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IDENTIFICATION OF MISSING NEUTRALS IN $e^+e^- \rightarrow \gamma+X$
WITH POLARIZED $e^\pm$ BEAMS

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A B S T R A C T

We study the production of missing neutrals in $e^+e^- \rightarrow \gamma+X$ for the four cases $X = \nu\bar{\nu}$, $\tilde{\nu}\tilde{\nu}$, $\tilde{\gamma}\tilde{\gamma}$ and $\tilde{H}^0\tilde{H}^0$ at future e^+e^- colliders. We show how longitudinal polarization can help in identifying the nature of the missing neutrals.

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I. INTRODUCTION

Although supersymmetry is an essential feature of most of the theories extending the standard model, there is presently no experimental manifestation of its presence. Limits on masses of supersymmetric partners to ordinary particles have been progressively pushed upwards by PEP, PETRA and SppS experiments:

$$M_{\tilde{e}_R} = M_{\tilde{e}_L} > 50 \text{ GeV} \text{ or } M_{\tilde{e}_{L,R}} \gg M_{\tilde{e}_{R,L}} > 41 \text{ GeV if } M_{\tilde{g}} = 0 \text{ Ref. [1]}$$

$$M_{\tilde{g}} > 17 \text{ GeV if } M_{\tilde{e}_R} = M_{\tilde{e}_L} = 0 \text{ or } M_{\tilde{e}_{L,R}} \gg M_{\tilde{e}_{R,L}} = 0 \text{ Ref. [1]}$$

$$M_{\tilde{d}, \tilde{q}} \geq 20 \text{ GeV} \text{ Ref. [2]}$$

$$M_{\tilde{W}} \geq 22 \text{ GeV} \text{ Ref. [2]}$$

$$M_{\tilde{q}, \tilde{g}} \gtrsim 0(40-60 \text{ GeV}) \text{ Refs. [3-4]}$$

These limits exclude the simplest models [5] based on spontaneous supersymmetry breaking; they still allow for many other possibilities (for example, supergravity models) in which supersymmetry breaking is associated to the Fermi scale of weak interactions, i.e., the supersymmetric partners would have masses of the order of M_W . Most of the supersymmetry models conserve R-parity which implies decays by cascade down to the lightest particle. This one would most probably be either a photino $\tilde{\gamma}$, a sneutrino $\tilde{\nu}$ or a higgsino $\tilde{H}^0$. There is presently no experimental limitation on the masses of such particles which could even be equal to zero. Their experimental signal would just be missing energy-momentum in some reactions. The simplest example is probably the reaction $e^+e^- \rightarrow \gamma + \text{missing energy-momentum}$ which had already been suggested [6] for the counting of neutrino species ($e^+e^- \rightarrow \gamma \nu \bar{\nu}$).

The question is then how to distinguish between these various types of missing neutrals $\nu \bar{\nu}$, $\tilde{\nu} \tilde{\nu}$, $\tilde{\gamma} \tilde{\gamma}$, $\tilde{H}^0 \tilde{H}^0$. In a given experimental set-up there is also the background due to missing charged or other observable particles escaping detection because of experimental inefficiencies (e.g., $e^+e^- \rightarrow \gamma e^+e^-$, $\gamma \gamma \gamma, \dots$). We will not discuss these effects in this paper. They correspond to well-known processes which can be exactly computed [7] for each apparatus in terms of its acceptance and detection efficiencies. Moreover, longitudinal asymmetries are expected to be very small for these backgrounds.

Cross-sections for $e^+e^- \rightarrow \gamma\tilde{\gamma}\tilde{\gamma}$ were first estimated in Ref. [8] and computed in Refs. [9] and [10]. Used by PEP experiments they allow one to give limits in the $(M_{\tilde{\gamma}}, M_{\tilde{e}})$ plane. However, for $\sqrt{s} \gtrsim 40$ GeV - 60 GeV, depending on the model chosen for selectron mass degeneracy, the $\gamma\nu\bar{\nu}$ background will overwhelm the $\gamma\tilde{\gamma}\tilde{\gamma}$ contribution and forbid the search for high photino masses. The process $e^+e^- \rightarrow \gamma\nu\tilde{\nu}$ has been discussed and computed in Refs. [11] and [12]. Finally, the possibility that $\tilde{H}^0$ is the lightest supersymmetric particle was discussed in Ref. [13], but no calculation of the $e^+e^- \rightarrow \gamma\tilde{H}^0\tilde{H}^0$ process had yet been performed. In this paper we first calculate unpolarized photon energy-momentum distributions and integrated cross-sections with cuts corresponding to dedicated experiments [14] at future machines (SLC, LEP) for the four processes $e^+e^- \rightarrow \gamma\nu\bar{\nu}$, $\gamma\tilde{\nu}\tilde{\nu}$, $\gamma\tilde{\gamma}\tilde{\gamma}$ and $\gamma\tilde{H}^0\tilde{H}^0$. We then consider the effects of longitudinal (||) and transverse (⊥) $e^\pm$ beam polarizations. The use of longitudinally-polarized beams as a way to increase the chances of discovering supersymmetry for $e^+e^- \rightarrow \gamma\tilde{\gamma}\tilde{\gamma}$ was first discussed in Ref. [10]. We were motivated by the results obtained [15] in the case of $e^+e^- \rightarrow \tilde{f}\tilde{f}$ and $e^+e^- \rightarrow \tilde{\chi}\tilde{\chi}$ ($\tilde{\chi}$ = charged or neutral gaugino) where spectacular effects on asymmetries $A_{||}$ and $A_{\perp}$ were found. From helicity conservation of the $e\tilde{e}\gamma$ couplings, one can expect [16] that $e^+e^- \rightarrow \gamma\tilde{f}\tilde{f}$ and $e^+e^- \rightarrow \gamma\tilde{\chi}\tilde{\chi}$ will present similar properties which will be helpful for the identification of the processes. In fact, this will be partly the case up to effects of diagrams where the photon is not emitted by $e^\pm$ lines and kinematical effects of the integration over invisible particles.

