HIGH VOLTAGE BREAKDOWN IN VACUUM

by

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1. Introduction

The phenomena that occur when high voltages are applied between electrodes in a vacuum have been studied for many years. However, the progress in this field has only recently been speeded up under the spur of the increasing number of technical applications of vacuum insulation. During the course of the last decade there has been a considerable increase in the number of people involved and, of course, in the number of papers they have published and conferences they have attended. As a result of this increased effort a progressively clearer picture is emerging, at least on some aspects of the problem, but there is still much to do before we can obtain a coherent, clear and complete explanation of all the relevant observed phenomena.

Let us first examine what we mean by electrical breakdown in vacuum. We shall call vacuum a gas which is so rarefied that the mean free path of its molecules is much larger than the significant distances between electrodes. In these conditions the gas molecules are not involved (to a first approximation) in the production and transport of charged particles which occur when an electrical current is established between electrodes. In other words this is the domain where the product of pressures and distances is so small that Paschen's law is no longer applicable. As the voltage is raised a steady current of very low intensity ($\lesssim 10^{-9}$ A), progressively increasing, first appears and then, in the case of large gaps, small bursts of self-extinguishing current (called microdischarges), are superimposed on this steady current, until finally a high intensity current ($\sim$ A) is developed if enough energy is available to vaporize a sufficient amount of electrode material. At this stage electrical conduction no longer takes place in a vacuum but in metal vapour and the voltage between electrodes at this stage is very low compared to its initial value.

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Except in very short pulses a high voltage cannot be maintained between electrodes together with a high current since we no longer have a vacuum between electrodes. We can thus understand how vacuum breakdown and gas discharge are related and there is an intermediate region where curious phenomena occur as we shall see it later.

Since we know that residual gas molecules are not involved in the initiation of vacuum breakdown, what kind of limitation are we going to encounter with the electrodes upon raising the voltage between them? At first sight we might expect an appreciable current to appear when field values of $10^7$ to $10^8$ V/cm are reached because electron field emission from the cathode metal becomes significant at this field intensity according to Fowler-Nordheim theory. At about the same field intensity the mechanical stress on the electrodes, which is proportional to the square of the field intensity, becomes comparable to the bulk tensile strength of the electrode material. A mechanical stress of about $5 \times 10^8$ pascals (i.e. $\sim 50 \text{ kg/m}^2$) is generated by a field intensity of $10^8$ V/cm. Therefore at a field of about $10^7 - 10^8$ V/cm there should be a catastrophic breakdown between electrodes for these reasons. Is this the limitation encountered in equipment in which high voltages are applied in a vacuum? Anyone who has spent hours or event days trying to bring up the performance of an electrostatic separator or of an accelerating column to its nominal value will tell you that there is a "performance gap" of about three orders of magnitude!!

In order to understand the reasons for this enormous discrepancy we must look again more closely into conditions at the electrodes:

1) The average or macroscopic field, which is voltage/gap length, is the relevant parameter for particle separator performance but for determining the current between the electrodes the surface field is the critical factor. This true microscopic field on the electron emitting cathode points is much larger than the average field because of the enhancement produced by the point geometry. The enhancement factor may reach one or two orders of magnitude.

2) The microgeometry of the electrodes, that is the shape of the points, is continuously modified under the combined action of the electric field and of the surface curvature gradient; this modification is
facilitated when the local temperature is raised. Therefore perfect polishing of the surface will not permanently reduce the field enhancement factor. Sputtering of the electrode surface also has to be taken into account in some cases.

3) In large scale practical devices such as separators, where ultrahigh vacuum technology and high temperature baking are not used, the surface of the electrodes is not pure metal but is contaminated by all kinds of organic materials which have a much smaller work function than perfectly clean metals. Impurities and dislocations in the metal also enhance electron field emission.

A fairly satisfactory theory of the electrical breakdown of small gaps (≤ 1 mm), based on a detailed analysis of the electron emission from the cathode and on the current interaction with either electrode, has been developed by several authors for the case of very clean metal electrodes 1,2). However, this theory does not fit, in its present form, the experimental data accumulated on the electrical breakdown of large gaps. Other phenomena may become prominent in large gaps and cause the onset of breakdown at lower voltage values than expected from this theory. We only mention briefly the clump hypothesis which we shall discuss in more detail later. No satisfactory theory has yet been produced for large gap breakdown and a completely new line of attack may perhaps be necessary in order to develop a successful theory.

