SOME TOPICS IN THE LOW-ENERGY PHYSICS FROM SUPERSTRINGS^{*)}

L.E. Ibáñez

CERN - Geneva

ABSTRACT

I discuss several aspects of the class of supersymmetric models which could form the low-energy physics of the $E_8 \times E_8$ heterotic string. The topics treated include supersymmetry breaking, Peccei-Quinn symmetries, low-energy scale invariances, dilaton-axion physics and superstring-inspired low-energy phenomenological models. Models with two, one and no extra Z' 's are discussed.

^{*)} To appear in the proceedings of the NATO Workshop on "Super Field Theories", Vancouver, July 1986, to be published by Plenum Press.

INTRODUCTION

The recent revival of string theories¹⁻⁵ was mostly motivated by purely phenomenological reasons. The experimental results obtained during the last decades indicate that quarks and leptons come in chiral (complex) representations of a gauge group $SU(3) \times SU(2) \times U(1)$ (or some extension). The important point of the heterotic $E_8 \times E_8$ superstring⁴ is that, since it contains explicit gauge bosons, one may obtain chiral low-energy fermions which transform like the standard quarks and leptons under an $SU(3) \times SU(2) \times U(1)$ subgroup of $E_8 \times E_8$. Still, the theory has no gauge or gravitational anomalies as shown for the field theory limit by Green and Schwarz². To some extent, the parity-breaking world we observe now is related to the left-right asymmetric construction of the heterotic string. This asymmetric construction is required to have explicit gauge bosons, which in turn are required to get low-energy chiral fermions (quarks and leptons). It is obvious that the possible phenomenological applications of the $E_8 \times E_8$ string are at the root of the present interest for this theory. Other strings (e.g., the anomaly-free chiral type IIB superstring) are simpler and have less contrived constructions, but are certainly less popular, since their phenomenological uses do not seem very bright.

It is reasonable to expect that, in the same way that chirality and observed gauge interactions were important in selecting a superstring candidate, other low-energy phenomenological constraints (an obvious example is the number of generations) could give us further information

on the superstring dynamics. This is particularly the case for the compactification process. Understanding the enormous hierarchy of scales between the weak scale and the compactification mass probably requires as a technical ingredient the existence of an unbroken supersymmetry in four dimensions. If, on the other hand, we insist on maintaining an unbroken supersymmetry when compactifying the superstring⁵, the possible form of the six-dimensional compact manifold becomes severely restricted [Kähler, SU(3)-holonomy manifolds]. It is in fact interesting that there seem to be two independent reasons to require supersymmetry in our fundamental theory: it seems to be necessary at low energies in order to understand the hierarchy problem, but supersymmetry also gives us the simplest way to obtain tachyon-free string theories.

If one unbroken supersymmetry is left after compactification, the effective low-energy theory will be $N = 1$, $d = 4$ supergravity coupled to chiral matter and gauge bosons⁶. In the case of the $E_8 \times E_8$ heterotic string, all the presently-known gauge interactions should be included in one of the E_8 's due to the direct product structure. The other E_8 will interact with the known world only through gravitational interactions; it will be a "hidden sector" of the theory. Remarkably enough, this structure for the low-energy limit of the $E_8 \times E_8$ string is very reminiscent of what are called "low-energy supergravity models"⁷⁻⁹. These are $N = 1$, $d = 4$ supergravity models coupled to quarks, leptons and gauge bosons. The minimal model is just an $SU(3) \times SU(2) \times U(1)$ theory coupled to three usual families of chiral superfields plus Higgs particles. In these theories, supersymmetry breaking takes place in a "hidden sector" of the theory⁸ and it is transmitted to the observable quark-lepton-Higgs world only through gravitational interactions. After SUSY breaking, one is left at low energies with a theory with softly-broken supersymmetry having soft parameters m (universal scalar masses), M (universal gaugino masses) and other soft couplings proportional to the superpotential. A remarkable fact of this class of models is that the $SU(2)_L \times U(1)_Y$ symmetry is broken in a natural way as a radiative effect of supersymmetry breaking^{9,10}. It is certainly interesting that this phenomenologically successful structure seems to be embeddable inside the $E_8 \times E_8$ string.

Low-energy physics could give us further restrictions on the superstring (particularly the compactification) dynamics. If, in the next generation of accelerators (Tevatron, SLC, LEP, HERA, LHC, SSC),

supersymmetric particles are found, their masses could give us interesting information on the SUSY-breaking dynamics and about the symmetries of the compactifying vacuum. Thus, e.g, if $m \ll M$, it could be an indication of an approximate scale invariance in the original Lagrangian. Also the ratios between the masses of different sparticles (e.g., sleptons and squarks) would give independent tests of the unification idea. It is also important to search for the existence of possible extra Z' 's, whose couplings could tell us a lot about the compactification and gauge-symmetry-breaking dynamics. Of course, quark masses and mixings (Yukawa couplings) also contain potential information about the compactification dynamics, but it will probably be highly non-trivial to extract such information. However, qualitative features such as, for example, the observed hierarchy of fermion masses (or the size of the CP-violating phase) could guide the search for interesting compactifications. Thus a "superstring-inspired" phenomenology could be very fruitful in providing constraints on the superstring dynamics. While this is probably correct, we are far away from a situation (which may never come) in which the superstring gives us any definite prediction able to be tested at low energies. Hence, most probably, "experimental" support for the superstring idea will only come from "circumstantial evidence" based on the possibility of these theories containing all the observed dynamics. I am going to discuss here some topics in "superstring-inspired" phenomenology¹¹⁻¹³ considered in the spirit explained above. It is not the search for experimental consequences of the $E_8 \times E_8$ superstring, but an effort to understand how one could embed the low-energy physics inside it.