In Section II, we write the amplitudes corresponding to the various diagrams and define cross-sections and asymmetries which will be computed numerically except in the simplest case of $\gamma\tilde{H}^0\tilde{H}^0$, completely treated in an analytic form in Appendix .

In Section III, we give the results for cross-sections and asymmetries of the four processes $e^+e^- \rightarrow \gamma\nu\bar{\nu}$, $\gamma\tilde{\nu}\tilde{\nu}$, $\gamma\tilde{\gamma}\tilde{\gamma}$ and $\gamma\tilde{H}^0\tilde{H}^0$ up to $\sqrt{s} \approx 200$ GeV.

We summarize our conclusions on the observability and the disentangling of these processes in Section IV.

II. AMPLITUDES, CROSS-SECTIONS AND ASYMMETRIES OF $e^+e^- \rightarrow \gamma + X$

With the standard couplings and the supersymmetric ones (defined in Ref. [15]), one writes the invariant amplitudes of the four processes under consideration as follows. General kinematical notations correspond to the labels $e^+(\ell') + e^-(\ell) \rightarrow \gamma(\epsilon, q) + X_1(p_1) + X_2(p_2)$ with $Q^2 = (p_1 + p_2)^2$, $s = (\ell + \ell')^2$.

1. $e^+e^- \rightarrow \gamma \nu_e \bar{\nu}_e$

The amplitude is the sum of the five terms depicted in Fig. 1(abcde).

$$R_{fi}^{(a)} = - \frac{e^3 \bar{v}(\ell') \not{\epsilon} (\not{q} - \not{\ell}') \gamma^\mu (1 - \gamma^5) v(p_1) \bar{u}(p_2) \gamma_\mu (1 - \gamma^5) u(\ell)}{8 \sin^2 \theta_w ((\ell' - q)^2 - M_e^2) ((\ell - p_2)^2 - M_w^2)}$$

$$R_{fi}^{(b)} = - \frac{e^3 \bar{v}(\ell') \gamma^\mu (1 - \gamma^5) v(p_1) \bar{u}(p_2) \gamma_\mu (1 - \gamma^5) (\not{\ell} - \not{q}) \not{\epsilon} u(\ell)}{8 \sin^2 \theta_w ((\ell - q)^2 - M_e^2) ((\ell' - p_1)^2 - M_w^2)}$$

$$R_{fi}^{(c)} = \frac{e^3 \bar{v}(\ell') \gamma^\mu (1 - \gamma^5) v(p_1) \bar{u}(p_2) \gamma^\nu (1 - \gamma^5) u(\ell) [\epsilon_\mu (p_1 - \ell' - q)_\nu + \epsilon_\nu (\ell + q - p_2)_\mu - g_{\mu\nu} \epsilon \cdot (p_1 + \ell - \ell' - p_2)]}{8 \sin^2 \theta_w ((\ell - p_2)^2 - M_w^2) ((\ell' - p_1)^2 - M_w^2)}$$

$$R_{fi}^{(d)} = \frac{e^3 \bar{u}(p_2) \gamma^\mu (1 - \gamma^5) v(p_1) \bar{v}(\ell') \not{\epsilon} (\not{q} - \not{\ell}') \gamma_\mu (1 - \gamma^5) u(\ell)}{2 \sin 2\theta_w ((\ell' - q)^2 - M_e^2) D_Z(Q)}$$

$$R_{fi}^{(e)} = \frac{e^3 \bar{u}(p_2) \gamma^\mu (1 - \gamma^5) v(p_1) v(\ell') \gamma_\mu (1 - \gamma^5) (\not{\ell} - \not{q}) \not{\epsilon} u(\ell)}{2 \sin 2\theta_w ((\ell - q)^2 - M_e^2) D_Z(Q)}$$

The Zee couplings in ld and le are given by

$$a = \frac{-1 + 4 \sin^2 \theta_w}{2 \sin 2\theta_w} \quad b = -\frac{1}{2 \sin 2\theta_w}$$

$$e^+e^- \rightarrow \gamma \nu' \bar{\nu}' \quad (\nu' = \nu_\mu, \nu_\tau, \dots)$$

The amplitude is only the sum of $R_{fi}^{(d)}$ and $R_{fi}^{(e)}$ given above. There is no free parameter in these amplitudes except the number N_ν of neutrino species.

$$2. \quad e^+e^- \rightarrow \gamma \tilde{\nu}_e \tilde{\bar{\nu}}_e$$

This process is the supersymmetric analogue of the $\gamma \nu_e \bar{\nu}_e$ one with the five diagrams of Fig. 2(abcde):

$$R_{fi}^{(a)} = \frac{e^3 \bar{\nu}(l') \not{\epsilon}(q - l') (\not{p}_2 - \not{l}) (1 - \gamma^5) u(l)}{2 \sin^2 \theta_w ((l - q)^2 - M_e^2) ((l - p_2)^2 - M_{\tilde{W}}^2)}$$

$$R_{fi}^{(b)} = \frac{e^3 \bar{\nu}(l') (1 + \gamma^5) (\not{p}_1 - \not{l}') (\not{l} - \not{q}) \not{\epsilon} u(l)}{2 \sin^2 \theta_w ((l - q)^2 - M_e^2) ((p_1 - l')^2 - M_{\tilde{W}}^2)}$$

$$R_{fi}^{(c)} = \frac{e^3 \bar{\nu}(l') (1 + \gamma^5) ((\not{p}_1 - \not{l}') \not{\epsilon} (\not{l} - \not{p}_2) + M_{\tilde{W}}^2 \not{\epsilon}) u(l)}{2 \sin^2 \theta_w ((p_1 - l')^2 - M_{\tilde{W}}^2) ((l - p_2)^2 - M_{\tilde{W}}^2)}$$

$$R_{fi}^{(d)} = -\frac{e^3 \bar{\nu}(l') \not{\epsilon}(q - l') (\not{p}_2 - \not{p}_1) (a - b \gamma^5) u(l)}{\sin 2\theta_w ((q - l')^2 - M_e^2) D_Z(q)}$$

$$R_{fi}^{(e)} = -\frac{e^3 \bar{\nu}(l') (\not{p}_2 - \not{p}_1) (a - b \gamma^5) (\not{l} - \not{q}) \not{\epsilon} u(l)}{\sin 2\theta_w ((l - q)^2 - M_e^2) D_Z(q)}$$

$$e^+e^- \rightarrow \gamma \tilde{\nu}' \tilde{\nu}' \quad (\tilde{\nu}' = \tilde{\nu}_\mu, \tilde{\nu}_\tau, \dots)$$

The amplitude is also given by the above terms $R_{fi}^{(d)}$ and $R_{fi}^{(e)}$. In these processes the only free parameters are wino and sneutrino masses $M_{\tilde{W}}$ and $M_{\tilde{\nu}}$. We consider the extreme wino-higgsino mixing cases with the $e\tilde{W}\tilde{\nu}$ coupling constant $e/(\sqrt{2}\sin\theta_W)$ (see Refs. [4], [15]).

$$3. \quad e^+e^- \rightarrow \gamma\gamma\gamma$$

The complete amplitude is the sum of 12 terms, the three depicted in Fig. 3(abc), corresponding to left selectron exchange ($\tilde{e}_L$), the three with the right selectron exchanges ($\tilde{e}_R$) and the six terms obtained by $\gamma\gamma$ interchange.

$$R_{fi} = R_{fiL}^{(a+b+c)} + R_{fiR}^{(a+b+c)} - (p_1 \leftrightarrow p_2)$$

$$R_{fiL}^{(a)} = \frac{e^3 \bar{v}(\ell') \varepsilon(\ell - \ell') (1 + \gamma^5) v(p_1) \bar{u}(p_2) (1 - \gamma^5) u(\ell)}{2((\ell' - q)^2 - M_{\tilde{e}}^2)((\ell - p_2)^2 - M_{\tilde{e}_L}^2)}$$

$$R_{fiL}^{(b)} = \frac{e^3 \bar{v}(\ell') (1 + \gamma^5) v(p_1) \bar{u}(p_2) (1 - \gamma^5) (\ell - q) \varepsilon u(\ell)}{2((\ell - q)^2 - M_{\tilde{e}}^2)((p_1 - \ell')^2 - M_{\tilde{e}_L}^2)}$$

$$R_{fi}^{(c)} = \frac{e^3 \bar{v}(\ell') (1 + \gamma^5) v(p_1) \bar{u}(p_2) (1 - \gamma^5) u(\ell) \varepsilon \cdot (\ell + p_1 - \ell' - p_2)}{2((p_1 - \ell')^2 - M_{\tilde{e}}^2)((\ell - p_2)^2 - M_{\tilde{e}_L}^2)}$$

The free parameters here are $M_{\tilde{\gamma}}$, $M_{\tilde{e}_L}$ and $M_{\tilde{e}_R}$.

4. $e^+e^- \rightarrow \gamma \tilde{H}^0 \tilde{H}^0$

As the $\tilde{H}e\tilde{e}$ couplings are proportional to the electron mass and therefore negligible, we are only left with the Z formation diagrams of Fig. 4(ab)

$$R_{fi}^{(a)} = - \frac{e^3 \bar{v}(\ell') \not{\epsilon} (\not{q} - \not{\ell}') \gamma^\mu (a - b\gamma^5) u(\ell) \bar{u}(p_1) \gamma_\mu (a_0 - b_0\gamma^5) v(p_2)}{((\ell' - q)^2 - M_e^2) D_Z(Q)}$$

$$R_{fi}^{(b)} = - \frac{e^3 \bar{v}(\ell') \gamma^\mu (a - b\gamma^5) (\not{\ell} - \not{q}) \not{\epsilon} u(\ell) \bar{u}(p_1) \gamma_\mu (a_0 - b_0\gamma^5) v(p_2)}{((\ell - q)^2 - M_e^2) D_Z(Q)}$$

The situation with $Z\tilde{H}^0\tilde{H}^0$ couplings has been reviewed in Refs. [13] and [17].

In the two-Higgs model, the couplings are non-vanishing if the two vacuum expectation values are unequal. In the minimal model one gets $a_0 = 0$, $b_0 = -(\cos 2\beta)/\sin 2\theta_W$ where β is a free parameter which will be taken as $\beta = 0$ in the numerical applications. The Higgsino mass $M_{\tilde{H}}$ is the other free parameter whose minimal value can even be zero.

In numerical applications, we take $M_{W^\pm} = 81$ GeV, $M_Z = 92$ GeV. The Z propagator $D_Z(Q^2) = Q^2 - M_Z^2 + iM_Z\Gamma_Z$ involves the Z width, which can be modified by supersymmetric channels if the masses of the partners are light enough. In practice, we will just take into account the contributions of the lightest supersymmetric pair, i.e.,

$$\Gamma_Z = \Gamma_Z^{st} + \delta\Gamma_Z$$

with $\Gamma_Z^{st} = 2.92$ GeV and $\delta\Gamma_Z$ given by either $\sum_{\tilde{\nu}} \Gamma(Z \rightarrow \tilde{\nu}\tilde{\nu})$ or $\Gamma(Z \rightarrow \tilde{H}^0\tilde{H}^0)$ (this additional term is in fact important in the $\tilde{\nu}\tilde{\nu}$ case if $m_{\tilde{\nu}}$ is small enough):

$$\Gamma(Z \rightarrow \tilde{\nu} \tilde{\bar{\nu}}) = \frac{2\alpha}{3 \sin^2 2\theta_w} \left(\frac{p_{\tilde{\nu}}^2}{M_Z^2} \right)$$

$$\Gamma(Z \rightarrow \tilde{H}^0 \tilde{H}^0) = \frac{\alpha}{2M_Z} \left(1 - \frac{4M_{\tilde{H}^0}^2}{M_Z^2} \right)^{1/2} \left[(a_0^2 + b_0^2) \frac{M_Z^2}{3} \left(1 + 2 \frac{M_{\tilde{H}^0}^2}{M_Z^2} \right) - 2b_0 M_{\tilde{H}^0}^2 \right]$$

For each final state we compute $\Sigma |R_{fi}|^2$ and cross-sections partly analytically, partly numerically, with the help of Reduce [18] and Riwiad [19]. The photon spectrum is given by:

$$\frac{d\sigma}{dx d\cos\theta d\phi} = \frac{1}{(2\pi)^5} \frac{1}{64} N \int_{z_m}^{z_M} dz \int_{y_1}^{y_2} \frac{d\cos\bar{\theta}}{J} P_{fi}$$

$$\text{with } J = [\sin^2\theta - x^2 - \cos^2\bar{\theta} + 2x \cos\theta \cos\bar{\theta}]^{1/2}$$

$$X = \frac{2(1-x-z) + xz}{xz \left(1 - \frac{4M^2}{s z^2} \right)^{1/2}}$$

$$z_m = 1 - \frac{x}{2} \left[1 + \sqrt{1 - \frac{4M^2}{s(1-x)}} \right]$$

$$y_{1,2} = X \cos\theta \mp \sin\theta \sqrt{1-x^2}$$

P_{fi} is the sum of the values of $\Sigma |R_{fi}|^2$ integrated over $\bar{\phi}$:

$$P_{fi} = \int d\bar{\phi} \delta[\bar{\phi} - \phi \pm \text{Arccos} \left(\frac{X - \cos\theta \cos\bar{\theta}}{\sin\theta \sin\bar{\theta}} \right)] \Sigma |R_{fi}|^2$$

x, θ, ϕ are the photon reduced energy ($x = 2q^0/\sqrt{s}$) and angles in the e^+e^- c.m. frame. $z, \bar{\theta}, \bar{\phi}$ are the corresponding variables for one of the invisible particles (e.g., p_1). $N = 1$ for $\nu\bar{\nu}, \tilde{\nu}\tilde{\bar{\nu}}$ but $\frac{1}{2}$ for $\tilde{\gamma}\tilde{\gamma}, \tilde{H}^0\tilde{H}^0$ (Majorana particles).

For polarized e^+e^- collisions, these photon distributions take the general form [15], [20]:

$$\frac{d\sigma}{dx d\omega d\theta d\phi} = (1 - P_{||}^- P_{||}^+) X_1 + (P_{||}^+ - P_{||}^-) X_2 + \xi_1^+ \xi_1^- (X_3 \cos 2\phi + X_4 \sin 2\phi)$$

This follows explicitly from the computation of $\sum |R_{fi}|^2$ in terms of the $e^\pm$ density matrices $\begin{bmatrix} 2 & 0 \end{bmatrix}$:

$$\rho_{e^-} = (1 + \gamma^5 P_{||}^-) \not{\epsilon} - \gamma^5 \not{\epsilon} \xi_1^-$$

$$\rho_{e^+} = (1 - \gamma^5 P_{||}^+) \not{\epsilon}' - \gamma^5 \not{\epsilon}' \xi_1^+$$

λ , ξ_1^- , $P_{||}^-$ and λ' , ξ_1^+ , $P_{||}^+$ are e^- and e^+ momenta, transverse polarization vector and longitudinal degree respectively.

A common general feature of all photon spectra is the $1/(x \sin^2 \theta)$ peaking factor coming from photon emission by $e^\pm$ lines. The longitudinal asymmetry (suitable for SLC) is defined by:

$$A_{||} = - \frac{X_2}{X_1}$$

And the transverse polarization asymmetry (suitable for LEP) is defined by :

$$A_1 = \frac{2 \int \cos 2\phi \frac{d\sigma}{d\phi} d\phi}{\int \frac{d\sigma}{d\phi} d\phi}$$

In practice, X_4 is always much smaller than X_3 (it is controlled by Γ_Z instead of M_Z) so the measurement of A_1 is essentially given by the $\cos 2\phi$ dependence of $d\sigma/d\phi$.

Detailed analytic results (output of Reduce) will not be reproduced here as they are very lengthy (a copy of the program could be provided to

anyone willing to do their own numerical applications, for example with given x and θ cuts). We only give in the Appendix the results of the simplest case $e^+e^- \rightarrow \gamma \tilde{H}^0 \tilde{H}^0$.

III. RESULTS

In this section we will examine some specific applications of the general reaction $e^+e^- \rightarrow \gamma + \text{missing energy momentum}$. We will compute the total cross-sections and the asymmetries in the domains $5^\circ < \theta < 175^\circ$ and $P_\perp > 1 \text{ GeV}$, as this should correspond to realistic experimental possibilities [14].

1. $e^+e^- \rightarrow \gamma \nu \bar{\nu}$

This process is the basic background and the standard reference for any other process leading to $\gamma + \text{missing energy momentum}$. Due to Z formation, the total cross-section is important as soon as $\sqrt{s} \gtrsim M_Z$. As we can see from Fig. 5a, it culminates for $\sqrt{s} = 95 \text{ GeV}$, reaching $\sigma \approx 122 \text{ pb}$ (150 pb) for $N_\nu = 3$ (4). For higher energies, it drops down because the Z peak occurs in the hard photon domain which is obviously depressed with respect to the soft photon one. At $\sqrt{s} = 200 \text{ GeV}$, it is still of the order of 5 pb (6 pb for four generations). Compared to $\sigma(e^+e^- \rightarrow W^+W^-) \approx 20 \text{ pb}$, this is still a very interesting magnitude.

The longitudinal asymmetry given in Fig. 5b is always negative and increases in magnitude with $\sqrt{s}$ (i.e., -45% at $\sqrt{s} = 200 \text{ GeV}$). This reflects the contribution of -19% ($-2ab/a^2+b^2$) for $e^+e^- \rightarrow \nu'\bar{\nu}'$ and the one increasing in magnitude up to -100 % for $e^+e^- \rightarrow \nu_e \bar{\nu}_e$ because of the left-handed W exchange term.

The transverse asymmetry appears to be very small when one integrates over the whole (x, θ) domain. This is because for the four cases X_3/X_1 behaves like $x^2 \sin^2 \theta$ (the soft photon contribution dominates the unpolarized term X_1), as can be seen explicitly in the Z formation terms of the Appendix.

In order to increase $A_\perp$, one should either look at $d\sigma/(dx d\cos\theta d\phi)$ for high x ,

Θ values or at least integrate in a domain limited by larger minimal values $x_{\min}$ and Θ_{cut} . We illustrate this point in Fig. 5b, choosing the cuts $x_{\min} = 0.3$ and $|\cos\Theta| < 0.4$ for $A_{\perp}$. We get $A_{\perp}$ of the order of -10% for $\sqrt{s} \approx 130$ GeV (with $\sigma \sim 1.8$ pb) and -30% at $\sqrt{s} = 200$ GeV (with $\sigma \sim 0.4$ pb).

Our results for the unpolarized cross-section agree with previous amplitudes [6] up to very small effects due to the diagram 1c. We checked that the influence of this diagram is very small in the energy range we consider (i.e., not more than 5% up to $\sqrt{s}=200$ GeV). This means there is no hope of looking for anomalous magnetic or quadrupole $W^{\pm}$ moments in this process [21]. It seems that a mistake which affects the expression for the diagram 1a in Ref. [10] is probably the reason for the numerical departures from our results away from the Z peak.

2. $e^+e^- \rightarrow \gamma \tilde{\nu} \tilde{\nu}$

We first compute the cross-section of this supersymmetric process with the same cuts on the photon momentum. Taking the ASP cuts [22], we recover the results for $\sigma(e^+e^- \rightarrow \gamma \tilde{\nu} \tilde{\nu})$ obtained in Ref. [12]. As can be seen from Fig. 6a for massless neutrinos, the cross-section behaves similarly to the $e^+e^- \rightarrow \gamma \nu \bar{\nu}$ one, up to a factor $\frac{1}{2}$ for $\sqrt{s} \gtrsim M_Z$. This factor comes from the Z diagrams (supersymmetry on the $Z\tilde{\nu}\tilde{\nu}$ couplings and number of spin states). Away from the Z peak, the precise ratio depends upon the $\tilde{W}$ mass. If it is smaller than the W mass, $\sigma(e^+e^- \rightarrow \gamma \tilde{\nu} \tilde{\nu})$ is correspondingly larger. For example, with $M_{\tilde{W}} = 40$ GeV, there is a factor 1.5 at $\sqrt{s} = 70$ GeV. Massive sneutrinos will reduce the cross-section at low energy (due to phase space suppression and the $\tilde{W}$ propagator effect), as shown in Fig. 7a. If the general trends are the same, the expected rates depend on the acceptance. The results for different sets of cuts are given in Ref. [23].

The longitudinal asymmetry appears to be negative (like the $\gamma \nu \bar{\nu}$ one) but with a different dependence because of $M_{\tilde{W}}$ and $M_{\tilde{\nu}}$ effects. As shown in Fig. 6b for massless $\tilde{\nu}$, $A_{\parallel}$ is very large below the Z peak if $M_{\tilde{W}} = 40$ GeV, whereas it is small if $M_{\tilde{W}} = 100$ GeV. For massive sneutrinos (see Fig. 7b) $A_{\parallel}$ is larger. Its magnitude decreases with $M_{\tilde{W}}$.

The transverse asymmetry is again only appreciable at large x and Θ . Z diagrams alone would give $A_{\perp}$ typical of $e^+e^- \rightarrow \gamma Z$, independently of the nature of the particles into which Z decays (when one integrates over them). This explains why there is no strong difference with the $\gamma\nu\bar{\nu}$ case.

3. $e^+e^- \rightarrow \gamma\tilde{\gamma}\tilde{\gamma}$

This process differs from the other ones essentially by the absence of Z diagrams. Consequently, the cross-section is much lower even for massless photinos and varies only smoothly with energy (see Fig. 8a). We also recover previous results [9],[10],[12] when taking similar photon cuts. The precise magnitude depends upon $M_{\tilde{\gamma}}$ but also upon $M_{\tilde{e}_L}$ and $M_{\tilde{e}_R}$. We plot three cases $M_{\tilde{e}_L} \gg M_{\tilde{e}_R}$ (or $M_{\tilde{e}_L} \ll M_{\tilde{e}_R}$), $M_{\tilde{e}_L} = M_{\tilde{e}_R}$ with $M_{\tilde{\gamma}} = 0$ and an intermediate case $M_{\tilde{e}_L} = 54$ GeV, $M_{\tilde{e}_R} = 36$ GeV with $m_{\tilde{\gamma}} = 10$ GeV, as suggested by no-scale models [24].

As shown in Fig. 8b, drastic differences appear on $A_{\parallel}$ (similarly to the non-radiative case $e^+e^- \rightarrow \tilde{\gamma}\tilde{\gamma}$), $A_{\parallel} = 0$ in the degenerate case, $A_{\parallel} = \pm 100\%$ if $\tilde{e}_R$ or $\tilde{e}_L$ exchange dominates. The intermediate case gives a typical illustration of how $A_{\parallel}$ can evolve from one extreme case to the other. Since $M_{\tilde{e}_R} < M_{\tilde{e}_L}$, we get a positive asymmetry decreasing with energy.

Since the cross-section is very small ($\sigma \sim 0.15-0.4$ pb), there is no chance of detection of the transverse asymmetry with present luminosity machines.

4. $e^+e^- \rightarrow \gamma\tilde{H}^0\tilde{H}^0$

This is the simplest of the four processes we study as it only proceeds through Z formation. The cross-section given in Fig. 9 is important for $\sqrt{s} \gtrsim M_Z$, reaching 30 pb around $\sqrt{s} = 100$ GeV for massless Higgsinos. It decreases strongly with $M_{\tilde{H}^0}$, reaching 4 pb for $M_{\tilde{H}^0} = 40$ GeV. It drops quickly with $\sqrt{s}$ down to 1 pb around 200 GeV. Let us recall that the cross-section values depend strongly on $\cos^2 2\beta$ which we have chosen equal to one.

A typical feature of all pure Z formation processes is the value of the longitudinal asymmetry:

$$A_{||} = \frac{-2ab}{(a^2+b^2)} \simeq -19\%$$

at any s. This is a clear signature.