We shall start our review with a brief look at the theory of emission of electrons from metals; then we shall consider the energy exchanges between the electrons and electrodes and discuss the anode versus cathode control of breakdown. We shall show that this provides a rather satisfactory explanation of experimental data for electrical breakdown in small gaps or relatively low voltages (i.e. millimetre gap or hundred kilovolt region) whereas other hypotheses have to be considered for larger gaps and voltages.

Since there is clearly not enough time to mention all the important contributions in the literature, we have selected papers which are typical of the main trends of the present state of knowledge. We also have to apologize for some over-simplifications which are inevitable for the sake of a clearer presentation.
2. Emission of Electrons from Metals

The theories of thermionic and field emission of electrons from metals were worked out about half a century ago \(^3,4,5,6\), but until recently they were considered as separate theories, each one having its own range of temperature and field within which the corresponding expression of the current was valid. However, Murphy and Good \(^7\) in 1956, gave a unified treatment and obtained a general expression for the current as a function of electric field \(F\), temperature \(T\) and work-function \(\phi\), in the form of a definite integral. Then they applied approximating techniques for this integral and set up two distinct formulae for thermionic and field emission, with a characteristic dependence on \(F, T\) and \(\phi\) for each type of emission. An approximation for low fields and high temperatures leads to an extension of the Richardson-Schottky formula for field assisted thermionic emission. The validity of the approximation determines the range of temperature and field within which it applies. A similar treatment of the integral for high fields and low temperatures, gives an extension of the Fowler-Nordheim formula for field emission, with the corresponding range of temperature and field in which the approximation is valid. With another approximation they evaluated the integral in an intermediate region between thermionic and field emission. Outside these three regions the emitted current is obtained by numerical integration of the general expression for each set of values of the parameters.

Murphy and Good derived their general expression for the emitted current from the well established model of a metal: the Fermi-Dirac distribution for a free electron gas in the metal and the classical image force barrier at the surface. This model provides an expression for \(N(T, \zeta, \omega)\) \(d\omega\), that is the number of electrons per second per unit area incident on the barrier with an energy \(\omega\) within the range \(d\omega\). \(N(T, \zeta, \omega)\) is called the supply function, \(T\) is the absolute temperature, and \(\zeta\) is the Fermi energy, (see Fig. 1). This analysis also gives an expression for the penetration probability \(D(F, \omega)\), that is the probability that an electron incident with an energy \(\omega\) on the surface potential barrier emerges from the metal into the externally applied electric field \(F\). The emitted current density \(j(F, T, \zeta)\) is then obtained by integrating, over all accessible energies \(\omega\). The following expression is obtained:

\[
j(F, T, \zeta) = e \int_{-\infty}^{\infty} D(F, \omega) N(T, \zeta, \omega) \, d\omega
\]
in which \(e\) is the electronic charge.
Without going through the mathematical details of this interesting paper, we shall just examine the general expression for the emitted current density in figure 2. In the expression (2) \( y = (eF)^{0.5} / |W| \) and \( v(y) \) is a tabulated function. Within the regions of validity of the approximations we have mentioned, simpler expressions can be derived as shown in figure 2, expression (3) for thermionic emission and expression (4) for field emission. From these expressions the well-known Richardson-Schottky and Fowler-Nordheim formulae, written with the system of units and the symbols used by Murphy and Good, are respectively obtained within still more restrained regions of validity.

The boundaries for the three types of emission discussed by Murphy and Good are represented in figure 3 for a work function of 4.5 eV, a representative value for tungsten. At current densities of about \( 10^8 \) A/cm\(^2\), which are typically required for excessive heating of tungsten protrusions the emission of electrons is mostly due to tunneling through the barrier, i.e. the emission increases rapidly with the field and is relatively insensitive to temperature \( T \), as can be seen in figure 4.