SUPERSYMMETRY BREAKING, GAUGINO CONDENSATION, DILATONS, AXIONS, ETC.

The massless sector of the heterotic string consists of a ten-dimensional supergravity¹⁴ sector which includes bosonic (scalar ϕ ; antisymmetric B^{MN} ; g^{MN}) and fermionic (gravitino ψ^M ; spinor λ) fields, and a Yang-Mills sector (gauge bosons A^M and gauginos χ^α) in the adjoint of $E_8 \times E_8$. The bosonic Lagrangian contains the terms

$$E^{-1} \mathcal{L} = -\frac{1}{2} R + \frac{9}{16} \left(\frac{\partial_M \Psi}{\Psi} \right)^2 - \frac{1}{4} \Psi^{-3/4} F_{MN}^a F_a^{MN} + \frac{3}{4} \Psi^{-3/2} H_{MNP} H^{MNP} + \dots \quad (1)$$

plus higher derivative terms. The scalar "dilaton" ϕ plays the rôle of a coupling constant and H_{MNP} is the field strength for B_{MN}

$$H_{MNP} = \partial_{[M} B_{NP]} - (\omega_{MNP}^G - \omega_{MNP}^L) \quad (2)$$

where ω^G and ω^L are the gauge and Lorentz Chern-Simons symbols^{2,14}. The tree-level Lagrangian has a classical invariance under the rescalings¹⁵

$$\left. \begin{aligned} g_{MN} &\rightarrow \lambda g_{MN} \\ \psi &\rightarrow \lambda^{-1/3} \psi \\ \psi_A, \chi^\alpha &\rightarrow \lambda^{-1/4} \psi_A, \chi^\alpha \end{aligned} \right\} \rightarrow E^{-1} \mathcal{L} \rightarrow \lambda^{-1} E^{-1} \mathcal{L} \quad M, N, A = 1-10 \quad (3)$$

The whole effective action is not invariant under this symmetry, but all n-loop contributions scale in a definite way since the power of λ measures the number of loops. The existence of this classical scale invariance turns out to be interesting when analyzing the low-energy symmetries.

In order to study the low-energy physics, we have to compactify our ten-dimensional theory down to four dimensions. A first possibility to consider is a compactification at the pointlike field theory level. In this case, if we insist on conserving an unbroken supersymmetry⁵, the extra dimensions must curl into a compact Kähler manifold of SU(3) holonomy (Calabi-Yau or some other more or less related type of manifold). Alternatively, one could perhaps compactify the theory already at the string level. This is what is done when compactifying¹⁶ on an "orbifold" [some type of six-torus modded out by some discrete subgroup of SU(3)]. This latter approach is very promising since it keeps much of the simplicity of the original torus manifold. Whatever the compactification procedure may be, up to now no concrete example of compactification completely consistent with phenomenological constraints has been given in the literature (see, however, Ref. 17). Still, there are some general properties of the low-energy states expected to be present in any supersymmetry-preserving compactification, as argued in Ref. 15.

Amongst the $d = 4$ massless (at least at the tree level) states which one expects, there is a dilaton ϕ (from the original $D = 10$ dilaton ϕ) and another dilaton σ associated to the size of the compact manifold

$$E = \det |g_{MN}|^{1/2} = e^{-3\sigma(x)} e_4 ; \quad V_6 = e^{3\sigma(x)} \quad (4)$$

Since the low-energy theory is assumed to be supersymmetric, these scalar fields must have a couple of massless pseudoscalar partners. These zero modes $\theta(x)$ and $\eta(x)$ come in fact from the antisymmetric field B_{MN} :

$$H_{\mu\nu\rho} \sim \epsilon_{\mu\nu\rho\sigma} \partial^\sigma \theta(x), \quad \mu, \nu, \rho, \sigma = 1-4 \quad (5a)$$

$$B_{i,\bar{i}} = \epsilon_{i\bar{i}} \eta(x) \quad (5b)$$

$i = \text{SU}(3)$ tangent $O(6)$ index

The precise combination of dilaton fields which are partners of these pseudoscalars $\theta(x)$ and $\eta(x)$ may be shown to be¹⁵

$$\begin{aligned} S &= \varphi^{-3/4} e^{3\sigma} + i\theta \\ T &= e^\sigma \varphi^{3/4} + i\eta \end{aligned} \quad (6)$$

and they have supersymmetric fermionic partners from the $D = 10$ fields ