(\mu, N_f = 4) = \left\{ (9 - \frac{2}{3}) \ln \left( \frac{\mu^2}{\Lambda_3^2} \right) - \frac{2}{3} \ln \left( 1 + \frac{5m_c^2}{\mu^2} \right) - \frac{2}{3} \ln \left( \frac{\Lambda_3^2}{1 + 5m_c^2} \right) \right\}^{-1}$$ \hspace{1cm} (54)
Note that the second term on the \( \rho \) of eq. (54) for \( \mu^2 > 5 m_c^2 \) contributes less than \( \frac{2}{3} \ln 2 \) (the neglect of such a contribution produces only 3\% error in the determination of \( \Lambda \) ) and diminishes like \( \frac{10}{3} \frac{m_c^2}{\mu^2} \) with growing \( \mu^2 \). The third term is due to the fact that the \( i = 4 \) term of the sum in eq. (53) tends to \( \ln \left( \frac{\mu^2}{\Lambda_4^2} \right) \) as \( \mu^2 \to \infty \) rather than to \( \ln \left( \frac{\mu^2}{\Lambda_3^2} \right) \). To get rid of the constant additive term

\[
\frac{25}{3} \ln \left( \frac{\mu^2}{\Lambda_3^2} \right) - \frac{2}{3} \ln \left( \Lambda_3^2 \right) \approx \frac{25}{3} \ln \left( \frac{\mu^2}{\Lambda_4^2} \right)
\]

one should redefine \( \Lambda_4 \), or more precisely, use \( \Lambda_4 \) instead of \( \Lambda_3 \):

\[
\Lambda_4 = \Lambda_3 \left( \frac{\Lambda_3^2}{4 + 5 m_c^2} \right) \,
\]

Thus, in the region \( \mu^2 + 5 m_c^2 < Q^2 < \mu_0^2 + 5 m_0^2 \) one can approximate eq. (53) by the formula

\[
\alpha_s (Q^2) = \frac{4 \pi}{\frac{25}{3} \ln \left( \frac{Q^2}{\Lambda_3^2} \right)}
\]

Note that eq. (56) is equivalent to the relation

\[
\frac{4 \pi}{\frac{25}{3} \ln \left( \frac{1 + 5 m_c^2}{\Lambda_3^2} \right)} = \frac{4 \pi}{\frac{25}{3} \ln \left( \frac{1 + 5 m_c^2}{\Lambda_4^2} \right)}
\]

i.e., to the requirement that the lowest order "massless" formulas (25) for \( N_c = 3 \) and \( N_c = 4 \) should give the same value of \( \alpha_s \) at the (modified) mirror threshold \( Q^2 = 1 + 5 m_c^2 \). It is easy to derive also a general recurrence relation for the \( k \)-th threshold

\[
\Lambda_k = \Lambda_{k-1} \left( \frac{\Lambda_{k-1}^2}{4 + 5 m_c^2} \right)^{\frac{1}{3} - 2 k}
\]

and the formula relating \( \Lambda_k \) to the low-energy parameter \( \Lambda_3 \)

\[
\Lambda_k = \Lambda_3 \left[ \frac{k}{14} \frac{\Lambda_3^2}{4 + 5 m_c^2} \right]^{\frac{1}{3} - 2 k}
\]

The difference between different \( \Lambda_k \)'s is quite sizeable. For example, taking \( 4 + 5 m_c^2 = 10 \text{ GeV}^2 \), \( 4 + 5 m_c^2 = 100 \text{ GeV}^2 \), \( 4 + 5 m_c^2 = 10^4 \text{ GeV}^2 \) and \( \Lambda_3 = 100 \text{ MeV} \), one easily obtains from eq. (60) that \( \Lambda_4 = 76 \text{ MeV} \), \( \Lambda_5 = 50 \text{ MeV} \), \( \Lambda_6 = 24 \text{ MeV} \). If \( \Lambda_3 = 500 \text{ MeV} \), then \( \Lambda_4 = 431 \text{ MeV} \), \( \Lambda_5 = 328 \text{ MeV} \), \( \Lambda_6 = 190 \text{ MeV} \).

This does not mean, however, that quark-mass corrections to \( \alpha_s (Q^2) \) are large. That the differences between \( \Lambda_k \)'s have nothing to do with the variation of \( \alpha_s (Q^2) \) is most clearly displayed by eq. (58) that just states that the change \( \Lambda_3 \to \Lambda_4 \) is due to the fact that the same value
of $\alpha_s$ (i.e., $\alpha_s(1+5m_c^2)$) must be obtained from two different massless formulas corresponding to $N_f = 3$ and 4, respectively. Furthermore, simple estimates show that the deviation of the "exact" formula eq. (53) from the simplest $N_f = 3$ massless formula is, in fact, rather small. For example, the contribution of the term $9 \ln(\mu^2/\Lambda_3^2)$ in eq. (53) increases by $9 \ln(10) \approx 20.7$ when $\mu^2$ changes from 10 to 100 GeV$^2$ while the account of the c-quark mirror threshold amounts to decreasing this number by $\frac{2}{3} \ln(5.5) \approx 2.4$, i.e. by $5.5\%$. Thus, though the formula (25) with $N_f = 4$ and $\Lambda_4 = 76$ MeV (say) reproduces the corresponding exact formula (53) better than eq. (25) with $N_f = 3$ and $\Lambda_3 = 100$ MeV, the latter may serve as a good (within a few per cent) approximation to the exact formula not only in "its own" region ($Q^2 < 10$ GeV$^2$) but also in the neighbouring one ($Q^2 \leq 100$ GeV$^2$). In other words, one can use the $N_f = 3$ formula (without changing $\Lambda_3$) not only for $Q^2 < 10$ GeV$^2$, but also for $Q^2 \leq 100$ GeV$^2$. If necessary, one can treat the above-mentioned few per cent deviation as a small perturbation. The threshold effects in higher orders were analyzed in refs. 57-59). These effects at accessible energies are also very small.

3.7 Effective coupling constant in the timelike region

As it has been emphasized above, the asymptotic - freedom formula (25) (and its higher-order generalizations) describes the behaviour of $\alpha_s(Q^2)$ in the spacelike region $Q^2 = -q^2 > 0$. However, in most of processes one deals with large, but timelike momentum transfer $q^2 > 0$. So, it is important to know which formulas for $\alpha_s$ should be used in this region. A formal analytic continuation

$$\frac{1}{\ln(Q^2/\Lambda^2)} \rightarrow \frac{1}{\ln(-q^2/\Lambda^2)} = \frac{1}{\ln(Q^2/\Lambda^2)} + i\pi = \frac{\ln(q^2/\Lambda^2)}{\ln(Q^2/\Lambda^2) + \pi^2} \equiv \text{Re } \alpha_s + \text{Im } \alpha_s$$

produces imaginary contributions $\text{Im } \alpha_s$. Of course, the observable quantities (like cross sections) should be real, and one may expect that a possible expansion parameter for $\alpha_s$ in the timelike region is $\text{Re } \alpha_s$ 50). This corresponds to the change

$$\frac{1}{\ln(Q^2/\Lambda^2)} \rightarrow \frac{\ln(q^2/\Lambda^2)}{\ln(q^2/\Lambda^2) + \pi^2}$$

(62)

However, the imaginary contributions may disappear not only because all terms containing $\text{Im } \alpha_s$ cancel. One imaginary contribution multiplied by another produces a real contribution just like $(i\pi)$ -term produced the $\pi^2$ one in eq. (61). So, another possibility may be $|\alpha_s(-q^2)|$, i.e. the change 50, 51)

$$\frac{1}{\ln(Q^2/\Lambda^2)} \rightarrow \frac{1}{\sqrt{\ln(q^2/\Lambda^2) + \pi^2}}$$

(63)
Notice, that both eqs. (62) and (63) correspond to the diminishing of the expansion parameter, e.g., by 50% for $\hat{b}_n(q^2/\Lambda^2) \approx 3$ and by 10-20% for $\hat{b}_n(q^2/\Lambda^2) \approx 7$.

It should be emphasized that there is no universal way for the analytic continuation into the timelike region, such a continuation has an essential dependence on the quantity or process investigated. For many quantities related to essentially timelike kinematics it is not even clear whether they can be related to any spacelike quantity or not. In fact, so far only the simplest case, the total cross-section of the $e^+e^-\to\mu^+\mu^-$annihilation into hadrons was investigated in some detail $^{60,62}$. The Euclidean (i.e., spacelike) quantity in this case is the logarithmic derivative

$$\mathcal{D}(Q^2) = Q^2 \frac{d}{dQ^2} \mathcal{T}(Q^2)$$

of the (properly normalized) hadronic contribution $\mathcal{T}(Q^2)$ to the vacuum polarization. It is connected with the experimentally observable quantity $R(s)$ (the ratio $\sigma_{tot}(e^+e^-\to hadrons)/\sigma(e^+e^-\to\mu^+\mu^-)$) by the dispersion relation

$$\mathcal{D}(Q^2) = -Q^2 \int_{\frac{\Lambda^2}{m^2}}^{\infty} \frac{R(s)\, ds}{(s+Q^2)}$$

(65)

The perturbative QCD calculation gives the $(\frac{d\sigma}{d\Omega})^n$ - expansion for $\mathcal{D}(Q^2)$ :

$$\mathcal{D}(Q^2) = \sum_q g_q \left\{ 1 + \frac{d_3(Q^2)}{s} + d_3 \left( \frac{d_2}{s} \right)^2 + d_3 \left( \frac{d_3}{s} \right)^3 + \ldots \right\}$$

(66)

Our problem now is to find an expansion for $R(s)$ that being substituted into eq. (65) will reproduce eq. (66). Using the definitions (64), (65) one can get the formal inverse transformation

$$R(s) = \frac{1}{2\pi i} \int_{-s-i\epsilon}^{s+i\epsilon} \frac{d\sigma}{s} \mathcal{D}(\sigma)$$

(67)

Integration here goes below the real axis from $-s-i\epsilon$ to zero and then above the real axis to $-s+i\epsilon$. In some practically important cases the integral (67) can be calculated explicitly $^{62,62}$

$$\frac{1}{s} \to 1$$

(68)

$$\frac{1}{L_{\sigma}} \to \frac{1}{L_{\sigma} \tan \left( \frac{\pi}{L_{\sigma}} \right)}$$

(69)

$$\frac{1}{L_{\sigma}^2} \to \frac{1}{L_{\sigma}^2 + \pi^2}$$

(70)
\[
\frac{\ell_1}{L_0} \rightarrow \frac{\ln (L_0^2 + \frac{\pi^2}{L_0})}{L_0^2 + \frac{\pi^2}{L_0}} + \frac{L_0}{\tan (\frac{\pi}{L_0})} + 1
\] (71)

\[
\frac{A}{L_n^{\eta}} \rightarrow (-1)^{n-\frac{\pi}{L_0}} (\frac{d}{dL_0})^{n-2} \frac{1}{L_0^{\frac{\pi}{L_0}} + \frac{\pi^2}{L_0}}
\] (72)

where \( L_0 = \ln(\frac{\sigma}{A^2}) \), \( L_0 = \ln(\frac{\sigma}{A^2}) \) and \( L_0 \) is a constant depending on the \( \eta \) -choice (see eqs. (28)-(31)). Using the \( \eta/L_0 \) -expansion for \( \alpha_s(\tau) \) and eqs. (66)-(72) (as well as their generalizations for \( L_0^2 L_0/L_0^3 \), \( L_0 L_1/L_0^3 \), etc.) produces the expansion

\[
R(\tau) = \sum_{\eta} \epsilon_\eta \left\{ 1 + \sum_{k=1}^{\infty} d_k \left[ \Phi \left( \frac{\alpha_s}{\tau} \right)^k \right] \right\}
\] (73)

It should be noted that the expansion (73) is not an expansion in powers of some particular parameter since the \( \Phi \) -operation normally violates nonlinear relations: \( \Phi [\eta/L_0] \neq (\Phi [\eta/L_0])^2 \), etc. However, there are no grounds to believe that a power expansion"a priori better than any other. In fact, the expansion over \( \Phi [\alpha_s^k] \) converges better than the generating expansion (66) for \( D(\tau) \) because \( \Phi [\alpha_s^\infty] < \alpha_s^\infty \).

Eq. (69) may be interpreted in the following way: if the coupling constant in the spacelike region behaves like \( \frac{4\pi}{\ln(\frac{\sigma}{A^2})} \), the corresponding behaviour in the timelike region will be

\[
\bar{\alpha}_s(\tau) = \frac{4}{\tilde{b}_0} \frac{\pi}{\ln(\frac{\sigma}{A^2})}
\] (74)

Note that \( \bar{\alpha}_s(\tau) \ll \frac{2\pi}{\tilde{b}_0} \approx 0.7 \) (for \( N_t = 3 \)). There are no infinities for \( \bar{\alpha}_s \) in the timelike region. Still, in the region where one may rely on perturbation theory the difference between \( \bar{\alpha}_s \) and \( \alpha_s \) is not very large: according to eqs. (68)-(72) we have \( \frac{1}{3} \rightarrow \frac{1}{3} \frac{9}{16} \); \( \frac{1}{6} \rightarrow \frac{1}{6} \frac{5}{9} \). For the most practically important values of \( \alpha_s \) (\( \alpha_s = 0.2 - 0.35 \)) its timelike counterpart \( \bar{\alpha}_s \) is smaller than \( \alpha_s \) by about 10%.

4. QCD PARTON PICTURE

4.1 "Old" parton model

Basic assumptions of the pre-QCD parton model (PM) 63) are essentially the following:

1) Fast-moving hadrons can be treated as being composed of pointlike constituents, the partons.

2) The constituents of the hadron are described by parton distribution functions \( f_{x/A}(x) \) which have the meaning of probability to find the parton \( A \) with (longitudinal) momentum \( x \rightarrow A \) (\( x \leq 1 \)) inside the
hadron $A$ having momentum $\vec{P}_A$. The distribution functions depend in general on the type of the parton, e.g., for the proton one has distributions $U_0(x), d(x), s(x), \bar{u}(x)$ etc.

3) Inclusive cross sections $\sigma^{\text{hadr}}$ are given by convolutions of the parton cross sections $\sigma^{\text{part}}$ with the distribution functions

$$\sigma^{\text{hadr}}_A(\vec{P},\ldots) = \sum_a \int_0^1 dx \, f_{q/A}(x) \sigma^{\text{part}}_a(x,\vec{P},\ldots)$$

A famous example is deep inelastic scattering. In this case

$$\sigma^{\text{part}} \sim 2 x (P_q) \delta\left((q+\vec{x} P)^2\right) e_a^2 = \omega x \delta(1-\omega x) e_a^2$$

where $\omega = 2(P_q)/Q^2$, $Q^2 = -q^2$. As a result, the structure function $W(\omega)$ is given by a linear combination of quark distribution functions

$$W(\omega) = \sum_a e_a^2 f_a\left(\frac{\lambda}{\omega}\right)$$

Another example is the $\mu^+\mu^-$ pair production process $h_1 h_2 \rightarrow \mu^+\mu^- \chi$. In this case

$$\frac{\partial \sigma}{\partial Q^2} = \int_0^1 dx_1 \int_0^1 dx_2 \frac{4 \pi a^2}{3 Q^2 S} \delta(x_1 x_2 - \frac{Q^2}{S}) \cdot \left[ f_{q/h_1}(x_1) f_{\bar{q}/h_2}(x_2) + (q \leftrightarrow \bar{q}) \right]$$

where $Q^2 = (P_{\mu^+} + P_{\mu^-})^2$, $S = (P_{h_1} + P_{h_2})^2$.

Strictly speaking, the $1/N_c$ factor in eq. (78) is of the QCD rather than pure FM origin: there was no colour in the original formulation of FM. The appearance of $(1/N_c)$ can be understood in the following way. The distribution function $f_{q/A}$ is the sum of those for quarks with definite colour

$$f_{q/A} = \sum_{i=1}^{N_c} f_q^i = N_c f_q^k$$

The last relation is based on the fact that the colour symmetry is exact. Note now that in the Drell-Yan process one deals with the sum

$$\sum_{i=1}^{N_c} f_q^i f_{\bar{q}^i} = \frac{\lambda}{N_c^2} \sum_{i=1}^{N_c} f_q f_{\bar{q}} = \frac{\lambda}{N_c} f_q f_{\bar{q}}$$

in which the colours of quark and antiquark are strongly correlated since the $\mu^+\mu^-$ pair should be colourless.

To describe semi-inclusive processes (i.e., those with some hadrons detected in the final state) like $e^+ e^- \rightarrow h \chi$, one should use also the
parton fragmentation (decay) functions $D_{AQA}(x)$ which give probability to get the hadron $A$ with momentum $P_A$ from the parton $Q$ produced in the hard subprocess and having momentum $zP_A (z>1)$.

4.2 Modification of FM in QCD

Asymptotic freedom of QCD explains the basic postulate of the parton model that in a deep inelastic process one can treat the hadron constituents as pointlike non-interacting particles. It is clear, however, that in QCD this is only an approximation, and the parton interactions should be included in the QCD description of these processes. An important ingredient of the QCD picture is also the colour degree of freedom which, as discussed above, should be sometimes taken into account even at the $(a_q)^0$ level, i.e. in the absence of the QCD interactions. Furthermore, the list of partons should now include not only quarks but also gluons that possess their own distribution and fragmentation functions $f_g(x)$ and $D_g(x)$.

The QCD interactions violate the Bjorken scaling (this was observed first within the operator product expansion approach $^{12,65}$), and one may expect that the QCD parton distributions $f(x)$, $Q(x)$ should also depend on the scale $Q$ characterizing the resolving power of the probe. Finally, the QCD picture must also include more complicated distributions describing, e.g., the multiparton correlations, transverse momenta of partons, etc.

4.2 QCD interaction and classic picture

The most naive approach is just to include QCD interactions as radiation corrections to the FM contributions. The relevant lowest-order diagrams for deep inelastic scattering are shown in fig. 1. An important thing to note now is that the standard parton model corresponds in fact to a classic (i.e., non-quantum-mechanical) description in terms of probabilities (not probability amplitudes). However, most of the 1-loop diagrams (e.g., $A^*B$) have an evident interference structure: they look like $A^*A$ rather than $A^*A = |A|^2$. Thus, there arises a natural question: does the classic picture survive in QCD? The answer is that at the leading power level (i.e., for all $(Q^2)^N (\ln Q^2)^M$ terms in deep inelastic scattering) the classic picture is stable with respect to the radiation corrections. The only effect is the $Q^2$ - evolution of the probabilities. A basic observation here is that the interference terms producing the leading power $(Q^2)^0$ for DIS contributions for separate diagrams correspond to nonphysical (longitudinal) polarizations of the gluons. Consider as an example the diagram shown in fig. 2. Its contribution is proportional to

$$\frac{\left[ (x+\lambda Q) p^\mu + \lambda' \right] g_{\mu\nu} \left( x' p^\nu + \lambda' \right)}{(2P_{q1}(x+\lambda Q)-Q^2) (2P_{q1}(x-Q^2))} <P|\bar{\psi} \ ... \ A_{\mu} \ ... \ \psi |P>$$

(61)
It behaves like \((Q^2)^0\) only if \(\langle P | \bar{\psi} \ A_{\mu} \ \psi | P \rangle\) brings in the \(P_\mu P^\mu\) factor. Note now that the vector \(e_\mu\) multiplying the \(\chi^\mu\) matrix related to the quark-gluon vertex should be interpreted according to the standard Feynman rules (see, e.g., (45)) as the gluon polarization vector. In our case \(e_\mu = P_\mu\), and, hence, the gluon is longitudinally polarized, i.e., it does not, in fact, correspond to a physical particle. In particular, one can rotate it away by a proper gauge transformation. One can verify, that, say, in the axial gauge \(A^\mu = 0\) the diagram shown in fig. 2 gives only \(1/Q^2\) contribution, and there are no \(O(\alpha_s)\) interference contributions in this gauge, all \(O((Q^2)^0)\) terms result from ladder diagrams (like fig. 1b) which obviously have a probabilistic interpretation.

4.3 Factorization of short- and long-distance contributions

One may ask now whether there is any sense in explicitly displaying the quark-gluon ladders. The distribution functions are supposed to absorb all information about the quark-gluon interactions preceding the hard parton subprocess, and the natural idea is to include the ladders inside the blob related to the distribution function to avoid the double-counting. However, this reasoning is only partially correct: the distribution functions accumulate information only about soft interactions, not about all of them. For example, if the quark momentum \(k\) in fig. 1b is soft, \(|k^2| < \mu^2\), then the ladder must be absorbed into the lower blob, but if \(|k^2| > \mu^2\), then diagram 1b should be treated as a radiation correction to \(\sigma^{\text{part}}\). The distribution function and \(\sigma^{\text{part}}\) now must be, of course, \(\mu\) - dependent quantities \(\bar{f}^{(x)} \rightarrow \bar{f}^{(x, \mu^2)}\), \(d\sigma^{\text{part}}(x P, \nu) \rightarrow d\sigma^{\text{part}}(x P, \nu; \mu^2)\):

\[
\sigma^{\text{hadr}} = \int_0^1 d\chi \ \sigma^{\text{part}}(x P, \nu; \mu^2) \bar{f}^{(x, \mu^2)} \tag{82}
\]

The parameter \(\mu\) \((\mu_s\) is the boundary between large and small momenta (short and long distances). Of course, the hadron cross section should be \(\mu\) - independent. This can be realized, e.g., if \(\sigma\) and \(\bar{f}\) have the \(\mu^2\) - dependence of the form \(\sigma^{\text{part}} \sim (\mu^2/\mu_s^2)^{\gamma}, \bar{f} \sim (\mu^2/\mu_s^2)^{\delta}\). Of course, for a combination like eq. (82) the compensation of the \(\mu^2\) - dependences might occur in a much more complicated way. Note, however, that by taking moments one can reduce the convolution present in eq. (82) to a simple algebraic product. For the deep inelastic structure functions this gives

\[
M_N \equiv \int_2^\infty W(\omega, Q^2) \frac{d\omega}{\omega N^2} = W^{\text{part}}(Q^2, \mu^2) \int_2^\infty f_N(\mu^2) \tag{83}
\]

Now the only possibility is that the above mentioned \(\chi, \bar{f}\) depend on \(N\) and \(Q^2\) (and, hence, on \(\mu^2\) if one uses \(\mu^2\) also as the renormalization scale for the coupling constant). One can show that \(f_N(\mu^2, Q^2)\) satisfies the
denormalization group equation \cite{12,65-66}

\[(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\bar{q}) \frac{\partial}{\partial \bar{q}}) f_N(\mu^2, \bar{q}) = \gamma_N(\bar{q}) \frac{\partial}{\partial \bar{q}} f_N(\mu^2, \bar{q}) \tag{84}\]

the solution of which is just of the type suggested above

\[f_N(\mu^2) = \exp \left\{ \int_{\mu_0^2}^{\mu^2} \frac{dt}{t} \gamma_N(\bar{q}(t)) \right\} f_N(\mu_0^2) \tag{85}\]

The only requirement on \(\mu^2\) is that the momenta \(|k^2| > \mu^2\) should be inside the asymptotic freedom region. The standard choice is \(\mu^2 = Q^2\) and in this way one arrives at the \(Q^2\)-dependent distribution functions

\[f(x) \rightarrow f(x, Q^2) \tag{86}\]

### 4.4 Field-theoretic treatment of parton distributions

The standard QCD analysis \cite{12,65} of deep inelastic scattering is based on the operator product expansion technique \cite{67,68}. The distribution functions of quarks and gluons in this approach are introduced \cite{69,55,70} by identifying \(f_N\)'s as reduced matrix elements of local composite operators, e.g.,

\[\frac{i^n}{n!} \sum_\sigma \langle P, \sigma | \left\{ \gamma_{d_1} D_{d_1} \ldots D_{d_n} \gamma_{d_n} \right\} | P, \sigma \rangle = \left\{ \bar{P}_{d_1} \ldots \bar{P}_{d_n} \right\} \left[ f_n^q(\mu^2) + (-1)^n f_n^g(\mu^2) \right] \tag{87}\]

for quarks and

\[\frac{i^n}{n!} \sum_\sigma \langle P, \sigma | T_r \left\{ G_{d_1} D_{d_2} \ldots D_{d_n} G_{d_n}^* \right\} | P, \sigma \rangle = \left\{ \bar{P}_{d_1} \ldots \bar{P}_{d_n} \right\} \frac{1 + (-1)^n}{2} f_n^g(\mu^2) \tag{88}\]

for gluons. The well-known sum rules

\[\int_0^1 \sum_\alpha e_\alpha ( f_\alpha(x) - f_\alpha^\gamma(x) ) dx = e_A \tag{89}\]

\[\int_0^1 \left( f_g(x) + \sum_\alpha ( f_\alpha(x) + f_\alpha^\gamma(x) ) \right) x dx = 1 \tag{90}\]

stating that the total electric charge (also strangeness, isotopic spin, etc.) and momentum of partons equal to those of the hadron immediately follow from the fact that \(n = 1\) and \(n = 2\) operators form the vector current and the energy-momentum operators, respectively.

Note also that the covariant derivatives \(D_\mu = \partial_\mu - ig A_\mu\) contain the gluon potential \(A_\mu\). This is just the consequence of that the diagrams of fig. 2 type give the leading power contribution. Physically,
the presence of \( A_\mu \) in eqs. (87), (86) can be interpreted so that the QCD partons (quarks, gluons) are always accompanied by their static (Coulomb-like) fields which do not correspond to physical particles. The latter are described by the field-strength operator \( \mathcal{G}_{\mu\nu} \) which in contrast to \( A_\mu \) cannot be rotated away by a gauge transformation. The physical gauges in which fig. 2 gives only a power \( \mathcal{W}q^2 \) suppressed contribution (e.g., axial gauge \( q_\mu A_\mu = 0 \) or Fock-Schwinger gauge \( x_\mu A_\mu(x) = 0 \)) are just the gauges for which \( A_\mu \) can be expressed in terms of \( \mathcal{G}_{\mu\nu} \):

\[
A_\mu(x) = n^\nu \int_0^\infty \mathcal{G}_{\nu\mu}(xs) e^{-xs} ds
\]

(91)

for the axial gauge \( A_\mu = 0 \) and

\[
A_\mu(x) = x^\nu \int_0^4 t dt \mathcal{G}_{\nu\mu}(tx)
\]

(92)

for the Fock-Schwinger gauge.

4.3 QCD evolution of parton densities

The variation of the coefficients \( f_n(\mu^2) \) with increasing \( \mu^2 \) (see eq. (85)) means, of course, that the distribution functions \( f(x,\mu^2) \) also evolve. Using the connection between \( f(x,\mu^2) \) and \( f_n(\mu^2) \)

\[
f_n(\mu^2) = \int_0^1 \frac{dx}{x} x^n f(x,\mu^2)
\]

(93)

we get from eq. (84) the integro-differential equation \( 74, 75 \)

\[
\left( \frac{\mu^2}{2} \frac{\partial}{\partial \mu^2} + \beta(q) \frac{d}{dq} \right) f_a(x,\mu^2,q^2) = \sum_b \int_0^1 \frac{d\xi}{\xi} P_{ab}(\xi) \int_0^1 \frac{d\eta}{\eta} f_b(\eta,\mu^2,\eta^2 q^2)
\]

(94)

the solution of which gives the evolution law of the distribution functions. By definition, \( P_{ab}(x,\mu^2,q^2) \) is a function whose moments are equal to \( \gamma_{a\xi}(n) \)

\[
\int_0^1 P_{a\xi}(\xi) \xi^n \frac{d\xi}{\xi} = \gamma_{a\xi}(n)
\]

(95)

In the leading logarithm approximation, i.e., when only \( O(q^2) \) terms in \( P_{a\xi} \) are taken into account, eq. (94) has a simple parton interpretation \( 76 \). As we have already pointed out, the functions \( f_a(x,\mu^2) \) describe the parton momentum distribution for a situation in which the hadron structure is probed at distances of order \( 1/\mu \). If \( \mu \), i.e., the resolution power of our "partonometer" is less than the characteristic hadron scale \( 1/R_{had} \), we shall not see at all that within the hadron (for example, a proton) there are smaller constituents. However, beginning with a certain \( \mu^2 \approx \mu_0^2 \) the three valence quarks begin to be seen in the proton, and at a certain \( \mu_1^2 \approx N \mu_0^2 \) so do the virtual gluons emitted by the valence quarks. If \( \mu^2 \) is increased by a further \( N \) times, the
partonometer will see the virtual $\bar{q}q$ pairs surrounding the valence quarks; at $\mu^2 = \mu_0^2 N^3$ the gluons emitted by the virtual quarks will become visible and so forth. Thus, at $\mu^2 = \mu_0^2 N^k$ the k-th level of hadron structure is seen. In such a picture, $P_{a\ell}(x/y)$ for $x \neq y$ characterizes the probability that parton $b$ having longitudinal momentum $y_P$ goes over (by emitting a gluon or producing a $\bar{q}q$ pair) into a parton $a$ with longitudinal momentum $x_P$. Probing the hadron with a partonometer with resolution $\mu^2 = \mu_0^2 N^{k+1}$ we obtain the distribution function $f_{a}(x, \mu_0^2 N^k)$ that differs from the function $f_{a}(x, \mu_0^2 N^k)$ measured for $\mu^2 = \mu_0^2 N^k$. This difference is due to the fact that at $\mu^2 = \mu_0^2 N^k$ a certain parton $b$ with momentum $y_P$ is observed as a pointlike particle, but at $\mu^2 = \mu_0^2 N^{k+1}$ one can, with probability $P_{a\ell}(x/y)$, observe in it parton $a$ with momentum $x_P$. It should also be taken into account that if $P_a$ is the total probability that parton $a$ is observed after an increase in resolution (by $N$ times) as consisting of "smaller" partons, then with probability $(1-P_a)$ we shall observe at $\mu^2 = \mu_0^2 N^{k+1}$ the same parton as at $\mu^2 = \mu_0^2 N^k$. Now we can write the balance equation

$$f_{a}(x, \mu_0^2 N^{k+1}) = f_{a}(x, \mu_0^2 N^k) (1-P_a) + \int_{x-a}^{1} \frac{dy}{y} P_{a\ell}(x/y) f_{\ell}(x, \mu_0^2 N^k).$$ \hspace{1cm} (96)$$

Replacing the difference

$$f_{a}(x, \mu_0^2 N^{k+1}) - f_{a}(x, \mu_0^2 N^k)$$

by the derivative $\frac{d}{d\ln \mu^2}$ and noting that $\mu = \mu_0 (\mu^2/\mu_0^2)$ we obtain from (96) the evolution equation (94) in which $P_{a\ell}(x/y)$ for $x=y$ must be taken equal to $[\delta_{a\ell} \delta(a-x/y)]$. According to their definition, the coefficients $P_{a}$ are not quantities independent of $P_{a\ell}$. Explicit expressions for $P_{a}$ in terms of $P_{a\ell}(x/y)$ (for $x \neq y$) can be obtained by using the conservation laws for the energy-momentum and the vector current. In particular, from the equation

$$\frac{d}{d\ln \mu^2} \left( \sum_{\alpha=q,\bar{q}} \int_0^1 dx \ f_{\alpha}(x, \mu^2) \right) = 0 \hspace{1cm} (97)$$

it follows that for quarks and antiquarks

$$P_a = \int_0^1 dx \ P_{a\ell}(x) \hspace{1cm} (98)$$

In QCD the functions $P_{a\ell}(x)$ were calculated first by Altarelli and Parisi \hspace{0.5cm} (77) and by Dokshitzer \hspace{0.5cm} (78)

$$P_{a\rightarrow \ell}(x) = P_{q\rightarrow \ell}(1-x) = \frac{4}{3} \frac{ds}{2m} \left( \frac{1+4(1-x)^2}{x} \right)$$ \hspace{1cm} (99)
\[ \mathcal{P}_{q \to q}(z) = 6 \left( \frac{z}{4 - z} + \frac{4 - z}{2} + z(4 - z) \right) \]  

(100)

The evolution of parton distributions predicted by QCD is in good agreement with experimental data (for details see the lectures by G. Giacomelli at this School).

4.6 QCD parton picture of hadron-hadron processes

One can also try to calculate the QCD radiation corrections to the PM approximation for other processes, say, for the massive lepton-pair production \( AB \to \mu^+ \mu^- \Xi \) process. The relevant diagrams (fig. 3) have the structure essentially different from that of the deep inelastic scattering, and there arises a natural question whether the QCD corrections generate in this case the same \( Q^2 \) - evolution of parton distributions as in DIS. Another problem is whether the timelike nature of momentum transfer in the DY process (contrasted to the spacelike one in DIS) would produce new effects.

That the \( Q^2 \) - evolution in the DY process is the same as in DIS was first established from an explicit calculation in Feynman gauge \(^79-81\), but much simpler way to observe this is to use a physical gauge, e.g., the planar gauge \(^15\) \((p_A^\mu + k p_B^\mu) A_\mu = 0 \) (where \( k \sim 1 \) is some arbitrary number, \( k \neq 0 \), \( k \to \infty \)). Then only generalized ladders give the leading power contributions, and it is now evident that \( DY \sim (DIS)^2 \).

Though the \( Q^2 \) - evolution is the same, there appear new contributions due to the time-like-ness of \( Q^2 \). For instance, the virtual gluon exchange diagram (fig. 3b) contains the double-logarithmic term proportional to

\[ - \ln^2 \left( \frac{Q^2}{\mu^2} \right) = - \ln^2 \left( \frac{Q^2}{\mu^2} \right) = 2 \ln^2 \left( \frac{Q^2}{\mu^2} \right) + \pi^2 \]  

(101)

The double logarithm \( \ln^2 (Q^2/\mu^2) \) is cancelled by the real gluon emission contribution, the imaginary term disappears after one includes the conjugate diagram, but the \( \pi^2 \) - term remains \(^82,83\). If one assumes\(^84-85\) that since the Sudakov double logarithms exponentiate, so do the \( \pi^2 \)'s associated with them, and as a result,

\[ C_F \cdot \frac{\alpha_s}{2\pi} \cdot \pi^2 \Rightarrow e \times (C_F \frac{\alpha_s}{2\pi} \pi^2) \]  

(102)

in higher orders producing thus the QCD K-factor which is 1.8 for \( \alpha_s = 0.3 \) and 1.5 for \( \alpha_s = 0.2 \) partially compensating the \( 1/N_c \) colour factor \(^83\).

General arguments similar to those presented above were given \(^86-90\) that the factorization of short- and long-distance contributions, i.e., the generalized parton representation for \( d\sigma^{hadr} \) in term of \( d\sigma^{parton} \) is valid for a large class of hadron-hadron processes. The original proofs of the factorization theorem \(^86-90\), however, were not sufficiently careful with some subtle problems related to soft gluon interactions.
between the spectator particles (which do not participate in a hard short-distance subprocess). Importance of such exchanges at the diagram level was emphasized in ref. 91). Later, however, it was established that additional contributions not taken into account in refs. 86-91 cancel when summed over all diagrams 92).

There are also processes for which \( d\sigma_{\text{parton}} \) starts with \( \alpha_s \) (i.e., \( d\sigma_{\text{parton}} \neq 0 \)) in the "old" parton model where \( \alpha_s = 0 \). A simple example is the high-

\[ Q_T \] muon-pair production. To get the \( \mu^+\mu^- \) -pair with high momentum transverse to the beam axis, one needs some particle with opposite \( Q_T \) which may be either quark or gluon. In principle, the \( Q_T \) of the \( \mu^+\mu^- \) -pair may be compensated by total transverse momentum of many gluons each having a smaller \( \rho_T \). However, emission of each additional gluon is damped by the \( (1/s_\mu) \) -factor, and for high \( Q_T \sim Q \) the hard \( O(s_\mu) \) subprocesses dominate. The situation changes for \( Q_T \) small compared to the pair mass \( Q \) \( (Q_T^2 \ll Q^2) \) when the \( (1/s_\mu) \) suppression is compensated by large logarithm \( \ln (Q_T^2/Q^2) \). In this case one should perform the summation over the (relatively) soft gluons to get the \( Q_T \) -distribution of the \( \mu^+\mu^- \) -pairs. The results obtained in this way 195,95-98) are in good agreement with experimental data (see 99).

4.7 High-

\[ \mu_T \] hadron production

Next in complexity is the process of high-

\[ \mu_T \] inclusive hadron production \( AB \rightarrow CX \) with \( A, B, C \) being hadrons. The relevant cross section in this case is given by (see, e.g., 100)

\[
\frac{d\sigma}{d^2p_T} (AB \rightarrow CX) = \sum_{a \bar{b} \rightarrow c \bar{d}} \int_0^1 dx_a \int_0^1 dx_b \int_0^{s_\mu} d\ln \frac{x_c}{x_b} \frac{d\sigma}{d^2x} (a \bar{b} \rightarrow c \bar{d}) D_{c/c} (x_c)
\]

(103)

where \( d\sigma/d^2x = \pi \alpha_s^2 A^2/s^2 \) are the \( a \bar{b} \rightarrow c \bar{d} \) subprocess cross sections first calculated in ref. 101). For example, the \( \Lambda \)-amplitude for the scattering of quarks \( q_1 q_2 \) of different flavours is

\[
A(q_1 q_2 \rightarrow q_1 q_2) = \frac{4}{3} \frac{s_{12} u_{12}}{t^2}
\]

(104)

and that of gluons is

\[
A(gg \rightarrow gg) = \frac{9}{2} \left( 3 - \frac{u_t}{s^2} - \frac{t_s}{t^2} - \frac{s_t}{u^2} \right)
\]

(105)

In principle, the QCD analysis of these processes is similar to that of the DY process, but the higher-order correction (necessary, e.g., for making the optimal choice of the \( \mu \) - scale) are much more difficult to calculate (see, e.g. ref. 102).
4.8 Jets

The presence of the fragmentation function $D_{q/g}(z_e)$ in eq. (103) manifests a very essential feature of perturbative QCD calculations; they are performed in terms of quarks and gluons while experimentally one observes only hadrons. The hadrons observed in the final state result from a parton produced in a short-distance subprocess. Between the short-distance stage and observation of the hadrons there is a mysterious (from a perturbative QCD standpoint) soft fragmentation process in which the parton converts into hadrons with small ($\lesssim 300$ MeV) transverse momentum to the parton axis. Thus, for a very energetic parton one observes a highly collimated jet of hadrons. This jet may be treated just as a track made by quark or gluon. Recall that the tracks of ordinary particles observed, say, in bubble chambers are also formed by secondary structures (bubbles, say), and one observes just these structures ("bubble jets") not the hadrons themselves. In this sense observing jets one (in) directly observes quarks and gluons, and the problem is to extract information about quarks and gluons by studying their "jet tracks". Originally, jets were observed in $e^+e^-\rightarrow$ hadron collisions. It was established, in particular, that the jets have just the $(1+\cos^2\Theta)$ angular distribution specific of the spin-1/2 particles. As a more QCD-related test one can mention the observed $\sim 10\%$ ratio of three-jet to two-jet events, which agrees well with the QCD expectation that the emission of a hard gluon is damped by $\alpha_s/m\sim 0.1$ factor. The analysis of the angular distribution of the third jet definitely favours the spin-1 nature of the particle generating this jet 103).

4.9 Jets in hadronic collisions

The cross section of producing a jet in the hadron-hadron collision differs from eq. (103) by the absence of the fragmentation function $D(z)$:

$$ E \frac{d\sigma}{d^3p_T} = \sum_{ij} \int dx_1 dx_2 \int \frac{d\sigma}{d\tau} F_{g}(x_1) F_{g}(x_2) \frac{\lesssim \tau}{\pi} \frac{d\sigma}{d\tau} \epsilon(1+S+u) \ (106) $$

where $S=(p_1+p_2)^2$, $t=(p_1-p_2)^2$, $u=(p_1-p_3)^2$ are the subprocess invariant variables. The transverse momentum $p_T$ of the c-particle is related to the $u, t, s$ - variables by $p_T^2 = ut/s$. It is usual to introduce the dimensionless variable $x_T = 2p_T/\sqrt{s}$. The scaling of the parton cross sections leads one to expect that

$$ \frac{d\sigma}{dp_T^2} \propto \frac{1}{p_T^4} f(x_T, \alpha_s(p_T^2)) \ (107) $$

and so the cross section should fall like $p_T^{-4}$ at fixed $x_T$, with an additional logarithmic decrease due to the $\alpha_s(p_T^2)$ evolution. However, the $p_T$ -dependence at fixed $S$ is much steeper $\sim p_T^{-9}$ due to a rapid variation of the distribution functions with $x$. 

From the analysis of the cross sections for various parton subprocesses (see, e.g. ref. 104) it was established that they all have to a good accuracy the same angular dependence $(1 - \cos \Theta)^2 \approx 1 / \sin^4 \frac{\Theta}{2}$ which is just the angular distribution predicted by Rutherford for the scattering of classical pointlike charged particles. Such an angular behaviour was observed experimentally, and this definitely rules out the scalar gluons (in that case the $q \bar{q}$-scattering cross section would have $d\sigma / d \cos \Theta \rightarrow \text{const}$ behaviour while that for $q \bar{q}$ and $q \bar{q}$ scattering would have the $(1 - \cos \Theta)^{-1} \approx 1 / \sin^2 (\Theta/2)$ dependence). Another observation is that over a wide angular range the subprocesses contribute in the ratio

$$g g \rightarrow q \bar{q} : q g \rightarrow q \bar{q} : g g \rightarrow q \bar{q} = 1 : \frac{4}{9} : \frac{4}{9} : \left(\frac{4}{9}\right)^2$$

(108)

so one effectively sees the effective distribution function

$$f(x) = \frac{1}{2} f_{g g}(x) + \frac{4}{9} f_{q g}(x)$$

(109)

Hence, if $A$ is the proton and $B$ the antiproton, then

$$\frac{d\sigma}{dx d\Theta} \sim f_g(x_A) f_{g g}(x_B)$$

(110)

This factorization is also supported by experimental data.

Note that before comparing the shape of $f(x)$ extracted from the $P\bar{P}$-collider data with the DIS structure functions one should evolve $f(x)|_{\text{DIS}}$ from $Q^2 \sim 100 \text{ GeV}^2$ to $Q^2 \sim 1000 \text{ GeV}^2$. The agreement between $f(x)$'s obtained in the above two ways is good (see, e.g. ref. 105). A very important observation is that the gluon contribution to $d\sigma^{\text{jet}} / dP_T^2$ is rather large, especially in the region $x_\tau \lesssim 0.15$ where the $q \bar{q}$-scattering dominates.

4.10 Jet fragmentation

The perturbative QCD analysis unfortunately, is not directly applicable to study the final stages of jet formation. Various Monte-Carlos are used, based either on purely classic models 106,107 or on QCD improved 108 in which the early stages (quark-gluon cascades) are modelled by QCD fragmentation functions. Assuming that multiplicity of hadrons in the real jet fragmentation is proportional to the total quark/gluon multiplicity calculated perturbatively one can calculate the energy dependence of the hadron multiplicity 109-111

$$N(W) \sim \exp \left[ \frac{\kappa W}{\Theta_0} \right]$$

(111)

where $\Theta_0 = \Lambda - \frac{2}{3} M_1 = g$, $M_1 = 3$

4.11 QCD analysis of high-$Q^2$ elastic processes

The (approximate) scaling observed in deep inelastic scattering, as emphasized earlier, was an unambiguous indication that there are pointlike
particles inside the hadrons. At the same time, it was clear that these pointlike entities are not the hadrons themselves since their elastic form factors were observed to be fast varying functions of the momentum transfer $Q^2$. This gave one of the strongest arguments in favour of the hypothesis that the hadrons are composite particles made of some pointlike constituents. Furthermore, it was established empirically that the elastic form factors have a very simple power-like behaviour for $Q^2 \approx 1$ GeV$^2$:

$$ F_{\pi} (Q^2) \sim \frac{\Lambda}{Q^2} \tag{112} $$

for the pion EM form factor,

$$ G^n_{M}, G^\pi_{M}, G^n_{E} \sim \frac{\Lambda}{Q^4} \tag{113} $$

for the proton and neutron form factors. In 1973 Matveev, Muradyan and Tavkhelidze$^{112}$ noticed that there exists a remarkable correlation between the $Q^2$-powers in eqs. (112), (113) and the quark content of the relevant hadron:

$$ \mathcal{F}^{(n)} (Q^2) \sim (\frac{\Lambda}{Q^2})^{n-4} \tag{114} $$

where $n$ is the number of quarks inside the hadron as dictated by the quark model. This "quark counting rule" is dynamically realized for large $Q^2$ in any field - theoretical model with a dimensionless coupling constant. To convince oneself, one should consider the hard rescattering diagrams$^{113}$ (fig. 4) the large - $Q^2$ behaviour of which is given by just counting the "hot" gluonic propagators (the effect of quark propagators $\sim Q^2/Q^2$ is compensated by the Dirac spinors $u(p)\bar{u}(p')\sim \mathcal{D}$ related to the corresponding quark line).

The simple parton-like picture for the high-$Q^2$ asymptotics of the hadronic form factors was justified later within the framework of perturbative QCD$^{114-117}$. It was demonstrated, in particular, that the QCD description in this case is equivalent to a parton picture of a new type in which the hadrons are described by the "parton wave functions" $\Psi(x_1, \mu^2)$ (for pion), $\Psi(x_1, x_2, x_3\equiv 1-x_1-x_2; \mu^2)$ (for nucleons), etc. describing, e.g., the splitting of pion with momentum $P$ into a $q\bar{q}$-pair with momenta $xP$ and $(1-x)\bar{P}$. The formal definition of these functions is quite analogous to that of the distribution functions. Their moments are given by the reduced matrix elements of local composite operators. For instance, the pion wave function $\Psi(x, \mu^2)$ is defined by$^{114}$

$$ \frac{1}{2} \langle 0 | \{ \bar{u} \gamma_5 \gamma_\mu D_{\mu_4} \ldots D_{\mu_n} d \} | P \rangle = \{ P^T_{\mu_1} \ldots P^T_{\mu_n} \} \frac{\Lambda(x)}{2} \tag{115} $$

where

$$ \mathcal{G}_n (\mu^2) = \int_0^1 x^n \Psi(x, \mu^2) dx \tag{116} $$
The pion form factor asymptotics can be now written in terms of $\varphi(x,\mu^2)$:

$$F_\pi(Q^2) = \frac{8\pi}{9} \int^1_0 dx \int^\Lambda dy \frac{d^2\eta}{xyQ^2} \varphi(x,\mu^2) \varphi(y,\mu^2) \left[ 1 + O(\alpha_s) + O(1/Q^2) \right]$$

(117)

The parton wave functions $\varphi(x,\mu^2)$ evolve with increasing the normalization scale $\mu^2$. The relevant evolution equation has the form

$$\left( \frac{d^2}{d\mu^2} + \beta(q) \frac{d}{dq} \right) \varphi(x,\mu^2) = \int^\Lambda_0 V(x,y) \varphi(y,\mu^2) dy$$

(118)

where the evolution kernel $V(x,y)$ can be calculated either directly or from the corresponding anomalous dimension matrix $\tilde{Z}_{nk} \equiv \frac{\partial}{\partial \log q}$ appearing in the renormalization-group equation

$$\left( \frac{d^2}{d\mu^2} + \beta(q) \frac{d}{dq} \right) \varphi_{nk}(\mu^2,\tilde{q}) = \sum_{n=0}^n \tilde{Z}_{nk} \varphi_n(\mu^2,\tilde{q})$$

(119)

In the lowest order

$$V(x,y) = \frac{d_s}{2\pi} \left\{ \frac{x}{y} \left( 1 - \frac{1}{x-y} \right) \Theta(x<y) + (x \rightarrow y, y \rightarrow y) \right\}$$

(120)

where the action of the "plus"-operation

$$\left[ V(x,y) \right]_+ = V(x,y) - \delta(x-y) \int^\Lambda_0 V(x,y) dy$$

(121)

is analogous to subtraction of $p_a$ in the ordinary evolution equation (see eq. (96)).

It was established that asymptotically $\varphi_{\pi}(x,\mu^2)$ tends to a very simple form

$$\varphi(x,\mu^2 \rightarrow \infty) = C_f_{\pi} x(1-x)$$

(122)

where $f_{\pi} \approx 133$ MeV is the pion decay constant

$$\langle 0 | \bar{u} \gamma_5 v | p \rangle = i f_{\pi} P_{\mu} = i P_{\mu} q_0$$

(123)

appearing due to the definition (115), (116) and eq. (123). Substituting eq. (122) into eq. (117) one obtains that asymptotically

$$F_\pi(Q^2) = \frac{8\pi \alpha_s(Q^2)}{9 f_{\pi}^2} \left[ 1 + O(\alpha_s) + O(1/Q^2) \right]$$

(124)

Thus, asymptotically the quark counting rule (114) is reproduced in the form
\[ \mathcal{F}^{(m)}(Q^2) \sim \left( \frac{\alpha_s(Q^2)}{Q^2} \right)^{n-1} \] (125)

implying an additional \( \left( \frac{1}{\beta_0 Q^2} \right)^{n-1} \) decrease due to the coupling-constant evolution. The nucleon form factors were analyzed in refs. 116, 17, 118.

A very important question, however, is how large should be the momentum transfer to be considered as an asymptotically large one? By analogy with inclusive processes one may expect that \( Q^2 \) is large if \( \sqrt{Q^2} \gg M^2 \), \( M^2 \) being a typical hadronic scale like \( m_P^2 \) (\( m_P^2 \approx 0.6 \text{ GeV}^2 \)). But note that the asymptotic QCD chains involve the \( \alpha_s^{-n+1} \) suppression (see eq. (125)) which in fact can be shown to be even \( \left( \frac{\alpha_s}{\pi} \right)^{n-1} \), i.e. \( 1/4\pi \) for pion and \( 1/40\pi \) for nucleons. Thus, the asymptotic QCD contribution has a sizeable suppression compared to that of soft diagrams (fig. 5) containing no gluon exchanges at all. The soft contributions, however, have a faster fall-off \( (1/Q^2) \) for \( F_T(Q^2) \) and \( 1/Q^2 \) for \( G_T^M(Q^2) \), and asymptotically the hard rescattering dominates. This dominance according to the above arguments is expected to be observable only for \( Q^2 \gtrsim 10 \text{ GeV}^2 \) for \( F_T(Q^2) \) and for \( Q^2 \gtrsim 100 \text{ GeV}^2 \) for \( G_T^M(Q^2) \), i.e. outside the experimentally investigated region. This means that to understand the existing data on hadronic form factors, one should be able to calculate the contributions of soft diagrams (fig. 5). This was made recently 119-123 within the framework of the QCD sum rule approach 124.

5. QCD SUM RULES

5.1 Introductory remarks

QCD sum rule approach is an attempt to calculate various low-energy hadronic characteristics (masses, decay widths, form factors at intermediate and low momentum transfers, etc.) using the asymptotic freedom property of QCD and some nonperturbative information about the structure of the QCD vacuum. It should be emphasized that the very idea to incorporate asymptotic freedom for calculating the low-energy parameters of real hadrons is quite nontrivial since the perturbative QCD calculations are reliable in the deep space-like (sometimes referred to as Euclidean) regions. To illustrate this point, let us consider the correlator of two electromagnetic currents

\[ \Pi_{\mu \nu}(q) = i \int d^4x < T \ J^\mu(x) J^\nu(0) > e^{iqx} \] (126)

In perturbation theory \( \Pi_{\mu \nu}(q) \) is given by diagrams shown in fig. 6, and the resulting \( \alpha_s \)-expansion is reliable for \( q^2 \) negative and furthermore \( q^2 \lesssim -1 \text{ GeV}^2 \).

On the other hand, physical states, i.e., the hadrons, contribute to the imaginary part of \( \Pi(q) \) in the region of positive \( q^2 \). In our case \( \Re \Pi(q) \) is proportional to the total cross section of the \( e^+e^- \)-annihili-
lation into hadrons. It contains the \( \rho \), \( \omega \) and \( \phi \) peaks and also some background smooth contribution (e.g., the \( \pi^{+}\pi^{-}\) contribution for \( q^2 \ll m_{f}^2 \) ) which tends for large energies to the parton model prediction \(^{124}\))

\[
\sigma_{\text{tot}} \left( e^+ e^- \rightarrow \text{hadrons} \right) \bigg|_{\text{PM}} = \mathcal{N} c \sum_{q} e_{q}^{2}
\]

(127)

The logic behind the PM prediction is the following: for high energies, when many hadronic channels are open, the \( q \bar{q} \) -pair produced in the \( e^+ e^- \) -annihilation process converts into some hadronic state with the 100% probability, and, hence, the cross section of the hadron production equals to that of the \( q \bar{q} \) -pair production. However, for low energies only a few channels are open, and one should remember that quarks are really quantum rather than classic particles, and hence the reasoning in terms of probabilities is not always adequate: for energies close to the \( \rho \)-mass, e.g., one should say that the probability of \( \rho \)-production is higher than that of \( q \bar{q} \)-pair production whereas for energies below \( \rho \) (and also slightly above) the probability of \( e^+ e^- \rightarrow \text{hadrons} \) is smaller than that of \( e^+ e^- \rightarrow \text{quarks} \).

Thus, the question is: can one tell anything about the behaviour of \( \text{Im} \tau(q^2) \) in the resonance region being able to calculate only \( \tau(q^2) \) in the deep space-like region? A straightforward analytic continuation of the contributions corresponding to the perturbation theory diagrams (fig. 6) from the space-like \( q^2 < 0 \) to the timelike \( q^2 > 0 \) region would produce just the smooth parton model cross section modified only by the \( \alpha_s \) -corrections. The latter, however, are small and even if summed over all orders will not produce anything similar to the \( \rho \)-state. This observation might suggest that the behaviour of \( \tau(q^2) \) in the deep space-like region contains no information about the resonance structure for time-like \( q^2 \). However, this is impossible because \( \tau(q^2) \) is an analytic function and its behaviour for space-like \( q^2 \) is correlated with that of \( \text{Im} \tau \) for timelike \( q^2 \).

5.2. Dispersion relations

The connection between \( \tau(q^2) \) and spectral density \( \text{Im} \tau(q^2) = \rho(s) \) (where \( s \approx q^2 \)) is given by the dispersion relation

\[
\tau(q^2) = \frac{\Lambda}{\pi} \int_{0}^{\infty} \frac{\rho(s)}{s-q^2} \, ds
\]

(128)

We are interested in negative \( q^2 \) 's, so it makes sense to denote \( q^2 = -Q^2 \) with \( Q^2 > 0 \). Then the weight function in eq. (128) becomes

\[
\frac{\Lambda}{s-q^2} = \frac{\Lambda}{s+Q^2}
\]

(129)

According to eq. (128) if one knows \( \tau(q^2) \) for space-like \( q^2 \), one can calculate integral characteristics of the hadronic spectrum (e.g., weighted cross sections). Note that the higher \( Q^2 \), the more reliable the perturbative QCD calculation. But the higher \( Q^2 \) the broader the weight function and the more hadronic states contribute to the dispersion integral. In
other words, the further one goes to the left along the $Q^2$ axis, the wider region on the timelike side can be seen, but at the same time the vager is the picture of each peak. On the other hand, for small $Q^2$ one sees only the first peak but $\Sigma(Q^2)$ is known worse. Recall also that the perturbative contribution $\Sigma^{pert}(Q^2)$ also satisfies the dispersion relation

$$\Sigma^{pert}(Q^2) = \frac{A}{\pi} \int_0^\infty \frac{\rho^{pert}(s)}{s+Q^2} ds$$  \hspace{1cm} (130)$$

where instead of $\rho(s) = \rho^{hadr}(s)$ one has $\rho^{pert}(s) = \rho^{pert} (1 + O(a_s))$. Hence, the "exact" correlator $\Sigma(Q^2)$ should differ from its perturbative counterpart because $\rho^{hadr}(s)$ differs from $\rho^{pert}(s)$. The difference is most pronounced just for small $Q^2$ where the first resonance dominates the dispersion integral. An obvious consequence is that there must exist some contributions to $\Sigma^{exact} \approx $ hard responsible for the difference between $\Sigma^{exact}$ and $\Sigma^{pert}$. Since all the perturbation theory contributions are supposed to be included into $\Sigma^{pert}$, the additional contributions must be of a nonperturbative origin.

5.3 Quark and gluon condensates

The basic idea is very simple and natural: the exact quark and gluon propagators for small momenta essentially differ from the perturbative ones. The difference should disappear for large momenta to restore the asymptotic freedom. Mathematically, this may be most conveniently formulated in the configuration space. Consider the quark propagator

$$\Sigma(x) = \langle \bar{\Psi}(x) \Psi(0) \rangle$$  \hspace{1cm} (131)$$

where $\langle \rangle$ corresponds to averaging over the exact vacuum. Using the Wick theorem (see, e.g., 45) one can write

$$\bar{\Psi}(x) \Psi(0) = \bar{\psi}(x) \tilde{\psi}(0) + \bar{\psi}(x) \tilde{\psi}(0)$$  \hspace{1cm} (132)$$

where the pairing $\psi \tilde{\psi}$ corresponds to substituting $\psi(x) \tilde{\psi}(0)$ by the perturbative propagator $\Sigma^{pert}(x)$ and $\bar{\psi} \tilde{\psi}$ denotes the normal product. By definition the latter vanishes after averaging over the perturbative vacuum but gives nonzero terms after averaging over other states (e.g., $\langle \phi \bar{\psi}(x) \tilde{\psi}(0) | \phi \rangle$ is general nonzero). It is easy to realize now that the statement that $\Sigma^{exact} \neq \Sigma^{pert}$ just means that $\langle \bar{\psi}(x) \tilde{\psi}(0) \rangle \neq 0$. The coefficients of the Taylor expansion

$$\langle \bar{\psi}(x) \tilde{\psi}(0) \rangle = \langle \bar{\psi}(0) \tilde{\psi}(0) \rangle + \sum_{n=1}^\infty \frac{x^{\mu_1} \cdots x^{\mu_n}}{n!} \langle \bar{\psi}(0) D_{\mu_1} \cdots D_{\mu_n} \tilde{\psi}(0) \rangle$$  \hspace{1cm} (133)$$

are examples of the nonperturbative vacuum averages characterizing (and parametrizing) the structure of the QCD vacuum. The most important is the first term $\langle \bar{\psi}(0) \tilde{\psi}(0) \rangle \langle \bar{q} q \rangle$ usually referred as the quark condensate.

The relevant corrections to $\Sigma(Q^2)$ can be rather easily calculated using Wick's theorem and expansion (133). The resulting diagrams are just
the ordinary Feynman diagrams with some external lines having zero momenta. In a similar way one arrives at the gluon condensate \( \langle G_{\mu \nu}^a G_{\mu \nu}^a \rangle \). Note that the gluonic field appears there only through the field \( G_{\mu \nu}^a \) rather than potential. This is a consequence of the gauge invariance.

5.4 Sum rules

As a result one obtains the modified expansion for \( \tilde{\Pi}(Q^2) \)

\[
\tilde{\Pi}(Q^2) = \tilde{\Pi}^{pert}(Q^2) + A \left\langle \frac{\langle G\rangle}{Q^6} \right\rangle + B \frac{\langle G G \rangle}{Q^6} + C \frac{\langle \sigma q \rangle}{Q^6} + \ldots
\]

(134)

in which the nonperturbative contributions appear as power corrections \( (\ln Q)^N \) to \( \tilde{\Pi}^{pert}(Q^2) \sim \ln Q^2 \). The relevant power is given by dimensional counting \( \left[ \langle \sigma q \rangle \right] \sim m^3 \), \( \left[ \langle G \rangle \right] \sim m^4 \), etc. Note that the power corrections have just the desired behaviour: they blow up for small \( Q^2 \), thus signaling about the presence of sizeable deviations from \( \tilde{\Pi}^{pert}(s) \) for small \( s \).

The deviation satisfies the following sum rule

\[
\frac{1}{\pi} \int_0^\infty \frac{\rho_{\text{had}}(s) - \rho_{\text{pert}}(s)}{s + Q^2} ds = A \left\langle \frac{\langle G\rangle}{Q^6} \right\rangle + B \frac{\langle G G \rangle}{Q^6} + C \frac{\langle \sigma q \rangle}{Q^6} + \ldots
\]

(135)

where the coefficients \( A, B, C, \ldots \) can be calculated from the (condensate-modified) Feynman diagrams. The last problem is to find \( \rho_{\text{had}}(s) \) that provides the best agreement between the sides of the sum rule. Of course, it is impossible from the integral (and approximate) relation to calculate all the details of \( \rho_{\text{had}}(s) \), but one can extract information about the gross features of the spectrum. Normally the precision of QCD sum rule results is about 10-15%.

As a rough picture of the spectrum for the correlator with the \( \rho \)-meson quantum numbers one can take the "\( \rho \) + continuum" ansatz

\[
\rho_{\text{had}}(s) = \rho_\rho \delta(s - m_\rho^2) + \Theta(s - s_0) \rho_{\text{pert}}(s)
\]

(136)

implying that the higher states are well described by the parton cross section. Then the sum rule (135) takes the finite-energy form

\[
\int_0^{s_0} \left( \rho_{\text{had}}(s) - \rho_{\text{pert}}(s) \right) \frac{ds}{s + Q^2} = A \left\langle \frac{\langle G\rangle}{Q^6} \right\rangle + \ldots
\]

(137)

In fact, instead of eq. (137) the borelized sum rule \(^{123}\)

\[
\int_0^{s_0} \left( \rho_{\text{had}}(s) - \rho_{\text{pert}}(s) \right) e^{-\frac{s}{M^2}} \frac{ds}{M^2} = A \left\langle \frac{\langle G\rangle}{M^6} \right\rangle + \ldots
\]

(138)

is used with the exponential weight. It has the merits that the higher states are better damped (and, hence, the roughness of our ansatz (136) less seriously affects the precision of the sum rule) and, moreover, that coefficients for the higher-dimensional condensates are smaller. To get eq.
(138) from eq. (137), one should apply the borelization operator $B(Q^2 \rightarrow m^2)$ acting only on $Q^2$

$$B(Q^2 \rightarrow m^2) \left\{ \frac{1}{s+Q^2} \right\} = \frac{e^{-S/m^2}}{m^2}$$  \hspace{1cm} (139)

The use of the borelized sum rules also removes the problem of possible divergences in the dispersion integrals (these appear, e.g., if $q_{\text{part}}(s) \sim \text{const.}^3$). 

5.5 Quark-hadron duality

Notice that since there are no condensate of $(mass)^2$ dimension, there are no $1/m^2$ terms in the l.h.s. of the sum rule (138). So, taking $M^2 \rightarrow \infty$ one obtains the global duality relation

$$\int_0^{s_0} \left( \rho_{\text{had}}(s) - \rho_{\text{part}}(s) \right) \, ds = 0$$  \hspace{1cm} (141)

that states that the deviation of $\rho_{\text{had}}$ from its parton counterpart is zero if averaged over the whole energy range. In other words, the "excess probability" of hadron production in the region of the $p$-peak is exactly compensated by its deficiency in other regions: probability is conserved in this sense. It does not disappear, it is only redistributed between the states.

Furthermore, if one adheres to the ansatz (136) the global duality is reduced to the local one

$$\int_0^{s_0} \rho_{\text{part}}(s) \, ds = \int_0^{s_0} \rho_{\text{had}}(s) \, ds \equiv \mu q_{\rho}$$  \hspace{1cm} (142)

It should be emphasized that eq. (142) is satisfied only for some specific value of $s_0$, the effective onset of the continuum.

The duality is rather old concept. It appeared first in the studies of high-energy hadron-hadron scattering within the framework of the finite-energy sum rules\textsuperscript{125).} The quark-hadron duality concept was used first just in the studies of $e^+e^-$ annihilation into hadrons\textsuperscript{126-127).}

In our case the local duality gives the relation between $q_{\rho}$ and $s_0$ :

$$q_{\rho} = \frac{\alpha}{\pi} \int_0^{s_0} \rho_{\text{part}}(s) \, ds = \frac{3s_0}{8\pi^2} \left( 1 + \frac{\alpha}{\pi} \right)$$  \hspace{1cm} (143)

The remaining parameters (e.g., $s_0$ and $m_{\rho}^2$) can be extracted from the sum rule using the requirement of the best agreement between its sides.

5.6 Extraction of the hadron parameters from sum rules

The sum rule in our case is

$$q_{\rho} e^{-m_{\rho}^2/m^2} = \frac{3}{2} M^2 \left( 1 + \frac{\alpha}{\pi} \right) \left( 1 - e^{-S/m^2} \right) + \frac{M^2}{2m^2} \frac{\alpha}{\pi} \langle G_G \rangle$$  \hspace{1cm} (144)
\[ - \frac{224}{27} \pi^3 \frac{\alpha_s <q^4>^2}{M^4} + \ldots \]

Differentiating it with respect to $\lambda M^2$ one obtains the "daughter" sum rule

\[ q^2_{\perp} \frac{m_t^2}{M^2} \left( 1 - \left( 1 + \frac{m_t^2}{M^2} \right)^{-2} \right) <q^2> \]

\[ = \frac{3}{2} M^4 \left( 1 + \frac{\alpha_s}{\pi} \right) \left[ 1 - \left( 1 + \frac{s_0}{M^2} \right)^{-2} \right] \]

\[ - \frac{\pi^2}{2} \frac{\alpha_s}{\pi} <G G> + \frac{4 \alpha_s}{27} \pi^3 \frac{\alpha_s <q^4>^2}{M^4} \]

(145)

Dividing eq. (145) by eq. (144) and using the values of quark and gluon condensates \(^{123}\)

\[ \alpha_s <q^4> = 1.8 \times 10^{-4} \text{ GeV}^6 \]

(146)

\[ (\frac{\alpha_s}{\pi}) <G G> = 0.012 \text{ GeV}^4 \]  

(147)

one obtains \( m^2_t \) as a function of the weight function parameter \( M^2 \) and the continuum threshold \( s_0 \). The latter is fixed from the requirement that \( m^2_t(M^2, s_0) \) should be maximally close to a constant in the "reliable" region \( 0.5 \leq M^2 \leq 1 \text{ GeV}^2 \) where both the continuum contribution and power corrections are smaller than 30%. This requirement fixes \( s_0 \) at \( 1.5 \pm 0.2 \text{ GeV}^2 \), and \( m^2_t \) is then \( 0.6 \pm 0.05 \text{ GeV}^2 \) (compared to the experimental value \( m^2_t = 0.59 \text{ GeV}^2 \)).

In a similar way one can get the sum rule for the pion decay constant \( f_\pi \) defined by the matrix element of the axial current

\[ <0| \bar{u} \gamma_\mu \gamma_5 d | \pi > = \frac{i}{f_\pi} P_\mu \]

(148)

where \(| \pi >\) is a pion state with momentum \( P \). In this case one should analyze the correlator of two axial currents. The relevant sum rule looks as follows

\[ f^2_\pi = \frac{1}{4\pi^2} \left( 1 + \frac{\alpha_s}{\pi} \right) \left( 1 - e^{-S_0/M^2} \right) + \frac{\alpha_s}{12\pi} \frac{<G G>}{M^2} + \frac{4 \alpha_s}{81 \pi} \frac{<q^4>^2}{M^4} \]

(149)

(the pion mass is approximated by zero, that is the true value for the massless quarks). Proceeding just as described above one obtains that \( s_0 = 0.7 \pm 0.1 \text{ GeV}^2 \) and \( f_\pi = 130 \pm 10 \text{ MeV} \) \(^{123,128}\) compared to \( f_\pi^{exp} = 133 \text{ MeV} \). To get information about the low-energy parameters of the nucleons one should analyze the correlators of the $j$-quark currents, e.g. for the proton one can take the current

\[ \eta_{\gamma} = (u \bar{c} \gamma_\mu d) \]

(150)

with the change \( u \leftrightarrow d \) for the neutron. The QCD sum rules give in this case \(^{129-131}\): \( m^2_N = (1.0 \pm 0.1) \text{ GeV}^2 \)

\[ s_0^N = (2.3 \pm 0.2) \text{ GeV}^2 \]  

(151)

for the nucleon mass and threshold parameter, and
\[ \lambda_N^2 \approx 6 \cdot 10^{-4} \text{ GeV}^6 \approx \frac{S_0}{\sqrt{12} \langle \sigma^3 \rangle} \tag{152} \]

for the nucleon decay constant \( \lambda_N \)

\[ \langle 0 | \eta_d | P \rangle = \lambda_N \langle P \rangle \tag{153} \]

5.7 Local quark-hadron duality and soft hadronic wave functions

A very suggestive observation (that helps us to understand the physical reasons for the successes of the QCD sum rules) is that incorporating the local duality ansatz (142) is analogous to fixing the soft pion wave function. Indeed, taking into account that \( \rho_{\eta \eta} \langle s, Q^2 \rangle \) is the squared amplitude for the \( j \rightarrow q\bar{q} \) transition, it is easy to realize that the prescription (142) reduces to substituting the \( q\bar{q} \pi \) vertex by the local one corresponding to transition of the \( j \) -current into free massless quarks with a subsequent averaging of the invariant mass of the 2-quark system over the region \( 0 \leq s \leq S_0 \). In terms of the infinite momentum frame variables \( x, k_\perp \) this means that

\[ \Psi_{\pi q\bar{q}} (x, k_\perp) \sim \theta \left( \frac{k_\perp^2}{\frac{x}{1-x} S_0} \right) \tag{154} \]

This wave function should be compared with the Gaussian \( \Psi^{(G)}_\eta (x, k_\perp) \sim \exp \left( -R^2 k_\perp^2 / x \right) \) and power-law \( \Psi^{(P)}_\eta (x, k_\perp) \sim (\mu^2 + k_\perp^2 / x)^{-\Delta} \) model wave functions considered in ref. [132]. All the wave functions have a common property: the cut-off for large \( x^2 \) values (where \( x^2 = k_\perp^2 / (1-x) \)). Of course, the sharp cut-off \( x^2 \leq S_0 \) dictated by the local duality is unrealistic; the exact pion wave function should be a smooth function of \( x^2 \) like \( \Psi^{(G)} \) or \( \Psi^{(P)} \). This means that \( \Psi_{\pi q\bar{q}} \) can reliably reproduce only the most general (integral) properties of the exact pion wave function. However, the predictive power of \( \Psi_{\pi q\bar{q}} \) is higher than that of the model wave functions since \( S_0 \) is not a free parameter.

The local duality wave function is evidently the relativistic generalization of the good old-transverse momentum cut-off. Using eq. (154) one can calculate the average momentum of the valence quarks inside the pion

\[ \langle k_\perp^2 \rangle_{\pi} \bigg|_{L \perp} = \frac{S_0}{10} \approx (260 \text{ MeV})^2 \tag{155} \]

which is just what one may expect. In a similar way one calculate \( \langle k_\perp^2 \rangle \) for quarks inside the nucleon

\[ \langle k_\perp^2 \rangle_{N} \bigg|_{L \perp} = \frac{4S_0}{k_3} \approx (360 \text{ MeV})^2 \tag{156} \]

which is also in the expected range.

5.8 QCD sum rules and hadronic form factors

To get information about the hadronic form factors, one should consider the 3-point correlators containing, in addition to the 2-point case,
e.g., the electromagnetic current corresponding to the probe. The 3-point invariant amplitudes $T_i$ are the functions of external momenta, say, $p_1^2$, $p_2^2$ and $q^2 - Q^2$. Instead of eq. (128) one should use now a double dispersion relation

$$T(p_1^2, p_2^2, Q^2) = \frac{A}{\pi^2} \int_0^\infty ds_1 \int_0^\infty ds_2 \frac{\rho(s_1, s_2, Q^2)}{(s_1 - p_1^2)(s_2 - p_2^2)}$$

(157)

or its (doubly) borelized version. As a rough ansatz for the spectral function one can use the 2-dimensional generalization of eq. (136), e.g. for the pion form factor related density one can take $^{119,120}$

$$\rho(s_1, s_2, Q^2) = \pi^2 \int_0^\infty F_{\pi}(Q^2) + \rho^{pert}(s_1, s_2, Q^2) \left[ 1 - \Theta(s_1 < s_0) \Theta(s_2 < s_0) \right]$$

(158)

This reduces to assuming that everywhere outside the square $s_1, s_2 \leq s_0$ the hadronic density coincides with the quark one. The resulting sum rule for the pion form factor is $^{119-120}$

$$\int_0^\infty F_{\pi}(Q^2) = \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \exp \left\{ - \frac{s_1 + s_2}{m^2} \right\} \rho^{pert}(s_1, s_2, Q^2) +$$

$$+ \frac{2 \delta}{2\pi} \frac{\delta s \leq q \times q^2}{8\pi M^4} \left( 1 + \frac{2}{13} \frac{Q^2}{M^2} \right)$$

(159)

where the perturbative (quark) density is given by

$$\rho^{pert}(s_1, s_2, Q^2) = \frac{3}{4} Q^4 \left\{ \left( \frac{d}{dq^2} \right)^2 + \frac{Q^2}{3} \left( \frac{d}{dq^2} \right)^3 \right\}$$

$$\left[ (s_1 + s_2 + Q^2 - 4 s_1 s_2)^{-1/2} \right]$$

(160)

The procedure of extracting $\int_0^\infty F_{\pi}(Q^2)$ from this sum rule is just the same as for $f_{1\pi}^+$. The threshold parameter $s_0$ extracted from the sum rules for $Q^2 = 1, 2$ and 3 GeV$^2$ coincides within 15% with the 2-point value $s_0 = 0.7$ GeV$^2$. This ensures one that the whole approach is fairly self-consistent. The values of $F_{\pi}(Q^2)$ obtained from the best agreement requirement are well reproduced by the local duality formula

$$\int_0^\infty F_{\pi}(Q^2) = \frac{A}{\pi^2} \int_0^{s_0} ds_1 \int_0^{s_0} ds_2 \rho^{pert}(s_1, s_2, Q^2)$$

(161)

$$= \frac{s_0}{4\pi^2} \left( 1 - \frac{1 + 6 s_0 Q^2}{(1 + 4 s_0 Q^2)^3} \right)$$

and are in a very good agreement with existing experimental data$^{134}$ for $Q^2 \geq 0.6$ GeV$^2$.

The local duality estimates for the nucleon form factors are also very successful in the region $1 \leq Q^2 \leq 20$ GeV$^2$ $^{133,135}$. 

\[\text{End of document.}\]
In the region of small $Q^2$ one should modify the operator product expansion because there appear new contributions related to the long distances in the $Q^2$ -channel. The technique of such a modification was developed in refs. 121, 122, 136-139. Among the results obtained in this field I want to mention a successful calculation of the proton and neutron magnetic moments 136, 138)

$$\mu_p = 3 \pm 0.3 \quad , \quad \mu_n = 2 \pm 0.2$$

(exp.: $\mu_p = 2.79$, $\mu_n = 1.93$), pion charge radius$^{121}$

$$\langle r^2_p \rangle = (0.44 \pm 0.04) \frac{1}{m^2} \quad (163)$$

(exp.: 0.43$^{+0.03}_{-0.03}$, 140, 141); pion 121) and nucleon 122, 142) form factors for $Q^2 < 1$ GeV$^2$ and the constant $g_A$ 137, 139)

$$g_A = 1.4 \pm 0.2 \quad (164)$$

(exp.: 1.20$^{+0.02}_{-0.02}$).

The use of the QCD sum rules was also very successful in the calculation of masses and decay widths of heavy quarkonia (see 123, 143) and references therein), the most remarkable result here being the prediction of the $\eta_c$ mass 144).

REFERENCES

39) P.N. Bogoliubov, JINR reprint P2-3115, Dubna (1967);
49) G. 't Hooft. Nucl. Phys. 61B (1973) 455.
Fig. 1 First terms of the QCD expansion for the structure functions of deep inelastic scattering.

Fig. 2 Diagram producing interference contributions.

Fig. 3 QCD expansion for the Drell-Yan process.

Fig. 4 Hard rescattering diagram for proton form factor.

Fig. 5 Soft diagrams for a) pion and b) nucleon form factors.

Fig. 6 Perturbative contributions to the 2-point correlator $\Pi_{\mu\nu}(q)$. 
PROTON–ANTIPROTON COLLIDER PHYSICS

P.1.P. Kalmus
Queen Mary College, London, UK

The following is a reproduction of the set of transparencies used for the lectures. The topics treated were:

1. The CERN $p\bar{p}$ Collider
2. Description of experiments at the CERN $p\bar{p}$ Collider
3. Total and elastic cross-sections and some ‘minimum-bias’ physics
4. Physics of W and Z particles
5. Evidence for top quark
6. Jet physics
### Why Build a Collider?

<table>
<thead>
<tr>
<th>Fixed Target</th>
<th>High Interaction Probability, But Low C.M. Energy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^* = \sqrt{2E_{\text{cm}}}$</td>
<td></td>
</tr>
<tr>
<td><strong>SPS</strong></td>
<td></td>
</tr>
<tr>
<td>$E = 450 \text{ GeV}; E^* = 29 \text{ GeV}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Colliding Beams</th>
<th>High C.M. Energy, But (So Far) Low Int. Prob. Per Pass</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E^* = 2E$</td>
<td></td>
</tr>
<tr>
<td>$E_{\text{cm}}$</td>
<td></td>
</tr>
<tr>
<td>$270 + 270 \text{ GeV} E^* = 540 \text{ GeV}$</td>
<td></td>
</tr>
<tr>
<td>Later measured as 273 GeV!</td>
<td></td>
</tr>
<tr>
<td>$315 + 315 \text{ GeV} E^* = 630 \text{ GeV}$</td>
<td></td>
</tr>
<tr>
<td>EQUIV. $E = 211 \text{ TeV}$</td>
<td></td>
</tr>
<tr>
<td>$450 + 450 \text{ GeV} E^* = 900 \text{ GeV}$</td>
<td></td>
</tr>
<tr>
<td>EQUIV. $E = 432 \text{ TeV}$</td>
<td></td>
</tr>
</tbody>
</table>

Need high c.m. energy so that hard collisions between partons can create $W$ and $Z$.

(Also need high luminosity – see later lecture)

Only 1 ring of magnets at SPS (protons).

Need antiprotons.
THE CERN COLLIDER

To obtain intense, collimated, monoenergetic beams: →

Protons: no problem
Antiprotons: big problems!

Target: 110 x 3 mm copper or tungsten

26 GeV
Protons from PS
～ $10^{13} / 2.4 s$

Mixed beam of many types
All angles, all momenta
Low $\bar{p}$ yield

Need to

Store
Purify (this is free!)
Accumulate
Cool

2 ways: electron cooling

Stochastic

Used in present schemes, CERN, Fermilab

Antiproton cooling requirements

$10^{13} \bar{p} \ (26 \text{ GeV}) \rightarrow 1-2 \ 10^7 \bar{p} \ \text{into a phase space volume}$

To get $3 \ 10^7 \bar{p}$
Need ~ 2-4 $10^4 \bar{p}$ pulses (~1 day)

$A_H = 100 \pi (\text{mm}^2 \text{mrad})$
$\Delta p/p = \pm 0.75\%$

Total phase-space ~ $3.6 \ 10^7 \% (\text{mm}^2 \text{mrad})^2$
This is about a 10^8 greater than injection acceptance of SPS after allowing for acceleration in PS.

Need reduce by factors:
- ~10^2 radial
- ~10^2 vertical
- ~10^5 momentum

10 each pulse
10^4 stack

Needed to build Antiproton Accumulator (AA)
- 156 m circumference (= Ps + A)
- Wide aperture

Stochastic cooling

Illustrate transverse (radial) cooling

If infinite bandwidth, immediate response, no signal degradation

→ Get correction individually for every antiproton → Genuine Maxwell demon!

In practice average over centroid of beam

→ Improve some, worsen others → "Stochastic"

Under correct conditions get overall improvement
TECHNIQUE USED IN AA

1. SHUTTER CLOSED
2. CENTRE OF MACHINE
   INJECT FIRST PULSE
   \( \sim 1.2 \times 10^7 \) p
3. 2.0 s PRECOOL \( \sim 10^7 \)
4. 2.3 s MOVE INTO STACK POSITION
5. 2.4 s INJECT SECOND PULSE
6. 4.7 s MOVE INTO STACK AFTER PRECOOLING \( \sim 2 \times 10^9 \)
7. 4 m STACK INTENSITY NOW \( 10^9 \)
8. 2 h DENSE CORE FORMING
9. \( \sim 1 \) day \( \sim 3 \times 10'' \) p

EJECT DENSE CORE
LEAVE REMAINDER
FOR MORE COOLING

POSITION IN CM
-40 -20 0 +20 +40
2 experimental areas
underground
"LSS 5" and "LSS 4" 1/6 apart
"minimal requirement \( \rightarrow 2(p) \text{ bunches} \times 1(\bar{p}) \)

Normal operation \( 3p \times 3\bar{p} \) bunches

Bunch size \( \sim 30 \text{ cm} \times \approx 1 \text{ mm diameter} \)

Time between bunch crossings = 7.6 \( \mu s \) use to receive data

And decide whether to record
COLLIDER PERFORMANCE

COLLISION RATE = LUMINOSITY x CROSS SECTION

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<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>LUMINOSITY</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( (10^{29} \text{cm}^{-2}\text{s}^{-1}) ) PEAK</td>
<td>0.5</td>
<td>1.7</td>
<td>3.5</td>
<td></td>
</tr>
<tr>
<td>AVERAGE START OF RUN</td>
<td>0.1</td>
<td>0.5</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>INTEGRATED LUMINOSITY</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( (\text{nb} \equiv 10^{33} \text{cm}^{-2}) ) PER DAY PER YEAR</td>
<td>0.4</td>
<td>1.8</td>
<td>5.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>2.8</td>
<td>153</td>
<td>3.95</td>
</tr>
</tbody>
</table>

Note: Luminosity falls off exponentially during a run. Typical (1/2) life \( \approx 18 \) hrs

"GOOD RUNNING CONDITIONS"
Set up machine, some pilot shots \( \approx 4 \) hours
Fill with dense shot, then coast \( \approx 20 \) hours
Repeat each day (meanwhile AA refilled)

PULSED COLLIDER RUNS. MARCH 1985
Cycle between 100 \& 450 GeV per beam.
Luminosity \( 1 - 3 \times 10^{26} \text{cm}^{-2}\text{s}^{-1} \)
No low beta. Lumin life \( \approx 2 \) hrs.
FERMILAB TEVATRON COLLIDER

Eφ

Fφ

Dφ ← 2⁰ COLLIDER HALL

Cφ

Bφ

CDF COLLIDER HALL

Aφ

P SOURCE

DEBUNCHER ACCUMULATOR

TO FIXED TARGET EXPERIMENTS.
FERMI LAB TEVATRON COLLIDER

ADVANTAGES OVER CERN

1. COLLISION ENERGY $2 \text{ TeV}$ ($0.63 \text{ TeV}$)
   - Possibility of new thresholds
   - Higher xsects. interesting events
   - More beam shrinkage.

2. HIGHER $\bar{p}$ PRODUCTION ENERGY
   - $120 \text{ GeV}$ protons $\rightarrow 8 \text{ GeV} \bar{p}$
     (CERN $26 \text{ GeV} p \rightarrow 3.5 \text{ GeV} \bar{p}$)
   - Smaller phase space (more accept. less cooling)

   Need fewer $\bar{p}$ and can get them in less time. $10^{30} \sim 1.8 \times 10^{11} \bar{p}$
   TeV: Design 2 hours
   CERN must cool for 1 day and needs $\sim 6 \times 10^{11} \bar{p}$

DISADVANTAGES

1. 4 years later starting

2. More experience at CERN
   - of collider machine
   - of collider experiments
   CERN must upgrade to remain competitive
**Luminosity**

\[
L_{av} = \frac{f_{rev}}{\pi} \left( \frac{N_p}{n} \right) \frac{1}{\pi \sigma_{x} \sigma_{y}} \frac{1}{n_{trans}} \left( \frac{1}{\tau} - 1 \right)
\]

Possible Improvements

1. **Mean Life \( \tau \)**: Dominated beam-beam, worsened by \( N_p, N_f \) imbalance. Might improve by stochastic cooling of SPS beam. Difficult for bunched beams. Need V.H.F. system (16 GHz?). Being studied.

2. **Transmiss. Eff. \( \eta \)**: Good on best shots, worse at high intensity? Try for higher reliability.

3. **Relativistic Beam Shrinkage \( J \)**: Automatic with energy: 
   - 310 GeV: \( \frac{1.15}{1.30} \)
   - 350 GeV: \( \frac{1.15}{1.30} \)

Even more important is rate increase caused by cross section rise.

4. **Increase no. of bunches**: From 3 to 6. Use E-S separators to prevent collisions in wrong regions.
(5) **DECREASE MACHINE BETA FUNCTION $\beta^*$**

\[
\beta^* = \sqrt{1 \times 2} = 1.4 \\
= \sqrt{1.3 \times 0.65} = 0.9 \\
= \sqrt{1 \times 0.5} = 0.7
\]

**Gain 1.3**

(6) **IMPROVE ANTIPROTON ACCUMULATION RATE A**

(a) **MORE PROTONS ON TO TARGET $\sim 1.2$**

(b) **FOCUSBING TARGET AND LITHIUM LENS**

**BETTER MATCH PHASE SPACE INTO TRANSFER CHANNEL AND NEW ANTIPROTON COLLECTOR.**

**A COL.**

**MOMENTUM ACCEPT.** $\frac{\Delta P}{P} = 6\%$ **GAIN FACTOR 4**

**HORIZ X VERT ACCEPT** **GAIN FACTOR 4**

**BUNCH ROTATION** $\frac{\Delta P}{P} = 6\% \rightarrow 1.5\%$

**BETATRON COOLING** $200\pi (\text{mm} \text{ ms})^2 \rightarrow 3\pi$

**MOMENTUM COOLING** $1.5\% \rightarrow 0.2\%$

**THEN TRANSMITS COOLED BEAM EVERY 24h TO PRESENT AA FOR FURTHER COOLING AND LONG-TERM STORAGE**
PREDICTED PERFORMANCE COMPARISON

BY PIPK - VERY UNCERTAIN!

1985 CERN RUN: SEP → DEC, CONDITIONS LIKE END OF 1984 → 600 ab⁻¹
FERMILAB HAVE TEST COLLISIONS

1986 CERN RUN FOR SHORTER TIME → 400 ab⁻¹
FERMILAB FIRST PHYSICS → 100 to 200 nb⁻¹

CERN SHUT DOWN 1 YEAR, LEP & ACOL

1987 AUTUMN, FIRST USE OF ACOL.
1988 "YEAR OF THE CERN COLLIDER"
1989 "YEAR OF LEP"

<table>
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<tbody>
<tr>
<td>CERN</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>S+S</td>
<td>600</td>
<td></td>
<td></td>
<td></td>
<td>? LEP</td>
<td>?</td>
</tr>
<tr>
<td>(nb)⁻¹</td>
<td></td>
<td>400</td>
<td>2000</td>
<td>10000</td>
<td></td>
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<tr>
<td>FERMI-</td>
<td></td>
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<tr>
<td>LAB</td>
<td>Test</td>
<td></td>
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<tr>
<td>TeV</td>
<td>100</td>
<td>400</td>
<td>1000</td>
<td>3000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(nb)⁻¹</td>
<td>-</td>
<td>XF</td>
<td>XF</td>
<td>XF</td>
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</tr>
</tbody>
</table>
**Some Definitions**

\[ \theta = \text{polar angle} \quad \phi = \text{azimuth} \]

**Rapidity**
\[ y = \frac{1}{2} \ln \frac{E + p_L}{E - p_L} \]

**Pseudo-rapidity**
\[ \eta = -\ln \tan \frac{\theta}{2} \approx y \]

**Transverse Energy**
\[ E_t = E_i \sin \theta_i \]

Energy flow can be used as a vector quantity in direction \( \theta \)

Scalar or vector sums of \( E_t \) over various particles \( i = 1, 2, \ldots \) are used for different purposes.
GENERAL PURPOSE DETECTOR, "4π" SOLID ANGLE

- CENTRAL DRIFT CHAMBER 6 m x 2.4 m dia
  Image readout: space points 1 cm, ionization
  RMS 290 μm; Δp/p ~ 0.005 GeV/c for 1 m track & field

- 48 SEMI-CYLINDER E-M CALORIMETERS +
  32 RADIAL SECTOR E-M CALORIM. AT EACH END
  LEAD-SCINTILL. ΔE/E ~ 0.15/ΔE IN GeV
  EFFECTIVE CELL SIZE ~ 5° x 5°

- ALL THIS INSIDE UNIFORM 0.7 T HORIZ. DIPOLE
  MAGNET, WHICH IS INSTRUMENTED WITH
  SCINTILLATOR SHEETS WHICH ACTS AS A
  HADRON CALORIMETER ΔE/E ~ 0.8/ΔE

ABOVE CENTRAL CALORIMETRY 5° < θ < 175°
-3 < y < 3

- ADDITIONAL FORWARD CALORIM.
  (+ SOME CHAMBERS): TOTAL ~ 0.2° < θ < 179.8°
  -6 < y < 6

- ROMAN POTS AT ± 22 m (EARLY PHASES OF EXPT.)

- ADDITIONAL IRON (SOME MAGNETISED) AND
  MUON CHAMBERS CROSSED DRIFT TUBES (8 PLANES)
  AND PLANES OF IAROCCHI CHAMBERS
UA1
Cross section of detector transverse to beam
UA1
Part of Calorimeter
UA2

GENERAL PURPOSE DETECTOR

FINE GRANULARITY CALORIM. CELLS. POINTING TOWERS
240 TOWERS (EM + HADR.) CENTRAL. 40° < θ < 140°
120 + 120 (EM ONLY + HAD. VETO) 20°-40°; 140°-160°

TOROIDAL MAG FIELD ONLY OVER 20°-37.5°; 145°-160°
(FOR W → q̄q ASYMMETRY)

VERTEX DETECTOR: 2 "JADE" TYPE DRIFT CHAMBERS
+ 5 PROPORTIONAL CHAMBERS + BARREL SCINT. HOELOSCOPE
INCLUDES 1.5 RAD. LENGTH TUNGSTEN CONVERTER IN
FRONT OF OUTER PROPORTIONAL CHAMBER ("PRE-SHOWER")

REMOVABLE 60° (ϕ) WEDGE SPECTROMETER (1981,82)
T.O.F. MAGNET + CHAMBERS + SCINTILL + LEMP GLASS
1983 → REPLACED BY STANDARD CALORIM. CELLS
UA2

P-P experiment UA2
<table>
<thead>
<tr>
<th>ADVANTAGES</th>
<th>DISADVANTAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>UA1</strong></td>
<td></td>
</tr>
<tr>
<td>✓ CENTRAL IMAGE CH. WITH MAG. FIELD</td>
<td>✗ NO $\phi$ GRANULARITY OF &quot;GONDOLAS&quot;</td>
</tr>
<tr>
<td>✓ CLOSED GEOMETRY OF CALORIMETERS (MISSING $E_T$)</td>
<td>✗ NO POSITION DETECTOR FOR GONDOLA SHOWERS</td>
</tr>
<tr>
<td>✓ DEPTH-SEGMENTATION OF EM CALOR. (4 SAMPLINGS)</td>
<td>✗ NO $\theta$ GRANULARITY OF &quot;BOUCHONS&quot;</td>
</tr>
<tr>
<td>✓ MUON DETECTION</td>
<td>✗ ENERGY CALIB. DIFFICULT</td>
</tr>
<tr>
<td><strong>UA2</strong></td>
<td></td>
</tr>
<tr>
<td>✓ GOOD GRANULARITY OF CALORIMETERS</td>
<td>✗ NO MAGNETIC FIELD IN CENTRAL REGION</td>
</tr>
<tr>
<td>✓ EM POSITION DETECTOR IN CENTRAL REGION</td>
<td>✗ NO CALORIMETRY IN END REGIONS (BAD FOR $E_T$)</td>
</tr>
<tr>
<td>✓ ALL CELLS CAN BE CALIBRATED IN BEAM</td>
<td>✗ NO MUON DETECTION</td>
</tr>
</tbody>
</table>

CALIBRATION OF EM CALORIMETERS:

**UA1**: $^{60}\text{Co} + \text{LASER} + \text{BEAM}$

**UA2**: BEAM + XENON FLASHTUBES + $^{60}\text{Co}$
UA3 MAGNETIC MONOPOLE SEARCH

- 125 μm KAPTON FOILS SURROUNDING UA1 CENTRAL IMAGE CHAMBER, ALSO INSIDE AND OUTSIDE COLLIDER VACUUM CHAMBER

\[
\frac{(dE/dx)_{\text{mono}}}{(dE/dx)_{\text{ion. min.}}} = \beta^2 \left( \frac{g_m^2}{e^2} \right)
\]

\(\beta\): MONOPOLE VELOC
\(g_m\): MAGNETIC CHARGE

FOR DIRAC MONOPOLES \(\left( \frac{g_m^2}{e^2} \right) \sim 5 \times 10^3\)

- CHEMICAL ETCHING TO LOOK FOR RADIATION DAMAGE "HOLES",Insensitive to MIN. ION. PARTICLES. THRESHOLD \(\sim 2 \times 10^3\)

- NONE FOUND

B. AUBERT ET AL., P.L. 120 B, 445, 1983

UA3 (NOT DRAWN TO SCALE)

---- INDICATES KAPTON FOILS PLACED INSIDE AND OUTSIDE VACUUM PIPE AND AROUND THE 6 HALF CYLINDERS OF UA1 CENTRAL DETECTOR
UA 4

ROMAN POTS

ELASTIC SCATTERING LAYOUT

SIDE VIEW OF A POT

INELASTIC DETECTOR

\[ \eta \text{ acceptance} \]
\[
\begin{align*}
T_1 & \quad 25 - 35 \\
T_2 & \quad 30 - 48 \\
T_3 & \quad 44 - 56
\end{align*}
\]
UA5

STREAMER CHAMBER - SURVEY EXPERIMENT

(PHYSICA SCRIPTA 23, 642 (1981))

2 CHAMBERS (6.0 m x 1.25 m x 0.5 m) ABOVE + BELOW VAC.PIN

GEOM. ACCEPT ~ 95% |η| < 3 FALING TO READ AT |η| = 5

90° CALORIMETER (LATEST RUN) FOR HIGH E_k TRIG

CANNOT CO-EXIST WITH UA2
Elastic Scattering & Total Cross Section

Total = Elastic + Inelastic

- Falls rapidly with angle $\theta$
  - $|1-t| \sim e^{-16t}$

- Single Diffraction (S.D.)
  - Non S.D.
  - One colliding particle dissociates, the other remains intact

Total Scatt. Rate $N = N_{el} + N_{inel} \equiv \sigma_T \ell$

Optical theorem relates $\sigma_T$ to forward differential elastic rate

$$\sigma_T^2 = \frac{1}{\ell^2} \frac{16\pi (4\pi)^2}{1 + \rho^2} \frac{dN_{el}}{dE} \bigg|_{t=0}$$  

Where $\rho$ is ratio real/imaginary part of forward elastic scattering amplitude. $\ell^2$ = luminosity

Hence

$$\sigma_T = \frac{16\pi (4\pi)^2}{1 + \rho^2} \cdot \frac{dN_{el}/dE}{N_{el} + N_{inel}}$$

UA1 measured elastic scatt. in 1981 & 1982 runs and used machine luminosity from wire scan ($\pm 8\%$) equation 1

$\sqrt{s} = 546$ GeV

UA4 measured $N_{el}, N_{inel}$ (1981, 82, 83, 84 runs) and obtain $\sigma_T$ indep. of $\ell$

Both assume $\rho = 0.15$ from dispersion rel. fit, lower energy data

$\sqrt{s} = 630$
Early results (1982) UA1, UA4 agreed, but have been superseded by later UA4 value

$$\sigma_T = 61.9 \pm 1.5 \text{ mb at 546 GeV}$$

UA5 during 1985 ramping run measure cross section ratios $$\frac{\sigma_{400 \text{ GeV}}}{\sigma_{200 \text{ GeV}}}$$ and use extrapolated UA4 ratios $$\frac{\sigma_{200 \text{ GeV}}^{e2}}{\sigma_{400 \text{ GeV}}}$$ to higher energy. Calculate $$\sigma_T = 66.5 \pm 1.8 \pm 1.6 \text{ mb at 900 GeV}$$

UA4 compare $$\sigma_T$$ with values at ISR, Fermilab & Serpukhov. Find $$(\ln s)^2$$ dependence. Actually fit to

$$\sigma_T = \sigma_0 + C_1 e^{-\alpha_1 + C_2 e^{-\alpha_2}} + C_3 \ln s$$

for pp

$$E = \text{equiv. lab energy.} \quad T \text{ parameter fit}$$

$$T = 1.9 \pm 0.1$$

Froissart bound: cannot rise faster than $$\ln^2 s$$
TOTAL XSECT.
UA4
 AND LOWER ENERGY EXPTS.
ELASTIC SCATTERING
USE 4 TELESCOPES, EACH OF 2 "ROMAN POTS"
SCINT. HORO. + WIRE CH.
MOVE NEAR TO BEAMS

~ 40m FROM BEAM CROSSING

Look for up-down (down-up) coincidence
demand collinear tracks extrapol. to vertex
demand vertical coord correct (≠ zero) at vertex
reject events with multiple tracks

FOR 4-MOM TRANSFER, \(- t < 0.15 \text{ GeV}^2\) data well
described by simple exponential \(e^{bt}\).
UA4 slope param. \(b = 15.2 \pm 0.2 \text{ GeV}^{-2}\).

For larger mom. transfer \(- t \) up to 1.6 GeV,
get more complicated behaviour
UA4 \(\sigma_{\text{tot}} / q_t = 0.215 \pm 0.005 \text{ 546 GeV}\),
compared to \(0.185 \pm 0.005 \) at ISR

The proton is getting bigger and blacker
with energy
more opaque

Totally black proton \(\frac{\sigma_{\text{tot}}}{q_t} = 0.5\)
ELASTIC SCATTERING

Up to 0.15 fit by single exponential (see straight line).

At larger values of $-t$ → more complicated.

UA4
ELASTIC SCATTERING

SLOPE PARAM.

b

GeV^{-2}

s (GeV^2)

10^2 10^3 10^4 10^5 10^6

\(-t\leq 0.05 \text{ GeV}^2\)

pp \bar{p}p
• Amaldi
○ Ambrosio
● Ams
○ Ayres
+ Baksay
• Barbieri
x Bartenev
= UA4

ISR

UA4
SINGLE DIFFRACTION DISSOCIATION \((\bar{p}p \rightarrow \bar{p}X)\)

UA4 DETECTED INELASTIC \(\bar{p}\) AND MEASURED \(\bar{p}, \theta\)

\[ t = - (p_\perp - p_\perp^0)^2 \]

\[ x = |p_\perp|/|p_\perp^0| \]

\[ -t = \frac{m^2 (1-x)^2}{s} + 2x p_\perp^0 (1-\cos \theta) \]

\[ M^2 = (1-x)s \]

RECOILING SYSTEMS WITH MASSES > 100 GeV OBSERVED

SCALING OF IN Variant CROSS SECTION \(s \frac{d^2 \sigma}{d t d m^2}\)

OCCURS, i.e. AT FIXED VALUES OF

\(M^2/s\) AND \(t\) IT IS INDEP OF C.M. ENERGY (WITHIN 20%)

APART FROM A KINEMATIC EFFECT AT LOW \(M^2/s\)

MIN. MASS THAT CAN BE PRODUCED IS INDEP OF \(s\)

THERE IS A \(1/M^2\) BEHAVIOUR FOR

\(s \frac{d^2 \sigma}{d t d m^2}\) AT FIXED \(t\)

UA5 IN PULSED COLLIDER RUN MEASURED RATIO

\[ \sigma_{SD}/\sigma_{ND} \quad \text{AT} \quad 900 \text{ GeV} \]

\[ 0.161 \pm 0.015 \pm 0.05 \]

\[ \text{AT} \quad 200 \quad \text{GeV} \]

\[ 0.125 \pm 0.015 \pm 0.04 \]
UA4
SINGLE DIFFRACTION DISSOCIATION

\[ \frac{d^2 \sigma}{dM^2 dt} \text{ (mb/GeV)} \]

\[ \sqrt{s} \]
- 23 GeV
- 38 GeV
- 540 GeV

\[ -t = 0.55 \text{ GeV}^2 \]

\[ -t = 0.75 \text{ GeV}^2 \]

\[ M^2 / s \]

\[ 10^{-1} \]
\[ 10^{-2} \]
\[ 10^{-3} \]
\[ 10^{-4} \]
\[ 10^{-5} \]

\[ \frac{d^2 \sigma}{dM^2 dt} \text{ (mb/GeV)} \]

\[ M^2 (\text{GeV}^2) \]

\[ \frac{1}{M^2} \]

\[ \sqrt{s} \]
pp-pX
- 23 GeV
- 45 GeV
\bar{p}p-pX
- 540 GeV

\[ -t = 0.55 \text{ GeV}^2 \]
SUMMARY OF TOTAL ELASTIC DIFFRACTIVE XSECTS

1. $\sigma_T$ COMPATIBLE WITH $(\ell u_s)^2$ INCREASE WITH $S$

2. ELASTIC SLOPE PARAMETER $b$ $(\frac{d\sigma}{d\ell} \propto e^{\ell b})$ INCREASES AS $\ell u_s$

3. HENCE $\frac{\sigma^{el}}{\sigma_T}$ IS STILL VARYING WITH ENERGY AND IS LARGER THAN AT 53 GeV (ISR)

4. WE ARE THEREFORE NOT YET IN ASYMPTOTIC REGION

5. SINGLE DIFFRACTIVE PRODUCTION OF LARGE MASSES (>100 GeV) OBSERVED WHICH SCALES WITH ISR TO ~ 20%

6. PRELIMINARY RESULTS FROM PULSED COLLIDER CONSISTENT WITH THE ABOVE

FOR A GOOD REVIEW, SEE G. MATTHIAE
PARTICLE PRODUCTION (MINIMUM BIAS PHYSICS)

PSEUDO RAPIDITY DENSITY

Feynman (PRC 23, 1415, 1969) conjectured that if transverse mom. was limited, there should be a uniform distribution of particles as a function of pseudo rapidity.

\[ \frac{dn}{d\eta} \quad (= \frac{1}{\alpha_{\text{inel}}} \frac{d\sigma}{d\eta}) \quad \text{should} \]

be independent of \( s \) "Feynman Scaling”

The available range of rapidity increases as \( \sqrt{s} \) hence the average multiplicity should also increase as \( \sqrt{s} \).

However at ISR \( \frac{dn}{d\eta} \) increases by 40% from \( \sqrt{s} = 23 \text{ GeV} \) to \( \sqrt{s} = 63 \text{ GeV} \), violates Feynman.

A continued rise of plateau observed in going to collider as \( \sqrt{s} \).

Hence mean no. of charged particles \( \langle n \rangle_{\text{charged}} \)

per NSD event

\[ \langle n \rangle_{\text{charged}} \sim 29 \text{ at } 560 \]

\sim 35 \text{ at } 900
MULTIPICITY DISTRIBUTIONS

IF PARTICLES WERE PRODUCED RANDOMLY AND INDEPENDENTLY
MULTIPICITY SHOULD OBEY

\[ P_n = \frac{e^{-\langle n \rangle} \langle n \rangle^n}{n!} \]

A POISSON DISTRIBUTION

PROB. OF GETTING \( n \), IF MEAN IS \( \langle n \rangle \)

THIS WOULD GET NARROWER (AS \( \sim \frac{1}{\sqrt{\langle n \rangle}} \))
WHEN ENERGY AND HENCE MULTIPICITY IS INCREASED
(IF FINAL PARTICLES COME FROM DECAY OF RESONANCES
THIS IS MODIFIED. CORRELATIONS BETWEEN PARTICLES
RESULTS IN A BROADER DISTRIBUTION)

KOBA, NIELSEN, OLESEN (NUC-PHY. 840, 117, 1972) SHOWED
THAT ON THE CONTRARY, SHAPE OF DISTRIBUTION
SHOULD TEND TO BECOME CONSTANT AS \( S \to \infty \)
"KNO SCALING": IT ASSUMES FEYNMAN SCALING

EARLY COLLIDER RESULTS (1981) FROM UA1
AND UA5, WHEN COMPARED WITH ISR DATA
WERE CONSISTENT WITH KNO SCALING, IN SPITE
OF VIOLATION OF FEYNMAN SCALING

LATER UA5 SHOWED A DEVIATION PARTICULARLY
AT HIGH MULTIPICITIES. HOWEVER, SCALING
APPROX CORRECT FOR RESTRICTED RANGE OF \( n \)

KNO \( \to \) PERHAPS A LIMITED NUMBER OF SUB-PROCESSES
GIVES PARTICLES, THIS NUMBER VARY LITTLE
WITH ENERGY
MULTIPICLITY DISTRIBUTIONS

UA1 1981 SHOWS KNO SCALING OVER LIMITED RAPIDITY

Probability for \( n \times \frac{\text{charged particles}}{\text{mean charged multiplicity}} \)

\( Z = \frac{n_{\text{ch}}}{<n_{\text{ch}}>} \)

\( \sqrt{s} (\text{GeV}) \)
- 23.6
- 30.8
- 45.2
- 53.2
- 62.8
- 540.0 \( p\bar{p} \) \( |\eta| < 1.3 \) COLLIDER

UA5 1985 SHOWS DEVIATIONS

FITS TO "NEGATIVE BINOMIAL"
SINGLE PARTICLE $p_t$ SPECTRUM

UA1 (PL 1186, 167, 1982) UNIDENTIFIED CH. HADrons 5.46 GeV
1. $\langle p_t \rangle$ INCREASES FROM 0.35 GeV/c TO 0.42 GeV/c
2. $\langle p_t \rangle$ INCREASES WITH INCREASING $dN/dy$ AND SPECTRUM BECOMES LESS STEEP
3. PRELIMINARY INDICATIONS THAT TENDENCY CONTINUED AT 900 GeV

IDENTIFIED PARTICLE PRODUCTION

UA2 (PL 1158, 59, 1982; PL 1228, 322, 1983)
HAVE IDENTIFIED $K^+, p, \pi$ USING WEDGE
LOOKED FOR FREE QUARKS (NOT FOUND) (CERN EP/85-62 MARCH 1985)

UA5 (PL 1158, 71+65, 1982)
STUDIED $K^0, \Lambda^0, \Sigma^0$
KAONS HAVE HIGHER $\langle p_t \rangle$ THAN PIONS
$\sim 0.7$ GeV/c
UAS, UAI SEARCH FOR CENTAUROS

6 SPECTACULAR COSMIC RAY EVENTS
(C.M.G. LATTES ET AL., PHYS. REPORT VOLS, NO. 3, 1980)
V. HIGH ENERGY, V. HIGH MULTIPLICITY OF HADRONS
"ZERO" MULT OF E.M. SHOWER PARTICLES
≈ 1.700 TeV LAB [1.8 TeV C.M. FF pp COLLISION]

UAS: ASSUME CENTAURUS IS COLLISION OF A
PRIMARY HADRON WITH ATMOSPHERIC NUCLEON.
LOOK FOR EVENTS WITH LOW PHOTON MULT
AND HIGH CHARGED MULTIPLICITY
→ NO EVENTS FOUND IN 3600 MIN. BIAS AT 546
(P.L. 115B, 71, 1982)

UAI: ASSUME 2 HYPOTHESES
(1) PRIMARY HADRON COLLISION
(2) DECAY OF HEAVY OBJECT

PLOT DEPOSITED E.M. ENERGY V. HADRON ENERGY
AFTER MAKING CUTS < p_T > > 1.0 GeV
CHARGED MULTIPLICITY > 10
→ NO EVENTS FOUND IN 48,000 MIN. BIAS AT 546
\[ \sigma_{\text{CENTAURUS}} \approx \frac{1}{\mu b} \] (P.L. 122B, 1983)

UAS \{ 1985, LOOKING AT 900 GeV IN PULSED
UAI \} COLLIDER RUN: NONE SEEN YET.
UA1 CENTAURO SEARCH

a) $5^\circ < \theta < 30^\circ$

b) $30^\circ < \theta < 150^\circ$

SAME AFTER CUTS

a) $5^\circ < \theta < 30^\circ$
with $N_a > 10$

Pt $> 1$ GeV/c

b) $30^\circ < \theta < 150^\circ$
with $N_a > 10$

Pt $> 1$ GeV/c

COLLISION HYPOTHESIS

1a MIXED HADRONS

16 BARYON ANTIBARYON

DECAY HYPOTHESIS

2a MIXED HADRONS

2b BARYON ANTIBARYON
SUMMARY ON PARTICLE PRODUCTION

1. WIDTH OF RAPIDITY PLATEAU GROWS FROM ISR → COLLIDER BUT LESS THAN MIGHT BE EXPECTED FOR CONSTANT $<p_T>$

2. HOWEVER $<p_T>$ RISES FROM 0.35 TO 0.42 GeV/c

3. HEIGHT OF PLATEAU RISES AS $L_{ms}$ (CONTINUED VIOLATION OF FEYNMAN SCALING)

4. $<n_{ch}>$ AS A CONSEQUENCE REQUIRES A ($L_{ms}$) TERM

5. DEVIATION FROM KNO SCALING AT LARGE MULTIPLICITY. FOR $|y| < 5$, BUT SCALING OBSERVED FOR $|y| < 1.5$

6. $<p_T>$ RISES WITH MULTIPLICITY

7. TYPICAL COLLISION AT $\sqrt{s} : 546$ GeV CONTAINS $\sim 43$ PARTICLES OF WHICH 29 ARE CHARGED $\sim 8\%$ KAONS; $\sim 5\%$ BARYON+ANTIBARYON

8. SO FAR NO INDICATIONS OF BIZARRE PROCESSES SUCH AS CENTAURS
**W and Z Physics**

**Mass Estimate from Beta Decay**

\[ n(d) \rightarrow p(u) + G + e^- \]

\[ G = \text{Fermi constant} \]

**Now replace point interaction by W coupling to quark pair and to lepton pair each with strength } g \]

\[ d \rightarrow u + W^- + e^- \]

\[ G \text{ becomes } \frac{g^2}{q^2 + m_W^2} \]

\[ q \text{ in a momentum of } W \]

**At low momentum, we can neglect } \frac{q^2}{m_W^2} \]

\[ G \sim \frac{g^2}{m_W^2} \]

**Now for electroweak unification } g \sim e \]

\[ e = \sqrt{\frac{4\pi}{3\alpha}} ; \alpha = \frac{1}{137} \]

\[ \therefore \quad m_W \sim \frac{g}{\sqrt{G}} = \sqrt{\frac{4\pi}{\alpha}} \sim 100 \text{ GeV/c}^2 \]
STANDARD MODEL

\[ m_W = \left( \frac{\pi \alpha}{\sqrt{2} \, G} \right)^{\frac{1}{2}} \sin \Theta_W \]

\( \alpha^{-1} = 137.03604 \pm 0.00011 \) [ATOMIC PHYSICS]

\( G = (1.16632 \pm 0.00002) \times 10^{-5} \text{ GeV}^{-2} \) [FROM MUON]

FROM THIS \( m_W = \frac{37.28}{\sin \Theta_W} \text{ GeV} \)

NEED MODIFY BY 2 EFFECTS

1. RADIATIVE CORRECTIONS, CHANGE VALUE OF \( \alpha \) FROM ITS LOW ENERGY VALUE \( \frac{1}{\alpha} = \frac{1}{137.036} \rightarrow \frac{1}{127.5} \)

2. VALUE OF \( \sin \Theta_W \) NOT DIRECTLY FROM NEUTRAL CURRENT NEUTRINO EXPERIMENTS, WHICH ARE "LOW ENERGY" i.e. \( q^2 \ll m_W^2 \), BUT RENORMALIZED \( \theta_W \)

THEN \( m_W = \frac{38.65}{\sin \Theta_W} \)

LATEST AVERAGE FROM LOW EN. EXPERIMENTS (BARI CONFERENCE) \( \sin^2 \Theta_W = 0.216 \pm 0.006 \)

\[ \therefore \text{ EXPECT } m_W = 83.2^{+1.5}_{-1.0} \text{ GeV} \]
**Mass of Z**

\[ M_Z = \frac{m_W}{\cos \theta_W} \quad \text{MINIMAL HIGGS SCHEME} \]

More generally, introduce parameter \( \rho \)

\[ \rho = \frac{m_W^2}{(m_Z^2 \cos^2 \theta_W)} = \frac{1}{(2\phi)} \]

Where \( \phi \) is isospin of Higgs scalar boson \( \phi \)

(\( \rho \) is 1 in standard model, doublet \( \phi \))

\[ \therefore M_Z = 93.9 \pm 0.9 \text{ GeV} \]
DECAYS OF $W$

$W^\pm$ PARTIAL WIDTHS NEGLECTING LEPTON, QUARK MASSES

$\Gamma_w(q\bar{q}) = \Gamma_w(q\nu) = \Gamma_w(q\bar{\nu}) \approx \frac{G M_w^3}{6 \pi m^2} \approx 250 \text{ MeV}$

APPROX, $\Gamma_w(q\bar{q}) = 3 \Gamma_w(q\nu) : 3$ COLOURS
d$\bar{u}$; s$\bar{c}$; b$\bar{f}$

HENCE $\frac{\Gamma_w(q\bar{q})}{\Gamma_w \text{ TOTAL}} \approx \frac{1}{12} (4.3\%)$ ASSUMING 3 GENERATIONS OF QUARKS + LEPTONS

THERE ARE SOME CORRECTIONS

1. WE HAVE OMITTED CABIBBO TYPE QUARK MIXING [KOBAYASHI-MASKAVA QUARK MIXING MATRIX]
   NOT IMPORTANT FOR CALCULATING DECAY WIDTHS

2. QCD CORRECTION $\times \left(1 + \frac{\alpha_s}{\pi}\right)$

3. PHASE SPACE FACTOR FOR $b\bar{f}$ SINCE $m_c$ HIGH

$$\left(1 - \frac{m_c^2}{M_W^2}\right)\left(1 + \frac{m_f^2}{M_W^2}\right) = 0.66 \quad m_c = 4.6 \text{ GeV}$$

[TOTAL WIDTH $\approx \frac{3}{11}$]

TOTAL WIDTH $\approx \left\{3 + 3 \left[2 + 0.66\right]\right\} \approx 10.9 \times 2.52 \text{ MeV}$

3 LEPTONS 3 COLOURS 3 QUARKS $= 2.77 \text{ GeV}$

BRANCHING RATIO $\frac{\Gamma_w(q\bar{q})}{\Gamma_w} = 9.1\%$ 3 GENERATIONS

$W$ DECAY: PURE $V-A$ $\rightarrow$ DECAYS INTO LEFT-HANDED LEPTONS OR QUARKS

PRODUCED BY L-H QUARKS OR OF COURSE R-H ANT QUIRKS OR ANTI-LEPTONS
DECAY OF $Z^0$. MORE COMPLICATED 'I' MIXING OF PROTON WHICH MODIFIES VECTOR COUPLING. CONSEQUENTLY $V \neq A$ EXCEPT FOR $\bar{q}q$ DECAYS AND IT COUPLES TO BOTH LH AND RH Q. + LEFT.

$$\Gamma_2 = \frac{G_m z^3}{24 \pi J_2} \left[ 3 \frac{\Sigma (q_3^2 + V^2)}{q} + \frac{\Sigma (q_3^2 + V^2)}{q} \right]$$

WHERE $V_q (V_A)$ = VECTOR COUPLING TO QUARKS (LEPTONS)
$q_A (q_L)$ = AXIAL " " " "
AND FACTOR 3 AGAIN FOR COLOUR

FROM STANDARD MODEL (USING DEFINITION $q_L = V_A = 1$)

$V_L = 2 I_{3L} - 4 Q_L \sin^2 \theta W$; $q_A = 2 I_{3A}$
SAME FOR $q_L$

$V_L = 2 I_{3L} - 4 Q_L \sin^2 \theta W$; $q_L = 2 I_{3L}$

$I_3$ = 3RD COMP OF WEAK ISO SPIN
$q_L, q_L$ = LEPTON AND QUARK CHARGES

$$\Gamma_2 (\bar{q}q) = \frac{G_m z^3}{12 \pi J_2} = 181 \text{ MeV}$$

$$\Gamma_2 (l\bar{l}) = 92 \text{ MeV}$$

NOTE: WE DO NOT GET $Z \rightarrow q\bar{q}'$ WHERE $q \neq q'$
NEUTRAL CURRENTS DO NOT CHANGE FLAVOURS
ALLOWING FOR $M_W$ AND QCD FACTOR

\[ \Gamma_z (\text{hadrons}) = 2.12 \text{ GeV} \]

\[ \Gamma_z (\text{all}) = 2.94 \text{ GeV} \]

ASSUMES 3 GENERATIONS OF NEUTRINOS

BRANCHING RATIO

\[ \frac{\Gamma_z (e^+e^-)}{\Gamma_z} = 3.1\% \]
Production of $W, Z$

Analogous to Drell-Yan mechanism

\[ \gamma \rightarrow W, Z \]

The $\gamma$ is then replaced by $W$ or $Z$

Monodirectional proton (antiproton) beams = Broad band beam of quarks (antiquarks) and gluons

A quark of a particular momentum must collide with an antiquark of correct mom. to give exactly $W$ (or $Z$) mass within width. Since quark mom. need not be equal the $W(Z)$ can have significant longit. momentum

Use kinematic variable $S_{\text{TOTAL}} \times$

Fraction of hadron mom. carried by active (struck) quark in the overall $(pp)$ C.M

In C.M.S. we have longit. momenta

\[ x_1 \sqrt{s} / 2 \quad \text{and} \quad -x_2 \sqrt{s} / 2 \]

\[ \uparrow \quad \text{Active quark} \quad \text{Active antiquark} \]
ASSUMING MASSLESS QUARKS, ENERGIES ARE SAME AS MOMENTA.
EQUATING 4-MOMENTA OF ANNihilating PAIR TO 4-MOM OF W (EW, PW), WE HAVE

\[ \mathbf{E}_W = (x_1 + x_2) \frac{\sqrt{s}}{2} \quad \text{AND} \quad \mathbf{p}_W = (x_1 - x_2) \frac{\sqrt{s}}{2} \]

\[ m_W^2 = E_W^2 - p_W^2 = x_1 x_2 \sqrt{s} \]

ALSO \[ x_W \equiv \frac{2 \mathbf{p}_W}{\sqrt{s}} = x_1 - x_2 \]

FEYNMAN X = LONGIT. MOM = MAX VALUE

NOTE: FOR CERN COLLIDER, \( \sqrt{s} = 630 \text{ GeV} \)
GEOM. MEAN X, \( \sqrt{x_1 x_2} = \frac{m_W}{\sqrt{s}} \sim 0.13 \quad (\sim \frac{1}{7}) \)
HENCE VALENCE QUARKS DOMINATE W (Z) PROD

TEVATRON 1, \( \sqrt{s} = 1600 \text{ to } 2000 \text{ GeV} \)
\( \sqrt{x_1 x_2} \sim 0.05 \text{ to } 0.04 \). HENCE IMPORTANT CONTRIBUTION FROM SEA QUARKS.

ALSO NOTE: IN pp COLLISIONS THE ANTIQUARKS MUST COME FROM THE SEA

W PRODUCTION \[ \bar{u}d \rightarrow W^+ \]
\[ \bar{u}d \rightarrow W^- \]
WITH SOME CONTRIBUTION FROM CABBibo-UNPREDICTED STRANGE QUARK CHANNELS \[ \bar{u}s \rightarrow W^+ \quad \{ \sin^2 \theta_c \leq 0.05 \] 
\[ \bar{u}s \rightarrow W^- \]
ASSUMING NO TRANSVERSE MOTION OF $W$, THE TRANSVERSE MOMENTUM OF LEPTON $p_T$ HAS A JACOBIAN PEAK AT $\frac{m_W}{2}$, SMEARED BY FINITE WIDTH OF $W$ (AND BY EXPERIMENTAL RESOLUTION)

IN PRACTICE, INSTEAD OF MOMENTUM, WE OFTEN USE "ENERGY FLOW VECTOR". MEASURE ENERGY OF ELECTRON (CALORIM.) $\sim \pm 2\%$ FOR $W$ AND ITS DIRECTION CAN THEN CONSIDER COMPONENTS OF VECTOR $E_T = E \cos \theta$ $E_T = E \sin \theta$ "TRANSVERSE ENERGY"
**W Production**

\[ \frac{d^2 \sigma}{dx_1 dx_2} = \frac{\pi G_{F}^2}{3} \left[ \left\{ u_1(x_1) \bar{d}_2(x_2) + \bar{d}_1(x_1) u_2(x_2) \right\} c_{W}^2 \delta_{c}^2 \\
+ \left\{ u_1(x_1) \bar{s}_2(x_2) + \bar{s}_1(x_1) u_2(x_2) \right\} c_{W}^2 \delta_{c}^2 \right] \]

**Total Cross Section given by Convolution over Parton Densities**

\[ \sigma_W = \int \int \frac{d^2 \sigma}{dx_1 dx_2} \delta(x_1 x_2 - \frac{m_W^2}{s}) \ \text{d}x_1 \text{d}x_2 \]

**Z Production**

\( u \bar{u} \), \( d \bar{d} \) (and \( s \bar{s} \)) contribute

**Bit More Complicated Expressions, But Similar Calculations Done (to lowest order)**

By F. Paige, BNL 27066 1979

Get at \( \sqrt{s} = 546 \)

\( \sigma_{W^+W^-} = 2 \sigma_W \approx 4 \ \text{nb} \)

\( \sigma_{Z^0} \approx 1.3 \ \text{nb} \)

At \( \sqrt{s} = 630 \) both increase \( \approx 25\% \)

Putting in Branching Ratios, and noting that \( \sigma_{\text{tot}} (p^p \to \text{anything}) \approx 60 \ \text{mb} \)

Expect \( W \to e\nu \approx 1 \) in \( 2 \times 10^8 \) collisions

\( Z \to \mu\nu \approx 1 \) in \( 2 \times 10^9 \) collisions
HIGHER ORDER CORRECTIONS

EXAMPLES OF NEXT-TO-LEADING ORDER (IN $\alpha_s$)

![Diagram of particle interactions]

EFFECT IS TO INCREASE CROSS SECTION

$\gamma \rightarrow k\nu$ "K FACTOR" $\sim 1.3$

THEY ALSO GIVE TRANSVERSE MOMENTUM TO W OR Z, $<p_T^W> \sim 7 \text{ GeV/c}$ AT $\sqrt{s} = 630 \text{ GeV}$

$p_T$ DISTRIBUTION OF ELECTRON OR MUON SMERRED FURTHER, BUT NOT TOO BADLY
EXPERIMENTAL DETERMINATION \( W \rightarrow q \bar{q}, Z \rightarrow e^+e^- \)

**UA1**

TRIGGERING (TO REDUCE RATES, FACTOR > 10³)

1e \( E_T > 10 \text{ GeV} \)

2e \( E_T > 6 \text{ GeV} \)

\( E_T^{\text{IMBAL}} > 17 \text{ GeV} \)

1e \( E_T > 8 \text{ GeV} \)

2e \( E_T > 4.5 \text{ GeV} \)

ELECTRON IDENTIFICATION (\( E_T > 15 \text{ GeV} \)) APPLY SERIES OF CUTS

SMALL ENERGY LEAKAGE INTO HADRON CALORIMETRY

LONGIT. SHOWER SHAKE (4 SAMPLINGS)

SMALL LATERAL DIMENSIONS OF CLUSTER (FINE GRANUL.)

**UA2**

TRACK - CALORIMETER MATCHING

REQUIRE HIGH \( p_T \) TRACK (\( > 7 \text{ GeV/c} \)) FROM CURV.

IN MAGNET

USE 1.5 RAD. LENGTH

CONVERTER + PROP. CHAMBER TO OBSERVE BEGINNING OF SHOWER. USE MAG.

FIELD \( 20° < \theta < 37.5° \)

ISOLATION

OVER CONE \( \Delta R \leq \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2} \)

\( \Delta R = 0.7 \leq E_T < 3.0 \text{ GeV} \)

OVER CALORIM. CELLS AROUND TO CELL HIT BY ELECTRON

\( \sim 1300 \) CANDIDATES \[1982-3\] \( \sim 860 \) CANDIDATES

SAMPLES CONTAIN \( W \rightarrow q \bar{q}, Z \rightarrow e^+e^-, W \rightarrow \nu \bar{\nu}, W \rightarrow e^+e^- (\bar{\nu} \rightarrow \text{hadrons}) \)

+ BACKGROUND (MISIDENTIFIED HADRONS)
TO SELECT $W ightarrow q
\nu$ TRY TO IDENTIFY NEUTRINO

CANNOT RECONSTRUCT LONGIT. MOMENTUM COMPONENT
(LOSS OF PARTICLES DOWN BEAM PIPE)

$$E_T^o = E_T^{\text{missing}} = -E_T^e - \sum_{\text{all other particles}} E_T^i$$

**UA1**

CALORIMETER COVERAGE
DOWN TO $\sim 0.2^{\circ}$

**UA2**

NO COVERAGE BELOW 20$^{\circ}$
ONLY E.M. 20$^{\circ} < \theta < 40$°

MUON DETECTORS $\vec{P}_T^m$
DO NOT MEASURE E.NE. NON-SHOWERING

$E_T^{\text{miss}}$ RESOLUTION IS
GAUSSIAN (EXCEPT WHEN
$E_T^{\text{miss}}$ VERTICAL, $\Delta \phi = \pm 15^{\circ}$)

$E_T^{\text{miss}}$ MAY COME FROM HIGH
$E_T$ JET ESCAPING
$\rightarrow$ NON-GAUSSIAN TAIL
IN $E_T^{\text{miss}}$ DISTRIBUTION

PLOT ELECTRON CANDIDATES $E_T^e$ VERSUS $E_T^o$
$\int \mathcal{L} dt = 399 \text{ nb}^{-1}$
\~\ 
$\sim 1300$ events

$\int \mathcal{L} dt = 452 \text{ nb}^{-1}$
\~\ 
$\sim 850$ events
$E_T^\nu$ Distributions

UA1

CUT $E_T^\nu > 15$ GeV
SELECTS GOOD SAMPLE OF $W \rightarrow q\bar{q}$ CANDIDATES

172 EVENTS

OF WHICH 17 ± 2 ARE BACKGROUND

$W \rightarrow \tau\nu$ JET-JET

2.7 9.1 5.3

HADR. 0.00

UA2

APPLY TOPOLOGY CUT IN TRANSVERSE PLANE

$\Delta \phi = 120^\circ$

IF JETS IN THIS SECTOR REQUIRE

$- \frac{E_T^\tau \sum E_T^{\text{jet}}}{E_T^\tau} < 0.2 E_T^\tau$
$W \rightarrow e\nu$

172 EVENTS
ELECTRON $E_T$ DISTRIBUTIONS AFTER
$E_T^e > 15$ GeV CUT (UA1)
TOPOLOGICAL CUT (UA2)

EVENTS WITH $\not{E}_T^e, \not{E}_T^\nu$ WITHIN $\pm 15^\circ$ FROM VERTICAL REJECTED

$W \rightarrow e\nu$ SAMPLE
148 EVENTS
$E_T^e > 15$ GeV

$W \rightarrow e\nu$ SAMPLE
119 EVENTS
$E_T^e > 25$ GeV

--- BACKGROUND MISIDENTIFIED HADRONS

TOTAL CONTAINS
$W \rightarrow e\nu, W \rightarrow e\nu$
$Z \rightarrow e^+ e^-_{\text{LOST}}$
+ BACKGROUND
DETERMINATION OF W MASS

LONGITUDINAL COMPONENT OF NEUTRINO MOM. NOT MEASURED. ESCAPE OF PARTICLES NEAR BEAM

DEFINE "TRANSVERSE MASS" VARIABLE FOR W; $m_T$

$$m_T^2 = 2 \frac{p_e^+}{p_e^-} (1 - \cos \Delta \phi)$$

SEPARATION BETWEEN $p_e^+, p_e^-$

$W$ IS THE END POINT OF THE $m_T$ DISTRIBUTION, APART FROM SHEARING AND RESOLUTION EFFECTS

HOWEVER A MUCH BETTER VALUE CAN BE OBTAINED BY TAKING A $W$ DISTRIBUTION FROM STANDARD MODEL (QCD) AND DECA PHY ANGULAR DISTAIB (V-A) AND USING $MW$ AS A FREE PARAMETER, FIND VALUE OF $MW$ WHICH BEST FITS DATA

UA1 USE 86 EVENTS WITH

$E_e^+ > 30, E_e^+ > 30$ GeV

(ALMOST PURE $W \rightarrow \ell \nu$ SAMPLE)

UA2 USE ALL 119 EVENTS. TAKE INTO ACCOUNT CONTRIBUTIONS FROM HADRONIC BACKGROUND $W \rightarrow \ell \nu, \ell \rightarrow e \nu \bar{\nu}$

TOTAL OF $W \rightarrow \ell \nu, Z \rightarrow e \nu \bar{\nu}$ EVENTS

12.5 ± 2.2
$$UAI \quad m_W = 83.1^{+1.3}_{-0.8} \pm 3 \text{ GeV}$$

$$UA2 \quad m_W = 81.2 \pm 1.1 \pm 1.3 \text{ GeV}$$

STATISTICAL

SYSTEMATIC

(UNCERTAINTY ON CALORIM. CALIBRATION)

THEORY \quad m_W = 83.2^{+1.5}_{-1.0} \quad (\frac{38.65}{\sin^2 \theta_W})

\sin^2 \theta_W = 0.216 \pm 0.006

CROSS SECTION \times BRANCHING RATIO

$$\sigma (p\bar{p} \to W^\pm + \text{anything}) \times B (W \to e\nu)$$

<table>
<thead>
<tr>
<th></th>
<th>$UAI$</th>
<th>$UA2$</th>
<th>THEORY ALTAFFELI ET AL.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_s = 546$</td>
<td>$0.55 \pm 0.08 \pm 0.09$ (nb)</td>
<td>$0.49 \pm 0.09 \pm 0.05$ (nb)</td>
<td>$0.36^{+0.11}_{-0.05}$ (nb)</td>
</tr>
<tr>
<td>630 GeV</td>
<td>$0.63 \pm 0.05 \pm 0.09$ (nb)</td>
<td>$0.53 \pm 0.06 \pm 0.05$ (nb)</td>
<td>$0.46^{+0.16}_{-0.08}$ (nb)</td>
</tr>
<tr>
<td>RATIO</td>
<td>$546 \quad 630$</td>
<td>1.15 $\pm 0.17$</td>
<td>1.06 $\pm 0.23$</td>
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$UAI$, $UA2$, THEORY ARE CONSISTENT

$\sigma \cdot B \sim 2\%$ LEVEL

$\frac{m_W}{\text{GeV}} \sim 30\%$ LEVEL
$Z^0 \rightarrow e^+ e^-$

**Require 2 energy clusters in calorims. Consistent with electrons. This channel is extremely clean (essentially free of background).**

To investigate background from jet-jet, $\pi^+ \pi^0$ overlap etc., relax electron definition. Low energy pairs will now appear.
TO DETERMINE $M_Z$ : MAXIMISE LIKELIHOOD USING BRET-WIGNER + DETECTOR RESOLUTION

$\text{UA1} \quad [15] \quad M_Z = 93.0 \pm 1.6 \pm 3 \text{ GeV}$

$\text{UA2} \quad [13] \quad M_Z = 92.5 \pm 1.3 \pm 1.5 \text{ GeV}$

$\uparrow \quad \uparrow \quad \text{WELL-MEASURED EVENTS}$

$\text{STATIST. SYSTEM.}$

THEORY $M_Z = 93.9 \pm 0.9 \left[= \frac{38.65}{\sin^2\theta_W 0.216}\right]$ \hspace{1cm} $\sin^2\theta_W = 0.216 \pm 0.006$

CROSS SECTION $\times$ BRANCHING RATIO

$\sigma' \left( p\bar{p} \rightarrow Z^0 + \text{ANYTHING} \right) \times B \left( Z^0 \rightarrow e^+e^- \right)$

<table>
<thead>
<tr>
<th></th>
<th>UA1</th>
<th>UA2</th>
<th>THEORY AL'TARELLI ET AL.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N/546$ GeV</td>
<td>$40 \pm 20 \pm 6 \quad (pb)$</td>
<td>$101 \pm 37 \pm 15 \quad (pb)$</td>
<td>$41^{13}_{-7} \quad (pb)$</td>
</tr>
<tr>
<td>$630$ GeV</td>
<td>$79 \pm 21 \pm 12 \quad (pb)$</td>
<td>$56 \pm 20 \pm 9 \quad (pb)$</td>
<td>$51^{16}_{-8} \quad (pb)$</td>
</tr>
<tr>
<td>RATIO $630 \quad / \quad 546$</td>
<td>$2.0 \pm 1.1$</td>
<td>$0.6 \pm 0.3$</td>
<td>$1.23$</td>
</tr>
</tbody>
</table>

UA1, UA2, THEORY ARE CONSISTENT

$M_Z \sim 2 \%$ LEVEL

$\sigma', B \sim \text{FACTOR 2}$

LOW STATISTICS
STANDARD MODEL PARAMETERS

1. Using radiatively corrected formula and $m_W$
\[ \sin^2 \hat{\theta}_W^r = (38.65 \text{ GeV} / m_W)^2 \]

   \begin{align*}
   \text{UA1} & : \sin^2 \hat{\theta}_W^r = 0.216 & \pm 0.008 & \pm 0.016 \\
   \text{UA2} & : \sin^2 \hat{\theta}_W^r = 0.227 & \pm 0.006 & \pm 0.007 \\
   \text{World average from low energy experiments} & : 0.216 & \pm 0.006 \\
\end{align*}

2. \[ \rho = \frac{m_W^2}{(m_Z^2 - m_W^2)} \]

   \begin{align*}
   \text{UA1} & : \rho = 1.018 & \pm 0.041 & \pm 0.021 \\
   \text{UA2} & : \rho = 0.996 & \pm 0.024 & \pm 0.009 \\
\end{align*}

3. Assume $\rho = 1$ and define $\sin^2 \hat{\theta}_W^s = 1 - (m_W/m_Z)^2$

   \begin{align*}
   \text{UA1} & : \sin^2 \hat{\theta}_W^s = 0.202 & \pm 0.036 \\
   \text{UA2} & : \sin^2 \hat{\theta}_W^s = 0.229 & \pm 0.030 \\
   \text{UA1 + UA2 weighted mean} & : \sin^2 \hat{\theta}_W^s = 0.218 & \pm 0.023 \\
   \text{Very good agreement with low energy experiments} &
\end{align*}
Decay Asymmetry of $W \rightarrow q\bar{q}$

The weak interaction (V-A) produces W's that are longitudinally polarised, provided a quark comes from proton and $\bar{q}$ from antiproton (e.g.). Because W couples only to L.H. quarks and R.H. $\bar{q}$

At collider energies W production involves at least one valence quark. i.e. above is correct.

$W$ then decays into L.H. lepton & R.H. antilepton

\[
\begin{align*}
\text{P} & \rightarrow q \quad u \quad \bar{d} \quad \bar{p} \\
\text{e}^+ & \rightarrow \text{e} \quad \nu_e \quad \bar{\nu}_e
\end{align*}
\]

$W$ spin is always along $\bar{p}$ momentum.
Lepton $e^+$ or $ve$.
$W$ spin.

$W$ decay in its rest frame

\[
\frac{dN}{d\cos \theta} = \frac{3}{8} \left(1 + \cos^2 \theta \right)^2
\]

i. $e^-$ predominantly emitted opposite to W spin (L.H. lepton).

ii. $e^+$ predominantly emitted (R.H. antilepton).

NOTES
1. V+A would give same all helicities flip.
2. Results different at TeVatron where sea quarks important.
$W \rightarrow q\bar{q}$

**UA1** MAG. FIELD OVER FULL SOLID ANGLE TAKE ALL EVENTS WITH UNAMBIGUOUS SIGN ELECTRON

→ EXCELLENT FIT TO $(1 + \cos \theta^*)^2$, REQUIRED BY V-Å. OR V+Å

**NOTE:** THERE IS A SIGN AMBIGUITY FOR $\cos \theta^*$. $E_{\text{longit}}$ NOT MESS.

→ 2 SOLUTIONS FOR $xW$

However, $\sim \frac{1}{3}$ UNPHYSICAL ($xW > 1$)

$\sim \frac{1}{3}$ RESOLVE EN. + MOM. IN OVERALL EVENT

NEARLY ALWAYS LOWER VALUE

IN REMAINING 30% TAKE LOWER VALUE

**UA2** MAG. FIELD ONLY $2^\circ < \theta < 37.5^\circ$

28 EVENTS GIVE ASYM. $A = 0.43 \pm 0.17$

(V ± A) EXPECT $0.53 \pm 0.06$

HENCE CONSISTENT
Jacob (Nuovo Cim. 9, 816, 1958) has shown that for a particle with arbitrary spin $J$, we expect

$$\langle \cos \theta^x \rangle = \frac{\langle \lambda \rangle \langle \mu^* \rangle}{J(J+1)}$$

$\langle \mu^* \rangle = \text{global helicity of production system (W)}$

$\langle \lambda \rangle = \text{average W polarization at production}$

$\langle \nu \rangle = \text{global helicity of decay system (e, \nu)}$

$\frac{1}{2}(\text{helicity of } \bar{\nu} - \text{helicity of } \bar{\nu})$

Obviously if $J$ were zero, $\langle \cos \theta^x \rangle$ would be 0 (uniform angular distribution).

If $J > 2$, then $|\langle \cos \theta^x \rangle| \leq \frac{1}{6}$

UA1 found $\langle \cos \theta^x \rangle = 0.4 \pm 0.1$

This shows W must have spin 1 and support maximal helicity states at production and decay, for which $\langle \cos \theta^x \rangle = 0.5$
Having solved for $x_w$ we can determine quark distributions in proton and antiproton sampled by $W$. $x_1 x_2 = m_W^2/s$ and $x_w = x_1 - x_2$.

UA1 & UA2 both do this and get results consistent with expectations from structure functions, Eichten et al., Rev. Mod. Phys. 56, 579 (1984).

UA1 only also identify charge of $q$, hence of $W$, hence identify type of quark (u or d).
EXPERIMENTAL DETERMINATION \([W \rightarrow \mu^+\mu^-; Z \rightarrow \mu^+\mu^-]\)

UA1 ONLY

TRIGGERING

\[ \begin{align*}
\text{1} & \quad \mu^- \\
\text{2} & \quad \mu^+
\end{align*} \]

Muon chamber hits pointing to vertex within 200 - 300 m rad

Muons identified by property of crossing the calorimeter (and additional iron) without interacting

Additional criteria: Isolated track

No back-to-back jet

Main background: Medium energy \(K^\pm \rightarrow \mu^\pm\) decay kink may be opposite to magnetic deflection → track appears straight like a high momentum muon

Requires careful visual examination on Megatek graphic display

W selection: \(p_T > 15 \text{ GeV} / c\); \(E_T > 15 \text{ GeV}\)

Momentum not energy

Missing transv. energy after correcting for muon energy

47 events \(W \rightarrow \mu^+\mu^-\) [Note angular cover and "running time" less than for electron triggers]
Momentum resolution is poor so do not see Jacobian peak, but \( \mu \) transverse mom. and \( W \) transv. mass distributions correspond to Monte Carlo which includes resolution effects.

\[ p_T^{M_1} > 3 \text{ GeV/c}; \quad p_T^{M_2} > 3 \text{ GeV/c}; \]
\[ m_{\mu\mu} > 6 \text{ GeV/c}^2 \]

10 \( Z^0 \rightarrow \mu\mu \) candidates

<table>
<thead>
<tr>
<th></th>
<th>UA1 Expt. ((\sigma \cdot B)) nb</th>
<th>Theory ((\sigma \cdot B)) nb</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W \rightarrow \mu \nu ) ( \sqrt{s} = 546 \text{ GeV} )</td>
<td>(0.67 \pm 0.17 \pm 0.15)</td>
<td>(0.36 \pm 0.11)</td>
</tr>
<tr>
<td>( W \rightarrow \mu \nu ) ( \sqrt{s} = 630 \text{ GeV} )</td>
<td>(0.61 \pm 0.11 \pm 0.12)</td>
<td>(0.46 \pm 0.14)</td>
</tr>
<tr>
<td>( Z \rightarrow \mu\mu ) combined runs</td>
<td>(0.051 \pm 0.017 \pm 0.009) (\text{(prelim.)})</td>
<td>(0.046 \pm 0.015)</td>
</tr>
</tbody>
</table>

Test of lepton universality

\[ \frac{\sigma \cdot B (W \rightarrow \mu \nu)}{\sigma \cdot B (W \rightarrow e \nu)} = 1.07 \pm 0.17 \pm 0.13 \]

Mass of \( Z \) from \( Z \rightarrow \mu\mu \)

\[ m_Z = 88.8^{+5.5}_{-4.6} \text{ GeV} \]

PRELIMINARY
ARE THERE ADDITIONAL TYPES OF NEUTRINOS?

\[ \nu_1, \nu_2, \nu_3 \quad \text{HOW ABOUT } \nu_4, \nu_5 \text{ etc.?} \]

IF KINEMATICALLY ALLOWED, DECAYS TO ADDITIONAL LEPTON OR QUARK DOUBLETs, INCREASES \( \Gamma_W \) AND \( \Gamma_Z \).

HOWEVER IT IS LIKELY THAT ANY NEW QUARK, \( q_4 \) etc., WOULD HAVE \( m_{q_4} \gg m_{top} \) \( \sim 40 \text{ GeV} \).

\( W, Z \) COULD NOT DECAY INTO NEW QUARKS.

\( Z^0 \) COULD DECAY: \( Z^0 \rightarrow \nu_4 \bar{\nu}_4 \) etc.

EACH NEW PAIR WOULD ADD 181 MeV TO \( \Gamma_Z \).

A PRECISION MEASUREMENT OF \( \Gamma_Z \) WOULD TELL US HOW MANY NEUTRINO TYPES EXIST, ACTUALLY TELL US THE NUMBER OF LIGHT PARTICLE PAIRS IT DECAYS TO.

EXPERIMENTALLY AT THE COLLIDER THIS IS DIFFICULT BECAUSE \( \Gamma_Z \) IS COMPARABLE TO EXPERIMENTAL RESOLUTION.

UA2: \( \Gamma_Z < 4.6 \text{ GeV} \) (90% C.L.), \( N_0 \leq 13 \)
AN ALTERNATIVE METHOD

\[ \text{DEFINE RATIO } R = \frac{\Gamma (W \to q\bar{q})}{\Gamma (Z \to e^+e^-)} \]

\[ \text{THEN } R = \frac{\Gamma (W \to q\bar{q})}{\Gamma (Z \to e^+e^-)} \]

\[ \text{WHAT WE WANT} \]

\[ \text{FROM Q.C.D. ALTARELLI ET AL.} \]

\[ 3.3 \pm 0.2 \]

\[ \text{Z. PHYS. 23, 618, 1985} \]

WE KNOW THAT ANY NEW CHARGED LEPTONS MUST BE HEAVY (\( m > \) PETRA BEAM ENERGY, PROBABLY > MW OTHERWISE SEEN)

IF \( m_{\ell A} > m_W \) THEN \( W \to \ell_A \bar{\nu}_A \)

IF LIGHTER, STILL SUPPRESSED BY PHASE SPACE

ASSUME ADDITIONAL NEUTRINOS INCREASE THE WIDTH OF \( Z^0 \), NOT OF \( W \)

DESHPANDE ET AL. (P. R. L. 52, 753, 1985)

USING (SLIGHTLY) DIFFERENT VALUES FROM ABOVE

\[ N_Y = R (1.73 \pm 0.10) - 12.56 \]

\[ \text{THEORETICAL UNCERTAINTY} \]

PROPER TREATMENT REQUIRES CAREFUL USE OF STATISTICS, PARTICULARLY IF COMBINE UA1 & UA2

L. DI LElla (RAPPORTEUR AT BARI, JULY 85)

\[ R = 8.0 \pm 1.5 \] (UA1 + UA2 WEIGHTED AVER.)

\[ N_Y = 1.3 \pm 2.7 \]

\[ N_Y < 5.4 \pm 1.0 \] 90% C.L.

VERY INTERESTING, BUT WAIT FOR PROPER PUBLISHED RESULTS LATER THIS YEAR!
Evidence for Top Quark

Search for $W^+ \rightarrow t \bar{b}$ (or $W^- \rightarrow \bar{t}b$)

The $t$ quark might then hadronize into a jet containing a $T$ meson ($\pi^0 t\bar{t}$) amongst fragments, and $\bar{b}$ into a jet containing $\bar{B}$ meson.

This would not be a clean signature because of QCD background giving 2 jets.

A good signature occurs when $t$ quark decays semi-leptonically $W \rightarrow t \bar{b}$ should occur ~ $\frac{1}{3q}$ for $e^+ \nu \bar{b}$ ($\mu$)

Hence in a plane $\perp \overrightarrow{pp}$ beam axis

$W \rightarrow b\bar{b}$ jet 1

$\rightarrow t$ quark

$\nu$ jet 2

$M_W$ is large (> 22 GeV) $\rightarrow b$ jet 2

$\therefore$ its decay products will have large relative transverse momenta

$\therefore e(\mu)$ will be isolated from jets

Signature: $e^- \nu \rightarrow J_1 J_2$
The $b$ quark would show a Jacobian peak

But remember:
Jets measured in apparatus are not identical to basic quarks.

Lepton spectrum would be as shown.

Problems (compared to $W \rightarrow e \nu$):

1. Average lepton energy down factor $\sim 3$
2. Presence of 2 jets increases background from hard parton-parton scattering
3. Number of events (after appropriate cuts) expected to be down factor $\sim 40$
UAI (P.L. 1478, 293, 1984) 1983 data ($^{120}_{\text{nb}}$)

**Formidable task**: require full rejection power of apparatus.

Look in both channels: $e$ and $\mu$

**Electron Selection**

$E_E > 12$ GeV; at least 1 jet $E_J > 10$ GeV $\rightarrow$ 152 events

43 electrons identified as $\pi$ conversion $\rightarrow 109$

Now apply tighter selection than in $W \rightarrow e\nu$ search

Raise $E_E$ to 15 GeV

Require energy (calorim.) to match mom (field)

Require correct longitudinal shower development

Stricter isolation: $E_E < 1$ GeV in cone $\Delta R = 0.4$

Around electron track

$\rightarrow e + 1$ jet 7 events

$\rightarrow e + 2$ jets 3 $\leftarrow$ main candidates

$\rightarrow e + > 2$ jets 2

**Background**: expected from $Q \bar{Q}$ jets which

fake electrons by fragmenting so that 1 energetic

charged pion overlaps with 1 or more $\pi^0$

**Find**: share (distribution) of background

in various kinematic parameters, very different.
UA1: Top Quark Candidate

$W^- \rightarrow \bar{t} b$

EVENT 7443/509
MUON SELECTION

\( p_T^\mu > 12 \text{ GeV/c} \), GOOD GEOM. MATCHING IN MUON CHAMBERS
\( E_T^j > 10 \text{ GeV} \)  \rightarrow 279 EVENTS

ISOLATION: IN CONE \( \Delta R = 0.4 \) AROUND MUON, REQUIRE
\( \sum E_T^j < 0.2 \ p_T^\mu \)  \rightarrow 40 EVENTS

DOMINANT BACKGROUND: DECAY OF \( \pi, K \) IN C.D.