3. Exchanges of Energy between Electrons and Electrodes

3.1. Protrusions on the cathode surface

Upon raising the voltage \( V \) between plane parallel electrodes a current is emitted at very localized points on the cathode surface. The value \( F \) of the field that determines the intensity \( I \) of the electron beam is not the average value of the field in the gap \( d \), \( F_0 = V/d \), but \( F = yF_0 \) at the apex of the emitting point. The enhancement factor \( y \) is determined by the shape of the point and by its height \( h \) expressed with respect to \( d \). Even when there are several emitting points, the factor \( y \) is determined, to a first approximation, by the sharpest one \( 11 \).

If the emitted current follows the Fowler-Nordheim formula, then the expression \( \log (I/V^2) \) is a linear function of \( 1/V \) in which the factor \( y \) enters in a simple way. In fact the current measured as a function of \( V \), in any experiment not interrupted by a spark which would destroy the emitting point, appears generally to follow the Fowler-Nordheim formula quite well so that a straight line is obtained in the F-N graph, that is when \( \log (I/V^2) \) is represented versus \( 1/V \) as in figure F. From the slope of this straight line \( m = A \phi^{1.5} d/y \), where \( A \) is a known constant, an experimental determination of the value \( y \) of the emitting point is easily obtained, provided \( \phi \) is known.
By making some assumptions on the geometrical shape of the protrusion, for example by considering an ellipsoid or a cylindrical rod with a hemispherical cap \(^8,9\), the corresponding \(\gamma\) values can be calculated from electrostatic principles. Experimental values of \(\gamma\), deduced from \(F-N\) graphs, are currently observed in the range 1 (for extremely small gaps) to a few hundreds (for millimetre gaps) \(^9,10\). When comparing the theoretical values with the experimental values obtained as a function of the gap \(d\) the geometrical parameters of protrusions are obtained: values of about 1 \(\mu\)m and 0.1 \(\mu\)m, within an order of magnitude each, are generally obtained for the height and the diameter of the protrusions respectively. These dimensions pertain to protrusions of simple geometrical shapes but direct geometrical measurements by using electron microscopes have confirmed that the dimensions thus estimated are of the right order of magnitude \(^11,12\).

When the emitted current density exceeds \(10^7\) \(A/cm^2\) electron space-charge effects reduce the true field \(F\) at the emitting surface appreciably below the value calculated from electrostatics. The field emission formula is still valid provided the true value of the field is used. The space-charge effects at current densities of about \(10^8\) \(A/cm^2\), which are often attained in practice, strongly reduce the influence of variations in the work function on the emitting surface \(^8\).

In general the presence of protrusions on the surface cannot be avoided and is primarily responsible for the limited voltage strength of vacuum gaps. Even perfect polishing of the surface would not be of great help since the surface condition can deteriorate quite quickly with time under the action of temperature, applied field and ion bombardment. It has also been shown by Little and Smith \(^13\) that projections are removed from the anode by the electric field and these produce craters and sharp field-enhancing protrusions on impact with the cathode. The growth of the protrusions has been studied by Charbonnier et al. \(^14\), using Herring’s theory, whose fundamental postulate is that the flux of atoms being transported past a certain point on the surface of a protrusion is proportional to the product of a diffusion coefficient and the local gradient of chemical potential at that point. When the average field exceeds a certain value the protrusion will grow indefinitely and produce a breakdown. They have also studied the formation and growth of protrusions by the action of sputtering.
3.2 Electron-cathode energy exchanges

The field-emission induced heating of cathode protrusions has been studied theoretically in detail by Lee [15] and by Charbonnier et al. [8,16]. Resistive heating alone cannot explain the stability of field emission at current densities high enough to raise the tip temperature to 2000 °K or higher: a balance of Joule heating with thermal conduction and radiation losses is impossible, leading to an unstable situation. Furthermore with a low work function coating on the tungsten tip one can draw a higher current density while maintaining a lower tip temperature which is impossible with heating by Joule effect alone. For these reasons the Nottingham effect clearly comes into account in the thermal balance of the protrusion tip.

