CERN-TH.4443/86

LIGHT-FRONT DYNAMICS OF TOPOLOGICALLY MASSIVE SUPERGRAVITY

C. Aragone
CERN - Geneva

ABSTRACT

The unconstrained light-front action of higher order (third order bosonic and second order fermionic) topologically massive supergravity is given. The generator of the light-front time evolution is determined as a functional of the two unique physical variables the system has: one (real) for the bosonic massive spin two excitation and other (Grassmannian, real) for massive spin-3/2 degree of freedom. The analysis is performed in the natural torsion sensitive supersymmetric generalization of the recently introduced dreibein light-front gauge. It shows in a particular transparent fashion how the dynamics of the fermionic sector, whose first layer consist of the non (locally) conformal invariant Rarita-Schwinger action is driven by the (local) conformal invariance of the supersymmetric Chern-Simons layer of the full action.

*) On leave of absence from Departamento de Física, Universidad S. Bolívar, Apartado 80659, Caracas 1080 A, Venezuela.

1. - INTRODUCTION

The quantum consistent coupling of fermions to bosons in the context of realistic low energy 10D point-like field theories derived from superstrings requires the presence of a local Lorentz Chern-Simons (LCS) type of coupling between the antisymmetric second-rank tensor and the vielbein gravity field¹⁾. This local LCS coupling generates a sixth-order modification to the basic ingredient of the action, the standard Einstein's second-order gravity action. One can interpret the presence of this topological contribution as the needed slight quantum increase of the classical D-dimensions of the bosonic spacetime one starts with, required by the presence of fermions which are intrinsically quantum entities.

In the 3D case, the LCS term²⁾ gives rise to a conformally invariant contribution to the full action which now has room enough to permit the propagation of a massive spin two excitation. It is known that Einstein's 3D action leads to a locally trivial dynamics.

Since the 3D LCS term originates an action (instead of a local coupling as it happens in $D = 10$ and, a fortiori, in $D = 4$), topological massive gravity is a third-order theory (instead of being a sixth-order one), and it is very likely that the qualitative modifications it will bring to the dynamics of the spin-two field will be milder than those which occur in $D = 4$ and in $D = 10$ when one has a local LCS coupling.

Presently there are three locally supersymmetric theories containing LCS terms: two in $D = 4$ ^{3), 4)} and one in $D = 3$ ⁵⁾.

In the following we shall study the dynamical properties of $D = 3$ topologically massive supergravity⁵⁾, which is the simplest case where an exact locally supersymmetric LCS layer of terms contributes to the initial, but dynamically trivial, 3D-standard supergravity action. Similarly, as it was already shown to happen for the pure bosonic sector of this action⁶⁾, we shall see here that actually it is this supersymmetric LCS layer of the action which provides the system with the possibility of having a non-trivial dynamics for both sectors. In particular we shall see how the fermionic conformal invariance of the supersymmetric LCS part of the action⁷⁾ drives the dynamical structure of the tightly coupled fermionic sector.

In 3D there is no chirality, and since we will have to express the dynamics of the topologically massive spin-3/2 excitation in terms of only one variable, we will see that the projection of the real Majorana spinors by means of $z^{-1}(1 \pm \gamma_1)$ comes in a natural way. These projectors turn out to be just the two light-front projections of a Majorana 2D spinor, so the fermionic sector is suggesting us the convenience of looking at the system which follows a light-front point of view, as it is our aim here.

As supergravity⁸⁾ requires the presence of torsion, here we shall have to extend in an appropriate way the dreibein light-front choice of gauge⁷⁾ for the whole system, since the fermionic and the bosonic sector talk to each other. We will clarify these points in the third section of the article. The next section is devoted to describing the system and establishing the covariant and light-front notation.

The fourth section, which is the central one, is dedicated to the computation of the action and to its reduction process. We find the constraints (in the light-front representation) and after solving them we cast the initial covariant expression in the final, light-front, unconstrained form. In the last section we discuss the results. In the Appendix, we have included the light-front 3D analysis of both the massive spin- $\frac{1}{2}$ Dirac equation and of the topologically massive spin-3/2 action, both for real Majorana spinors.

2. - TOPOLOGICALLY MASSIVE SUPERGRAVITY. LIGHT-FRONT KINEMATICS

The action of topologically massive supergravity⁵⁾ is the addition of two separately locally supersymmetric actions

$$I_{\mu, SG} \equiv I_{SG} + I_{SCS}, \quad (1a)$$

where I_{SG} corresponds to 3D massless supergravity and I_{SCS} is the supersymmetric completion of the gravitational LCS action,

$$I_{SG} = I_E + I_{RS} = \frac{1}{k^2} \langle e_{pa} \epsilon^{pmn} (\partial_m \omega_n^a - \partial_n \omega_m^a + \epsilon^{abc} \omega_{mb} \omega_{nc}) \rangle + i \langle \bar{\psi}_m \epsilon^{mnp} D_n \psi_p \rangle, \quad (1b)$$

$$I_{SCS} \equiv I_{CS} + I_{TF} = \frac{1}{\mu\kappa^2} \langle \omega_{pa} \epsilon^{pmn} \partial_m \omega_n^a \rangle + \frac{1}{3\mu\kappa^2} \langle \epsilon^{mnp} (1c) \\ \epsilon^{abc} \omega_{ma} \omega_{nb} \omega_{pc} \rangle - \frac{i}{2\mu} \langle \bar{D}_p \psi_q [e g^{ps} g^{qr} - \gamma^p \epsilon^{qrs}] (D_s \psi_r - D_s \psi_r) \rangle.$$

In 3D it is more convenient to work with the dual Lorentz affinity $\omega_m^a \equiv 2^{-1} \epsilon^{abc} \omega_{mbc}$, $\epsilon^{012} = +1$, the flat metric η^{ab} has the signature +1 ($\eta^{00} = -1$) and Lorentz covariant derivatives make the two actions (1b) and (1c) co-ordinate invariants. Since D_r^ω only appears as a curl, we need only its effect on Lorentz (including spinor) indices, e.g., $D_r^\omega \psi_s \equiv (\partial_r + \frac{1}{2} \omega_r^a \gamma_a) \psi_s$. The Rarita-Schwinger spin-3/2 field ψ_s is a 3D Majorana real spinor having two spinor components which are anticommuting Grassmann quantities. The γ -matrices are in the real representation, $\gamma^a \gamma^b = \eta^{ab} + \epsilon^{abc} \gamma_c$, the graviton is carried by the dreibein 1-forms $e^a \equiv dx^r e_r^a$, $e \equiv \det e_r^a$, the contravariant form metric of the tensor is made out of the inverse dreibein $g^{rs} \equiv \eta^{ab} e_a^r e_b^s$, $e_a^r e_r^b = \delta_a^b$.

The action (1) is of third order in the bosonic field e_r^a and second with respect to its supersymmetric partner ψ_s . The affinities are not independent quantities. They have their value determined by first order supergravity $I_{SG}(e, \psi, \omega)$, e.g., they are the solutions of the torsion field equations,

$$\frac{1}{2} \kappa^2 \bar{\psi}_n \gamma^a \psi_p = T_{np}^a = D_n^\omega e_p^a - D_p^\omega e_n^a = \partial_n e_p^a - \partial_p e_n^a + \epsilon^{abc} (\omega_{nb} e_{pc} - \omega_{pb} e_{nc}). \quad (2)$$

This massive action has three local gauge invariances. Their infinitesimal transformation laws are⁹⁾

$$\delta e_r^a = D_r^\omega \xi^a; \quad \delta \psi_r = e^{as} \xi_a (\omega_r \psi_s - D_s^\omega \psi_r); \quad (3a)$$

$$\delta e_r^a = \epsilon^{abc} l_b e_{rc}; \quad \delta \psi_r = \frac{1}{2} l^b \gamma_b \psi_r, \quad \delta \omega_r^a = D_r^\omega l^a \quad (3b)$$

and

$$\delta e_r^a = i\kappa \bar{\xi} \gamma^a \psi_r, \quad \delta \psi_r = \frac{2}{\kappa} D_r^\omega \xi, \quad (3c)$$

which are the co-ordinate, local Lorentz and local supersymmetric invariances respectively. In $D = 3$ bosons have dimensions $M^{\frac{1}{2}}$, fermions $\sim M^1$ and the gravitational coupling constant $\kappa \sim L^{\frac{1}{2}}$. It is convenient to absorb κ into ϕ_r (and κ^2 into each action) $\kappa\phi_r \equiv \phi_r^{\text{New}}$ since here we will not take weak limits. We do so from now on. The three most typical gauge transformations are those having a covariant derivative acting upon the infinitesimal parameter, i.e., $\delta e_r^a = D_r^{\omega} \xi^a$; $\delta \omega_r^a = D_r^{\omega} \lambda^a$ and $\delta \psi_r = 2D_r^{\omega} \zeta$. These transformation laws will suggest (see next section) how to choose a natural dreibein light-front gauge.

Using the torsion field Eqs. (2), the action of supergravity can be written in a simpler form [where the ω 's are given by Eq. (2)]

$$I_{SG} = -\langle \epsilon^{mnp} \epsilon^{abc} e_{ma} \omega_{nb} \omega_{pc} \rangle + i \langle \bar{\psi}_m \epsilon^{mnp} \partial_n \psi_p \rangle. \quad (1b')$$

Fermions θ^α are in the real anticommuting Majorana two-dimensional spinor representation, their indices are lowered (and raised) with the real metric $\epsilon_{\alpha\beta}^* = \epsilon^{\alpha\beta}$, $\epsilon_{12} = +1 = \epsilon^{12}$, $\bar{\chi}\psi \equiv \chi_\alpha \psi^\alpha = \chi^\beta \epsilon_{\beta\alpha} \psi^\alpha = \chi_\alpha \epsilon^{\alpha\gamma} \psi_\gamma$. If one takes the light-front co-ordinates $x^\pm \equiv 1/\sqrt{2}(x^0 \pm x^2)$, then it is convenient to represent the $\gamma^{\alpha\beta}$ by:

$$\gamma^{\bar{\alpha}\alpha}_\beta = i\sigma_2, \quad \gamma^{\bar{1}\alpha}_\beta = \sigma_3, \quad \gamma^{\bar{2}\alpha}_\beta = -\sigma_1. \quad (4a)$$

(barred numbers indicate Lorentz indices). In the light-front local Lorentz frame the basic γ matrices are

$$\gamma^{\pm\alpha}_\beta = \frac{1}{\sqrt{2}} (\gamma^0 \mp \gamma^2)^\alpha_\beta = \sqrt{2} \begin{pmatrix} 1 & \\ & \end{pmatrix}; -\sqrt{2} \begin{pmatrix} & \\ 1 & \end{pmatrix} \quad (4b)$$

while $\epsilon^{-+1} = +1$. In spite that the rotation from $(e_0, e_2) \rightarrow (e_+, e_-)$ is not a Lorentz transformation, it is an element of $SO(2)$ and it is linear, so all tensor Lorentz equations keep their validity in these co-ordinates. For instance, taking $a, b = +, -(-, +)$ in $\gamma^a \gamma^b = \eta^{ab} + \epsilon^{abc} \gamma_c$, one obtains the two expressions which show by inspection why $-\frac{1}{2} \gamma^- \gamma^+ = \frac{1}{2} (1 - \gamma^1) \equiv P_+$ and $\frac{1}{2} \gamma^+ \gamma^- = \frac{1}{2} (1 + \gamma^1) \equiv P_-$ are the two spinor-light-front projectors. The representation (4a) tells us that while P_+ kills the first component of any spinor, P_- takes care of the second one,

$$P_+ \psi = (\dot{\psi}_2) \quad ; \quad P_- \psi = (\dot{\psi}_1). \quad (4c)$$

This is relevant to the light-front dynamics of a supersymmetric system, since once one has chosen x^+ to be the light-front time, the supersymmetry commutation relations define which is the associated spinorial "time"-like co-ordinate playing the role of $\sim \sqrt{x^+}$. From the cloning of the basic supersymmetry relation for the spinorial derivatives

$$(D_\alpha, D_\beta)_+ = 2 \tau_{\alpha\beta}^r p_r, \quad (5)$$

one obtains $D_2^2 = 2^{\frac{1}{2}} p_+$ which indicates that $\theta^2 \sim \sqrt{x^+}$ is the spinorial light-front "time" co-ordinate (likewise $\theta^1 \sim \sqrt{x^-}$). In the curved 3D space-time, we denote co-ordinates by (u, v, x) to remind they are a distortion of the (local) Lorentz light-front orthonormal frames. The choice of the light-front gauge dreibein enforces that one of them, u , has to become an exact light-front co-ordinate, i.e., the surfaces $u = \text{const.}$, are characteristics of the curved D'Alembertian field equation $\sim g^{uu} = 0$, as we shall see next.

3. - LIGHT-FRONT DREIBEIN GAUGE

We generalize to the supergravity case the gauge choice made in⁶⁾ by requesting the three gauge potentials e_r^a , ω_r^a , ϕ_r to simultaneously satisfy

$$e_v^a = e_u^+ \delta_-^a \equiv m \delta_-^a, \quad \omega_{va} = \omega_{v+} \delta_a^+, \quad (6ab)$$

and

$$\psi_v = 0. \quad (6c)$$

These conditions seem rather natural (where one is taking into account that the three-volume element cannot vanish) to be imposed on the gauge potentials if they were independent. They are here, however, linked through the torsion equations (2) which are essential to the proper working of the Chern-Simons terms. One has then to check whether the initial choice (6) is consistent as it stands and if it brings additional consequences to the behaviour of the system, as we shall see is the case.

Notice that the light-front gravitino gauge choice (6c) kills six components

of the torsion. The only three non-vanishing are $T_{u1}^a = (i/2)\bar{\psi}_u \gamma^a \psi_1 \equiv (i/2)\bar{\chi}\gamma^a \psi$. From $T_{uv}^+ = 0 = T_{v1}^+$ we get that

$$m'^0 = e_1^{+'}.$$

We then take $m = 1$, $e_1^+ = 0$ as an additional complement of Eqs. (6) which we also assume for the sake of simplicity. [In case one does not wish to lose generality, one should continue with $m = m(u, x)$, $e_1^+ = mN^+(u, x)$.] The dreibein field has then the aspect

$$\begin{aligned} e^+ &= du & ; & \quad e^- = du(n + \frac{1}{2}p^2 e^2) + dv + dx(N + p e^2), \\ e^{\bar{1}} &= du e p + dx e. \end{aligned} \quad (7a)$$

This parametrization has been chosen to obtain the maximum of polynomiality for both e_r^a and its inverse e_b^s . The associated metric has the form

$$ds^2 = -2ndu^2 - 2dudv - 2Ndu dx + e^2 dx^2. \quad (7b)$$

There is no dependence upon p which shows that this variable is a Lorentz degree of freedom, which we are then allowed to set to zero from now on.

The inverse dreibein vectors $e_a \equiv e_a^r \partial_r$ have the form

$$e_+ = \partial_u - n \partial_v \quad ; \quad e_- = \partial_v \quad ; \quad e_{\bar{1}} = -e^{\bar{1}} N \partial_v + e^{\bar{1}} \partial_x \quad (7c)$$

showing that $g^{uu} = 0$ and consequently that the surfaces $u = \text{const.}$ are characteristics of the system, i.e., light-fronts.

The remaining seven components of the torsion equation (2) determine the seven non-vanishing components of the Lorentz affinity in terms of the graviton and gravitino components. One finds

$$\omega_{v,+} = -\frac{1}{2e} (N' - T_{u1}^+), \quad (8a)$$

$$e\omega_{u,+} = -\dot{N} + n_{,1} + \frac{1}{2}nN' - n'N + T_{u1}^- + \frac{1}{2}nT_{u1}^+, \quad (8b)$$

$$\omega_{1,+} = \dot{e} - ne' - \frac{1}{2e} N(N' - T_{u1}^+) - T_{u1}^{\bar{1}}, \quad (8c)$$

$$e\omega_{u,-} = \frac{1}{2} (N' - T_{u1}^+) \quad , \quad \omega_{1,-} = -e' \quad , \quad (9ab)$$

$$\omega_{u,I} = n' , \quad \omega_{L,I} = \frac{1}{2}N' + \frac{1}{2}T_{uL}^+ , \quad (10ab)$$

where dots indicates light-front time derivatives, $\dot{f} \equiv \partial f / \partial u$ and primes stand for derivatives with respect to v , $f' \equiv \partial f / \partial v$.

Having obtained the values of ω_{ra} in terms of the two basic independent fields e^a , ϕ , we can proceed to evaluate the full action and analyze its behaviour.

4. - THE UNCONSTRAINED ACTION

We will deal first with massless supergravity as written in (1b'). The two Majorana spinors non-vanishing components ϕ_u , ϕ_L of the gravitino are denoted $\chi = (\chi^1, \chi^2) \equiv \phi_u$ and $\psi = (\psi^1, \psi^2) \equiv \phi_L$ respectively, where the indices stand for the spinorial components. The three non-vanishing components of the torsion are

$$T_{uL}^+ = -\frac{i}{\sqrt{2}}\chi^2\psi^2 , \quad T_{uL}^- = -\frac{i}{\sqrt{2}}\chi^1\psi^1 , \quad T_{uL}^I = \frac{i}{2}(\psi^1\chi^2 - \chi^1\psi^2). \quad (11)$$

After introducing the values of the affinities (8),(9) and (10) into I_{SG} one finds that

$$I_{SG} = 2 \langle n e'' - \frac{1}{4e} [N' + \frac{i}{\sqrt{2}}\chi^2\psi^2]^2 \rangle + 2i \langle \psi^1\chi^2' - \chi^1\psi^2' \rangle. \quad (12)$$

The system does not have any driving dynamical term which may generate a time evolution, neither for the graviton⁶⁾ nor for its supersymmetric completion. No time derivative appears in (12). χ^1 and ϕ^1 are Lagrange multipliers enforcing the dynamical triviality of the spin-3/2 carrier ϕ^2 and of χ^2 , while n constrains the graviton not to propagate. There is a fermionic contribution to the bosonic energy of the system which appears in a form that shows up its explicitly non-negative character, since $i\chi^2\psi^2$ is Hermitian.

The dynamics of the massive spin two variable e stems from the LCS part of the action. In terms of the variables we are left with, it has the value:

$$\begin{aligned} \mu I_{CS} = 2 \left\langle \frac{e''}{e} \dot{N} + n \left\{ \frac{1}{2} (N' + T_{u1}^+)'' - (\frac{e''}{e} N)' - \frac{e''}{e} N' + (\frac{e''}{e})_{,1} \right\} \right\rangle_{(13a)} \\ - 2 \left\langle \frac{e''}{e} T_{u1}^- \right\rangle + 2 \left\langle T_{u1} \bar{I} \left[\frac{1}{2e} (N' - T_{u1}^+) \right]' + \frac{1}{8e^2} N'^2 T_{u1}^+ \right\rangle. \end{aligned}$$

The key dynamical part of the full action is I_{TF} , the gravitino Chern-Simons supersymmetric partner of I_{CS} which has been shown⁷⁾ to need the presence of I_{CS} in order to keep having in a curved space the local fermionic conformal invariance it has on flat space. In terms of our dreibein light-front variables (7)-(10) it has the form:

$$\begin{aligned} \mu I_{TF} = & \left\langle i\sqrt{2} e^2 \psi^2 \psi^{2'} \dot{N} \right\rangle + 2i \left\langle \psi^{2'} e^{-1} [\psi^1 - \frac{e}{\sqrt{2}} \chi^2]' \right\rangle + i \left\langle n \cdot \left[\psi^1 - \frac{e}{\sqrt{2}} \chi^2 \right]' \right. \\ & \cdot 2\bar{e}^1 \psi^{2'} + (\psi^1 \bar{e}^1 \psi^{2'})' + (\chi^{2'} \frac{1}{\sqrt{2}} \psi^2)' + (\sqrt{2} \bar{e}^2 \psi^2 \psi^{2'})_{,1} - \\ & \left. - (N\sqrt{2} \bar{e}^2 \psi^2 \psi^{2'})' - N' \sqrt{2} \bar{e}^2 \psi^2 \psi^{2'} \right\rangle + i \left\langle \chi^1 \cdot [\sqrt{2} \psi^{1'} + \bar{e}^1 (e^2 \chi^{2'})' \right. \\ & - 2(e^{-1} \psi^{2'})_{,1} + \frac{1}{2} (N' e^{-1} \psi^2)' + 2(N e^{-1} \psi^{2'})' + \frac{1}{2} N' \bar{e}^1 \psi^{2'} \right. \\ & \left. - \frac{i}{4} \sqrt{2} e^{-1} \chi^2 \psi^2 \psi^{2'} + i (\psi^1 - \frac{e}{\sqrt{2}} \chi^2) \bar{e}^2 \psi^2 \psi^{2'} \right\rangle - \\ & - i \left\langle (2e)^{-1} (N' - T_{u1}^+) (\chi^2 \psi^1)' \right\rangle - \left\langle (2e)^{-2} N'^2 T_{u1}^+ \right\rangle. \end{aligned} \quad (13b)$$

We observe the presence of a term $\sim \dot{N}$ which gives a fermionic contribution to the dynamical germ of the graviton and we receive the indication that the gravitino should be better described by the new set of Grassmann variables $\chi^1, \psi^2, \psi^1 + 2^{-\frac{1}{2}} e \chi^2$. In fact we will learn from the flat case that it must be so due to the dynamical dominance in the full gravitino action of the locally (fermionic) conformal invariance of I_{TF} which enforces the right number of degrees of freedom - one - for the full massive 3D gravitino gauge invariant action. On flat space the massive gauge invariant gravitino action is the sum of the Rarita-Schwinger and of the fermionic Chern-Simons term⁵⁾,

$$I_{\mu, 3/2}^0 \equiv I_{RS}|_{\eta} + I_{TF}|_{\eta} = i \langle \bar{\psi}_m f^m(\psi) \rangle - \frac{i}{2\mu} \langle \bar{f}_m(\psi) \gamma^n \gamma^m f_n(\psi) \rangle_{(14)},$$

where, for simplicity, the 3D-dual field strength $f^m(\psi) \equiv \epsilon^{mrs} \partial_r \psi_s$ has been introduced. In the light-front gravitino gauge $\psi_v = 0$, when written in terms of the remaining four Grassmann numbers $\chi^{1,2} = \chi \equiv \psi_u$ and $\psi^{1,2} = \psi \equiv \psi_1$, it takes

the dynamically transparent form

$$\begin{aligned} \mu I_{\mu, 3/2}^0 = & 2i \langle \psi^{2'} \dot{\lambda}^1 \rangle + \frac{i\mu}{\sqrt{2}} \langle -\lambda^1 \lambda^{1'} + \lambda^2 \lambda^{2'} \rangle - \\ & - 2i \langle \lambda^1 [(\mu + \partial_1) \psi^{2'} + \frac{1}{\sqrt{2}} \lambda^{1''}] \rangle, \end{aligned} \quad (15a)$$

where one has been led to introduce two new variables $\lambda^1 \equiv \phi^1 - 2^{-\frac{1}{2}} \chi^2$, $\lambda^2 \equiv \phi^1 + 2^{-\frac{1}{2}} \chi^2$, which we will see in a moment also have a covariant meaning.

Inspection of (15a) tells us that χ^1 is the Lagrange multiplier associated with the differential constraint

$$(\mu + \partial_1) \psi^{2'} + \frac{1}{\sqrt{2}} \lambda^{1''} = 0 \quad (16a)$$

whose solution provides the value of ψ^2 in terms of the non variable λ^1 : $\psi^{2'} = -2^{-\frac{1}{2}} \Delta_\mu^{-1} (\mu - \partial_1) \lambda^{1''}$, $\Delta_\mu \equiv \mu^2 + \partial_{11}^2 > 0$. After substituting this value of $\psi^{2'}$ into $I_{\mu, 3/2}^0$ we get the unconstrained action in terms of $\lambda^{1,2}$, or going a step further redefining λ^1 , $\lambda^1 \rightarrow \tilde{\lambda}^1 = \mu \Delta_\mu^{-\frac{1}{2}} \lambda^1$, we reach the unconstrained action written in a canonical light-front way:

$$I_{\mu, 3/2}^{0, \text{unconstr}} = -\frac{i\sqrt{2}}{\mu^2} \langle \tilde{\lambda}^{1''} \dot{\tilde{\lambda}}^1 \rangle - \frac{i}{2\sqrt{2}\mu^2} \langle \tilde{\lambda}^1 \Delta_\mu \tilde{\lambda}^{1'} \rangle + \frac{i}{2\sqrt{2}} \langle \lambda^2 \lambda^{2'} \rangle \quad (15b)$$

The second variable λ^2 decouples from the $\tilde{\lambda}^1$ -part of the initial action, it is an auxiliary field which does not propagate. On the other hand, $\tilde{\lambda}^1$ obeys a massive Klein-Gordon propagation law (with an additional non-timelike derivative in it) as one can recognize after doing independent variations of it in Eq. (15b) leading to:

$$(\square - \mu^2) \tilde{\lambda}^{1'} = 0. \quad (16b)$$

A similar analysis can be made by solving from (16a) λ^1 in terms of ψ^2 and of ϕ such that $\mu\phi' = \psi^2$. One obtains the same action (15b) with ϕ in the place of $\tilde{\lambda}^1$.

The unconstrained gravitino action (15b) coincides with the light-front unconstrained action for a massive spin- $\frac{1}{2}$ Majorana fermion, as we show in the Appendix, where it is also shown how does the covariant structure of $I_{\mu, 3/2}^0$ look

like both in terms of 3D covariant gauge invariant variables and as a function of $\phi \equiv \gamma^r \phi_r$ and of the γ -transverse part of ϕ_r , $\phi_r^- \equiv \phi_r - 1/3 \gamma_r \phi = 2/3 \phi_r + 1/3 \epsilon_r^{s_1 s_2} \gamma_{s_1} \phi_{s_2}$. In the light-front gravitino gauge these spinors have the respective values

$$\psi = \begin{pmatrix} \psi^1 + \frac{1}{2} \chi^2 \\ -\psi_2 \end{pmatrix}, \quad \psi_{\mp} = \begin{pmatrix} \chi^1 \\ -\frac{\sqrt{2}}{3} \chi^1 \end{pmatrix}, \quad \psi_{\pm} = -\frac{\sqrt{2}}{3} \begin{pmatrix} \psi^2 \\ \cdot \end{pmatrix}, \quad \psi_{\mp} = \frac{2}{3} \begin{pmatrix} \chi^1 \\ \psi_2 \end{pmatrix} \quad (17a)$$

The components of ϕ_r^- are precisely given in terms of the variables we have been induced to choose by dynamical reasons $(\chi^1, \psi^2, \lambda^1, \lambda^2)$. This suggests what happens in the curved case. The right dynamical variables will be the components of $\phi_a^- = e_a^r \phi_r - 1/3 \gamma_a^b \gamma_b^r \phi_r$, which turn out to be $[\phi_+ = \chi, \phi_- = 0, \phi_1^- = e^{-1} \psi]$, the double bar on subindices indicating components of ϕ_a^-

$$\psi_{\mp} = \begin{pmatrix} \chi^1 \\ -\frac{\sqrt{2}}{3e} [\psi^1 - \frac{e}{2^{1/2}} \chi^2] \end{pmatrix}; \quad \psi_{\mp} = 2(3e)^{-1} \begin{pmatrix} \psi^1 - \frac{e}{\sqrt{2}} \chi^2 \\ \psi_2 \end{pmatrix}; \quad \psi_{\pm} = -2^{1/2}(3e)^{-1} \begin{pmatrix} \psi_2 \\ \cdot \end{pmatrix} \quad (17b)$$

reinforcing our belief in the convenience of generalizing for the curved case the flat definition given after Eq. (15a). $\lambda^{1,2}$ become

$$\chi^1 \equiv \psi^1 - \frac{e}{\sqrt{2}} \chi^2, \quad \chi^2 \equiv \psi^1 + \frac{e}{\sqrt{2}} \chi^2. \quad (17c)$$

We are now in the position of collecting the two parts (13) of the supersymmetric Chern-Simons action in a closed expression, using the new variables (17c). We find that it becomes

$$\begin{aligned} \mu I_{SCS} = & 2 \langle \epsilon N \rangle + 2i \langle \bar{\epsilon}^1 \psi^{2'} \chi^1 \rangle + 2 \langle n \{ \frac{3i}{2} \chi^1 \bar{\epsilon}^1 \psi^{2'} + \\ & + \frac{1}{2} \chi^1 (\bar{\epsilon}^1 \psi^{2'})' + \frac{1}{2} N'' - 2 \epsilon^{\frac{1}{2}} [\epsilon^{\frac{1}{2}} N]' + \epsilon_{,1} \} \rangle + \\ & + 2i \langle \chi^1 \{ -2^{-\frac{1}{2}} \chi^{1''} + 2^{\frac{1}{2}} \epsilon \chi^1 - (\bar{\epsilon}^1 \psi^{2'})_{,1} + \frac{3}{2} N' \bar{\epsilon}^1 \psi^{2'} + N (\bar{\epsilon}^1 \psi^{2'})' \} \rangle \end{aligned} \quad (18a)$$

where we have introduced the superquantity $\epsilon \equiv e^{-1} e'' + i 2^{-\frac{1}{2}} e^{-2} \psi^2 \psi^{2'}$.

Notice that the first two terms provide the possibility of a dynamical evolution of both the bosonic and fermionic physical variables, in a supersymmetric extension of the behaviour observed in Ref. 6). Another peculiarity appears in its supersymmetric version: the action (18a) does not have energy; neither for the spin-2 sector nor for the gravitino. The energy of the system is stored in the massless supergravity action, as it already can be read by looking at the reduced expressions (12) and (18a). Both $\sim(N'+i2^{-\frac{1}{2}}\chi^2\phi^2)^2$ and $\sim\psi^1\chi^{2'}$ contribute to the energy of the system, its final form will be given later on for the unconstrained action.

The sum of the supergravity with the supersymmetric LCS action yields the light-front form of the full topological massive supergravity action. It reads

$$\begin{aligned} \mu I_E + \mu I_{SCS} = \mu I_{SG,\mu} = & 2 \langle \epsilon \dot{N} \rangle + 2i \langle \phi \chi^{2'} \rangle + \\ & + 2i\mu \langle (\chi^1 + \frac{\epsilon}{\sqrt{2}} \chi^2) \chi^{2'} \rangle + \langle n \{ N''' - 4\epsilon^{\frac{1}{2}} [\epsilon^{\frac{1}{2}} N]' + 2\epsilon_{,1} + 2\mu e'' + 3i\chi^1 \phi' (18b) + \\ & + i\chi^1 \phi' \} \rangle - 2i \langle \chi^1 \{ \mu e \phi + \phi_{,1} + 2^{-\frac{1}{2}} \chi^{1''} - 2^{-\frac{1}{2}} \epsilon \chi^1 - N^{-\frac{1}{2}} [N^{\frac{3}{2}} \phi]' \} \rangle \end{aligned}$$

where $\phi \equiv e^{-1}\phi^2$. In this form it is immediate to realize that n, χ^1 constitute two Lagrangian multipliers, respectively associated with one bosonic and one fermionic differential constraint,

$$C_n \equiv N''' - 4\epsilon^{\frac{1}{2}} [\epsilon^{\frac{1}{2}} N]' + 2\epsilon_{,1} + 2\mu e'' + 3i\chi^1 \phi' + i\chi^1 \phi' = 0, \quad (19a)$$

$$C_\chi \equiv [\mu e + \partial_1 - N^{-\frac{1}{2}} [N^{\frac{3}{2}} \cdot]'] \phi + 2^{-\frac{1}{2}} \chi^{1''} - 2^{-\frac{1}{2}} \epsilon \chi^1 = 0. \quad (19b)$$

This is a very tightly coupled system of ordinary differential equations. One might think of it as allowing to obtain $\lambda^1(e, \phi^2, N)$ through (19b) and then, after insertion of this value in the first equation, solving for N . The solution $N(e, \phi^2)$ will then come out as a function of (e, ϕ^2) , the two physical variables of the system.

Observe that since λ^1 is present in (19a), this equation, regarded as an equation for N , has a very complicated structure, its complexity entirely arising

from the higher derivative structure of the LCS action. Anyhow, we regard this system (19) as providing the values of $N(e, \phi^2)$, $\lambda^1(e, \phi^2)$ where it is worth realizing there is no dependence upon χ^2 which only appears in $I_{SG, \mu}$ through the two kinetic terms. Hence variations with respect to it lead to the third differential constraint the system has, which is the curved version of the flat expression obtained by independent variations of λ^2 in action (15b), i.e.,

$$\lambda^{2'} \equiv (\lambda^1 + \sqrt{2} \chi^2)' = 0.$$

The gravitational coupling converts this equation into

$$2e^{\frac{1}{2}} [e^{\frac{1}{2}} \chi^2]' = -2^{\frac{1}{2}} \lambda^{1'} + (2e)^{-1} N' \psi^2, \quad (19c)$$

obtained from variations of χ^2 in action (18b). Once one would get $\lambda^1(e, \phi^2)$, $N(e, \phi^2)$, this equation can easily be integrated providing the solution $\chi^2(e, \phi^2)$ to this differential constraint. This solution $\chi^2(e, \phi^2)$ can be introduced back into the constrained light-front action (18b), providing the unconstrained action of topological massive supergravity. It turns out to be:

$$\begin{aligned} I_{SG, \mu}^{unconstr} = & \frac{2}{\mu} \langle e N \rangle + \frac{2i}{\mu} \langle \bar{e}^1 \psi^{2'} \lambda^1 \rangle - \\ & - \langle \frac{1}{2e} [N' - i e^{-1} \lambda^1 \psi^2]^2 \rangle - 2^{\frac{3}{2}} i \langle \bar{e}^1 \psi^1 \psi^{1'} \rangle, \end{aligned} \quad (20a)$$

where

$$\psi^1 = \lambda^1(e, \psi^2) + \frac{e}{\sqrt{2}} \chi^2(e, \psi^2).$$

This action only depends upon (e, ϕ^2) , the two variables needed for the propagation of a pseudospin-2 plus a pseudospin-3/2 locally supersymmetric massive multiplet. The light-front energy of the system is the addition of a typically non-negative bosonic contribution plus the standard light-front spinor contribution of the Rarita-Schwinger field. The two main constraints (19ab) are rather tightly coupled if one compared them with, i.e., 4D-supergravity. This increased complexity of the system is due to the presence of the LSC terms in the full action.

The above unconstrained action is not in the canonical light-front form¹⁰⁾ which is mandatory in order to study the functional quantization¹¹⁾ of this

system in the ghost-free gauge we have chosen (without including BRS auxiliary fields). It is straightforward to bring this action to a canonical form by adding appropriate Lagrange multipliers. In its canonical form $I_{SG,\mu}$ has the aspect:

$$\begin{aligned}
 I_{SG,\mu}^{can, light-front} = & \frac{2}{\mu} \langle e \dot{N} \rangle + \frac{2i}{\mu} \langle \phi \dot{\lambda}^1 \rangle + \langle n C_n(e, \psi^2, \phi, \lambda^1, N) \rangle - \\
 & - 2i \langle \chi^1 C_\chi(e, \psi^2, \phi, \lambda^1, N) \rangle + \langle v_1(\phi - \bar{e}^1 \psi^2) \rangle + \\
 & + \langle r_1(\sigma - \theta^1) \rangle + \langle r_2(\vartheta - \bar{e}^1 e^1) \rangle + \langle r_3[e - (\sigma + \theta^2 + \frac{i}{\sqrt{2}} \bar{e}^1 \psi^2 \phi)] \rangle \\
 & + \langle v_2[(e^{\frac{1}{2}} \chi^2)' - (2e)^{-\frac{1}{2}} \lambda^{1'} + 4^{-1} e^{-\frac{3}{2}} N' \psi^2] \rangle \\
 & - \langle (2e)^{-1} [N' - i \bar{e}^1 \lambda^1 \psi^2]^2 \rangle - i 2^{\frac{3}{2}} \langle \bar{e}^1 \psi^1 \psi^{1'} \rangle
 \end{aligned} \tag{20b}$$

where the higher order structure of the action has obliged us to introduce several Lagrange multipliers, some bosonic as the r_i and others Grassmannian v_j .

It is worth pointing out that the quantity ϵ can be written in a Lorentz invariant form

$$\epsilon \equiv \bar{e}^1 e^1 + i 2^{\frac{1}{2}} \bar{e}^2 \psi^2 \psi^{2'} = D_- \vartheta + \vartheta^2 - \frac{2}{8} i \bar{\psi}_I \gamma^+ D_- \psi_I, \tag{21a}$$

where

$$\vartheta \equiv - e^{\bar{I} r} D_- e_I^r. \tag{21b}$$

This last quantity ϑ is one of the Newman-Penrose coefficients¹²⁾ whose connection with the light-front dynamics (in 4D) has been studied time ago¹³⁾.

Since a very nice and direct interpretation of the light-front energy in terms of the NP-coefficients was shown in such a work¹³⁾, it seems quite a plausible conjecture that for the present gravitational system, its light-front energy will also have a simple interpretation in terms of the supersymmetric generalization of the bosonic NP coefficients.

5. - DISCUSSION AND COMMENTS

Making use of the gauge character with respect to the three local invariance groups of the three sets of quantities required to define topologically massive supergravity $(e_{\mu}^a, \omega_r^a, \psi_r)$, we introduced the light-front dreibein gauge. The associated metric tensor then appears automatically to be in the usual metric light-front representation.

We then performed the dynamical analysis of this locally supersymmetric system, checking in our way that 3D-massless supergravity is locally trivial, both for the graviton and for the gravitino.

The most interesting part of the action turned out to be the fermionic LCS one, due to its local conformal gauge invariance which permits that χ^2 , one of the four Grassmann components contained in $(\psi_u, \psi_l) = (\chi, \phi)$ remains essentially trivial through a very mild coupling to the rest of the system.

The dynamical behaviour of the gravitino part of the system was first analyzed on flat space-time. It turns out that already at this level the good physical variables are those associated with the γ -transverse part of the Rarita-Schwinger pseudospin-3/2 field.

When we went to the full massive supergravity action we observed the same dynamical architecture (now supersymmetrized) found for pure topological massive gravity⁶⁾. The supersymmetric Einstein's sector provides the kinetic energy for both the graviton and the gravitino while the supersymmetric LCS contribution provides the dynamics generating terms which make non-trivial their evolution along the light-front time. These two pieces of action, the dynamical generating terms $\sim p\bar{q}$ and the kinetic ones $\sim 2^{-1}q'^2$ are tightly coupled through the two basic differential constraints c_n, c_{χ} the system presents. Both the unconstrained and the light-front canonical forms of the action were given.

The unconstrained form of the action only depends, as it must on physical grounds, on the unique two degrees of freedom the theory has, one - e - for the massive graviton and the second - ϕ^2 (a real Grassmann quantity) - for the massive gravitino. It is interesting to observe the very different role played in the action by the two components of ϕ_l , the one world component of the covariant gravitino field. While $\phi^2 = P_- \phi_l$ is the spin-3/2 dynamical variable, $\phi^1 = P_+ \phi_l$ (thought as composed by $\lambda^1 + 2^{-\frac{1}{2}} e \chi^2$) provides the fermionic part of the light-front energy, as can be read from Eq. (20a).

Since the passage from massive $N = 1$ supergravity to $N \geq 2$ ¹⁴⁾ does not contain additional qualitative changes in the structure of the action, we believe the present results may be extended to this interesting case with extended local supersymmetries.

APPENDIX

First we want to show what is the unconstrained light-front form of Dirac's action for a standard massive spin- $\frac{1}{2}$ Majorana fermion. The standard covariant massive 3D-spin- $\frac{1}{2}$ action is

$$I_{\mu, \frac{1}{2}} \equiv i \langle \bar{\phi} \not{\partial} \phi \rangle - i \langle \bar{\phi} \mu \phi \rangle. \quad (A1)$$

When written in the light-front co-ordinates, it looks like

$$I_{\mu, \frac{1}{2}} = \frac{i}{\sqrt{2}} \langle \phi^2 \not{\partial}^2 + \phi^1 \phi^{1'} + 2 \frac{1}{2} \phi^1 (\partial_1 + \mu) \phi^2 \rangle, \quad (A2)$$