VISUAL SCAN FOR KINKS \( \rightarrow \) LOOK AT TRACK DIGITIZINGS. ALSO ENHANCE ISOLATION

\( \Delta R (J, \mu) > 1 \)

\rightarrow \mu + 1 JET  \quad 6 \ \text{EVENTS}

\rightarrow \mu + 2 JETS  \quad 3 \ \text{"}  \quad \leftarrow

\rightarrow \mu + > 2 \text{JETS}  \quad 1 \ \text{"}
TOP QUARK CANDIDATES

3-BODY Versus 4-BODY mass distribution

\[ e^+ e^- J_z \quad e^+ e^- J_1 J_2 \]

\[ W \rightarrow 1B \]
\[ m_t = 40 \text{ GeV/c}^2 \]

EVENTS/5 GeV/c^2

\[ 6 \text{ EVENTS} \]

\[ m (l \nu, J_1 J_2) \text{ GeV/c}^2 \]
EXPECTED NUMBER OF EVENTS

ASSUME $m_t = 40$ GeV  \[ t \rightarrow b e \nu \] \[ t \rightarrow a l l \] = 11%

BEFORE CUTS: EXPECT 18 EVENTS

CUTS: $p_T^l$ (LEPTON) AND $E_T^j$ (JET) \[ \rightarrow 2.7 \, e + 2.3 \mu \]

EFFICIENCIES: $E_e = 0.6$ ; $E_\mu = 0.34$
(MEASURED FROM $W \rightarrow e \nu$ AND $W \rightarrow \mu \nu$ SAMPLES)

\[ \therefore \text{EXPECT} \]

\[ 2.7 \times 0.6 + 2.7 \times 0.34 = 2.6 \text{ EVENTS} \]

OBSERVE 6 " \[ \text{STATISTICALLY COMPATIBLE.} \]

SITUATION: AUTUMN 1984

EXCITING RESULT, EVIDENCE IN 2 EXPERIMENTALLY QUITE DISTINCT CHANNELS.
BUT CONFIRMATION REQUIRED.

SITUATION: SUMMER 1985

1984 RUN ADDED $\sim 270$ nb$^{-1}$ \[ \text{[i.e. TOTAL STATISTICS}
\text{NOW INCREASED BY FACTOR 3.2]}

AMUSING SITUATION \[ \rightarrow \text{WHICH IS BETTER} \]
EVIDENCE?

(a) GET MORE TOP CANDIDATES \[ \text{CANNOT WIN!!} \]
(b) GET NO MORE
UA2

Do not confirm signal, but also cannot exclude existence of 40 GeV t quark
[No muon capability, no momentum measurement
over most detector, no longit.-electron sampling]

UA1

[PRELIMINARY - ANALYSIS IN PROGRESS]

1. Find $e + 2\text{jet} \rightarrow \text{another 6 events}$
   $\mu + 2\text{jet} \rightarrow \text{still scanning}$
   (look for decay kinks)

2. Investigate (by computing) alternative channel
   Direct QCD production
   \[
   p\bar{p} \rightarrow t\bar{t} + \ldots \quad \xrightarrow{\text{hadrons (}J_1\text{)}} \quad l_{\nu} b_{(}J_2\text{)}
   \]

From Eurojet Monte Carlo $\rightarrow$

If $M_t = 40$ GeV, we expect rate from this process to be $\sim 2$ times rate from W
$\text{Mass}(l\nu_1 J_2)$

- $W \rightarrow l\nu_1$
- $m_1 = 4.0 \text{ GeV}/c^2$

Events/5 GeV/c$^2$

- $e \rightarrow 2$ Jets
- $\mu \rightarrow 2$ Jets

$\text{Mass}(l\nu_2 J_\tau J_3)$

- $W \rightarrow l\nu_2$
- $m_1 = 4.0 \text{ GeV}/c^2$

12 Events

- $e + 2$ Jets
- $\mu + 2$ Jets
Possible Interpretation of Lepton + 2 Jets Events

\[ \frac{1}{3} W \rightarrow t \bar{b} \quad \frac{2}{3} p \bar{p} \rightarrow t \bar{t} + \ldots \]

\[ 30 < m_t < 50 \text{ GeV} \]

1. Why not \( m_t = 40 \pm 5 \text{ GeV} \)?
   - Mass plot \( m(A \cup J_2) \) distorted by cuts
   - Definition of jet not identical to quark

2. Why do \( t \bar{t} \) events still agree with \( W \) mass?
   - Kinematic constraints. \( m_t = \frac{m_W}{2} \)

3. Do we need top?
   - Consider \( p \bar{p} \rightarrow b \bar{b} q \left( c \bar{c} q \right) \)
     as a possible source of isolated leptons + 2 jets.
     2 reasons to eliminate this:
     1. Isolation criterion removes 95% of \( b \bar{b} q \)
     2. \( c \bar{c} J_2 \) distribution is wrong.

Has top been found?
Evidence not yet stronger than last year.
I am optimistic – but wait for full analysis.
JETS

SOFT COLLISIONS \equiv SMALL p_t, E_t

HARD \equiv LARGE p_t (OR LARGE EFFECTIVE MASSES PRODUCED)

HARD COLLISIONS REPRESENT ONLY A SMALL FRACTION ($\leq 10^{-3}$) OF TOTAL AT COLLIDER
BUT SENSITIVE TO INTERNAL STRUCTURE OF PROTON (ANTIPROTON)

EXAMPLE (PRE-HISTORY, 1911 $\alpha$-NUCLEUS SCATT.)

THOMSON ATOM: STATISTICAL EFFECT OF MANY SMALL COLLISIONS IN "PLUM PUDDING"
EXPECTED FRACTION OF $\alpha$ SCATT. AT $\theta > 90^\circ \leq 1$ IN $10^{14}$?

EFFECT OF "POINT NUCLEUS" $\sim 10^{-15}$ $\ll 10^{-8}$ NUCLEUS ATOM
EXPECTED FRACTION $\theta > 90^\circ \sim 1$ IN $10^{8}$

GEIGER, MARSDEN RUTHERFORD

BECAUSE MULT. SCATT. DOES OCCUR, MUST LOOK IN CORRECT REGION (SUFFICE, LARGE ANGLES OR $p_t$)
HISTORY (1971-81: 10 years of ISR + some fixed target)

First observation of high $p_T$ single particles interpreted within parton model.

We now know partons = quark, antiq., gluon
Free partons not observed. Confinement.
Final state interactions result in production of conventional hadrons $\rightarrow$ "hadronisation" or "fragment."
This involves mostly low momentum transfer $\rightarrow$ collimated system of hadrons ("jets")

Jet 4-momentum $\cong$ 4-mom. of parent parton

Incident partons have low $p_T$ .
Jets coplanar with beam, but need not be collinear with each other ($x_1, x_2$)
ISR → Initially Single Particle Triggers (Bias)

Correlations between 2 large pT hadrons suggest jets

→ Calorim. triggers best way unbiased ??

Problems

1. Limited Solid Angle, Fermilab E270

Δη x Δφ ~ 1 x 1 rad ~ Jet Size

Limited Solid Angle + mom. conservation force 2-Jet Structure

2. Integration over too large solid angle, CERN

Σ E_0, Full Azimuth Φ = 2π, 40° < θ < 140°

Result: trigger on high multiplicity events, consisting of low pT particles! No Jet Evidence

Situation: End of 1981

→ Confusion in hadron collisions, no convincing evidence for jets

→ However, hadronic jets seen from e+e- collisions
PROBLEM: RELATIONSHIP BETWEEN "PARTONS"
AND EXPERIMENTAL SIGNATURE OF "JETS"

ANALOGY: BUBBLE CHAMBER

EXTERNAL COUNTER Hodoscope (Poor Resolution)

"MEDIUM-HARD"
PARTON COLLISION

To Apparatus
Earth-Sun Distance
On This Scale!!
REQUIRE
1. LARGE ANGLE (HIGH $E_t$) SCATTER TO MOVE PARTONS AWAY FROM BEAM FRAGMENTS
2. HIGH ENERGY SCATT. PARTONS $\rightarrow$ COLLIMATE JETS $\rightarrow$ COLLIDER IS GOOD
3. "GOOD" JET ALGORITHMS

**UA2 JET = CLUSTER OF ADJACENT CELLS**

SIMPLE: 1 FINE GRANULARITY, SAME GEOM.

FOR E-M & HADRON CELLS

240 CELLS, EACH $15^\circ \times 10^\circ$

CLUSTER $n (\geq 1)$ ADJACENT CELLS

EACH WITH $E > 0.4$ GeV

CLUSTER $E_t = \sum E_{t,i}$ OVER CLUSTER

CLUSTER ORDERING:

$E_t^{(1)} > E_t^{(2)} > \ldots E_t^{(n)}$

FOR 2-JET EVENT, EXPECT

$E_t^{(1)} \approx E_t^{(2)} \gg E_t^{(3)} \gg \ldots E_t^{(n)}$

$\Delta \phi_{1,2} \approx 180^\circ$ (COPLANARITY WITH BEAM AXIS)
To each "effective" cell (EM or hadron) associate an energy flow vector pointing from vertex to cell center. Order those cells with $E_T > 2.5$ GeV ("initiator") jets are defined as clusters of fixed angular size in ($\eta, \phi$) space.

$$\Delta r \equiv \left[ (\Delta \eta)^2 + (\Delta \phi)^2 \right]^{1/2} = 1.0$$

Value comes from a study of jet width.

(Cone of ~40° half angle when near $\phi = 90°$)
WE CAN NOW UNDERSTAND RESULTS OF EXPERIMENTS WITH $E_T$ CALORIM. TRIGGERS OVER $\Delta \phi = 2\pi$

--- HARD (PARTON-PARTON) COLLIS. 2 JETS

--- SOFT (TAILS OF MULTIPlicity)

---

NA5 CANNOT SEE JETS

$$\frac{d\sigma}{d(E_T)}$$

$|\eta| < 1$

SPS $\sqrt{s} = 25$ GeV

ISR FINALLY SEE THEM (AFS) 1983

$$\frac{d\sigma}{d(E_T)}$$

$|\eta| < 1$

ISR $\sqrt{s} = 63$

UA2 FIND JETS PARIS 1982

$$\frac{d\sigma}{d(E_T)}$$

$|\eta| < 1$

SPS $\sqrt{s} = 540$

UA1 INITIALLY TRIGGER ON TOO LARGE $\Delta \eta$

$$\frac{d\sigma}{d(E_T)}$$

$|\eta| < 3$

$\sqrt{s} = 540$
ARE QUARKS COMPOSITE?

RUTHERFORD SCATT. → "POINT" NUCLEUS INSIDE ATOM
JETS AT COLLIDER → "POINT" PARTONS INSIDE PROTON
↑ [ALREADY KNOWN, DEEP INELASTIC LEPTON-PROT. SCATT.]
WORLD'S HIGHEST ENERGY → CAN WE SEE IF
PARTONS HAVE A SUB STRUCTURE?

SUPPOSE A HYPOTHETICAL SUPER STRONG FORCE
BINDS PREONS INSIDE QUARKS
→ INDEPENDENT OF EXACT DETAILS, WE WOULD
EXPECT TO SEE MORE SCATTERING AT LARGE P_T

IF THE ENERGY SCALE OF THIS NEW INTERACTION
IS Λ_c, THE EFFECTS WOULD SHOW UP AT
CONSIDERABLY LOWER MOM. TRANSFERS [GICKEN ET AL.]

INCLUSIVE JET AND 2-JET CROSS SECTIONS
→ CROSS SECTIONS INCREASE WITH √S , 546 → 630 GeV
→ Λ_c = 00 IS A GOOD FIT
→ Λ_c = 460 GeV IS BEST FIT
\{ \text{INCLUDING ALL THEORET. & EXPERIMENTAL UNCERTAINTIES} \}
Λ_c > 370 GeV
95% C.L.
**Parton-Parton Scattering**

If \( \frac{d\sigma}{d\cos \theta} \) is differential cross-section for a particular parton-parton sub-process, we can write contribution to 2-jet cross-section

\[
\frac{d^3\sigma}{dx_1 dx_2 d\cos \theta} = \left[ \frac{F(x_1)}{x_1} \right] \left[ \frac{F(x_2)}{x_2} \right] \frac{d\sigma}{d\cos \theta}
\]

\( F \) is a structure function representing number density of appropriate partons in anti-proton and proton

\[ \uparrow \quad \uparrow \]

scaled longitudinal parton mom. \( x_1 \quad x_2 \)

Differential cross sections for the possible subprocesses have been calc. to leading order in QCD

[COMBRIDGE ET AL. P.L. 70 B, 224, 1977]

\[ \leftrightarrow + MAXWELL NUC. PHYS. B229, 429, 1984 \]

**Elastic Processes**

- Gluon-Gluon
- Gluon-Quark [Antiquark]
- Quark-Antiquark

Have a similar angular dependence and are expected to dominate like Rutherford scattering
Consider some 1st order diagrams [OTHERS EXIST]

\[ g g \rightarrow g g \quad g q \rightarrow g q \quad g g \rightarrow g g \quad q \bar{q} \rightarrow q \bar{q} \]

**Vector Gluon Exchange**

Cross sections will depend on $x_s$.

Structure function can be factorised

\[ F(x) = G(x) + \frac{4}{9} \left[ Q(x) + \bar{Q}(x) \right] \]

Relative strength of $ggg$ and $gqg$
predicted by QCD

Cross sections will be in ratio $1 : \frac{4}{9} : \left( \frac{4}{9} \right)^2$

$gg, gq, gq$

To the approximation that angular distributions have exactly same shape, we can extract a combined structure function from the data.
VECTOR GLUONS (QCD) \{ q\bar{q} \text{ & } g\bar{g} \text{ & } gg \}

ABELIAN SCALAR GLUONS DO NOT FIT DATA

COLLINE + SOPER F.R. 16, 2217 1977

DATA CONSISTENT WITH VECTOR GLUONS (QCD)

SIMILAR RESULTS FROM UA2
UAI
STRUCTURE FUNCTION
\[ F(x) = G(x) + \frac{4}{q}[Q(x) + \bar{Q}(x)] \]

QCD PARAMETRIZATION OF CDF\nNEUTRINO MEASUREMENTS + SCALING
ADJUSTMENT FOR Q^2

WITHOUT GLUONS
\[ \frac{4}{q}[Q(x) + \bar{Q}(x)] \]

SIMILAR RESULTS FROM UA2

SHOWS DIRECTLY, FOR FIRST TIME (1983)
VERY LARGE FLUX OF GLUONS IN THE
PROTON AT LOW x (x < 0.3)
3 JET : 2 JET COMPARISON

QCD STRONG INTERACTION RADIATIVE EFFECTS → CAN GIVE MULTIJETS

AT HIGH (SUB-PROCESS) ENERGIES $\sqrt{s}$

QCD BREMSSTRAHLUNG DOMINANT FOR 3 JETS

\[ \sum J \]

THIRD VERTEX

\[ \gamma \]

EXAMPLE

CROSS SECTIONS:

\[ \sigma_{2J} = C_{2J} \frac{\alpha_s^2}{\sqrt{s}} \]

\[ \sigma_{3J} = C_{3J} \frac{\alpha_s^3}{s} \]

DIMENSIONLESS

DEPEND ON SUB-PROCESSES

$gg \rightarrow gg ; gg \rightarrow JJJ$ ETC

AND ON EXPERIMENTAL CUTS

NUMERICAL VALUES CAN BE CALCULATED FOR PARTICULAR

EXPERIMENTAL CUTS USING EXACT LEADING-ORDER

THEORETICAL FORMULA


CAMBRIDGE: P.L. 708, 22A, 1977

RESULT: 2 JET CROSS SECTIONS FOLLOW 1 : 4 : 16 : (64)$^2$ RULE

DEPENDING ON INCOMING PARTONS.

BUT, REMARKABLY, SO DO 3 JETS !! (±25%)

IMPORTANT CONSEQUENCE: $3J/2J$ INDEPENDENT OF PARTON COMBINATION
Hence can measure $\alpha_s = \frac{\sigma_{3J}}{\sigma_{2J}} \frac{C_{23}}{C_{3J}}$ calculated in leading order.

Experimentally: For a given amount of data ($N_{\text{jet}} = 5600$)

$\rightarrow$ Find 2-jet events and 3-jet events

$\rightarrow$ Decide on cuts for "clean" events

$\rightarrow$ Need scan all (173) 3-jet candidates

$\rightarrow$ Decide how to match (in $Q^2$) the 3 jet with corresponding 2 jet events

Theoretically: Calculations only leading order

$\rightarrow$ Higher order effects have not yet been calculated (but could be)

$\rightarrow$ Hence for now insert k factors

UA1 (P.L. 158 B, 494 5 Sep. 1985)

Result $\alpha_s \left( \frac{K_{3J}}{K_{2J}} \right) = 0.16 \pm 0.02 \pm 0.03$

\[ \text{Anticipate statistical systematic} \]

UA2 (preliminary - Autun meeting)

$0.21 \pm 0.02 \pm 0.04$
UA1 Measurement of $\alpha_s$

(P.L. 1588, 494, 1985)
SUMMARY OF JET PHYSICS

HIGH $E_T$ JET PRODUCTION DOMINANT IN HARD COLLISIONS AT $p\bar{p}$ COLLIDER

JET PRODUCTION CAN BE STUDIED BY (ALMOST) IGNORING FRAGMENTATION (AS IF HIGH $E_T$ PARTONS DETECTED)

DATA IN GOOD AGREEMENT WITH QCD:
  INCLUSIVE CROSS SECTIONS
  ANGULAR DISTRIBUTION OF PARTON-PARTON SCATTERING STRUCTURE FUNCTIONS

VALUES OF STRONG COUPLING CONSTANT OBTAINED AT HIGH ENERGIES (STILL REQUIRE CALCULATION OF HIGHER ORDER DIAGRAMS)

QUARKS ARE POINT-LIKE (AT LEAST UP TO AN ENERGY SCALE OF $\sim 370$ GeV)

[ NOTE: FRAGMENTATION FUNCTIONS NOT COVERED IN THIS LECTURE ]
LATTICE QUANTUM FIELD THEORIES

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ABSTRACT

An elementary introduction to basic ideas forming the modern
theory of strong interaction, QCD, is given. The lattice
formulation of QCD as well as applications of the numerical
Monte Carlo method is discussed. Some results on calculations
of physical quantities obtained by using this approach to QCD
are presented.

PREFACE

Recently, there has appeared a lot of lecture courses and surveys
where both a systematic introduction and a review of the present status of
lattice gauge theories are given. However, most of those are written on
an advanced level.

The present lectures are addressed to an experimentalist working on
elementary particle physics who is not a professional, generally speaking,
in quantum field theory (and is not going to become one after reading
these notes). Due to this purpose, the main attention in these lectures
is paid to a schematic description of
1. the basic ideas founded a basis for QCD — the modern theory of strong
interaction;
2. the methods for numerical studies of QCD by using computers;
3. the obtained results that are of interest for elementary particle
physics.

The author attempted to consider a little bit more thoroughly the
points that might turn out to be related to current research work of
the reader (like glueballs of quark-gluon plasma). Few exercises for more
detail studies of the subject are presented at the end of each section.

The author apologizes for lacking of references to classical papers
on lattice gauge theory. This was done in order to not interrupt the
reading. The references are given only on those original papers whose re-
results are reproduced in the text. The complete list of references can be
found in lectures/surveys 1-10 .
INTRODUCTION

All that is spoken about below turned out to be possible due to the fact that fundamental equations describing strong interaction are known already. It is known that strongly interacting particles, hadrons, are composed of quarks as well as the law of interaction between quarks.

Nevertheless, it does not mean that theory of strong interaction is finished. Quantitative or even qualitative understanding of hadron physics, that would follow directly from the first principles, was lacking up to recent time. This is related to an extreme difficulty of the corresponding dynamical problem.

Some promising results along this line have been obtained by applying to the theory of strong interaction the numerical methods developed in statistical mechanics for solving somewhat similar problems. As a result of such a synthesis of quantum field theory, statistical mechanics and computer science, there arose a new subject called lattice quantum field theories or, more precisely, lattice gauge theories.

Any characteristic of strong interaction can be calculated, in principle, withing this approach. However, only simplest quantities have been computed up to now. An optimistic (maybe too) view is that limitations of the method are due solely to the power of present computer facilities.
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1. IDEAS (the modern view on the theory of strong interaction)

The modern theory of strong interaction, quantum chromodynamics (shortly QCD), is built on the fact that quarks possess the color charge which can emit and absorb gluons — mediators of strong interaction. Gluons possess the color charge as well. Due to this fact there appears their self-interaction whose shape is determined unambiguously by the local gauge invariance. The color interaction weakens at small distances so that perturbation theory is applicable to QCD in this region. However, the color interaction becomes strong at large distances. Therefore, the problem of large vacuum fluctuations arises that cannot be solved by perturbative methods.

1.1. The Era before QCD

The beginning was the work by H. Yukawa published in 1935. Yukawa put forward the hypothesis that strong interaction between nucleons originates by a meson exchange (not seen experimentally that time) like photon mediates the electromagnetic interaction.

The Yukawa theory lived till fiftieth when a lot of new strongly interacting particles had been discovered. These mesons can be emitted and absorbed by nucleons just as π-meson. Therefore, those are mediators of strong interaction in the same sense as π-meson. Do not think Yukawa theory to be wrong. It works well at distances about 1 fm = (200 MeV)\(^{-1}\) where heavy meson contributions are suppressed. However, those become essential at smaller distances, say 1 GeV, where mesons are no longer elementary pointlike objects but rather are composed of quarks.

The simplest version of the quark model (without color) had been contradicted to the Pauli exclusion principle since some baryons (like Ω\(^{-}\) or Δ\(^{++}\)) turned out to be composed of three identical quarks. The color had been invented to eliminate this contradiction. The number of different color quark states equals 3. The color degrees of freedom are described mathematically by the fact that quark field transforms as the fundamental representation of the color group SU(3).
1.2. QCD

The color is a first step in constructing QCD. The second one consists in the fact that color interaction between quarks results from exchanging quanta of corresponding field -- gluons -- that possess spin 1 like photons. Having imposed only the conservation of the color charge when emitting and absorbing gluons, one arrives at the interaction lagrangian \( \overrightarrow{\gamma} \gamma^\mu \overrightarrow{A}_\mu \overrightarrow{\gamma} \) where \( \overrightarrow{A}_\mu \) is an arbitrary 3x3 hermitian matrix while \( \gamma^\mu \) stand for the Dirac matrices. Since an arbitrary hermitian 3x3 matrix that contains 9 independent components can be represented as a linear combination of singlet (unit matrix) and octet (adjoint) representations of SU(3), the two possibilities arise: singlet versus octet gluons.

The theory with singlet gluons (that are called also white or colorless) would have only little differences from QED (short for quantum electrodynamics). The interaction between quarks should be diagonal w.r.t. color that would lead immediately to experimental consequences. In particular, a quark and an antiquark form 9 meson states which differ by their color only. The masses of all 9 mesons would be equal. Hence the number of meson states would be 9 times bigger than observed experimentally.

For the theory with octet gluons (called now simply as gluons), the color interaction is no more diagonal w.r.t. color. Hence there is no reason to expect equal masses of white and octet mesons. It is the theory that is known now as QCD. It agrees with all the experimental data on strong interaction and is considered to be its Theory.

1.3. Gauge Fields

The third fundamental principle, QCD, is based on, is that of local gauge symmetry of color interaction. This symmetry allows to fix, in part, self-interaction of gluon field.

The gluon field belongs to a class of gauge fields. The simplest example of a gauge field is the electromagnetic field whose transversal components describe photons. Otherwise, the longitudinal ones are related to gauging the phase of wave-function, i.e. permit to compare its values at different space-time points when an electron is placed in an external
electromagnetic field.

As is well-known from quantum mechanics, the wave-function phase itself is unobservable. Only the phase differences are observable, e.g. via interference phenomena. For the electron in an electromagnetic field, the current (gauged) value of phase of the wave function, $\Psi$, at the point $y$ of 4-dim. space-time is related to its value at some reference point, $x$, by the parallel transport

$$\Psi(y) = e^{ie \int_x^y d\gamma^\mu A_\mu(\gamma)} \Psi(x) \quad (1.1)$$

where $A_\mu(x)$ stands for the 4-vector potential of electromagnetic field. Integration in the exponent goes along some path, $C_{xy}$, that connects $x$ and $y$.

The exponential in (1.1) is called the phase factor. It is essential that phase factors are observable in the framework of quantum theory, contrarily to classical one. This is known as the Aharonov-Bohm effect. The corresponding experiment is depicted schematically in fig. 1. It allows to measure the phase difference between electrons passing along the two sides of solenoid. The fine point is that magnetic field is nonvanishing only inside the solenoid while electrons do not pass there. Hence electrons pass throughout the region where magnetic field strength vanishes! Nevertheless the vector potential, $A_\mu$, itself is nonvanishing that yields observable consequences.

For the non-abelian gauge group SU(3), a quark can alter its color under the parallel transport so that non-abelian phase factor is a unitary $3 \times 3$ matrix. An extension of (1.1) to the non-abelian case reads

$$\Psi(y) = P \exp \left( i g \int_x^y d\gamma^\mu A_\mu(\gamma) \right) \Psi(x) \quad (1.2)$$

The symbol, $P$, on the r.h.s. means the path-ordering. That is to construct the matrix of parallel transport at finite distance, one has to subdivide the path $C_{xy}$ into small parts and to form the ordered product of the parallel transports along these small parts:
\[ \text{P} \exp \left( i g \int_x^y d\lambda^\mu A_\mu (\tau) \right) = \prod_j \left( 1 + i g d\lambda^\mu A_\mu (\xi_j) \right). \quad (1.3) \]

A non-abelian analogue of the quantity measurable in the Aharonov-Bohm experiment is the trace of matrix of parallel transport along the closed path.

1.4. Dynamics of QCD

The quark and gluon fields fluctuate in space and time as it should be due to the quantum-mechanical uncertainty principle. Owing to this fact the quantum vacuum is not an empty space but is filled by virtual fields born at small time. Because those possess the color charge, the virtual particles participate in strong interaction.

It is convenient to characterize vacuum by the shape of potential between two heavy (static) charges against the distance between them. In QED the virtual fields do not change drastically the Coulomb law because the coupling, \( e \), of interaction of photon emitted by the test charge with virtual electrons is small ( \( e^2/4\pi = 1/137 \)). Corrections to the Coulomb law can be calculated as a series in \( e^2/4\pi \). The result is that QED-vacuum gains some weak dielectric properties at distances of order of Compton wave length of electron, i.e. screens the test charges.

Such a screening can be imagined as attraction of virtual electrons and positrons born in vacuum to the test charges (electrons to the positive charge and positrons to the negative one, respectively) that diminishes their absolute values. If the test charge is surrounded thoughtfully by a small sphere (whose centre coincides with the charge), the value of charge inside the sphere would depend on its radius. This quantity is called the effective charge at given distance. Therefore, the effective charge grows in QED as the distance is decreased.

An opposite situation occurs in QCD — the effective color charge diminishes with decreasing the distance. An opposite phenomenon — antiscreening — rather than screening occurs. That is to say the
smaller the distance the smaller the strength of color interaction is. At distances that are small compared to the characteristic scale of strong interaction ($\sim 1$ fm), the effective charge versus the distance, $r$, is given by the asymptotic freedom formula

$$g^2(r) = \frac{8\pi^2}{(11 - 2N_f/3) \log r \Lambda_{\text{QCD}}}.$$  \hspace{1cm} (1.4)

Here $N_f$ is the number of different quark flavors ($u, d, s, \ldots$). The scale parameter, $\Lambda_{\text{QCD}}$, is determined experimentally to be $200 \pm 400$ MeV. As is seen from eq. (1.4), strong interaction weakens at small enough distances so that perturbation theory to be applicable.

However, $g^2(r)$ becomes $\sim 1$ at $r \sim 1$ fm. In other words, the interaction becomes strong at such distances, indeed. As a result, gluon vacuum fluctuations of the size $\sim 1$ fm interact strongly that can lead to a drastic change of QCD-vacuum. There is no reason to expect the interaction of color charges at distances $\sim 1$ fm to be described by the Coulomb law.

The law of interaction between color charges is described usually making use of lines of force similar to those in electrodynamics. For the Coulomb law, the lines of force have a shape shown in fig. 2a. An attractive physical picture should appear if large gluon fluctuations would shrink the lines of force into a tube (see fig. 2b) like magnetic field in superconductor is shrunk into vortices. In this case the interaction energy would be linearly proportional to the distance between test color charges.

The linear potential between quarks would not allow them to be separated at macroscopic distances and, therefore, to exist as free particles. The idea of quark confinement is one of central in the modern theory of strong interaction. The major arguments in favor of confinement in QCD are obtained by lattice calculations.

**Exercises to sect. 1.**

To sect. 1.1.

Given the quark fields, $\Psi$, form the composite fields with the
quantum numbers of $\Omega^-$ and $\Delta^{++}$.

To sect. 1.2.

Expand the direct product of the quark field, $\Psi$, and the antiquark field, $\overline{\Psi}$, into singlet and octet representations of $\text{SU}(3)$.

To sect. 1.3.

1. How the phase difference measured in the Aharonov-Bohm experiment depends on the value of current?

2. How the non-abelian phase factor (1.3) transforms under the gauge transformation

$$A_\mu(x) \rightarrow \Omega^+(x) A_\mu(x) \Omega(x) - \frac{i}{g} \Omega^+(x) \partial_\mu \Omega(x)?$$

3. Show that sufficient and necessary conditions for the phase factor to be independent on the form of the path are vanishing of the strength

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + ig [A_\mu(x), A_\nu(x)].$$
2. METHODS (analogy between quantum field theory and statistical mechanics)

The placing of gauge theories onto the lattice provides, at first, a proper regularization of QCD-divergences in the ultraviolet region (i.e. at small distances or large virtual momenta). Secondly, the lattice formulation of QCD possesses some nonperturbative terms in addition to perturbation theory. Therefore, one has a nontrivial definition of QCD beyond perturbation theory which guarantees confinement. Thirdly, the use of lattice formulation makes clear analogy between quantum field theory and statistical mechanics. If offers a possibility to apply to QCD the nonperturbative methods well-developed in statistical mechanics, such as high-temperature expansion or numerical Monte Carlo method. However, the lattice in QCD is no more than an auxiliary tool to obtain results for the continuum limit. In order to pass to the continuum, the lattice spacing has to be many times smaller than the scale characterizing strong interaction.

2.1. Lattice Formulation of Gauge Theories

The lattice is defined as a regular set of space points as is shown in fig. 3. The points themselves are called the sites, the shortest distance between two neighbour sites — the link, while an elementary square bounded by four links — the plaquette. The length of the link is known as the lattice spacing, a. A site is characterized by the coordinate, \( x \) (which is an integer in units of \( a \)), a link — by the coordinate and the direction, a plaquette — by a coordinate and two directions. Usually, the lattice has finite extent and the periodic conditions are imposed at the boundary (see fig. 3).

Dynamical variables of the lattice gauge theory are the phase factors, \( U_{x/\mu} \), associated with the parallel transport along links. Those are called the link variables. For the given gauge theory, \( U_{x/\mu} \) belongs to the given gauge group, say \( U_{x/\mu} \in SU(3) \) for QCD or \( U_{x/\mu} \in U(1) \) for QED. To pass to the continuum, one lets \( a \to 0 \). In this limit

\[
U_{x/\mu} \to \exp (iga \Lambda_{\mu} (x)).
\]
The Wilson action for an euclidean formulation of lattice gauge theory is build of the traces of the matrices $U_p$ — the phase factors associated with plaquette boundaries (i.e. ordered products of 4 link variables $U_{x,\mu}$ along plaquette boundaries). The Wilson action reads

$$S = \sum_p \left( 1 - \frac{i}{N} \Re \text{tr} \ U_p \right)$$

(2.1)

where summing goes along all the plaquettes of 4-dim. lattice. The action (2.1) is written for arbitrary gauge group SU(N) or U(N). For SU(3), one has to substitute $N = 3$, while for U(1) $N = 1$ and eliminate the trace, $U_p \rightarrow \exp(i g a^2 F_{\mu}(x))$ as $a \rightarrow 0$ so that (2.1) reproduces the correct continuum Yang-Mills action.

The words "euclidean formulation" at the beginning of the previous paragraph means that time is replaced by 4-th (euclidean) coordinate $x_4 = i t$. The euclidean formulation is very convenient when calculating stationary (i.e. independent on time) quantities like hadron masses, width of their decays, interaction potentials, formfactors in space-like domain while a large part of information about space-time dynamics is lost.

The euclidean lattice formulation allows to present explicit and practically useful formulas for averaging over quantum fluctuations of $U_{x,\mu}$'s. For example the vacuum effects are determined by the partition function

$$Z(\rho) = \int \prod_{x,\mu} dU_{x,\mu} e^{-\beta S}.$$  

(2.2)

Here the product in the integration measure goes along all the links because $U_{x,\mu}$ fluctuates at each link. The connection between these fluctuations is provided by the weight factor, $\exp(-\beta S)$, which is similar to the Boltzman factor in statistical mechanics. The inverse "temperature", $\beta$, should be related to the coupling constant, $g$, by
\[ \beta = \frac{2N}{g^2} \tag{2.3} \]

in order that eq. (2.2) would reproduce the corresponding path integral for the continuum theory. Integration over \( U_{x,j} \) at each link goes over the group measure (known also as the Haar measure). This guarantees (2.2) to be invariant under local gauge transformation on the lattice.

The central role in the lattice approach is played by the averages of traces of phase factors associated with closed loops on the lattice. An example of such a loop is a rectangular of the size \( R \times T \). These averages are determined by the formulas of the type (2.2):

\[ W(R \times T) = \mathcal{Z}^{-f}(\beta) \int \prod_{x,j} dU_{x,j} \mathcal{E}^{-\frac{\beta S}{N} \text{tr} U_{R \times T}} \tag{2.4} \]

where \( U_{R \times T} \) stands for the phase factor associated with the rectangular. \( \mathcal{Z}^{-f} \) in front is necessary in order to define the physical quantities relatively the correct vacuum of quantum theory.

The integrals in (2.4) can be calculated as a power series in \( \beta \) which is similar to the high-temperature expansion in statistical mechanics. It is called the strong coupling expansion. To leading order in \( \beta \), one gets

\[ W(R \times T) = \left( \frac{\beta}{2N^2} \right)^{R \cdot T / a^2} \tag{2.5} \]

for \( N \geq 3 \).

The exponent contains the area enclosed by the given (rectangular) loop. This fact becomes clear by noticing the following relation between \( W(R \times T) \) and \( E(R) \), the potential energy between two static quarks at the distance \( R \) in colorless total state,

\[ W(R \times T) = \mathcal{E}^{-E(R) \cdot T} \tag{2.6} \]

\( T \gg R \)
Therefore the area law (2.5) is associated with the linear potential that leads to confinement. Otherwise, the perimeter (rather than area) law is associated with the Coulomb potential. These two assertions form the body of the Wilson criterion for quark confinement.

Up to now we do not consider quantum fluctuations of quark fields. To switch them on, one adds to (2.1) the quark action. Its simplest form is given by

$$S_\psi = -\frac{i}{2} \sum_{\vec{x}, \vec{\mu}} \left[ \bar{\psi}_\vec{x} \gamma_\mu U_{\vec{x}, \vec{\mu}}^\dagger \psi_{\vec{x} + \hat{\mu}}^\dagger - \bar{\psi}_{\vec{x} + \hat{\mu}} \gamma_\mu U_{\vec{x}, \vec{\mu}} \psi_{\vec{x}} \right] - M \sum_{\vec{x}} \bar{\psi}_\vec{x} \psi_{\vec{x}} \quad (2.7)$$

where $\vec{x} + \hat{\mu}$ stands for the site shifted from $\vec{x}$ by a lattice spacing in $\mu$-direction. The second term in the brackets is needed for hermiticity.

While (2.7) reproduces the Dirac continuum action as $a \to 0$ for smooth $\bar{\psi} \psi$, the momentum-space propagator of a lattice fermion reads

$$G(p) = \frac{a}{\sum_{\vec{p}} \frac{i\gamma_\mu \sin p_\mu a}{M}}. \quad (2.8)$$

This expression is symmetric under "reflection" of each momentum component w.r.t. $\pi/a$. Therefore the doubling of fermion souls occurs for each axis so that their total number to be $2^4 = 16$.

The enlarging of fermion states does not occur for the action

$$S'_\psi = \frac{i}{2} \sum_{\vec{x}, \vec{\mu}} \left[ \bar{\psi}_\vec{x} \left( \gamma_\mu - i \gamma_5 \gamma_\mu \right) U_{\vec{x}, \vec{\mu}}^\dagger \psi_{\vec{x} + \hat{\mu}}^\dagger + \bar{\psi}_{\vec{x} + \hat{\mu}} \left( \gamma_\mu + i \gamma_5 \gamma_\mu \right) U_{\vec{x}, \vec{\mu}} \psi_{\vec{x}} \right] - M \sum_{\vec{x}} \bar{\psi}_\vec{x} \psi_{\vec{x}} \quad (2.9)$$

Now only one relativistic fermion survives as $a \to 0$.

2.2. Monte Carlo Method

The series of strong coupling expansion becomes divergent with increasing $\beta$ so that methods of numerical integration are used. For the $L \times L \times L \times L$ lattice, the multiplicity of the integral is as large as $L^4 \cdot 4 \cdot (N^2 - 1)$ where additional factors are due to spin and color of a gluon. As will be explained in sect. 2.3, the larger $L$ the better
approximation of the continuum space can be achieved by the lattice. For 
SU(3) at \( L = 10 \), the multiplicity is more than 2 millions, while at 
\( L = 24 \) — more than 10 millions. The latter lattice is the biggest one 
used in calculations up to now.

It is hopeless to calculate such an integral exactly. To do it the 
numerical Monte Carlo method is applied.

As usual, this method is applied not to successive integral over \( U_{x,\mu} \) 
at each link but rather to the multiple integral as a whole, which can 
be represented as a sum over states of the system. One state consists of 
a gauge field configuration, i.e. the values of the link variables at all 
the links of the lattice

\[
\mathcal{C} = \left\{ U_{x,\mu}, \ldots \right\}.
\] (2.10)

Then the successive integral can be rewritten as

\[
\int \prod_{x,\mu} dU_{x,\mu} \ldots = \sum_{\mathcal{C}} \ldots.
\] (2.11)

The task of Monte Carlo calculations is not to construct all the 
possible configurations whose number is infinite. The task is to construct 
an ensemble of \( K \) configurations

\[
E = \left\{ \mathcal{C}_1, \ldots, \mathcal{C}_K \right\}
\] (2.12)

so that the given configuration, \( \mathcal{C}_K \), to encounter with the Boltzmann 
probability

\[
P(\mathcal{C}_K) \propto e^{-\beta S(\mathcal{C}_K)}.
\] (2.13)

Here \( S(\mathcal{C}_K) \) stands for the value of the action (2.1) calculated for 
the configuration \( \mathcal{C}_K \). Such a sample is called an equilibrium 
ensemble.

Given an equilibrium ensemble, the averages can be calculated as 
follows
\begin{equation}
W(R \times T) = \frac{1}{K} \sum_{\kappa=1}^{K} \frac{1}{N} \text{tr } U_{R \times T}(C_{\kappa})
\end{equation}

because each configuration "weighs" already as many as required.

\(U_{R \times T}(C_{\kappa})\) on the r.h.s. of eq. (2.14) stands for the value of the
phase factor for the given configuration \(C_{\kappa}\). If all configurations
in the ensemble (2.12) are independent, (2.14) will approximate the exact
value of (2.4) with an accuracy \(\sim 1/\sqrt{K}\).

The crucial point of Monte Carlo calculations is to construct the
equilibrium ensemble. It is not simple to do that, because the Boltzman
probabilities are not known in advance. A way out is to establish the
so-called Markovian process, i.e. one for which each new configuration
of the sequence (2.14) is obtained from the previous one by using a de-
finite algorithm but stochastically. The "direction" of the process is
determined at each step by a random number generator.

The most popular are two algorithms: the heat-bath method and the
Metropolis one. Both algorithms update, at each step, \(U_{x,M}\) at a link
only. That is to say, the new configuration, \(C'\), is obtained from the
previous one, \(C\), by changing one link variable.

For the heat-bath method, a new link variable \(U_{x,M}'\) is selected
randomly from the gauge group with a probability given by the Boltzman
factor
\begin{equation}
P(c \rightarrow c') \propto e^{-\beta S(c')}
\end{equation}

For the Metropolis algorithm, one calculates \(\Delta S = S(c') - S(c)\)
after that a random number, \(r\), distributed uniformly on the interval
\([0,1]\) is generated. If \(\exp(-\beta \Delta S) > r\), \(U_{x,M}\) is replaced
by \(U_{x,M}'\). Otherwise, the old value \(U_{x,M}\) is kept.

The new configuration \(C'\) obtained from \(C\) by one Monte Carlo
step will be correlated strongly with \(C\). One needs very many such
steps, either passing through the lattice successively or choosing the
updating link randomly, in order to "forget" the old configuration.
Such an obtained configuration can be included into the ensemble (2.12) and used for calculating the averages.

2.3. Passing to Continuum

As is discussed in sect. 2.1, the area law holds in the strong coupling region leading to the linear potential

\[ E(R) = K R \]  \hspace{1cm} (2.16)

The coefficient \( K \) here is called the string tension. The physical meaning of \( K \) is the energy of the tube (string) of gluon field, formed between quarks, per unit length. As follows from eq. (2.5) (for \( N=3 \))

\[ K = -\frac{1}{a^2} \log (\beta/18) = \frac{1}{a^2} \log (3g^2) \] \hspace{1cm} (2.17)

Eq. (2.17) establishes a connection between values of \( a \) and \( g \) as follows. Set \( K \) to be equal to its experimental value about 1 GeV/fm = (450 MeV)^2. Then variations of \( a \) and \( g^2 \) should be done simultaneously in a way that \( K \) be fixed. This procedure calls for \( a \to \infty \) as \( g^2 \to \infty \). That is to say, the lattice spacing is large in the strong coupling region, compared to 1 fm, as is shown in fig. 4a. It is evident that such a lattice cannot describe correctly the rotational symmetry.

In order to pass to the continuum, \( a \) has to be decreased. Eq. (2.17) shows that \( a \) decreases with decreasing \( g^2 \). However, this formula ceases to be applicable at \( a \sim 1 \) fm.

The recipe of further decreasing \( a \) is the same as in the strong coupling region: further decreasing of \( g^2 \). While no formulas exist at intermediate values of \( g^2 \), the expected relation between \( a \) and \( g^2 \) as \( g^2 \to 0 \) is dictated by the asymptotic freedom formula (1.4).

Substituting \( r = a \) since \( g \) has a meaning of an effective charge at the distance \( a \), one gets (for \( N_f = 0 \))

\[ a = \frac{1}{\Lambda_{QCD}} e^{-\frac{g^2}{\mu^2}} \] \hspace{1cm} (2.18)

Therefore, \( a \) can be given, by decreasing \( g^2 \), small enough compared to 1 fm in order that rotational symmetry restores.
For the values of $g^2$ where the continuum limit sets in, the relation between $a$ and $g^2$ will be the same, independently on what dimensionful quantity is held fixed. This fact is guaranteed by the general property of QCD referred to as the dimensional transmutation. This property provides an appearance of the QCD scale parameter, $\Lambda_{\text{QCD}}$, of order of hundreds MeV which does not enter the QCD lagrangian (the quark masses are out of the game because $m_u \approx 4$ MeV, $m_d \approx 7$ MeV). $\Lambda_{\text{QCD}}$ appears as a combination of the (dimensionful) cut-off and $g^2$. As $g^2 \to 0$, this combination is given by asymptotic freedom.

The phenomenon of dimensional transmutation consists in the fact that all the dimensionful quantities are proportional to same parameter, $\Lambda_{\text{QCD}}$. For example the hadron masses

$$M_i = C_i \Lambda_{\text{QCD}}$$  \hspace{1cm} (2.19)

where the numbers $C_i$ depend on the hadron quantum numbers but neither on $g^2$ not on the cut-off.

A task of the Monte Carlo calculations is to compute the numbers. For this purpose, the dimensionless values $M_i a$ are multiplied by $\exp \left( \frac{8\pi^2}{\mu g^2} \right)$ in order to compensate, at small $g^2$, the dependence of $a$ on $g^2$ given by eq. (2.18). The obtained values is plotted versus $1/g^2$ as is shown in fig. 5. Their asymptotics as $g^2 \to 0$ determines the values of $C_i$.

In fact, very small values of $g^2$, that correspond to small $a$, cannot be achieved because of computer limitations. For example $a \approx 0.1$ at $g^2 = 1$ so that one needs $L \geq 10$ in order to have the lattice which is larger than (1 fm)$^4$. The smaller $g^2$ the larger $L$ is required so that the number of degrees of freedom becomes soon too large.

Due to this reason, the values of $C_i$ are extracted for those finite values of $g^2$ when the values of $M_i a \exp \left( \frac{8\pi^2}{\mu g^2} \right)$ are practically independent on $g^2$. This domain is called that of asymptotic scaling. Fortunately, asymptotic scaling turned out to set in rather early, probably for $g^2 < 1$. It is the point that makes it possible to obtain physical results on relatively small lattices ($L = 8 \pm 16$).
Exercises to sect. 2.

To sect. 2.1.

1. For the \( d \)-dim. lattice of the size \( L \times L \times L \times L \) with periodic boundary conditions, obtain formulas for the numbers of sites, links and plaquettes.

2. Show that \( \beta S \), where \( S \) is given by (2.1) while \( \beta \) — by (2.3), reproduces the continuum Yang-Mills action as \( \alpha \to 0 \).

3. Show that integration over the Haar measure on \( U(N) \) can be represented as integration over arbitrary complex \( N \times N \) matrices, restricted by the unitary condition, :

\[
\int dU \ldots = \int \prod_{i,j} \left[ d \text{Re} \psi_{ij} d \text{Im} \psi_{ij} \delta^{(2N^2)} (\psi \psi^* - I) \right] \ldots .
\]

Here the delta-function provides vanishing of its matrix argument.

4. Prove eq. (2.6) for a rectangular \( R \times T \) loop in the continuum theory.

5. Prove that \( \int dU \ U_{ij} \ U_{k\ell}^* = \frac{1}{N} \delta_{ij} \delta_{k\ell} \) for the integral over the Haar measure on \( SU(N) \). Using this formula, obtain (2.5).

6. Show that the fermion lattice action (2.7) is invariant under the transformation of quark field

\[
\Psi_x \rightarrow \gamma^t \gamma^s (-1)^{t/q} \Psi_x ,
\]

where \( t \) stands for the coordinate along the 4-th axis. Find 14 more transformations that leave (2.7) invariant.

7. Show that (2.7) can be transformed to a diagonal w.r.t. spinor indices form by means of substitution of \( \psi \) by \( \Phi \):

\[
\Psi_x = (\gamma_1)^{x/q} (\gamma_2)^{y/q} (\gamma_3)^{z/q} (\gamma_4)^{t/q} \Phi_x .
\]

To sect. 2.2.

Using a concept of "distance" between ensembles \( E \) and \( E' \) defined by
\[ \|E - E'\| = \sum_C |P(C) - P'(C)| \]

where \(P(C)\) and \(P'(C)\) stand for the probability density of configuration \(C\) in \(E\) or \(E'\), respectively, show that an algorithm with \(P(C \rightarrow C')\) satisfying the detailed balance condition

\[ P(C \rightarrow C') e^{-\beta S(C)} = P(C' \rightarrow C) e^{-\beta S(C')} \]

brings an ensemble closer to equilibrium.

To sect. 2.3.

Starting from the renormalization group equation for the effective charge, \(g(r)\): \(-r \frac{dg}{dr} = \beta(g)\) show that \(\Lambda = \frac{1}{\alpha} \exp \int g \beta(g)\) is renorminviant. Obtain an explicit expression for \(\Lambda\) using the two-loop approximation to \(\beta(g)\): \(\beta(g) = -bg^3 - b_1 g^5\).
3. RESULTS (simulating strong interaction on computer)

The Monte Carlo method in lattice QCD allows to calculate dimensionful physical quantities. Monte Carlo data indicate that rotational symmetry restores, as the lattice spacing is decreased, so that the continuum limit sets in. This fact guarantees that obtained results refer to continuum QCD. A lot of such calculations have been performed since 1979 (the beginning of Monte Carlo era in QCD). The calculations of interquark potential showed that Coulomb potential is replaced by the linear one as the distance is increased. The calculations of string tension allowed to obtain the value of QCD scale parameter. There were performed evaluations of glueball spectrum, masses of mesons and baryons composed of quarks, the value of quark condensate and other quantities. A special role is played by Monte Carlo simulations of QCD at finite temperature. Here it is shown that QCD undergoes a phase transition from hadronic matter to quark-gluon plasma as the temperature is increased, obtained the value of this critical (deconfinement) temperature as well as that of latent heat which is eaten during the phase transition, studied relation between deconfinement and restoration of chiral symmetry.

3.1. String Tension

To study the phenomenon of rotational symmetry restoration with decreasing lattice spacing, it is convenient to consider the lines of equal potential between color charges. Since the string that connects charges is infinitely thin in the strong coupling limit, it can be laid along lattice links only, which yields the potential

\[ E = \kappa \left( |x| + |y| + |z| \right) \quad (3.1) \]

where \( x, y \) and \( z \) stand for the differences of coordinates of two charges. Otherwise, in the continuum, it reads

\[ E = \kappa \sqrt{x^2 + y^2 + z^2}. \quad (3.2) \]

Equipotential lines given by eq. (3.1) are depicted in fig. 6a. They look like squired rather than circles as it would be for eq. (3.2), so
that there is no rotational symmetry. Its restoration shows up as the fact that those become look more and more like circles, when lattice spacing is decreased, as is shown in fig. 6b,c \(^{11}\).

For such values of lattice spacing where the continuum limit has set in, one can calculate physical quantities. In fig. 7 it is shown the potential between color charges versus the distance, which is calculated by the Monte Carlo method \(^{12}\). As is seen from fig. 7, the Monte Carlo data are fairly described by a simple sum of linear and Coulomb potentials.

The results of Monte Carlo calculation of string tension for SU(3) group obtained in ref. \(^{13}\) without taking into account dynamical quarks are shown in fig. 8. As usually, it is depicted the ratio \(10^3 \Lambda_L / \sqrt{K}\), where

\[
\Lambda_L = \frac{\alpha}{\alpha} \left( \frac{16\pi^2}{4q^2} \right)^{s/12} \epsilon - \frac{8\pi^2}{11q^2}
\]  

(3.3)

is lattice scale parameter with the two-loop correction to the asymptotic freedom formula.

Usually one compares with experiment the value of scale parameter \(\Lambda_{\text{MOM}} = 84 \Lambda_L\) that corresponds to the continuum. As is seen from fig. 8, \(\Lambda_L = 10^{-2} \sqrt{K}\) which yields

\[
\Lambda_{\text{MOM}} \approx 0.84 \sqrt{K} = 340 \text{ MeV}
\]  

(3.4)

for \(\sqrt{K} = 400 \text{ MeV}\). This value of \(\Lambda_{\text{MOM}}\) is in agreement with experimental one.

It is also seen from fig. 8 that Monte Carlo data, being in agreement with asymptotic scaling for \(6.0 < \beta = 6/q^2 \leq 6.2\), violate asymptotic scaling as \(\beta < 6.0\). This fact does not mean, however, that continuum limit does not yet set in. Monte Carlo calculations of various dimensionful quantities show practically the same asymptotic scaling violation within errorbars. An interpretation of this phenomenon is that continuum limit has already set in but simple two-loop formula (3.3) for the scale parameter has to be modified since \(g^2\) is not small. Such a situation
is called scaling but not asymptotic.

One more verification that obtained quantities refer to continuum QCD has been done. There were considered various modifications of the simplest Wilson action (2.1) which possess the same limit as \( a \to 0 \). For this reason, the results should not depend on the form of lattice action (this property was called universality). It was shown by direct Monte Carlo calculations that universality takes place, indeed, within errorbars.

### 3.2 Hadron Masses

To calculate hadron masses, one consider the correlation function of two operators, \( O(\vec{x}, t) \), with quantum numbers of a given hadron:

\[
\Delta(t) = \sum_{\vec{x}} \left\{ \left\langle O(\vec{x}, t) O(\vec{0}, 0) \right\rangle - \left\langle O(\vec{0}, 0) \right\rangle^2 \right\}, \tag{3.5}
\]

where the sum goes over all the space lattice sites, \( \vec{x} \), within given "time" slice \( t = \text{const} \).

For large enough \( t \),

\[
\Delta(t) \propto e^{-M_t} \tag{3.6}
\]

determines the mass of lowest hadron with the given quantum numbers. Exponentially small value of \( \Delta(t) \) at large \( t \) complicates its evaluation because one needs big statistics to obtain reasonable signal to noise ratio.

To evaluate glueball masses, it is more convenient to use the variational method. Then \( O(\vec{x}, t) \) is taken as a linear combination of operators (with the same quantum numbers in the continuum limit). The main idea of the variational method is to minimize \( \log(\Delta(t-o)/\Delta(t)) \). In this case, reliable results for masses can be obtained already for \( t = 2a + 3a \).

The glueball spectrum obtained by the Monte Carlo method \(^{14,15} \) is represented in Table 1. The stars are used in the same sense as by Particle Data Group. The mass of \( \Upsilon^{**} \) was equal to a smaller value (700 \( \pm \) 750 MeV) for a time before ref. 15 appeared, because it was extracted from the region \( \sigma = 5.5 \pm 5.6 \) where \( M(\Upsilon^{**}) \) is abnormally small. The
value in Table 1 is extracted from the Monte Carlo data of ref. 15 for $\beta = 5.7 \pm 5.9$, where those agree with violation of asymptotic scaling seen for string tension. This number has, however, three- rather than four-star status for the reason that it can mix with $O^{\leftrightarrow}$ quark state which may change the mass. From this viewpoint, it is interesting to calculate masses of the so-called "oddballs" (e.g. $1^{\leftrightarrow}$ or $0^{-}$) that cannot mix with two-quark boundstates. Unfortunately, their masses, given in Table 1, are less reliable (two-star status) because the Monte Carlo calculations where performed at single value of $\beta$ only.

It is worthwhile noticing that mass ratios are more reliable than absolute values which depend on the scale parameter. The values in Table 1 (and below) are recalculated using eq. (3.4). This leads to minor differences from those in original papers.

The glueball masses have been calculated in pure SU(3) Yang-Mills, i.e. quarks were ignored completely. A useful approximation to full QCD is the so-called quenched (or valence) approximation, when only valence quarks are taken into account disregarding virtual quark loops. This approximation is supported by the experimental fact that widths of hadron decays are small compared to their masses.

The quenched approximation saves a lot of computer time required for Monte Carlo calculation with quarks. Relatively simple (compared with hadron mass calculations) are calculations of vacuum quark condensate, $\langle \overline{\Psi} \Psi \rangle$. Within perturbation theory, $\langle \overline{\Psi} \Psi \rangle$ vanishes due to chiral symmetry of QCD action.

Monte Carlo calculations lead to nonvanishing $\langle \overline{\Psi} \Psi \rangle$ which means breakdown of chiral symmetry due to properties of vacuum (spontaneous breakdown). Results are consistent with scaling \cite{16} and, therefore, refer to continuum QCD. The obtained value of $\langle \overline{\Psi} \Psi \rangle$ is in agreement with phenomenological estimate.

Calculation of hadron masses were performed (e.g. see \cite{17}) on lattices of different sizes, at various values of $\beta$ and for different types of lattice fermions as well as of lattice actions. Reliable results are obtained for $\rho$ - and $\pi$-meson masses, with $\rho$-mass being
consistent with scaling. Usually, one sets these two masses to be equal to their experimental values. This fixes the value of scale parameter and quark mass. Then masses of various mesons as well as baryons can be calculated. Within errorbars (which are still not small) there is agreement with experiment.

However, the proton mass turns out to be too big, while $\Delta$-proton splitting comes too small. Some improvement seems to be achieved by taking into account the virtual quark loops. Such a calculation is at present the most advanced application of the Monte Carlo method in QCD.

There exist Monte Carlo calculations of other quantities in QCD which are, however, a little bit less reliable than those discussed above. It is worthwhile mentioning Monte Carlo results on gluon condensates 18 as well as first attempts of calculation hadronic coupling constants 19.

3.3 QCD at Finite Temperature

QCD at finite temperature is formulated on an euclidean lattice with different extents in the spatial and "temporal" directions. Now the lattice has to have, by definition, a finite extent in the "temporal" direction with imposed periodic boundary conditions for the gluon field (antiperiodic for the quark one). Such a lattice can be imaged as a torus along the "time" axis.

If the spatial extent is infinite (many times larger, practically, than the "temporal" one), then the temperature, $T$, is related to the extent along "time" axis by

$$ T = \left( \alpha \beta \right)^{-1}. $$

This $T$ is a real physical temperature of the system (that can be measured by a thermometer) and should be distinguished from the coupling $\beta = g^2 / \beta$ which is called sometimes the "temperature" by analogy with statistical mechanics.

The partition function of QCD on the lattice at finite temperature is given by eq. (2.2) where integration goes along all the links on the asymmetric lattice. I would like to emphasize this formula, being euclidean, to be associated with an equilibrium system in the real
$3+1$-dimensional space-time (3 space and 1 time without quotation marks dimensions). The point is that nothing depends on time for an equilibrium statistical system.

The central role in QCD at finite temperature is played by the trace of the product of $U_{x_{M}}$'s along a line that is parallel to 4th axis. This trace is called Polyakov (or Wilson) line and is denoted by $L_{\vec{x}}$. $L_{\vec{x}}$ is gauge invariant due to the periodic boundary conditions. Its mean value is related to $\Delta F$ -- the shift of the free energy related to placing a static quark at the point $\vec{x}$ -- by the formula

$$\Delta F = -\frac{l}{\alpha L_{t}} \log \left\langle L_{\vec{x}} \right\rangle.$$  \hfill (3.8)

If $\left\langle L_{\vec{x}} \right\rangle = 0$, then the energy of isolated quark is infinite which leads to confinement. On the contrary, nonvanishing value of $\left\langle L_{\vec{x}} \right\rangle$ means that quarks are not confined. This criterion is equivalent at $T = 0$ to the Wilson one and extends it at $T \neq 0$.

Numerous Monte Carlo calculations of $\left\langle L_{\vec{x}} \right\rangle$ in the quenched approximation indicate that $\left\langle L_{\vec{x}} \right\rangle = 0$ in QCD for $T < T_{c}$ where $T_{c} \approx 200$ MeV. However, $\left\langle L_{\vec{x}} \right\rangle \neq 0$ for $T > T_{c}$, so that a deconfinement phase transition occurs at $T = T_{c}$. For $T > T_{c}$, the strongly interacting matter would exist in the nature in the form of quark-gluon plasma, rather than in the hadron phase as for $T < T_{c}$. At the phase transition, the interquark potential at large distances changes its shape. The linear potential (at $T < T_{c}$) is replaced by the Coulomb one (at $T > T_{c}$).

The value of the deconfinement temperature, $T_{c} \approx 200$ MeV, has been obtained by the Monte Carlo method. Some data for the ratio $T_{c}/\Lambda_{L}$ are shown in fig. 9. One can see the asymptotic scaling violation which is consistent with that of fig. 8 for string tension. This fact yields $T_{c} \approx 50 \Lambda_{L} = 200$ MeV. This quantity is not measured yet experimentally and is an interesting prediction for heavy-ion collisions.

The value of $T_{c}$ can be obtained in an alternative way by calculating the energy density, $\varepsilon$, versus $T$. The results have a shape shown in fig. 10. The first order phase transition occurs at $T = T_{c}$ so that
the energy density for quark-gluon plasma is bigger than that for hadrons by \( \Delta \varepsilon = 1 + 2 \text{ GeV/fm}^3 \). \( \Delta \varepsilon \) is about 1 GeV/fm\(^3\) for pure Yang-Mills, while it is increased due to the quark contribution up to \( 1.5 \pm 2.0 \) GeV/fm\(^3\). The true value of \( \Delta \varepsilon \) is very important for experiment since it is the energy that has to be created during collision in order to produce quark-gluon plasma.

The Monte Carlo calculations showed that \( \varepsilon(T) \) in the plasma phase begins to be well-described by the Stefan-Boltzman law for the values of \( T \) which are only slightly bigger than \( T_c \). This fact means that plasma is a gas of almost noninteracting quarks and gluons.

When the temperature increases from zero, QCD undergoes one more phase transition associated with chiral symmetry restoration. That is to say, the quark condensate is destroyed at some value \( T = T_{ch} \). Monte Carlo calculations indicate \( T_c = T_{ch} \) in the quenched approximation within errorbars.

An important question is how the results obtained in the quenched approximation are changed when dynamical, light quarks are taken into account. As I have mentioned already, such calculations are being performed. Except for pure technical difficulties related to anticommuting nature of quark variables, a conceptual difficulty arises which is due to the fact that a pair of virtual quarks can now be created from the vacuum screening the test color charges.

First Monte Carlo calculations with dynamical quarks \(^{23}\) tell us that most of qualitative results seems do not change. However, there is a minor shift of the values of dimensionful quantities. E.g. for the critical temperatures, this shift is less than 10 percents/flavor.

As I mentioned above, the deconfining phase transition is not seen yet in accelerator experiments. Nevertheless, an "experiment" has been done where the deconfining phase transition has occurred inversely. I mean the quark-hadron phase transition in the early universe. At early times (less than few \( \mu \)s \( ) \), the temperature was high enough for matter to be in a quark-gluon plasma state, while nucleation started to occur as the temperature had fallen down to the deconfinement value. This tempera-
ture was maintained during few tens $\mu s$ until the latent heat was transformed into the energy of the expansion. About the hundredth $\mu s$, the standard approach to the hadron era becomes applicable.

Unfortunately, most of cosmological consequences of the deconfining phase transition in the early universe are washed out during $\sim 10^{10}$ years. However, it might lead to production of quark-matter lumps which could be detectable (for review see $^{10}$ and refs. therein). Anyhow one consequence is that a chapter of textbooks on cosmology concerning the hadron era should be rewritten taking into account the deconfining phase transition.

**Exercises to sect. 3.**

To sect. 3.1.

For a string with massless quarks at the ends, obtain the relation $K = 1/2\pi \alpha'$ between the string tension, $K$, and the Regge trajectory slope, $\alpha'$.

To sect. 3.2.

1. Calculate the lightest glueball mass to leading order of the strong coupling expansion.

2. Obtain the following spectral representation for the correlator $\Delta(t)$ defined by (3.5):

$$\Delta(t) = \sum_{n \neq 0} \left\langle O(\delta,0) \right| n \right\rangle^2 e^{-M_n t}$$

where the sum goes over all states $|n\rangle$ with given quantum numbers (except for vacuum one).

3. It is known that the specific heat, $\beta^2 d^2 \log Z/d\beta^2$, where $Z(\beta)$ is defined by (2.2), has a pronounced peak at $\beta = 5.5 \pm 5.6$. Show that $0^{++}$-glueball mass has to be abnormally small in this region.

4. For QCD with the color group being $SU(N)$, show that hadron decay widths are of order $1/N$ at large $N$.

To sect. 3.3.

1. For a free boson field, $\varphi$, of mass $m$ at finite temperature $T$, obtain the following representation for the partition function, $Z$,
via the path integral

\[ Z = \int \mathcal{D} \varphi \exp \left( -\int_0^{1/T} dt \int d^3x \, \mathcal{L} \right) \]

with periodic boundary conditions \( \varphi(0) = \varphi(1/T) \), where the lagrangian density \( \mathcal{L} = \frac{i}{2} \varphi (\mathbf{- \Box} + m^2) \varphi \).

2. Find how the ordered product of \( U_{x, \mu} \)'s along a line which is parallel to \( t \)-axis changes under the global transformation \( U_{x, \mu} \rightarrow Z U_{x, \mu} \). \( Z \in \{ 1, \exp(\pm 2\pi i/3) \} \), i.e. \( Z \) belongs to the group \( \mathbb{Z}_3 \) -- the center of \( SU(3) \). Discuss a relation of this symmetry to confinement.

3. It is known that 4-dim. QCD is reduced to 3-dim. one as \( T \rightarrow \infty \). It is known as well that quarks are confined in 3-dim. QCD so that the area law holds for loop averages. On the other hand, deconfinement takes place in 4-dim. QCD at \( T > T_c \). How do these two (correct) statements agree?

4. Consider the following model description of the deconfinement transition: at \( T < T_c \) -- a gas of non-interacting (massless) \( \pi \)-mesons, at \( T > T_c \) -- an ideal gas of gluons and \( u \)- and \( d \)-quarks with the energy density shifted by some value \( B \approx 200 \text{ MeV/fm}^3 \) (the bag model). Obtain the following formulas for the deconfinement temperature, \( T_c \), and the latent heat, \( \Delta \mathcal{E} \), in this model:

\[ T_c^4 = \frac{g_0 B}{(g_\rho - g_h)^2} \quad , \quad \Delta \mathcal{E} = 4B \]

where the statistical weights \( g_h \) and \( g_\rho \) of the hadron and plasma phases equal, respectively, \( g_h = 3 \), \( g_\rho = 8 \cdot 2 + \frac{2}{8} \cdot 3 \cdot 2 \cdot 2 \cdot 2 = 37 \).
CONCLUSIONS

In these lectures, we have discussed the basic ideas and results of Monte Carlo calculations in QCD. Now there exist a lot of such calculations. The main problem is to check how reliable are the obtained results. This work is being done, basically, along two lines: increasing the lattice size as well as taking virtual quark contribution into account. These tasks stimulate inventing of new, more efficient algorithms for calculations. In particular, there are projects of special-purpose processors designed for numerical "experiments" with QCD, which are under construction. The time seems to come soon to think about, at first, what reliable predictions for physics of strong interaction can be obtained by numerical methods and, at second, to what new problems of the modern elementary particle physics can this approach be extended.
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### Table 1.

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>Value of Mass (MeV)</th>
<th>Reliability</th>
</tr>
</thead>
<tbody>
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<td>0$^{++}$</td>
<td>$\approx 1000$</td>
<td>...</td>
</tr>
<tr>
<td>2$^{++}$</td>
<td>$1620 \pm 100$</td>
<td>...</td>
</tr>
<tr>
<td>0$^{--}$</td>
<td>$1420 \pm 240$</td>
<td>...</td>
</tr>
<tr>
<td>1$^{-+}$</td>
<td>$1730 \pm 220$</td>
<td>...</td>
</tr>
<tr>
<td>0$^{--}$</td>
<td>$2880 \pm 300$</td>
<td>...</td>
</tr>
<tr>
<td>1$^{+-}$</td>
<td>$2980 \pm 300$</td>
<td>...</td>
</tr>
</tbody>
</table>

The values of glueball masses calculated in QCD by the Monte Carlo method. The mass of 0$^{++}$ is taken from ref. 14 while the others — from ref. 15.

![Diagram of electron beam and solenoid](image1)

**Fig. 1** A principal scheme of the experiment that demonstrates the Aharonov-Bohm effect. Electrons do not pass inside the solenoid where magnetic field is concentrated. Nevertheless, a phase difference arises between electron beams passing along the two sides of solenoid. Its value depends on the value of current.

![Lines of force](image2)

**Fig. 2** The lines of force for a) Coulomb and b) linear potentials.
Fig. 3  2-dim. lattice with periodic boundary conditions. The sites denoted by the same numbers are identified.

Fig. 4  A description of continuum field configurations by the lattices a) "rough" and b) "fine". The lattice a) can represent the given continuum fluctuation very roughly, while the lattice b) has the spacing which is small enough to represent its form.

Fig. 5  \( M \cdot \exp \left( \frac{8\pi^2}{11g^2} \right) \) versus \( 1/g^2 \). The points represent schematically Monte Carlo data. The dashed line corresponds to the asymptotic scaling.
Fig. 6 The behavior of equipotential lines at different values of the lattice spacing $\alpha$. Fig. a) corresponds to the strong coupling limit $\alpha \to \infty$. Figs. b) and c), taken from ref. 17, show how the rotational symmetry is restored as $\alpha$ is decreased.
Fig. 7  The potential between static color charges versus distance (measured in the units of $1/k^2$). The points represent the Monte Carlo data of ref. 12. The bold line corresponds to the linear-Coulomb fit to the data.

Fig. 8  The Monte Carlo data $^{13}$ for the string tension (measured in the units of $\Lambda_L$) versus $\beta$. The dashed line corresponds to asymptotic scaling.
Fig. 9  The Monte Carlo data by Gottlieb et al.\textsuperscript{20} for the deconfinement temperature. The dashed line corresponds to asymptotic scaling.

Fig. 10  The energy density of the Yang-Mills matter versus temperature. The gap at the deconfinement temperature $T_c$ is equal to $\Delta \varepsilon / T_c^4$ with $\Delta \varepsilon$ being the latent heat.
SUM RULES AND HADRON PROPERTIES IN QCD

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ABSTRACT

We consider the applications of the sum rules method to the following aspects of hadron physics: (i) radial excitations of the quark mesons, (ii) gauge theories with scalar quarks and properties of scalar containing hadrons, (iii) the generalization of sum rules method to the case of nonzero temperatures and quark-hadron phase transitions.

1. INTRODUCTION

Now, quantum chromodynamics (QCD) is considered as a true theory of the strong interactions. The quantitative and semiquantitative predictions of QCD are in a good agreement with the experiment.

QCD is a gauge theory \(^1\) of colour \(^2\) where SU(3) Yang-Mills fields interact with the quark fermion fields \(^3\). Improved by the renormalization group \(^4\) the perturbative QCD works well at large momentum transfers \(^5\); however, all physically interesting processes include the effects of large distances and hence the effects of strong coupling. The reason is that we always deal not with the fundamental fields of QCD (gluons and quarks) but with their bound states - hadrons formed at large distances. The most difficult (because of strong coupling) and interesting task is the description of the low energy strong interactions. In particular, this includes the determination of the hadron properties (i.e. their masses, widths, form-factors, etc) within the QCD framework.

Among various methods which are applied to the problem the following two stand out. The first one is the direct numerical calculations of the QCD path integrals in the lattice formulation of the theory \(^6\). The second method is based on the QCD sum rules \(^7,8\). In principle, this approach is not so unambiguous as the lattice calculations. However it is distinguished by its simplicity and provide us with more clear understanding of the physical picture of QCD at low energies. At the same time this approach allows one to perform analytical calculations.

Some aspects of sum rules method have already been discussed at this School \(^9\). We are going to consider the following questions: (i) different forms of the sum rules and the suitable choice of their form depending on the problem considered; (ii) sum rules approach to the description of the properties of the radial excitations of the quark mesons \(^10,11\); (iii) gauge theories with scalar fields and bound states of light (by light we mean, as usually, the particles with Lagrangian mas-
ses smaller than the scale parameter $\Lambda$ of QCD, $m \ll \Lambda$) coloured scalar quarks\(^{12,13}\). The consideration of this problem is of interest because of following reasons. Firstly, this allows one to consider the different phases of the gauge theories, namely the strong coupling (confinement) phase and small coupling regime ("spontaneously broken" or Higgs phase) on the same grounds of sum rules. Secondly, it may give answer to the question to what extent the confinement forces depend on the spin of coloured partons. Then, the knowledge of the scalar quarks dynamics is necessary for the constituent model building. At last, but not at least, it is not excluded that the bound states of light scalar quarks indeed exist in nature thus far escaping from the experimental hands. Such a situation may occur provided the expected masses of scalar-containing hadrons may be rather large (of order of tens GeV) because of unusually strong binding in the channels which contain scalar quarks; (iv) hadron matter at large temperatures and quark-hadron phase transitions\(^{14}\). This question has been considered in the lectures\(^{6}\) at this School in the context of the lattice-gauge theories. We try to show that the sum rules approach may be effective at this point too.

2. SUM RULES METHOD

QCD perturbative calculations of the Green functions are correct only in the deep Euclidean region of the momentum space while the physical processes take place at the time-like momentum and on the mass shell. Therefore, the analytical continuation of the theoretical calculations to the physical region is needed. In fact, the use of the sum rules provides us with the correct procedure.

Let us consider the derivation of the sum rules for the two-point Green functions on the example of the correlator of vector currents with $\rho$-meson quantum numbers ($\rho^{PC}=1^{-}\); \(I=1\), \(I\) is the isotopic spin):

\[
J_{\mu} = \frac{1}{2} \langle \bar{u} \gamma_{\mu} u - \bar{d} \gamma_{\mu} d \rangle; \quad \langle q_{\mu} q_{\nu} ^{2} - q_{\mu} q_{\nu} \rangle \Pi(q^2) = i \int d^4x \mathcal{E}^{q_{x}q_{y}} T \{ J_{\mu} (x) J_{\nu} (0) \} \gamma_{0}. \tag{1}
\]

The tensor structure \(q_{\mu} q_{\nu} ^{2} - q_{\mu} q_{\nu} \) is singled out by the conservation of the vector current.

The imaginary part of the function \(\Pi(q^2)\) is proportional to the experimentally observable cross-section of the reaction $e^+e^- \rightarrow \text{hadrons}$ in the $I=1$ channel. \(\Pi(q^2)\) is analytic on the complex plane $Q^2$ excluding real semiaxis and it obeys the standard Källen-Lehmann dispersion relation

\[
\Pi(q^2) = (4\pi \alpha)^{-1} q^2 \int_0^\infty dS R(S)/S (S-q^2), \tag{2}
\]

where $R(S) = \sigma(e^+e^- \rightarrow \text{hadrons}, I=1)/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. Due to the asymptotic freedom of QCD $\Pi(q^2)$ may be perturbatively calculated for the complex $q^2=S+i\Delta$ which lies sufficiently far from the real semiaxis.
\( \prod_{\text{th}}(q^2) \) be the theoretically calculated (up to the certain order of perturbation theory) value. Then at large \( S \) and \( |\Delta| \gg \Delta_0 \) where \( \Delta_0 \) is some quantity depending on \( S \) the following equality should hold:

\[
\prod(q^2) = \prod_{\text{th}}(q^2), \quad |\text{Im} q^2| \approx \Delta_0. \tag{3}
\]

Note that the function \( \prod_{\text{th}}(q^2) \) includes the usual perturbative terms as well as the nonperturbative corrections proportional to the so called condensates of the quark and gluon fields. The latter contributions are extremely important for the determination of the mass scales of low lying hadrons.

The last step in the derivation of the sum rules consists in the integration of the relation (2) over contour \( C \) (see Fig. 1). Taking into account Cauchy theorem and equality (2) it is easy to obtain

\[
\int_{C} R(S) \varphi(S) dS = \int_{C} R_{\text{th}}(S) \varphi(S) dS + \delta(S_1, S_2), \tag{4}
\]

where

\[
R_{\text{th}}(S) = \frac{1}{2\pi i} \left\{ \prod_{\text{th}}(S+i\epsilon) - \prod_{\text{th}}(S-i\epsilon) \right\}, \tag{5}
\]

\( \varphi(S) \) is an arbitrary analytic function. Of course, the equality (4) is not an exact one; the main error \( \delta(S_1, S_2) \) comes from the deviations from the relation (3) in the domain \( |\text{Im} q^2| < \Delta_0 \). The magnitude of \( \delta(S_1, S_2) \) is minimal for those values of \( S_1 \) and \( S_2 \) where \( R_{\text{th}}(S_1) \approx R(S_1) \), i.e., \( S_1, S_2 \) should lie on the plateau of \( R(S) \) far from the resonance positions. Information about resonances can be obtained by the following procedure. Firstly, one should parametrize the cross-sections by a known function of \( S \) depending on a small number of parameters which may be measured experimentally. Then, the unknown parameters should be adjusted in such a way that the sum rules (4) are saturated. For example, the low lying part of the spectrum in the \( J^{PC}=1^{--} \); \( I=1 \) channel may be well reproduced by the function

\[
R(S) = f m_{p}^2 \delta(S-m_{p}^2) + \frac{3}{2} \theta(S-S_{o}), \tag{6}
\]

where the first term gives the \( p \)-meson contribution (\( m_{p} \) is the mass of the \( p \)-meson and \( f \) is its residue proportional to the \( p \)-meson electronic width) and the second one is due to continuum with the effective threshold \( S_{o} \).

While studying the different parts of the spectrum it is useful to choose the different weight functions \( \varphi(S) \) in (4). Here we consider two widely used forms of the \( \varphi(S) \) and sum rules: (i) Borel sum rules are obtained by the setting \( S_1=0, S_2=\infty, \varphi(S) = \exp(-S/M^2) \), where \( M \) is an arbitrary parameter with the mass dimension; (ii) finite energy sum rules in the form FESR are characterized by the choice \( S_1=0, S_1\neq0, S_2\neq0, \)

\[
\int_{0}^{\infty} R(S) \varphi(S) dS = \int_{0}^{\infty} R_{\text{th}}(S) \varphi(S) dS + \delta(S_1, S_2), \tag{7}
\]

where

\[
R_{\text{th}}(S) = \frac{1}{2\pi i} \left\{ \prod_{\text{th}}(S+i\epsilon) - \prod_{\text{th}}(S-i\epsilon) \right\}, \tag{8}
\]

\( \varphi(S) \) is an arbitrary analytic function.
\( \psi(S) = S^k \) \((k=0,1,2)\). For the consideration of the ground states the Borel version of the sum rules seems to be the most convenient\(^3\). Indeed, the weight \( \exp(-S/M^2) \) suppresses the higher physical states contributing to the experimental spectral functions. Then, for the \( S_1 = 0, S_2 = \infty \) the error \( \delta \) is equal to zero. Moreover, the contribution of the condensates with high dimension is factorially suppressed. Note, however, that the FESR with the function \( \psi(S) = S^k \) in most cases gives practically the same results for the properties of the ground states of mesons, baryons, and glueballs\(^5\). In other words, as for these problems the Borel sum rules and FESR should be considered as the complementary methods.

For the consideration of the higher meson states FESR seems to be much more informative because the Borel function suppresses the contribution of the radial excitations of resonances to the spectral density. The next section is devoted to the study of the properties of these states by means of the FESR method.

3. FESR AND THE PROPERTIES OF RADIAL EXCITATIONS OF MESONS

For the consideration of the radial excitations of the mesons let us choose the following ansatz for the spectral function

\[
R(S) = \sum_{n=0}^{\infty} A_n S (S - m_n^2),
\]  

(7)

where \( m_n \) is the mass of the \( n \)-th excited state, and \( A_n \) is the corresponding matrix element. This rough model becomes exact in the leading order of the \( 1/N \) expansion (\( N \) is the dimension of the colour group). The approach is based on the sum rules (4) in the form

\[
\int_{S_n^0}^{S_n^1} R_{th}(S) S^k dS = \int_{S_n^0}^{S_n^1} R(S) S^k dS.
\]

(8)

These relations are usually called the local duality relations: \( n \)-th resonance is set to be dual to the perturbative spectrum in the interval \((S_n^0, S_n^1)\). With the choice \( m^2(n+1) - m^2(n) = \delta n^{10,11} \), which seems to minimize the error \( \delta \) \((S_n^0, S_n^1)\) in the sum rule (7), we obtain for the \( \rho - \text{meson channel} \)*:

\[
\frac{3}{2} \left( 1 + \frac{\alpha_s}{\pi} \right) (S_n - S_{n-1}) = A_n, \quad k = 0,
\]

\[
\frac{3}{2} \left( 1 + \frac{\alpha_s}{\pi} \right) (S_n^2 - S_{n-1}^2) = A_n m_n^2, \quad k = 1.
\]

(9)

Considering \( m_\rho \) as an input parameter from (9) it may be found that\(^{10,11}\)

\[
m_n^2 = m_\rho^2 (1 + 2n), \quad m_n^2 = (1 + 2n)^{1/3} m_\rho, \quad n = 0, 1, 2, \ldots
\]

(10)

From the above mass formula it follows that the first \( \rho - \text{meson radial ex-}

*Note that nonperturbative corrections are nonimportant here.
citation has the mass \( m_\rho(1) = 1.3 \text{ GeV} \). This result can be interpreted as a theoretical indication of the existence of the \( J^P(1260) \) meson, whose experimental status is not yet clear.\(^{16}\)

Finite energy sum rules can also be applied to other channels. We summarize the results\(^{11}\) of this consideration for the mesons built from the light quarks in the Table 1.

It is impossible to discuss here all the results for the resonance properties which have been obtained by the FESR method. The variety of the problems can be treated by this method including the electromagnetic form-factor of \( \Xi^- \)-meson and baryons\(^\text{17}\), charge radius of \( \Xi^- \) and \( K \)-mesons\(^\text{28}\), \( K^0-K^0 \) mixing\(^\text{19}\) properties of light and heavy mesons and baryons\(^\text{15,11}\) and so on. Besides, we consider the application of the sum rules for the determination of the light quark masses. The idea\(^\text{20,10,11}\) is in fact opposed to the method described above. One starts from the known hadron spectrum and then finds the fundamental parameters of QCD by the sum rules.

Consider the correlator \( \Pi_\Xi(q^2) \) of \( \Xi^- \)-meson currents \( j^\mu = i(m_u+m_d)(U\bar{u}d) \) which have nonzero matrix element with the \( \Xi^- \)-meson state \( \langle 0|j(\vec{x})|\Xi^-\rangle \) = \( f_\Xi m_\Xi^2 \), where \( f_\Xi = 130 \text{ MeV} \) is the \( \Xi^- \)-meson decay constant:

\[
\Pi_\Xi(q^2) = i \int d^4x e^{iqx} \langle T(\bar{j}(x)j(0))\rangle_0 . \tag{11}
\]

The main contribution to the spectral density gives the \( \Xi^- \)-meson state. The sum rule has the form

\[
\frac{2 \pi^2 m_\Xi^2}{(m_u+m_d)^2} = \frac{3}{8 \pi^2} \left( \frac{g_\pi^2}{f_\pi} \right)^2 \left[ \frac{4.46}{\Lambda} + 5\frac{\alpha_s^2}{\Lambda^2} \right] \frac{S_\Xi^2}{2} , \tag{12}
\]

where \( S_\Xi \) is the duality interval. Right-hand side of eq. (12) is the theoretical calculation up to three-loop corrections\(^\text{11}\). Taking the value of \( S_\Xi \) equal to \( m_\Xi^2/2 \) where \( \Xi' \) is the first radial excitation state of the \( \Xi^- \)-meson, the sum of the \( u \) and \( d \) quark masses may be obtained. The similar analysis of the \( K \) and \( \delta \) currents admits to find the absolute values of the light quark masses\(^\text{11}\)

\[
m_u \quad (1 \text{ GeV}) = 5 \text{ MeV} \\
m_d \quad (1 \text{ GeV}) = 11 \text{ MeV} \\
m_s \quad (1 \text{ GeV}) = 195 \text{ MeV} \tag{13}
\]

These are the current quark masses normalized in the standard way in the point 1 GeV. The uncertainties in the determination of the current quark masses have been discussed in details in Ref.\(^\text{21}\).

Now we turn to some nontraditional applications of the sum rules method, namely to the consideration of gauge theories with scalar fields in Section 4 and to analysis of the hot hadronic matter in Section 5.
4. GAUGE THEORIES WITH SCALAR QUARKS

The main purpose of this Section is to discuss the phase structure of
gauge theories with scalar fields from the point of view of the sum rules
method and consider the bound states of light scalar quarks. The phenome-
nology of scalar containing hadrons is also briefly discussed.

There are at least two different phases in the nonabelian gauge theo-
ries with scalar fields. Consider for definiteness SU(2) gauge theory with
doublet of scalar fields and $N$ doublets of left and right fermions.
The Lagrangian has the usual form

$$
\mathcal{L} = -\frac{1}{2} F_{\mu \nu}^a \tilde{F}_{\mu \nu}^a + (D_{\mu} \varphi)^* (D_{\mu} \varphi) + \sum_{i=1}^{N} \bar{\psi}_i \gamma^\mu \not{D} \psi_i - U(\varphi),
$$

$$
D_{\mu} = \partial_{\mu} - ig \frac{\sigma^a}{2} h^a_{\mu}, \quad F_{\mu \nu}^a = \partial_{\mu} h^a_{\nu} - \partial_{\nu} h^a_{\mu} + g \varepsilon^{abc} h^b_{\mu} h^c_{\nu},
$$

where $\sigma^a$ are the Pauli matrices, $\not{D} = \partial_{\mu} \gamma^\mu$. The Higgs phase (or "spontaneously broken" phase) is realized when the potential $U(\varphi)$ (or, strictly
speaking, the effective potential which includes the high order correc-
tions $^{22}$) has the minimum at some point $\langle \varphi \rangle \neq 0$. For the $\langle \varphi^* \varphi \rangle \gg \Lambda^2$, where $\Lambda$ is the "strong" scale of this theory, the weak coupling regime is
realized because the gauge coupling constant freezes out on the scale
$M^2 \sim g^2 \langle \varphi^* \varphi \rangle$. The spectrum consists of three massive vector bosons,
one scalar particle and $N$ left and right fermions.

If the potential $U(\varphi)$ has only the trivial minimum $\varphi = 0$, then it
is believed that the strong coupling or confining phase is realized. The
spectrum contains the bound states of scalar and fermion quarks and SU(2)
gluons.

Our first task is to show that the physical particles in the Higgs
phase are in fact the colour SU(2) singlets and that the spectrum in the
Higgs phase is similar (in a sense) to the spectrum in the confinement
phase.

Let us introduce the new variables with the help of the gauge trans-
formation depending on the scalar field $^{23,13}$:

$$
U = (\varphi^* \varphi)^{1/2}, \quad \varphi_i \rightarrow U \psi_i = \chi_i = \begin{bmatrix} \chi_1 \\ \chi_2 \end{bmatrix} = \begin{bmatrix} (\hat{\gamma}^* \psi_i)/\sqrt{2} \\ (\varphi^* \psi_i)/\sqrt{2} \end{bmatrix},
$$

$$
\varphi \rightarrow U \varphi = \begin{bmatrix} 0 \\ \sqrt{2} \end{bmatrix}/(\varphi^* \varphi)^{-1/2}, \quad \not{A}_\mu \rightarrow U (\hat{\not{A}}_\mu - i g \frac{\sigma^a}{2} \not{D}_\mu) U = \hat{Z}_\mu,
$$

$$
\tilde{Z}_\mu^a = \frac{i}{2} \frac{\varphi^* \not{D}_\mu \varphi}{\varphi^* \varphi}, \quad Z_\mu^1 = \frac{i}{2} \frac{\varphi^* \not{D}_\mu \varphi}{\varphi^* \varphi}, \quad Z_\mu^2 = \frac{i}{2} \frac{\varphi^* \not{D}_\mu \varphi}{\varphi^* \varphi}, \quad \hat{Z}_\mu = \frac{i}{2} \sigma^a \tilde{Z}_\mu^a,
$$

where $\hat{\psi}_i = \varepsilon_{ij} \psi^j$ is the dualy conjugated field. This change of vari-
ables is definitely possible in the Higgs phase where $\langle \varphi^* \varphi \rangle \neq 0$. In
fact, this transformation is just the fixing of the unitary gauge. It is easy to show that the Lagrangian can be written directly in terms of new variables *:

$$\mathcal{L} = -\frac{1}{4}(\mathcal{Z}_{\mu\nu}^a)^2 + (\phi_{\mu}^a)\xi_{\mu}^a + \frac{1}{4} \rho^2 \mathcal{R}^a (\mathcal{Z}_{\mu}^a)^2 + \sum \mathcal{L}_i (\iota \phi_{\mu}^a + \Phi^a_{\mu} \mathcal{Z}_{\mu}^a) \lambda_i - U(\mathcal{R}).$$  \tag{16}

From the eq. (15) it is clear that all the new fields are invariant under SU(2) gauge rotations, i.e., they are colourless. The role of the transformation (15) is that it separates the unphysical gauge degrees of freedom from the physical SU(2) invariant ones. Vector bosons are massive due to the existence of the condensate of the white scalar field $<\mathcal{R}> \neq 0$. It is clear that this condensate does not break local and global SU(2) gauge symmetry. Therefore from the symmetry considerations the Higgs phase and the confinement phase are undistinguishable. The consideration also shows that the question about the phase transitions between the Higgs and the confinement phases cannot be solved by the symmetry arguments only.

The statement on the colourlessness of the particles in the Higgs phase admits to find the properties of the white states $\mathcal{Z}, \mathcal{R}, \mathcal{L}$ by the standard sum rules method **. Consider, for example, the Green function of the vector currents $J_{\mu} = \psi^+ \Phi^a_{\mu} \psi$.

$$\Pi(q^2)(\mathcal{Z}_{\mu\nu}^a - q_{\mu} q_{\nu}) = \frac{i}{8} d^4x e^{i q x} <T (J_{\mu}(x) J_{\nu}(0))>_0. \tag{17}$$

In one loop approximation \(^{13}\)

$$\Pi(q^2) = \frac{1}{24 \pi^2} \sum_{n} \left( \mathcal{Z}_{\mu\nu}^a - q_{\mu} q_{\nu} \right) \left( 1 + \frac{3\mathcal{R}}{4\mathcal{R}} \right) + 2 <\psi^+ \psi> + \frac{1}{4} \frac{\mathcal{R}}{q^2} <\phi^+ \phi^a_{\mu} \phi^a_{\mu} > + \frac{1}{4} \frac{\mathcal{R}}{q^2} <\phi^+ \phi^a_{\mu} \phi^a_{\mu} > + \frac{1}{4} \frac{\mathcal{R}}{q^2} <\phi^+ \phi^a_{\mu} \phi^a_{\mu} > \tag{18}$$

Here we write only dominant contributions which come from the different scalar condensates, scalar fields are considered to be massless. Choosing the standard ansatzy

$$R(s) = A \delta(s - m^2) + \Theta(s - s_0)$$

the following finite energy sum rules may be obtained (we use FESR realization \(^9\) with $S_1 = 0$, $S_2 = S_0$, $\Upsilon(s) = s^k$, $k = 0, 1, 2$)

$$S_0 - A = -4 \mathcal{R} \mathcal{Z}^2 <\psi^+ \psi>, \quad 2m^2 A - S_0^2 = 76 \mathcal{R} \mathcal{Z}^2 <\psi^+ \phi^a_{\mu} > \tag{19}$$

$$3m^4 A - S_0^3 = 9216 \mathcal{R}^4 <\psi^+ \phi^a_{\mu} \phi^a_{\mu} > \tag{19}$$

*) Note that the index a have nothing to do with the gauge group SU(2).

The invariance of the Lagrangian (16) with respect to the global SU(2)\(_f\) transformation is connected with the invariance Lagrangian (14) with respect to the global rotations of the scalar fields only:

$$(\psi \phi) \rightarrow (\psi \phi) U_\xi, \quad U_\xi \in SU(2)\(_f\).$$

**) An application of the sum rules to the case of constituent models with scalar fields may be found in Ref. \cite{24}.\)
In the weak coupling regime the higher scalar condensates may be expressed through the condensate \( \langle \bar{n} \rangle \), e.g.,

\[
\langle \psi^+ \psi \rangle = \langle \bar{n}^2 \rangle = \langle \bar{n} \rangle^2, \quad \langle (\psi^+ \frac{s}{2} \psi) \rangle^2 = \frac{1}{4} \langle \bar{n} \rangle^4 = \frac{1}{4} \langle \bar{n} \rangle^4.
\]

With these relations the solution of the system (19) have the form:

\[
m^2_2 = 2 \frac{\Pi}{\alpha} \langle \psi^+ \psi \rangle, \quad A = \frac{24 \Pi}{\alpha} m^2_2, \quad S_0 = 0.
\]

These expressions for the mass and coupling of the resonance coincide exactly with those obtained directly from the Lagrangian (16) by the conventional methods.

Consider now the strong coupling regime. In this case we have no relations like eq. (20) and one should invoke some hypothesis about the higher condensates like vacuum dominance one\(^{3}\), where \( \langle \psi^+ \psi \rangle = (m^2 + 1) \langle \psi^+ \psi \rangle \). It is naturally to suppose also that there no quantitative difference in the spectrum in the \( \psi^+ \bar{n} \psi \) channel and, say, in the \( \rho \) -meson channel. This situation is realized if the main nonperturbative correction proportional to the scalar condensate is negative (like the four fermion correction in the \( \rho \) -meson channel). Thus, we have come to the statement that the scalar condensate \( \langle \psi^+ \psi \rangle \) should be negative in the strong coupling regime \(^{12,13}\).

Note that the positive definiteness of the operator \( \psi^+ \psi \) is violated by the renormalization procedure. The analysis of the sum rules (19) and analogous sum rules for the \( T \) and \( Y \) particles gives the result

\[
m^2_2 = 12 \frac{\Pi}{\alpha} \langle \psi^+ \psi \rangle, \quad m_\pi \simeq m_\rho \simeq m_2 / \sqrt{3},
\]

\[
A_2 = 1.2 \frac{m^2}{\alpha}, \quad S_0 = 1.5 \frac{m^2_2}{3}.
\]

The number \( \sqrt{3} \) comes from the comparing of the operator product expansion for the corresponding currents. This solution is similar to the spectrum in the \( \rho \) -meson channel; \( S_0 > m^2 \) and \( A_2 \) is relatively small (compare \( A_2 \) with the corresponding number in the weak coupling regime). Notice the large coefficient \( \sim 36 \) in front of the scalar condensate in the expression (22) for the \( Z \)-mass \(^{*)\}). It is explained by the suppression of the bare loop with respect to the tree graphs by the factor \( 1/16 \frac{\Pi^2}{\alpha} \) and by the lowest possible dimensionality of the scalar condensate \( \langle \psi^+ \psi \rangle \).

If one forgets for a moment about fermions then the total coincidence of the number of low energy degrees of freedom in the confinement and Higgs phases can be seen. This may imply that there are no two different phases in gauge theories with scalar fields but one unique Higgs-confinement phase. Note that analogous arguments have been presented in the Ref. \(^{23}\). The absence of the infinite boundary between the Higgs and confinement phases has been demonstrated also in the lattice calculations \(^{23,25}\).

\(^{*)\} \) Let me remind that ratio of the \( \rho \) -meson mass to the quark condensate is of the order of unity: \( m_\rho / \langle -\bar{q}q \rangle^{1/3} \sim 3 \).
When the massless fermions are introduced the situation changes cardinally. In the small coupling regime we have $2N$ left and $2N$ right massless white fermions. In the case of the negative scalar condensate the sum rules admit only the massive fermions $m^2 N = m_3^2 / \sqrt{3}$. Therefore, there is a doubling of the fermionic degrees of freedom in the confinement phase \(^{13}\). Due to the spontaneous breaking of the chiral symmetry $SU(N)_L \times SU(N)_R$ down to $SU(N)_{L+R}$ the degeneracy between the $(N,1)$ and $(1,N)$ states disappears. The resulting mass difference of the fermions with the opposite parity $x^2 = (\chi^{(m)}_L + \chi^{(m)}_R) / \sqrt{2}$ is of the order of chiral symmetry breaking fermion condensate, $\Delta m \sim \langle \bar{\psi} \psi \rangle / m_2^2$. Note that the change in the number of states is also seen on the example of the other particles. Indeed, there is no partner to the diquark meson (which definitely exist in the confinement phase) in the Higgs regime. The difference in the number of the fermionic degrees of freedom probably implies the existence of the phase transition between the Higgs and confinement phases in the Higgs-gauge-fermion system.

So, the analysis of the sum rules may clarify some subtle points in the questions concerning phase structure of the gauge theories.

Consider now standard quantum chromodynamics with the triplet of nearly massless coloured scalar quarks with the electric charge $Q(Q=2/3$ or $1/3)$. Such a theory can be the low energy limit of the supersymmetric theory. In this case the scalar is the superpartner of the fermion quark. It is clear that in addition to the bound states of quarks and gluons the hadrons built from the scalar quarks should exist. The scalar containing hadrons with the minimal mass are the meson-like states $\rho\bar{\sigma}$ (vector pho ninum with $J^{PC}=1^{--}$, $\rho_1 \sim \bar{q}^+ D^0 q^-$), $\Pi\bar{\pi}$ (scalar pho ninum with $J^{PC}=0^{++}$, $\Pi \sim \bar{q}^0 q^0$) and "white quarks" $x_1, x_2 \sim \bar{q}^+ q^-$. The masses of these particles are proportional to the scalar condensate $\langle \bar{q}^0 q^0 \rangle$.

$$m_{\rho_1} \approx (40-45) \langle -\bar{q}^0 q^0 \rangle^{1/2} = \frac{4}{\sqrt{3}} m_{\rho_1} \approx \frac{1}{\sqrt{3}} m_{\rho_2}, \quad \frac{\alpha_5(m)}{\pi} = \frac{1}{b \ln(m^2 / \Lambda^2)}.$$ \(^{(23)}\)

The factor $\alpha_5^{1/6}$ in front of the scalar condensate is introduced to ensure the renorm-invariance of the result. From the technical point of view the main specific feature of the scalar fields leading to the large coefficient in the expression for the $\rho_1$ mass is the lowest possible dimension of the scalar condensate $<\bar{q}^0 q^0> \sim (\text{GeV})^3$ (remind that the most important condensate in the $\rho$-meson channel has dimensionality $<\bar{q} q> \sim \text{GeV}^5$).

Thus, for a quite sensible value of the scalar condensate, say, $<\bar{q} q> \sim (1 \text{ GeV})^2$, the masses of scalar containing hadrons are very large and therefore these particles may escape the experimental detection up to now.

The other indication to the large mass scale of the new hadrons and
tight binding of scalar quarks comes from the analysis of $e^+e^-$ annihilation processes in the framework of perturbative QCD. In the $\overline{MS}$ scheme\(^{26}\),

$$R_q(S) = \sum Q_q^2 \left( 1 + \frac{\alpha_s}{\pi} + \left( \frac{\alpha_s}{\pi} \right)^2 \left( 1.92 - 0.12 n_f \right) \right),$$

$$R(\eta) = R_q(S) + R_q(S),$$

$$R_{q}(S) = \sum Q_q^2 \left( 1 + \frac{\alpha_s}{\pi} + \left( \frac{\alpha_s}{\pi} \right)^2 \left( 4.44 - 2.28 n_f \right) \right).$$

(24)

where \(n_f\) is the number of quark flavours. Comparing the scales\(^*\) in $\tilde{q} \tilde{q}$ and $\tilde{q} \tilde{q}$ channels (\(n_f = 5\))

$$\Lambda_{\tilde{q} \tilde{q}} = \Lambda_{\tilde{q} \tilde{q}} \exp \left( \frac{4.92 - 0.42 n_f}{2} \right) \approx 4 \Lambda_{\tilde{q} \tilde{q}},$$

$$\Lambda_{\tilde{q} \tilde{q}} = \Lambda_{\tilde{q} \tilde{q}} \exp \left( \frac{4.44 - 2.28 n_f}{2} \right) \approx 0 \Lambda_{\tilde{q} \tilde{q}},$$

we may see that the QCD perturbative expansion for the $\tilde{q} \tilde{q}$ channel becomes valid at much higher energies than for the $\tilde{q} \tilde{q}$ channel:

$$E_{\tilde{q} \tilde{q}} > \Lambda_{\tilde{q} \tilde{q}} \exp (-1/2 b x) \sim 0.83 \text{ GeV},$$

$$E_{\tilde{q} \tilde{q}} > \Lambda_{\tilde{q} \tilde{q}} \exp (1/2 b x) \sim 1.1 \text{ TeV}, \quad (x \approx 15\%),$$

where \(x\) is the admissible value of the correction to the bare loop. As soon as the breakdown of the perturbative expansion usually originates in the appearance of the bound state poles and production thresholds one can estimate from the above equations the ratio of the corresponding mass scales\(^{12, 26}\)

$$\frac{M(\tilde{q} \tilde{q} \text{- hadrons})}{M(\tilde{q} \tilde{q} \text{- hadrons})} \approx \frac{\Lambda_{\tilde{q} \tilde{q}}}{\Lambda_{\tilde{q} \tilde{q}}} \exp (3/2 b x) \approx \Lambda_{\tilde{q} \tilde{q}}^{2+1}$$

(27)

for \(x \sim 10-100\%). The above arguments have convinced ourselves that the masses of scalar containing hadrons built from the light scalars may be very large. Hence experimental search of these particles should be very important. Unfortunately up to now there are no any reliable estimates of the scalar condensates (the lattice calculations of this value may be extremely useful) and consequently we do not know the energy interval where new hadrons can be found.

Analysis of the experimental situation is given in Ref.\(^{12}\). Here we present some properties of the new hadrons which may be useful from the experimental point of view. The phenomenological upper bound on the value of scalar condensate will also be given.

(1) Properties of the vector phionium $\rho^0 = \varphi^0 D^0 \psi$ (\(J^{PC} = 1^{--}\)). This particle may be created as a resonance in $e^+e^-$ collisions provided the $\varphi$ charge is nonzero. The electronic width of $\rho^0$ is equal to\(^{12}\)

$$\Gamma_{ee} = 2.2 \frac{\alpha^2}{2 \pi} \left( \frac{3 Q_{\varphi}}{2} \right)^2 m_{\rho^0}. $$

(28)

For example, for $m_{\rho^0} = 30$ GeV and $Q = -1/3$, $\Gamma_{ee} = 10$ KeV. At this point the

* The effective scale may be found by minimizing of the three loop corrections by means of the redefinition $\Lambda_{\tilde{q} \tilde{q}} \rightarrow \Lambda_{\tilde{q} \tilde{q}}^{2+1}$.
bound states of light scalar quarks essentially differ from that of the heavy scalar quarks. The electronic widths of the heavy scalar quarkonium are of the order of eV.\(^2\) Because of the relation \(m_\rho < 2m_\rho\), the particle \(\rho_s\) is relatively narrow hadronic resonance, the main decay mode being\(^2\)
\[
\rho_s \to \eta_s + \text{light hadrons.}
\]
The total width of \(\rho_s\) is of the order of\(^2\)
\[
\Gamma_{\rho_s} \approx 0.02 \alpha \, \frac{m_{\rho_s}}{m_\rho} \approx 130 \text{ MeV}
\]
for \(m_{\rho_s} = 30 \text{ GeV}\). The decay rate is given by the formula\(^2\)
\[
\Gamma_{\rho_s \to \eta_s \gamma} \approx 0.4 \alpha \, \frac{Q_{\rho_s}^2}{m_{\rho_s}} \approx 2.5 \text{ MeV} \left( m_{\rho_s} = 30 \text{ GeV}, Q_\rho = -\frac{1}{3} \right).
\]
It does not seem that the PETRA experimental data on the \(e^+e^-\) collisions exclude the existence of \(\rho_s\) resonance in the region \(12 \text{ GeV} < S < 30 \text{ GeV}\). Of course, the masses of \(\rho_s\) larger than \(47 \text{ GeV}\) are not excluded too.

(ii) Properties of the scalar phionium \(\eta_s = \gamma^+ \gamma^- (J^{PC} = 0^{++})\). The decay width of the \(\eta_s\) is suppressed by the OZI rule. The dominant contribution to the \(\eta_s\) decay rate comes from the process \(\eta_s \to 2\pi \to \text{light hadrons.}\) Hence \(\eta_s\) is a narrow resonance with the hadron width\(^2\)
\[
\Gamma_{\eta_s} = \frac{\alpha_s^2}{12 \pi} \, m_{\eta_s} \approx 9 \text{ MeV} \left( m_{\eta_s} = 20 \text{ GeV} \right).
\]
The radiative width of the \(\eta_s\) is\(^2\)
\[
\Gamma_{\eta_s \to \gamma \gamma} = \frac{3 \alpha \, Q_{\gamma}^4}{8 \pi} \, m_{\eta_s} \approx \frac{4}{5} \text{ MeV} \left( m_{\eta_s} = 20 \text{ GeV}, Q_\rho = -\frac{1}{3} \right).
\]

(iii) Properties of the white quarks \(\lambda_s = \gamma^+ \gamma^-\). The decay modes of \(\lambda_s\) are model dependent in a great extent. In non-supersymmetric theories \(\lambda_s\) decays on baryon + mesons or on lepton + mesons and may be practically stable or have the lifetime of the order of, say, \(10^{-17} \text{ sec.}\) The lower bound on the mass of the stable \(\lambda_s\) (\(\gamma_s > 10^{-13} \text{ sec.}\)) is \(m_\lambda > 12 \text{ GeV}\).\(^12\) Unstable \(\lambda_s\) (\(\gamma_s \leq 10^{-13} \text{ sec.}\)) create the thresholds in \(e^+e^-\) annihilation (\(Q_\gamma = 2/3, \Delta R = 1/3\) and \(Q_\rho = -1/3, \Delta R = 1/12\)) and probably \(m_{\lambda_s} > 20 \text{ GeV}\) because no deviations from the QCD value of \(R\) have been observed at PETRA. Combining the different bounds on the \(\rho_s\) and \(\lambda_s\) masses and taking into account the mass relations (23) the following lower bounds on the value of the scalar condensate may be obtained \(< -\gamma^+\gamma^- > \approx (500 \text{ MeV})^2\) for the stable \(\lambda_s\) and \(< -\gamma^+\gamma^- > (1 \text{ GeV})^2\) for the unstable \(\lambda_s\).

If on a fine day the \(< -\gamma^+\gamma^- >\) is computed and proves smaller than \((500 \text{ MeV})^2\) this will definitely exclude the models with light coloured scalar particles.

The experimental search for the new hadrons may be carried out in \(e^+e^-\) collisions\(^12\), \(p\bar{p}\) reactions\(^2\) and in cosmic rays\(^12\). The observa-
tion of heavy hadrons with properties predicted by the strong coupling solutions of the QCD sum rules may signal existence of the light scalar quarks - the new flavour in particle physics.

5. HOT HADRONIC MATTER AND FINITE TEMPERATURE QCD SUM RULES

Continuing the review of the sum rules method applications consider now the analysis of properties of the hot hadronic matter.

The consideration of QCD at extreme conditions is interesting from the different points of view. It is expected that in the high energy hadron (or ion) collisions the dense and hot clots of hadronic matter are formed\(^{30}\). If the lifetime of these formations is large enough then in such a system the approximate thermodynamical equilibrium is established. Due to the large temperature and/or density the new state of hadronic matter, namely quark-gluon plasma is produced. The physical picture of the quark-hadron phase transition is quite transparent. At low temperatures hadronic matter looks like dilute gas of mesons and baryons. When the temperature and density rise the individual hadron wave functions become overlapping and quarks and gluons easily travel from one point to the other. The interaction between quarks and gluons is weak due to the small (in average) distance between them and because of the QCD asymptotic freedom. It is clear that the critical energy density should be of the order of nuclear one \( n_{cr} \sim 500 \text{ MeV/fm}^3 \) and critical temperature \( T_c \sim (n_{cr})^{1/4} \).

200 MeV. Because of this relatively small numbers the quark-hadron phase transition is the only transition in particle physics which may be studied experimentally. Besides, quark-hadron phase transition in the early universe may have observable consequences (see, e.g., Ref. \(^{31}\)). Here we shall not discuss the question of the experimental consequences of the quark-hadron phase transition. The discussion of this point may be found in Refs. \(^{30,32}\). Our aim is to consider the sum rules approach in the case of the hot hadronic matter*.

The motivation of the sum rules application to the nonzero temperatures \( T \) is very simple. Sum rules are very useful for \( T=0 \) and one should try to consider them at \( T \neq 0 \).

First of all, several words about the generalization of the sum rules to the case of nonzero temperatures. At \( T \neq 0 \) the retarded (or advanced) Green functions should be considered, because they possess the useful analytical properties:

*) A regular method for the investigation of the quark-hadron phase transitions is the numerical calculations in the lattice formulation of QCD. See lectures\(^6\) and references therein.
\[ G_{ab}^R(\omega, \vec{p}) = i \int d^4 x e^{ipx} \Theta(x^0) \ll [J_a(x), J_b(0)] \gg. \] (33)

Here \( \ll \ldots \gg \) stands for the Gibbs averaging of commutator of two currents:

\[ \ll \ldots \gg = \sum_n w_n \langle n | \ldots | n \rangle, \quad w_n = \exp(\Omega - E_n)/T, \] (34)

\( \Omega \) is the complete set of the eigenstates of the Hamiltonian \( H : H|n\rangle = E_n|n\rangle, \Omega = -T \ln \text{tr} \exp(-H/T). \) We distinguish the time component of the 4-vector \( \vec{p} = (\omega, \vec{p}) \) because of the lack of Lorentz invariance in the media with \( T \not= 0 \).

Function \( G_{ab}^R(\omega, \vec{p}) \) is analytic in the upper half-plane of complex variable \( \omega \) and is expressed through its own imaginary part by means of the standard spectral representation

\[ G_{ab}^R(\omega, \vec{p}) = \int_{-\infty}^{\infty} du \frac{\rho_{ab}(u, \vec{p})}{u - \omega - i\varepsilon}. \] (35)

The spectral density \( \rho_{ab} \) is determined by the matrix elements of currents \( J_{a,b} \) and gives the desirable effective spectrum of excitations of the hadronic matter

\[ \rho_{ab}(\omega, \vec{p}) = \sqrt{2}(\omega) \sum_{n,m} \langle n | J_a | m \rangle \langle m | J_b | n \rangle \delta(\omega - \omega_{mn}) \delta(\vec{p} - \vec{k}_{mn})(w_n - w_m), \] (36)

where \( \omega_{mn} = E_m - E_n, \quad k_{mn} = k_m - k_n \). Repeating the derivation of the sum rules presented in the Section 2 one obtains

\[ \int_{-\infty}^{\infty} d\omega \psi(\omega) \rho(\omega) = \int_{-\infty}^{\infty} d\omega \psi(\omega) \rho_{th}(\omega). \] (37)

Here \( \rho_{th} = \text{Im} G_{th}(\omega + i\varepsilon, \vec{p}) \). Thus, finite temperature sum rules looks like zero temperature ones.