The Nottingham effect is due to the difference between the average energy \( \langle E \rangle \) of the emitted electrons and that \( \langle E' \rangle \) of the replacement electrons which is assumed to be the Fermi energy in the simple Sommerfeld free electron model of metals. At room temperature \( \langle E \rangle \) is lower than the Fermi energy so that the cathode is heated by the Nottingham effect. As the temperature is raised the energy distribution of the emitted electrons is modified and becomes symmetrical with respect to the Fermi level at the transition temperature. At higher temperatures the Nottingham effect changes its sign as \( \langle E \rangle \) becomes higher than \( \langle E' \rangle \) and cooling of the cathode results from the absorption of energy by the electron beam. At these temperatures the Joule and Nottingham effects have opposite actions on the tip temperature. The transition temperature is \( T_1 = 5.35 \times 10^{-5} \frac{F}{\theta^{0.5}} \), that is \( T_1 = 1250 \, ^0K \) at \( F = 5 \times 10^7 \, V/cm \) for tungsten (\( \theta = 4.5 \, eV \)).

Experimental data confirm this analysis of the cathode protrusion heating process but a more detailed analysis, taking into account a possible decrease of the average charge carrier energy \( \langle E' \rangle \) with increasing temperature, is required to explain why the transition temperature is lower than expected from the formula for some metals in which the band structure does not conform to the simple Sommerfeld model [16].

The theory may be applied to estimate the tip temperature of projections of simple geometrical shapes, for example cones or cylinders with hemispherical caps. When computing the critical current which brings the tip to melting temperature the duration of the voltage pulse has to be taken into account. Three different cases may be considered:
1) The pulse is longer than a thermal time constant $t_1$. Then the steady-state distribution of the temperature is reached for the protrusion standing on the cathode surface.

2) The pulse is very short: shorter than a smaller time constant $t_2$. This is the adiabatic case where no appreciable heat diffusion can take place and the local temperature is governed by the local heat generation.

3) The pulse duration is intermediate between $t_1$ and $t_2$.

The values of $t_1$ and $t_2$ are given by the theory 8,16) and for pulses longer than $t_1$ or shorter than $t_2$ simple expressions for the critical currents as functions of the relevant parameters are obtained. From these the corresponding critical voltage may be calculated.

4. Breakdown across Small Gaps

4.1 Cathode initiation of breakdown

Field emission at high current density does not necessarily lead to breakdown as pointed out by Charbonnier 8), since dc emission densities in excess of $10^7$ A/cm$^2$ or pulsed emission densities in excess of $10^8$ A/cm$^2$ have been maintained without instability for operating periods of several thousand hours. However, excessive current density causes excessive heating of the tip and evaporation of cathode material. This metal vapour has a high probability of being ionized in the very high density, low-energy electron beam just outside the emitting surface. Production of positive ions in front of the tip reduces the electron space-charge effect and therefore increases the field emitted current. Furthermore the cathode tip is bombarded by these positive ions which can liberate more electrons so that these two regenerative processes lead to an extremely rapid development of a full vacuum arc in times of the order of 10 nanoseconds. The Nottingham effect does not markedly affect these general conclusions.

Experimental studies have shown the onset of rapid instability when the protrusion tip reaches a critical temperature corresponding to a metal vapour pressure of $10^{-4}$ Torr. The critical current density $j_c$ does not differ widely

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from one material to another and is about $10^8 \text{ A/cm}^2$ for a typical conical projection 16).

The rate of increase of the breakdown voltage, as pulse length $t_0$ becomes smaller, is quite low since, in the adiabatic limit, the permissible current density $j_M$ is proportional to $t_0^{-0.5}$. Now since $j$ in the region of $10^8 \text{ A/cm}^2$ varies approximately as $V^2$, the breakdown voltage increases as $t_0^{-1/6}$. Therefore the voltage increase is only 40 to 50% when the pulse duration is reduced by a factor ten.

4.2 Anode versus cathode control of breakdown

We have so far discussed the emission-induced heating of the cathode protrusions. However, there is an additional heating effect: the field-emitted electrons, accelerated by the field across the gap, cause anode heating on impact with the anode. Breakdown is likely to occur when the temperature at either electrode becomes high enough locally to cause significant evaporation of electrode material into the path of the electron beam. Whether this critical condition first occurs at the cathode or at the anode depends on the gap geometry, on the field enhancement factor $\gamma$, on the relative electrode materials and on whether the gap voltage is dc or pulsed, as discussed by Charbonnier 8).

Chatterton 17) has pointed out that in the case of dc voltages and parallel plane electrodes it is virtually certain that excessive heating will occur at the anode well before the electron current density reaches a level sufficient for excessive protrusion self-heating at the cathode.

From the radial spreading of the electron beam emitted from a protrusion in the case of parallel plane electrodes the beam power density at the anode may be calculated as a function of the current density and of the corresponding field on the protrusion $8,10,17,16$; the values of these two quantities should be smaller than the values obtained from the conditions of cathode-controlled breakdown since we are now interested in anode-controlled breakdown. If the values corresponding to cathode-controlled breakdown are used in the expression of the anode power density the maximum possible value of this power density is obtained. The breakdown is anode-controlled if this maximum possible value of the power density produces excessive heating of the anode.
We may define two thermal time constant values \( t_1 \) and \( t_2 \), corresponding respectively to the steady-state and adiabatic conditions of anode temperature distribution, in order to assess the critical or maximum admissible anode power density. The value of the voltage pulse duration \( t_0 \) relative to \( t_1 \) and \( t_2 \) determines which expression for critical anode power density has to be considered and then the corresponding criterion of anode-controlled breakdown is obtained simply by stating that the maximum possible value of the anode power density exceeds the critical value \( 8,19 \). This criterion turns out to be a simple expression for the maximum value \( \gamma_M \) of the cathode field enhancement factor \( \gamma \). If no protrusion on the cathode has a \( \gamma \) value larger than \( \gamma_M \), then the breakdown is induced by excessive heating of the anode. When the voltage pulse duration \( t_0 \) decreases the relevant value of \( \gamma_M \) also becomes smaller; the breakdown which is generally anode-controlled in dc conditions is finally cathode-controlled at very short pulse durations.

Utsumi and Dalman \( 10 \) have studied theoretically and experimentally what are the respective conditions for anode versus cathode control of dc breakdown between plane parallel electrodes as a function of their separation. The type of breakdown which is likely to occur can be determined from the dimensions of the projection and the thermal and electrical properties of the electrode materials. As the separation \( d \) is increased the dominant mode of breakdown is first the anode type when the value of the enhancement factor \( \gamma \) is small, then the cathode type with which is associated a critical current of constant value, and finally, the anode type again when \( d \) reaches the 0.1 - 1 mm region and the corresponding critical current varies as \( d^{-0.5} \).

The analysis of vacuum breakdown, which we have just presented in simplified form, is sufficient to provide estimates of the breakdown voltage and indication of the necessary electrode conditions for initiation, which are confirmed experimentally \( 10,19 \), at least for small gaps in clean vacuum conditions.

5. Breakdown across Large Gaps

5.1 Failure of the electron emission induced breakdown theory

The experimental data obtained for breakdown across large gaps \( 20 \) to \( 27 \), that is at higher voltages (say above 100 kV), do not agree well with the theory.
we have just presented, which is based solely on the heating power of the field emission-induced electron beam. The large gap studies with plane parallel electrodes have revealed several facts which cannot be explained by this theory at the present time; these are:

1) The breakdown field $F_{OB}$ is a decreasing function of the gap $d$. The experimental data can be fitted to the following empirical expression:

$$F_{OB} = A d^{-\alpha} \quad \text{with} \quad 0.3 < \alpha < 0.8$$

2) The steady current value just below the microdischarge threshold is strongly dependent upon the gap $d$ in the range 1 - 6 cm: Whereas in a 0.5 cm gap between stainless steel electrodes in ultrahigh vacuum conditions hundreds of microamps can be drawn without causing breakdown, it is impossible to obtain more than a few nanoamps when the gap is increased to 5 cm.

3) The breakdown field increases with pressure when gas is admitted to the vacuum chamber up to the region where Paschen's gas discharge law becomes applicable, which is usually in the millitor range. This effect, which was first demonstrated clearly for separators at CERN in 1959, had been noted in the early fifties by a few authors.

The effect of the gas pressure on the microdischarge threshold and on the breakdown fields is stronger the larger the gap, (figure 6).

When discussing the cathode initiation of breakdown we have established the existence, in this case, of a critical value of the cathode emitted current which is independent of the gap $d$. Since the value of the enhancement factor $\gamma$ is independent of $d$ for large gaps, the associated critical average field $F_{OB}$ should also be independent of $d$.

The first and second points just mentioned, as well as our earlier discussion of anode versus cathode control of breakdown showing that anode-controlled breakdown is dominant when $d$ is larger than 0.1 - 1 mm, rule out the possibility of breakdown controlled by the mere vaporization of cathode whisker tips in large gaps.
If we now consider the possibility of breakdown controlled by the vaporization of a small amount of anode material under the action of the field-emitted electron beam impinging on the anode, we have to remember, according to Utsumi and Dalman \(^{10}\), that in this case the critical current varies as \(d^{-0.5}\). This condition is obtained by imposing the condition that the temperature of the heated anode spot centre be kept constant, at the melting point for instance, when the separation between electrodes is increased. The experimental results mentioned in point (2) show that the microdischarge threshold current varies by several orders of magnitude instead of half an order when the gap is increased from 0.5 to 5 cm, which rules out also this theory for microdischarges initiation. This conclusion is valid also for breakdown initiation, since microdischarges develop into full discharges, or breakdown, at a higher voltage and can therefore be considered as being governed by the same law.

5.2. The microparticle breakdown initiation hypothesis

In 1952 Cronberg \(^{28}\) put forward the hypothesis that breakdown is initiated by tiny bits of material or clumps, impinging on the electrodes under the effect of the electric field. The breakdown, in this clump theory, is the consequence of the electrical gas discharge taking place in the vapour generated when a high-speed microparticle collides with one of the electrodes. Assuming that the micro-particle gets detached from one electrode, it carries along an electrical charge \(q\), which is proportional to the field, and it has an energy \(W = q \cdot V\) after crossing the gap when it impinges on the other electrode. A criterion for breakdown is obtained by simply stating that breakdown will take place when this energy is larger than a critical value \(W_{\text{crit}}\).

The breakdown field-gap relationship derived from this simple clump theory, in the case of plane parallel electrodes, is: \(F_{\text{OB}}^{2} \times d = \text{constant}\). When comparing this formula with the experimental results mentioned in point (1), above no discrepancy appears but the range of experimentally possible values for \(\alpha\) is too broad for this to be a strong confirmation of the clump theory. Furthermore by refining the clump theory, the theoretical value obtained for \(\alpha\) can be made to differ slightly from 0.5, so that some direct experimental confirmation not only of the existence of clumps but also of their triggering action in the breakdown process is necessary. Many attempts are made in that direction.
and we only mention briefly a method which was first proposed by C. Germain at CERN for detecting clumps during their flight across the gap in order to establish directly their triggering action in the breakdown process. This method is based on the detection of the light reflected by the clump when it crosses at right angles a wide laser beam in the gap between the electrodes.

The microdischarges we have mentioned in the introduction are observed at voltages lower than the breakdown voltage. They are essentially self-extinguishing bursts of current, that is they do not involve a complete collapse of the voltage across the gap. On the basis of the clump hypothesis, a micro-discharge and a full discharge can be caused by the same process, that is by a more or less complete vaporization of the microparticle when it impinges on the opposite electrode. In the case of a microdischarge one can assume that the amount of material vaporized do not generate a bubble of vapour large enough to induce a diverging gas discharge process.

In the clump theory the microdischarge and the full discharge are considered as being unrelated to the steady current that exists simultaneously between electrodes since no proposed clump extraction or generation process under the action of this steady current can account for the very large variation of the micro-discharge threshold current as a function of the gap in the centimetre range. The clump can rather be considered as detached by the action of the electrostatic pressure from the electrode to which it is loosely bound. The exact process of this extraction is not yet known and this is the first of a series of questions which must be asked about the theory of breakdown across large gaps. Little and Smith 13) have reproduced natural breakdown phenomena in a 0.25 mm gap by intentionally inserting nickel particles about 0.01 mm in size into a small hole on the anode surface and they evaluated the critical clump energy to be a few nanojoules. They demonstrated that in natural breakdown clumps had been extracted from anode pits located at impurity centres near grain boundaries and had produced sharp-field-enhancing protrusions on the cathode surface. No such experimental work has yet been performed for large gaps and high voltages.

A regenerative process is required for the creation of clumps, otherwise it would be possible to condition electrodes once and for all at a given voltage provided the working voltage is kept lower than this value in a constant gap.
The growth of protrusions under the action of the electrostatic pressure is a possible regenerative process which is well known experimentally but it is only valid for small gaps since it is associated with a given field or current level. Any proposed clump regeneration process, based on the effect of either the current or the field, must account for the respective variations of the critical values with the gap length.

The marked influence of the residual gas pressure on breakdown voltage mentioned earlier, which is of importance in some practical applications, is also difficult to explain completely on the basis of the clump theory because of the gap dependence of both the microdischarge threshold current and the breakdown field. The influence of the pressure on the breakdown voltage in small gaps has been investigated recently and it can be explained by the selective ion bombardment of the sharpest points of the cathode: the corresponding values of the field enhancement factor are thus reduced by sputtering so that a higher value of the average field can be sustained. Such an explanation was also attempted for large gaps but the mechanism of this influence is not yet completely understood.

6. Conclusion

The initiation and the development of electrical discharges in vacuum across small gaps between clean metal electrodes can be described satisfactorily in terms of the gap geometry, cathode surface microgeometry and the physical properties of the electrode material. The relevant theory is based on a detailed analysis of the electron field emission from the cathode and on the associated current thermal interaction with the electrode. However this theory fails to fit the experimental data obtained with large gaps: in this case the hypothesis of breakdown initiation by microparticles which are accelerated across the gap by the electric field, provides an explanation which has received some experimental confirmation, but is not fully satisfactory. Further investigations, both theoretical and experimental, are required in order to develop a completely quantitative theory of breakdown across large gaps.

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$V(x) = \text{Potential energy of an electron near the metal surface}$

$X = \text{Coordinate normal to the surface}$

$\xi = \text{Fermi energy}$

$\phi = \text{Work function}$

$-W_a = \text{Effective constant electron potential energy inside the metal}$

Energies are measured from zero for a free electron outside the metal, then $\phi = -\xi$

$-F = \text{Externally applied electric field}$

$V_{\text{max}} = -(e^3 F)^{1/2} = \text{Maximum value of the potential energy}$

$W_L = \frac{1}{\sqrt{2}} V_{\text{max}}.$
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General expressions:

\[ j(F, T, \xi) = e^{\int_{-W_a}^{\infty} D(F, W) N(T, \xi, W) \, dW} \]

\[ j(F, T, \xi) = \frac{kT}{2\pi^2} \left\{ \int_{-W_a}^{W_L} \frac{\log[1 + \exp\left(\frac{W - \xi}{kT}\right)] \, dW}{1 + \exp\left(\frac{4V/2}{3} \cdot \frac{v(y)}{F^{0.25} y^{1.5}}\right)} + \int_{W_L}^{\infty} \log\left[1 + \exp\left(\frac{W - \xi}{kT}\right)\right] \, dW \right\} \]

Approximate expressions:

Thermionic emission

\[ j = \left(\frac{kT}{\sqrt{2\pi t}}\right)^2 \frac{\phi d}{\sin \pi d} \exp \left(\frac{-\phi - F^{0.5}}{kT}\right) \]

Field emission

\[ j = \frac{1}{\phi} \left(\frac{F}{4\pi t}\right)^2 \frac{\pi c kT}{\sin \pi c kT} \exp \left(\frac{-4V/2 \phi^{1.5}}{3F}\right) \]

Current density \( j(F, T, \xi) \) as a function of electric field \( F \), temperature \( T \) and Fermi energy \( \xi \), in Hartree units, according to Murphy and Good 7)

Fig. 2

Fig. 3 - The three emission regions for a 4.5 eV work function (tungsten) according to Murphy and Good 7)
Fig. 4. Current density $j$ as a function of temperature $T$ and field $F$ for $\phi = 4.5\,\text{eV}$

(F. M. Charbonnier$^8$)

Fig. 5. Typical F.N. graph

(D. Alpert et al$^9$)
Fig. 6 — Typical voltage-gap graph for various values of the gap current and for two different pressures.