(ϕ^1, ϕ^2) are the two 3D-spinorial components of the basic Majorana spinor ϕ . Variations of ϕ^1 yield the differential constraint

$$(\mu + \partial_1) \phi^2 + \sqrt{2} \phi^{1'} = 0, \quad (A3)$$

[compare with Eq. (16a)], whose solution is $\phi^2 = -2^{\frac{1}{2}} \Delta_\mu^{-1} (\mu - \partial_1) \phi^{1'}$. After introducing this value of ϕ^2 into (A2), we obtain the unconstrained form of $I_{\mu, \frac{1}{2}}$,

$$I_{\mu, \frac{1}{2}}^{\text{unconstr.}} = \frac{i 2^{\frac{1}{2}}}{\mu^2} \langle \bar{\phi}^1 \not{\partial}^2 \phi^1 \rangle + \frac{i}{2 \mu^2} \langle \bar{\phi}^1 \Delta_\mu \phi^{1'} \rangle \quad (A4)$$

where we redefined $\phi^1 \rightarrow \tilde{\phi}^1 \equiv \mu \Delta_\mu^{-\frac{1}{2}} \phi^1$ in order to have a canonical structure. It is immediate to realize that variations of ϕ^1 in the unconstrained action lead to the Klein-Gordon propagation equation acting upon $\phi^{1'}$:

$$(\square - \mu^2) \phi^{1'} = 0. \quad (A5)$$

Notice the $I_{\mu, \frac{1}{2}}^{\text{unconstr.}}$ only depends upon one degree of freedom, as it must happen with a 3D-pseudospin $+\frac{1}{2}$ (or $-\frac{1}{2}$) particle. Since 3D-Majorana spinors possess two real components, and in 3D there is no chirality, the only natural way of decomposing ϕ is through the light-front "chirality" $\frac{1}{2}(1 + \gamma^1) = P_-$ defined in (4c). Secondly, we want to point out how appears the covariant structure of $I_{\mu, 3/2}^0$ given by Eq. (14). In terms of $\phi_r^- \equiv \phi_r - 1/3 \gamma_r \phi$, $\phi \equiv \gamma^r \phi_r$ [$\gamma^r \phi_r^- = 0$], this action takes the form

$$I_{\mu, 3/2} = i \langle \bar{\psi}_F \not{\partial} \psi_F \rangle + \frac{2}{3} i \langle \bar{\psi}_F \partial_r \psi_F \rangle - \frac{2}{3} i \langle \bar{\psi} \not{\partial} \psi \rangle + \quad (A6)$$

$$+ \frac{i}{\mu} \langle -\bar{\psi}_F \square \psi_F + \frac{3}{2} \bar{\psi}_F \partial_{rs} \psi_s \rangle,$$

where we see that the dynamical generating part does not depend upon ϕ , showing explicitly its local conformal fermionic gauge invariance under $\delta\psi_r = \gamma_r \beta(x)$.

If one introduces the 3D decompositions of ψ_r ¹⁵⁾:

$$\psi_r = \psi_r^{\tau t} + \frac{1}{2} \tau_r^T \psi^{\tau} + \not{s}_r \psi^L$$

where

$$\tau_r^T \equiv (\eta_{rs} - \not{s}_r \not{s}_s) \tau^s, \quad \not{s}_r \equiv \square^{\frac{1}{2}} \partial_r$$

one can calculate the form which action (A6) takes in terms of the (two) gauge invariant variables $\psi^{\tau t}$, ψ^{τ} . Its Rarita-Schwinger part becomes independent of ψ^{τ} too and, since the quadratic part is independent of ψ^L (gauge invariance) and $\psi = \psi^{\tau} + \not{s} \psi^L$ (conformal invariance) it also only depends upon $\psi_r^{\tau t}$. In terms of $\psi_r^{\tau t}$ the action (A6) achieves its reduced covariant expression

$$I_{\mu, 3/2} = i \langle \bar{\psi}_r^{\tau t} \not{\partial} \psi_r^{\tau t} \rangle - \frac{i}{\mu} \langle \bar{\psi}_r^{\tau t} \square \psi_r^{\tau t} \rangle \quad (A7)$$

which tells us that $\psi_r^{\tau t}$ will propagate with mass μ (since $\not{s} \psi_r^{\tau t} = 0$ being equivalent to $\epsilon_r^{st} \partial_s \psi_t = 0$ means the vanishing of $\psi_r^{\tau t}$).

REFERENCES

- 1) M.B. Green and J.H. Schwarz, Phys. Lett. 149B (1984) 117.
- 2) S. Deser, R. Jackiw and S. Templeton, Ann. of. Phys. (N.Y) 140 (1982) 372.
- 3) S. Cecotti, S. Ferrara, L. Girardello and M. Porrati, Phys. Lett. 164B (1985) 46.
- 4) L. Baulieu and M. Bellon, Ecole Normale Supérieure preprint LPTENS 85/32 (1985).
- 5) S. Deser and J. Kay, Phys. Lett. 120B (1983) 97.
- 6) C. Aragone, CERN preprint 4434/86 (1986).
- 7) C. Aragone and S. Deser, in "From SU(3) to Gravity", ed. by Gottsman and C. Taubes, Cambridge Univ. Press (1985).
- 8) S. Ferrara, D. Freedman and P. van Nieuwenhuizen, Phys. Rev. D13 (1976) 3214;
S. Deser and B. Zumino, Phys. Lett. B62 (1976) 335.
- 9) D. Boulware, S. Deser and J.H. Kay, Physica 96A (1979) 141.
- 10) P. Senjanovic, Ann. of Phys. (N.Y.) 100 (1976) 227.
- 11) M. Kaku, Nucl. Phys. B91 (1975) 99;
M. Kaku and P. Senjanovic, Phys. Rev. D15 (1977) 1019;
C. Aragone, Phys. Rev. D18 (1978) 2776.
- 12) E. Newman and R. Penrose, J. Math. Phys. 3 (1962) 89;
E. Newman and T. Unti, J. Math. Phys. 3 (1962) 891.
- 13) C. Aragone, R. Gambini and A. Restuccia, Phys. Rev. D13 (1976) 218.
- 14) M. Roček and P. van Nieuwenhuizen, Class. Quantum Grav. 3 (1986) 43.
- 15) C. Aragone and J. Stephany, Class. Quantum Grav. 1 (1984) 265.