The transverse asymmetry is now also a pure $e^+e^- \rightarrow \gamma Z$ one and is only appreciable in the strong cut domain ($x > 0.3$ and $|\cos\theta| < 0.4$).

IV. CONCLUSIONS

We have studied and compared four different sources of single γ production in polarized e^+e^- collisions : $\gamma\nu\bar{\nu}$, $\gamma\tilde{\nu}\tilde{\nu}$, $\gamma\tilde{\gamma}\tilde{\gamma}$ and $\gamma\tilde{H}^*\tilde{H}^*$. The standard process $\gamma\nu\bar{\nu}$ (first proposed for ν counting) has an important cross-section which can easily hide the supersymmetric particle production, even those like $\tilde{\nu}\tilde{\nu}$, $\tilde{H}^*\tilde{H}^*$ which have also a Z peak. Because of the relatively low level of the cross-sections and the expected luminosities at SLC or LEP, it will not be experimentally possible to make precise x and θ photon distributions which could help in identifying the actual processes.

We have shown that longitudinal polarization can largely help for this identification. As summarized in Fig. 10a, $A_{||}$ is negative, strongly s-dependent for $\gamma\nu\bar{\nu}$ and $\gamma\tilde{\nu}\tilde{\nu}$ (depending upon $M_{\tilde{W}}$ these dependences are different in each case). It is constant and equal to -19% for $\gamma\tilde{H}^*\tilde{H}^*$. It is zero or $\pm 100\%$ in $\gamma\tilde{\gamma}\tilde{\gamma}$ respectively if $M_{\tilde{e}_L} = M_{\tilde{e}_R}$, $M_{\tilde{e}_L} \gg M_{\tilde{e}_R}$, $M_{\tilde{e}_R} \gg M_{\tilde{e}_L}$. It is positive and decreasing with $\sqrt{s}$ for the set of masses suggested by no-scale models.

In the fully-integrated cross-section where the soft photon domain dominates the transverse asymmetry $A_{\perp}$ is very small. Larger asymmetries appear if one can restrict oneself to large x and θ . $A_{\perp}$ is negative and increasing in magnitude with s. Unfortunately, as shown in Fig. 10b, the predictions for $\gamma\nu\bar{\nu}$, $\gamma\tilde{\nu}\tilde{\nu}$ and $\gamma\tilde{H}^*\tilde{H}^*$ are very close since $A_{\perp}$ is essentially controlled by the Z formation.

We noticed other interesting features of these processes: the dependence of $e^+e^- \rightarrow \gamma \tilde{\nu} \tilde{\nu}$ upon the $\tilde{W}$ mass and the existence itself of $e^+e^- \rightarrow \gamma \tilde{H}^0 \tilde{H}^0$ depending upon the two-Higgs structure. Detailed experimental studies of these processes will, however, require the highest luminosities.

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A P P E N D I X

Cross-sections and asymmetries for $e^+e^- \rightarrow \gamma \tilde{H}^0 \tilde{H}^0$

From the amplitudes $R_{fi}^{(a)}$ and $R_{fi}^{(b)}$ given in Section II.4, and the phase space integration, one gets the following photon spectrum:

$$\begin{aligned} \frac{d\sigma}{dx d\omega d\theta d\phi} = & \frac{2\alpha^3}{3\pi} \sqrt{1 - 4 \frac{M_{\tilde{H}^0}^2}{S(1-x)}} \frac{1}{|D_Z(Q)|^2} [(a_0^2 + b_0^2)(S(1-x) - M_{\tilde{H}^0}^2) \\ & + 3 M_{\tilde{H}^0}^2 (a_0^2 - b_0^2)] \left\{ [(a^2 + b^2)(1 - P_{||}^- P_{||}^+) + 2ab(P_{||}^+ - P_{||}^-)] \right. \\ & \left. \left((1 - \frac{x}{2})^2 + \omega^2 \theta \frac{x^2}{4} \right) \frac{1}{x \sin^2 \theta} + \xi_{\perp} \xi'_{\perp} \cos 2\phi \frac{x}{4} (a^2 - b^2) \right\} \end{aligned} \quad (A.1)$$

with $Q^2 = S(1-x)$.

The longitudinal asymmetry defined in Section II is just

$$A_{||} = - \frac{2ab}{(a^2 + b^2)} = -19\% \quad (A.2)$$

and the transverse one is x, θ dependent as :

$$A_{\perp} = \frac{(a^2 - b^2)}{(a^2 + b^2)} \frac{x^2 \sin^2 \theta}{(2-x)^2 + x^2 \omega^2 \theta} \quad (A.3)$$

The factor $(a^2 - b^2)/(a^2 + b^2)$ is typical of $e^+e^- \rightarrow \gamma Z$ (see Ref. [20]). Notice the strong $x^2 \sin^2 \theta$ reduction factor for soft photons.