Right hand side of eq. (37) is determined by the theoretical calculations. It contains the results of the temperature (Watson-type) perturbation theory and nonperturbative corrections proportional to the temperature dependent condensates. Left hand side of sum rules (37) which is to be determined contains the information about the effective hadron spectrum at nonzero temperatures. One should expect the drastic modification of the spectrum at the point of quark-hadron phase transition.

Consider now the specific features of the finite temperature sum rules in more details.

(i) Ansatz for the temperature dependent spectral function.

The form of the \( T=0 \) spectral function is given by the expression (6) which in fact comes from the known experimental data. In our case we have no analogous information. To find the formula suitable for the description of the temperature dependent spectral function, consider perturbative
spectral density (we choose for concreteness $\rho$-meson currents and took $\hat{p} = 0$, thus dealing with the resonance which is at rest with respect to the medium):

$$\rho^4 = \frac{3}{2} \theta(\omega^2 - 4m^2) \omega \frac{\omega}{4T} \rho_0(\omega^2), \quad \rho_0(\omega^2) = \left( 1 + \frac{2m^2}{\omega^2} \right)^{4/2} \delta(\omega^2),$$

$$\rho^5 = \delta(\omega^2) \int d\omega^1 \beta F \left( \frac{\omega}{2T} \right) \rho_0(\omega^1), \quad n_F(\infty) = (e^x + 1)^{-1},$$

$$\rho_c(\omega) = \rho^4(\omega) \cdot \rho^5(\omega),$$

$m_q$ being the quark mass, $n_F$ being Fermi distribution. The quantity is proportional to the probability of the creation of the quark-antiquark pair by the virtual photon in the medium. The influence of the medium on this process manifests itself in the suppression factor $\frac{\omega}{4T} = (1 - 2n_F(\omega/2T))$ describing the relative number of unoccupied fermionic energy levels. The quantity $\rho^5$ describes the scattering of the virtual $\gamma$-quanta on the quarks from the medium.

At sufficiently large temperatures ( $T > T_c$, $T_c$ is the temperature of the quark-hadron phase transition) we have quasifree gluons and quarks, therefore the form of the spectral function should look like (38):

$$\rho_c = \frac{3}{2} \left[ \theta(\omega^2 - S_0) \omega \frac{\omega}{4T} + \delta(\omega^2) \int d\omega^1 \beta n E \left( \frac{\omega^1}{2T} \right) \right],$$

$S_0(T)$ being effective continuum threshold which is to be determined. This formula describes well the continuum contribution at low temperatures too. The first term at $T=0$ in (39) is exactly the same as the second in (6), and the second one effectively takes into account the scattering of the virtual $\gamma$-quanta on the hadrons. As for the resonance contribution, it takes the standard form $\rho_{res}(\omega) = f(\omega^2 - m^2)$ but $f$ and $m$ are now temperature dependent. So, the total spectral density has the form $^{14}$

$$\rho = \rho_c + \rho_{res}. \quad \text{For } T=0 \quad f = 4, \quad m = 0.77 \text{ GeV}, \quad S_0 = 1.2 \text{ GeV}^2. \quad \text{The spectrum of the quark-gluon plasma is described by the numbers } f=0, \quad S_0=0, \quad m^2 \text{ undetermined.}$$

(ii) At $T=0$ the condensates are known fixed numbers. Now they are temperature dependent and their values are unknown at large temperatures. In this paper we paid main attention to the case of the low temperatures $T \leq 150$ MeV. It may be shown $^{14}$ that if in this temperature interval the spectrum coincide with zero temperature one then the values of condensates are approximately independent. So, we consider sum rules with the constant condensates.

(iii) The case of nonzero temperatures may be characterized by two independent energy scales. The first one is connected with the momentum $^{14}$

\text{ For the case of low temperatures this quantity may be found more exactly in the model of the dilute hadron gas. However, the results of the sum rules treatment appears to be independent on the exact form of $\rho^5$.}
of the virtual photon $\omega_k$ and the second one is defined by the average 3-momentum $q_T^2 \equiv \sum_i N_i \langle k_i^2 \rangle$ of quarks and gluons in the medium. Hence the $\alpha_s q$ corrections for the correlator have the generic form $O(\alpha_s(q_T^2))$ and $O(\alpha_s(q_T^2))$. Perturbative calculations are correct for $\omega^2 \gg \Lambda^2$, $q_T^2 \gg \Lambda^2$. Estimating $q_T^2$ as
\[ q_T^2 = \frac{\sum_i N_i \langle k_i^2 \rangle}{\sum N_i} = 3T^2 \frac{7\xi(5)}{7\xi(3)} \sim 4T^2 \]
(40) (here $\langle k_i^2 \rangle$ is the thermodynamical average of the quarks $i=1$ and gluons $i=2$ 3-momentum square, $N_i$ is the total number of quarks $i=1$ and gluons $i=2$) one can see that the perturbative treatment of correlator should be correct for $T \gg \frac{1}{2} \Lambda \sim 30$ MeV ($\Lambda \sim 100$ MeV). Consider now the Borel version of the finite temperature sum rules in the $\rho$-meson channel. They have the form
\[ \frac{4m^2}{M^2} e^{-m^2/M^2} = \frac{3}{2} \int_0^{\omega_o} d\omega / M^2 e^{-\omega^2/M^2} \frac{d}{d\omega} \left( \frac{\omega}{q_T} \right) + \int_0^{\omega_o} d\omega / M^2 3n_F(\omega / q_T) \]
(41) (+ zero temperature condensate)

First of all, let us consider the temperature evolution of the spectrum parameters qualitatively. The "scattering density" $\rho^5$ enters the Borel sum rule as a power term $B/M^2$ with positive coefficient $B > 0$. As the temperature grows this coefficient rapidly increases $B \sim T^2$. The appearance of such a power term implies the enhancement of the spectral density in the lower part of the spectrum, which is naturally realized by decrease in both the resonance mass $m$ and continuum threshold $S_0$. An estimate of the critical temperature where spectrum must undergo some drastic modification is obtained by the comparing of the scattering contribution with the quark bare loop at $M^2 \sim m^2$. Thus $16m^2 / m_F^2; T_c \sim 200$ MeV. This value appears to be very close to the result of a more involved treatment.

The temperature dependence of the spectrum parameters $S_0$, $f$ and $m^2$ which follow from the sum rules (41) is shown in Fig. 2. The solid lines in the Fig. 2 ends at temperature $T \approx 135$ MeV. Up to this point the sum rules analysis does not differ from the analogous consideration at $T=0$.

The curves look approximately as a plateau up to the temperature $T \approx 130$ MeV and as an intensive decrease in the parameters $m$, $S_0$, $S_0/m^2$ at higher temperatures. The most characteristic is the falling of the parameter $S_0$ which implies the appearance of new unsuppressed modes in the, say, $e^+e^-$ annihilation in the matter. The natural (if not unique) candidates for the new particles, responsible for such a behaviour of the continuum are quasifree quarks with possibly nonzero dynamical mass $m^2$. Therefore, the modification of the spectrum in the vector channel $J^{PC}=1^{--}$ at the temperature $T \approx 130-150$ MeV is an evidence of the deconfinement at this temperature. Due to the sharpness of the changes in the spectrum it is natural to relate such a behaviour of $m^2$, $S_0$ and $f$ to the first order deconfinement quark-hadron phase transition.
Unfortunately, finite temperature sum rules do not allow one to determine an exact character of the transition from hadrons to quarks and gluons. In particular, the spectrum behaviour discussed above may proceed in the presence of the first or second order phase transition or some other process of, say, ionization type. Nevertheless, from the experimental point of view when studying fixed channels (e.g., dimuon spectrum) it is the behaviour of the spectral function that is most important.

Consider now the domain of high temperatures \( T \geq 140 \) MeV. Due to the drastic modification of the spectrum any information about condensates is absent. We consider two plausible scenarios of the condensates evolution:

(i) all the condensates \( \langle \bar{Q} Q \rangle, \langle \bar{q} q \rangle, ... \) are equal zero for \( T > T_c \) (i.e., the restoration of the chiral symmetry and deconfinement phase transition take place simultaneously). In this case the sum rules admit the unique solution \( f m^2 = 0, S_o = 0 \) (see curve 1 in Fig. 2). This is the spectrum of quark-gluon plasma with massless quarks. Any resonances including \( \rho \)-meson are absent;

(ii) the deconfinement phase transition \( (T_p) \) and chirality restoration \( (T_F) \) are substantially separated on the temperature axis, i.e., \( \langle \bar{q} q \rangle \neq 0 \) for \( T \geq 150 \) MeV. In this case it is naturally to suppose that at \( T_c < T < T_F \) the condensates are practically temperature independent. The analysis of the sum rules are shown by the broken line 2 in Fig. 3. The parameter \( S_o \) in this case is connected with the dynamical mass of the unconfined quarks \( 4 m^2 q_1^T = S_o \) because they dominate in such a medium. If \( \langle \bar{q} q \rangle_T \approx \langle \bar{q} q \rangle_0 \) then \( m_T Q \approx 250 \) MeV coinciding with commonly accepted value of the constituent quark mass. Besides \( \pi \) -mesons which are definitely present in the temperature interval \( T_c < T < T_F \) since \( \langle \bar{q} q \rangle \neq 0 \) sum rules admit the existence of \( \rho \)-meson. It looks like rather crumbly object \( (f m^2 \sim |\psi(0)|^2, f m^2(T = 150 \text{ MeV}) \sim 0.3, \psi(0) \) being a magnitude of the quark wave function at the origin). In fact, the "hot" \( \rho \)-meson may form rather wide bump at \( S \sim 600 \) MeV with the electronic width \( \Gamma_{ee} \sim 3 \) keV in the cross section against the background of continuum (at \( m^2_T > S_o \approx (2 m_0 T)^2 \) \( \rho \)-meson can decay into the quarks).

It is not excluded, however, that the \( \rho \)-resonance disappears completely whereas its presence in our analysis may be a consequence of the uncertainties in the sum rules method.

The analysis of the sum rules may be carried out for other channels too. In particular, for the members of vector nonet one obtains the same value of the transition temperature despite of the 20% variation of the spectrum parameters. This fact confirms the naive expectation of the universality of the deconfinement temperature.
6. CONCLUSION

The main conclusion from the above is that the sum rules are powerful and very informative method for studying the different aspects of particle physics.

The author is deeply indebted to V.A. Kuzmin, V.A., Matveev and A.N. Tavkhelidze for continuous interest in the work and useful suggestions. It is a pleasure to thank the members of the Theory Division of the Institute for Nuclear Research for fruitful discussions and criticism. I am grateful to the organizers of the CERN-JINR School of Physics for the opportunity to present this lecture.

* * *

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Table 1

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Fig. 1
The contour C used in the derivation of sum rules

Fig. 2
The temperature dependence of the effective spectrum parameters in the \(\rho\) -meson channel in the case \(T_c=T_F\) (curve 1) and \(T_F>T_c\) (curve 2): a) \(m\) is the \(\rho\) -meson mass; b) \(S_0\) is the effective continuum threshold; c) the ratio \(S_0/m\); d) \(m^2\) stands for the resonance contribution to the spectral density.
QUARK DISTRIBUTION IN NUCLEI

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ABSTRACT

Some salient features of the limiting fragmentation of nuclei in cumulative region substantiating extraction of the nuclear structure functions from hadron reactions are discussed. Complementarity of information provided by lepton and hadron probes is pointed out. The QCD-based approach to the nuclear quark distribution in the lepton deep inelastic scattering is outlined which necessitates additional sea in nuclei.

1. INTRODUCTION

A study of the quark degrees of freedom in atomic nuclei is of interest from many points of view. First and foremost, it has a direct bearing on most acute problem of modern theory - the behaviour of QCD at large distances. The interaction of nucleons provides a possibility of the formation in nuclei of the multi-quark clusters. The properties of these clusters define the short-range part of the NN-potentials and the nuclear wave functions, which are of prime importance for the nuclear physics as a whole. The experimental investigation of the spatial and momentum distributions of partons in the short-lived multiquark configurations is possible only in the reactions of leptons and hadrons with nuclei at high energies. It will give a unique information on the unusual objects that may be considered as an admixture of new phase cells - quark - gluon plasma - in the nuclear matter. There is now a great interest in this new phase of the hadronic matter and the main hopes of its formation in a "macro-volume" (of an order of the nuclear volume) are related to the ultra-relativistic heavy ion collisions. It is evident that a reliable diagnostics of this new phase of matter requires knowledge of all mechanisms of nuclear reactions with large energy-momentum transfers taking into account all existing in nuclei under the ordinary condition configurations of the fundamental constituents of the hadronic matter - quarks and gluons. Therefore, the detailed study of the quark-gluon composition of nuclei is an indispensable part of building the complete theory of hadrons on the basis of QCD.

2. HADRON REACTIONS AND STRUCTURE FUNCTIONS OF NUCLEONS AND NUCLEI. CUMULATIVE EFFECT

The deep inelastic scattering of leptons on hadrons is a traditional and most reliable means to measure the structure functions (i.e. the quark momentum distributions in hadrons). The nuclear targets were customarily used with the aim to extract the nucleon structure functions. A widespread belief about the nuclear effects was to consider them as not very
interesting and significant in most cases, or as rather reliably understood within the traditional notions (the impulse approximation, Fermi-motion etc.) stemming from the nonrelativistic quantum mechanics applied to the problem of the $A=Z+N$-interacting nucleons. This attitude seems to be distinctly changed when the results of the European muon collaboration (EMC) on the ratio of the heavy nucleus ($Fe$) structure function to that of the deuterium turned out in sharp disagreement with expectations derived from the standard theory. We wish to stress, however, that long before the EMC data, the point of view alternative to the above-mentioned beliefs was put forward and developed. Namely, on the basis of observing the limiting fragmentation of nuclei in the high-energy hadron-nucleus collisions the clear-cut statement was made that the structure functions of nuclei are the qualitatively new objects in the physics of hadrons not reducing to the nucleon ones and thus well-deserving the special studies (see, e.g., the lectures by A.M. Baldin and V.K. Lukyanov at the precedent JINR-CERN Schools). The parton model and approximate scaling in the hard ($Q^2 \gg m^2$, $\nu = E_{z} - E_{\ell} \gg m$, $0 \leq x = Q^2/2m\nu \leq 1$) lepton-nucleon interactions are derived from the mechanism of the incoherent scattering of leptons on the point-like quarks. What justifies the application of the parton picture and measurability of the structure functions in the "soft" hadronic inclusive reactions $\alpha + b \rightarrow c + X$ with small values of $<P_{c}\alpha > \approx 0.4$ GeV? It is natural to expect that the production cross-section of hadron $C$ should be proportional to the probability to find, in one of the initial hadrons, the group of partons $\{i\}$ with the quantum numbers of $C$ and total longitudinal and transversal momentum fractions $\Sigma x_i$ and $\Sigma P_{Ti}$, the same as $x_C$ and $P_{TC}$. One should also invoke the "soft hadronization" hypothesis which means that the colour neutralization does not result in significant energy-momentum redistribution between newly produced hadrons. These qualitative arguments underline a number of the recombination or fragmentation models relating the inclusive cross-section $E_c d\sigma/dx_C (\alpha + b \rightarrow c + X)$ and the structure function of a hadron $a$ (or $b$) in its fragmentation region ($x_C > 0.3$; $x_a = (a_1 + P_1^a)/(P_0^a + P_2^a)$). As $x \rightarrow 1$ the experimental distribution has the form $d\sigma/dx \sim (1-x)^\kappa$. The mean values of the exponent $\kappa$ for the meson production $p + h \rightarrow M(q\bar{q}) + X$ are $\kappa(p \rightarrow M(u\bar{u})) \approx 3.2$ and $\kappa(p \rightarrow M(d\bar{d})) \approx 4.0$ (one can obtain this by averaging over many experimental values collected in the review article). These values correspond to fragmentation of proton into meson $M$ containing the valence $\bar{u}$ - or $d$-quark respectively and by the soft colour neutralization assumption should be proportional to the $\bar{u}$ - and $d$-quark distribution in the parent proton: $u_{v}(x) \sim (1-x)^{\kappa(\bar{u})}$, $d_{v}(x) \sim (1-x)^{\kappa(d)}$. It turned out that both the ratio $d_v(x)/u_v(x) \sim (1-x)^{\kappa(\bar{u})}$ and the numerals $\kappa(\bar{u}) \approx 3$ and $\kappa(d) \approx 4$ are very close to values found in the $NN$ -reactions for the "current" quark distributions in proton. Now, we turn to the inclusive
hadron-nucleus reactions where the fast mesons $M$ and nucleons $N$ are emitted in the nucleus fragmentation region.

To explore the deep short-range nuclear structure, the cumulative particle production is most adequate, i.e., the particle production in the region kinematically forbidden for the reaction on the quasi-free nucleons. Exploring the cumulative reactions at high and intermediate energies uncovers a number of general and universal features which specify this new region of the hadron interaction physics $6-8$.

1. Starting from the relatively low energy of incident particles $E_{lab} \approx 4-5$ GeV the limiting fragmentation of atomic nuclei is observed in the reactions $\alpha + A \rightarrow c + X$, $a = \gamma$, $\ell$, $\pi$, $p$, $\Delta$, $\text{He}...$, $c = \pi, K, N$ i.e. the shape of the fragment distributions is independent of energy and the type of incident particle.

2. The inclusive cross-sections are parametrized for $P_{tc} \approx 0$ in the form

$$E_c \frac{d\sigma}{dP_{tc}} = \sum (A, x) = A^{\chi(x)} \cdot G(x)$$

$$G(x) = G_0^{(x)} \cdot \exp(-x/\langle x \rangle), \langle x \rangle \approx 0.14$$

where the scaling variable is, in high-energy limit, $x = A\left(\frac{p^c + p^X}{p^A + p^X}\right)$ with the Z-axis chosen along the incident particle momentum. By definition, $x \geq 1$ specifies the cumulative region.

For the meso-production in the "pre-cumulative" region $\chi(x) \approx x^{-1/3} \cdot (1-x) \left(\langle x \rangle \right)$, $\chi(x) = \left\{ \begin{array}{ll} 1, & x \leq 1 \\ 0, & x < 0 \end{array} \right.$, in Eq. (1), for $\chi > 0$.6.

The numerical values of $G_0$ may be very different for various reactions, e.g. $G_0^p \geq 10^2$, $G_0^n$ while $\langle x \rangle$ appears to be universal, thus providing the smooth fall of the $x$-distribution without observable steps at the border of different "cumulativity" order ($\chi = 1, 2, 3,...$).

3. The "cross-section per nucleon"$(E_c/A)(d\sigma/dP_{tc})$ increases for $P$ and $P_{tc}$ in a similar manner till $A \approx 20$, then the pion yield is freezing at $\chi(x) \approx 1$ while that of protons continues to grow (roughly as $x^{1.4}$, $\chi_\pi \approx 1.4$) up to heavy nuclear-target. The normalized cross-section $(E_n/G(A)^N)(d\sigma/dP_{tc}^N(aA \rightarrow N+X))$ has a similar $A$-dependence for any incident particle $A$ and to 20-30% - accuracy does not depend at all on the initial particle: $a = \gamma$, $\ell$, $\pi$, $p$, ...

4. For the cumulative nucleon yields the isotopic and isotonic effects have been observed, that is the independence of proton (neutron) yield of increasing the number of neutrons (protons) in the given isotope (isotonic). A kind of the "isosymmetrization" has also been observed, i.e. an approximate equality of the cumulative proton and neutron yields in sharp contrast with a relative number of neutrons and protons in heavy nuclei. The ratio of the sufficiently energetic $(P_{lab} \geq 300$ MeV) pion yields $R = \pi^-/\pi^+$ is also very close to 1.

Practically all attempts to explain the accumulated experimental
facts start from the general idea of existence in nuclei of the short-ranged (and short-lived) multiquark (or multibaryon, B \( \geq 2 \)) clusters, or configurations \( 9-13 \), the cumulative particles being just the fragmentation products of these clusters. The immediate task for theory now is to obtain the necessary properties of these objects and to embed them into a general structure of the traditional nuclear physics. The second aspect includes the calculation of the probability amplitudes \( C_\kappa \) to find the \( 3\kappa \) quark clusters in the nuclear wave functions:

\[
\Psi = \Psi (AN) + \sum_{\kappa=2}^{A} C_\kappa \Psi (3\kappa q; (A-\kappa)N) \tag{3}
\]

The modern searches for the solution of these problems are based on the same model concepts and methods that are applied in the derivation of the nucleon-nucleon potentials and the NN-scattering phases. In brief, the region of the nucleon interaction is divided into the "internal" and "external" parts. In the internal region all quarks of the system are localized and governed by the appropriately chosen dynamics while the external region is described with the traditional nuclear physics in terms of nucleons interacting via the pion exchanges and the corresponding Yukawa-type potentials. Information about the internal region can be introduced, e.g., into the logarithmic derivative of the nucleon-channel wave function at the surface separating two regions \( 14, 15 \). In this framework the internal quark dynamics has been approximated by models of the MIT-bag-type, and it was found \( 15 \) for the probability of the 6q-bag in deuteron \( \omega_{6q}^d \approx 2\% \).

An alternative class of models appears as the appropriate generalization of the interpolation methods used in describing the clustering phenomena in the nonrelativistic nuclear physics. Sewing up two regions with different dynamics is realized by the effective interpolating potential

\[
\mathcal{V}(\tau) = \mathcal{V}_{\text{int}}(\tau) \Theta(\tau_0 - \tau) + \mathcal{V}_{\text{ext}}(\tau) \Theta(\tau - \tau_0) = \sum_{i,j=1}^{6} \mathcal{V}_{\text{qq}}(i,j) \Theta(\tau_0 - \tau) + \left( \mathcal{V}_{NN}^m(\tau) + \sum_{i,j=1}^{3} \mathcal{V}_{\text{qq}}(i,j) + \sum_{i,j=4}^{6} \mathcal{V}_{\text{qq}}(i,j) \right) \Theta(\tau - \tau_0) \tag{4}
\]

where \( \mathcal{V}_{NN}^m \) is the long-range "tail" of the meson potentials, \( \mathcal{V}_{\text{qq}} \) are the qq-interaction potentials, \( \tau_0 = 2\tau_n \approx 0.8 \) fm ( \( \tau_n \) is the radius of the quark dimension of a nucleon). The calculation within the nonrelativistic constituent quark model, using Eq. (4), gives \( \omega_{6q}^d \approx 6-7\% \) \( 3, 11 \).

The detailed investigation and discrimination between the model-dependent structure of the internal, multiquark configuration wave functions is clearly of upmost importance and it may be provided only by the high-energy particle-nuclear reactions sensitive to short-distances between nucleons in nuclei. The first knowledge about the form of
the momentum distribution of quarks in the multibaryon clusters has just been gained from the limiting nuclear fragmentation studies. That we really deal with the quark-parton structure functions of nuclei is suggested by theoretical arguments supported by experiment. In addition to what we have known from the hadron-hadron reactions, one more important circumstance should be stressed here. The universality of cumulative particle spectra gives evidence of the relative suppression, in the hadron-nuclear reaction of those factors which reveal themselves in dependence on energy and quantum numbers of incident particles of the inclusive spectra observed in some hadron-hadron reactions, thus hampering the interpretation of these data in terms of the single and universal characteristic of fragmenting hadron - its structure function.

The prediction \(^{16}\) based on the nuclear fragmentation data of the essential features of the structure functions for the deep inelastic scattering of leptons on nuclei at \(x > 1\) was later confirmed by the experiment \(^{17}\). For large \(x > 1.5\) the hadronic reactions are the only source of information to date, and all findings from there may serve as the predictions to be verified in the forthcoming experiments with the lepton beams. The detailed comparison between the leptonic and hadronic data may also serve as the source of new information. As is well-known, there is a firmly established breaking of scaling, i.e. the quark distribution functions are \(Q^2\)-dependent: \(q_\perp(x, Q^2)\). The structure functions are extracted from data taken usually at high \(Q^2 \approx 10-100 \text{ GeV}^2\). But, what effective scale should the structure functions from the hadron reactions be referred to? Most reasonably, this scale should be of the "natural" order of magnitude \(Q^2_0 \sim \langle p T^2 \rangle \sim O(1 \text{ GeV})\) i.e., of an order of the mean transverse momentum of the current quarks in hadrons or multiquark clusters. If so, then to compare the similar quantities in the similar conditions, one should evolve the measured \(q_\perp(x, Q^2 \approx 10-100 \text{ GeV}^2)\) backward to \(Q^2_0\) with the help of evolution equations of QCD. In a sense, the momentum distributions of the spectator-quarks from the "soft" hadron-hadron or hadron-nucleus processes are more adequate to check the model wave function dominating the static properties of hadrons and, in turn, dominated by the nonperturbative mechanisms of QCD. Low effective scale \(Q^2_0\) should be reflected in more "hard" gluon distribution and it would be interesting to find its experimental implications.

3. DEEP INELASTIC SCATTERING OF LEPTONS AND NUCLEAR EFFECTS

Nuclear effects in deep inelastic scattering of leptons were communicated for the first time by the ZEUS collaboration \(^{17}\) at large \(x > 1\) and the EMC \(^{1}\) at \(0.05 \leq x \leq 0.65\). The results \(^{17}\) in cumulative region are still waiting for a confirmation and those of the EMC have largely (except, possibly, \(x < 0.2\) region) been confirmed by other experiments \(^{18}\) (recall the factor \(A^{\alpha(x)}\) in Eq. (1) showing
qualitatively the same phenomenon which was later pinpointed by the EMC). These results have initiated the multitude of theoretical works (see, e.g., the reviews 18-21) and references therein) in which the data are described (or fitted) within a broad variety of models. To clear up really essential factors for subsequent interpretation it seems reasonable to choose a starting point as close as possible to fundamental theory -- QCD. Nuclei are nothing but the quark-gluon systems and all methods developed for analysis of the nucleon structure functions (the moment method, sum rules, etc.) are entirely applicable to them. Having this in mind we shall follow basically the approach of Ref. 22.

At large $Q^2$ QCD defines evolution of the parton distributions

\[ \dot{q}_V(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int dy \, dy' P_{qV}(y) q_V(y, Q^2) \delta(y - x) \quad (5a) \]

\[ \dot{q}_c(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int dy \, dy' \left[ P_{qV}(y) q_c(y, Q^2) + P_{q6}(y) G(y, Q^2) \right] \delta(y - x) \quad (5b) \]

\[ \dot{G}(x, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \int dy \, dy' \left[ P_{q6}(y) q_5(z, Q^2) + P_{g6}(y) G(z, Q^2) \right] \delta(y - x) \quad (5c) \]

where $\dot{q}_V(x, Q^2) = dq_V(x, Q^2)/d \ln Q^2$, $\alpha_s(Q^2)$ is the running coupling constant, $P_{qV}, P_{q6}, P_{q4}, P_{g6}$ are the known "splitting" functions which are calculated via perturbation theory and are universal ones, i.e. independent of whether the partons reside in a nucleus or in a free nucleon. Universality of the $P$'s leads to an important relation for power moments of the valence quark distributions

\[ \frac{\mathcal{V}_A(n, Q^2)}{\mathcal{V}_A(n, Q^2)} = \frac{\mathcal{V}_V(n, Q^2)}{\mathcal{V}_V(n, Q^2)} \quad (6) \]

from where

\[ \mathcal{V}_A(n, Q^2) = T_A(n) \mathcal{V}_V(n, Q^2) \quad (7) \]

with

\[ q_V(n, Q^2) \equiv \mathcal{V}(n, Q^2) = \int dx \, x^{n-4} \left( q_V(x, Q^2) - \bar{q}_V(x, Q^2) \right) \quad (8) \]

$N(A)$ refers to the structure functions of a free nucleon or nucleus (divided by $A$), $i$ labels the flavour of quarks, the limits of integration in (8) are $0 \leq x \leq 1 (A)$, and $T_A$ is not yet specified. Returning from moments to functions we have

\[ \mathcal{V}_A(x, Q^2) = \int \frac{dp}{p} \int dy \, T_A(p) \mathcal{V}_V(y, Q^2) \delta(p \cdot y - x) = \int \frac{dp}{p} T_A(p) \mathcal{V}_V\left( \frac{x}{p}, Q^2 \right) \equiv T_A \otimes \mathcal{V}_V \quad (9) \]
Normalization of $\nu_N(x, Q^2)$ and $\nu_A(x, Q^2)$ to the unit baryon number gives
\begin{equation}
\int_0^A T_A(\beta) \, d\beta = 1
\end{equation}
(10)
Positiveness and normalization of $T_A(\beta)$ suggest to identify it with the effective distribution of the nucleon longitudinal momentum fraction in nuclei.

For the singlet distributions including the sea quarks and gluons the corresponding formulas are more complex due to "nondiagonal" transitions of sea quarks to gluons and vice versa. However, one can find such linear combinations $\bar{f}_A^\pm = q_{\bar{f}G} + C_{\bar{f}G} G$, $q_{\bar{f}G} = \sum_i (q_i + \bar{q}_i)$ for which the analog of Eq. (9) is written in the diagonal form
\begin{equation}
\int_0^A f_A^\pm = T_A^\pm \otimes f_N^\pm
\end{equation}
(11)
Extremely important relation is the total energy-momentum sum rule
\begin{equation}
\int_0^A (n = 2; Q^2) = \int d\vec{x} \cdot \vec{x} \left[ q_2(x, Q^2) + G(x, Q^2) \right] = 1
\end{equation}
(12)
which gives
\begin{equation}
\int_0^A T_A^+(\beta) \, d\beta = 1
\end{equation}
(13)
(We neglect terms of the order $\epsilon_N/m$, $\epsilon_N$ being the binding energy per nucleon). In the general case $T_A^{+} \neq T_A^{-}$, $T_A^{\pm} \neq T_A^{\pm}$. By analogy with (9) and (11) one can introduce the relations between "pure" $q_{\bar{f}}$ -sea and gluon distributions in nuclei and a free nucleon:
\begin{equation}
S_A(x, Q^2) = \int_x \frac{d\beta}{\beta} T_A(\beta) S_N \left( \frac{x}{\beta}, Q^2 \right) + S'_A(x, Q^2)
\end{equation}
(14)
\begin{equation}
G_A(x, Q^2) = \int_x \frac{d\beta}{\beta} T_A(\beta) G_N \left( \frac{x}{\beta}, Q^2 \right) + G'_A(x, Q^2)
\end{equation}
(15)
where $S'_A$ and $G'_A$ are additional, "collective" nuclear sea which can be shown to be absent only in the case $T_A^{+} = T_A^{-} = T_A$. But just this case is at variance with data if we require validity of exact sum rules (10) and (13). For nuclei taken as weakly-bound system of (basically) nonrelativistic nucleons $T_A(\beta)$ should look like the distribution sharply peaked at $\beta \approx 1$ with dispersion of an order of $O(p_F^2/m^2)$, $p_F$ is the Fermi-momentum. Now, in expression for the structure function
\begin{equation}
F_2^A(x, Q^2) = \int_x \frac{d\beta}{\beta} T_A(\beta) F_2^N \left( \frac{x}{\beta}, Q^2 \right)
\end{equation}
(16)
we expand $F_2^N(\frac{x}{\beta})$ around $\beta \approx 1$ and obtain for point $x_0$ of inter-
ception of ratio \( R = \frac{F_2^A}{F_2^N} \) with the value \( R = 1 \):

\[
x_0 = 2 \left( 1 + \frac{\delta}{\delta^2} \right) / \left( 1 + \kappa + \frac{\delta}{\delta^2} \right) \tag{17}
\]

where

\[
\delta = 1 - \int_0^A p \, T_A(p) \, dp = 1 - \frac{\langle x_v \rangle_A}{\langle x_v \rangle_N} \tag{18}
\]

\[
\delta^2 = \int_0^A (1 - p)^2 T_A(p) \, dp \tag{19}
\]

and we use, for simplicity, \( F_2^N \sim (1 - x)^\kappa \).

If \( \delta = 0 \) (following from \( T_A^+ = T_A \) and (13)) and \( \kappa = 3 \) we have \( x_0 \approx 0.5 \) in contradiction with \( x_0^{exp} \approx 0.85 \). One should evidently diminish the mean momentum fraction of valence quarks \( \delta > 0 \) which in turn requires an additional sea in nuclei. The needed values \( \delta \approx 0.04 - 0.05 \) are easily implemented within the simplest nuclear model - the "shifted" Fermi distribution

\[
T_A(p) = \frac{3}{4} \left( \frac{m}{P_F} \right)^3 \left\{ \begin{array}{ll}
P_F^2 / m^2 - (1 - p - \delta)^2, & \text{if } |1 - p - \delta| < P_F / m \\
0, & \text{otherwise}
\end{array} \right. \tag{20}
\]

which reproduces well the dependence of \( F_2^A / F_2^N \) on \( x \) in the range \( 0.3 \leq x \leq 1 \). The momentum fractions of the additional nuclear sea should compensate the momentum lost by valence quarks

\[
\Delta \langle x_v \rangle + \Delta \langle x_s \rangle + \Delta \langle x_g \rangle = 0 \tag{21}
\]

\[
\Delta \langle x_v \rangle = \langle x_v \rangle_A - \langle x_v \rangle_N = -\delta \cdot \langle x_v \rangle_N \tag{22a}
\]

\[
\Delta \langle x_s \rangle = \langle x_s' \rangle_A - \delta \cdot \langle x_s \rangle_N \tag{22b}
\]

\[
\Delta \langle x_g \rangle = \langle x_g' \rangle_A - \delta \cdot \langle x_g \rangle_N \tag{22c}
\]

that is

\[
\langle x_s' \rangle_A + \langle x_g' \rangle_A = \delta \tag{23}
\]

To have an impression of relative values of \( \langle x_s \rangle \) and \( \langle x_g \rangle \), we turn to the second moment of structure functions calculated in Ref. 22 according to the EMC data

\[
I_2^A - I_2^N = \int dx \left[ F_2^A(x, Q^2) - F_2^d(x, Q^2) \right] = \tag{24}
\]

\[
= \frac{2}{9} \langle x_s' \rangle_A - \frac{2}{3} \delta \cdot \left[ \langle x_s \rangle_N + \frac{5}{9} \langle x_v \rangle_N \right] \approx \frac{0.65 \pm 0.06}{10^{-2}}
\]
Using the values \( \langle x_A \rangle_N^0 = -0.12 + \frac{5}{4} \cdot 0.35 = 0.56 \) and \( \langle x_N \rangle = 0.045 \), one gets \( \langle x_A \rangle_{\Lambda} = 0.05, \langle x_0 \rangle_{\Lambda} = -0.005 \). If we want the additional qg-pairs and gluons to be "materialized" into pion's sea, it would be most natural to have \( \langle x_A \rangle_{\Lambda} = \langle x_0 \rangle_{\Lambda} = 0.5 \cdot 0.05 \). But in this case the integral (24) will acquire the zero value

\[
I_2^\Lambda - I_2^N \simeq 0 \tag{25}
\]

which appears to be more consistent with the SLAC data \(^{23a,b}\) and new BCDMS data for \(^{14}\)N-nucleus \(^{23c}\).

As was pointed out in a recent work \(^{24}\) the slight (\( \approx 5\% \)) renormalization of the EMC data at small \( x \lesssim 0.2 \) admissible by their experimental uncertainties would result in fulfilment of the asymptotic sum rule (25). In Figure 1 below we sketch a possible \( x \)-dependence of nuclear effects in the deep inelastic scattering.

![Figure 1](image)

**Figure 1**
Possible \( x \)-dependence of nuclear effects

4. **CONCLUSION**

The immediate perspectives for a future research work are more or less evident. A complete separation of sea, valence and gluon distributions in nucleons and nuclei using \( e, \mu, \nu, \bar{\nu} \) data, massive di-lepton production, \( J/\psi \) - photo-(lepto-) production remains to be the first priority task. The probing of the cumulative region by leptons should be accomplished. A detailed, varying \( Q^2 \) exploration of low \( x \) region including transition to the shadowing regime is needed. Further continuation and development of nuclear studies by incident hadron beams including the polarized beam-target facilities, cumulative jets and particle-correlation measurements, the resonance production (both the OZI allowed and forbidden) are indispensable. To conclude, the investigation of the quark-gluon structure of nuclei opens new horizons in both nuclear physics and strong interaction theory, focusing attention on the colour dynamics in multiparticle systems. A very broad spectrum of the proposed theoretical models, each claiming to be "QCD-motivated", speaks to only how much remains to be done for elaboration of the universal, selfconsistent, and more tightly bound to fundamental QCD, theory of the considered processes.
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