Since for the two-body case [15]

$$\sigma_2(s) = \frac{2\pi\alpha^2}{3|D_2(s)|^2} \sqrt{1 - \frac{4M_{H^0}^2}{s(1-x)}} \left[(a^2 + b^2)(1 - P_{11}^- P_{11}^+) + 2ab(P_{11}^+ - P_{11}^-) \right]_{(A.4)}$$

$$\cdot \left[(a_0^2 + b_0^2)(s - M_{H^0}^2) + 3M_{H^0}^2(a_0^2 - b_0^2) \right]$$

the radiative corrections to that cross-section are given by :

$$\frac{d\sigma}{dx d\omega \theta} = \frac{2\alpha}{\pi} \frac{1}{x \sin^2 \theta} \sigma_2(s(1-x)) \quad (A.5)$$

Expression (A.5) is a check of (A.1) after integration over ϕ .

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FIGURE CAPTIONS

- Figure 1 Diagrams contributing to $e^+e^- \rightarrow \gamma\nu\bar{\nu}$.
- Figure 2 Diagrams contributing to $e^+e^- \rightarrow \gamma\tilde{\nu}\tilde{\nu}$.
- Figure 3 Diagrams contributing to $e^+e^- \rightarrow \gamma\tilde{\gamma}\tilde{\gamma}$ via selectron left exchange.
- Figure 4 Diagrams contributing to $e^+e^- \rightarrow \tilde{H}^*\tilde{H}^*$.
- Figure 5a Cross-sections for $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ as a function of $\sqrt{s}$.
Full curve : 3 generations. Dashed curve : 4 generations.
- Figure 5b Spin asymmetries for $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ as a function of $\sqrt{s}$.
Full curve : $A_{\parallel}$ for 3 generations. Dashed curve : $A_{\parallel}$ for 4 generations. Dot-dashed curve : $A_{\perp}$ for 3 generations with the cuts $x > 0.3$ and $|\cos\theta| < 0.4$.
- Figure 6a Cross-sections for $e^+e^- \rightarrow \gamma\tilde{\nu}\tilde{\nu}$ as a function of $\sqrt{s}$.
Full curve : $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ for comparison. Dashed curve : $e^+e^- \rightarrow \gamma\tilde{\nu}\tilde{\nu}$ for $M_{\tilde{\nu}} = 0$ and $M_{\tilde{W}} = 40$ GeV. Dot-dashed curve : $e^+e^- \rightarrow \gamma\tilde{\nu}\tilde{\nu}$ for $M_{\tilde{\nu}} = 0$ and $M_{\tilde{W}} = 100$ GeV.
- Figure 6b The longitudinal asymmetry $A_{\parallel}$ for $e^+e^- \rightarrow \gamma\tilde{\nu}\tilde{\nu}$ as a function of $\sqrt{s}$. Dashed curve : $M_{\tilde{\nu}} = 0$ and $M_{\tilde{W}} = 40$ GeV. Dot-dashed curve : $M_{\tilde{\nu}} = 0$ and $M_{\tilde{W}} = 100$ GeV.
- Figure 7a The same as Fig. 6a with $M_{\tilde{\nu}} = 40$ GeV.
- Figure 7b The same as Fig. 6b with $M_{\tilde{\nu}} = 40$ GeV.
- Figure 8a Cross-sections for $e^+e^- \rightarrow \gamma\tilde{\gamma}\tilde{\gamma}$ as a function of $\sqrt{s}$. Full curve : $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ for comparison. Dashed curve : degenerate case $M_{\tilde{e}_L} = M_{\tilde{e}_R} = 50$ GeV, $M_{\tilde{\gamma}} = 0$. Dot-dashed curve : $M_{\tilde{e}_L} = 50$ GeV, $M_{\tilde{e}_R} = \infty$, $M_{\tilde{\gamma}} = 0$. Cross-dashed curve : $M_{\tilde{e}_R} = 36$ GeV, $M_{\tilde{e}_L} = 54$ GeV, $M_{\tilde{\gamma}} = 10$ GeV.
- Figure 8b Same as Fig. 8a for the longitudinal asymmetry $A_{\parallel}$.
- Figure 9 Cross-sections for $e^+e^- \rightarrow \gamma\tilde{H}^*\tilde{H}^*$ as a function of $\sqrt{s}$.
Full curve : $e^+e^- \rightarrow \gamma\nu\bar{\nu}$ for comparison. Dashed curve : $e^+e^- \rightarrow \gamma\tilde{H}^*\tilde{H}^*$ with $M_{\tilde{H}^*} = 0$. Dot-dashed curve : $e^+e^- \rightarrow \gamma\tilde{H}^*\tilde{H}^*$ with $M_{\tilde{H}^*} = 40$ GeV.

Figure 10a

The longitudinal asymmetry $A_{\parallel}$ as a function of $\sqrt{s}$ for the four reactions.

Figure 10b

The transverse asymmetry $A_{\perp}$ as a function of $\sqrt{s}$ for $e^+e^- \rightarrow \gamma\nu\bar{\nu}$, $\gamma\tilde{\nu}\tilde{\nu}$, $\gamma\tilde{H}^0\tilde{H}^0$ with the cuts $x > 0.3$ and $|\cos\theta| < 0.4$.

FIG. 5a

FIG. 5b

FIG. 6a

FIG. 6b

FIG. 7a

FIG. 7b

FIG. 8a

FIG. 8b

FIG. 9

FIG. 10 a

FIG. 10b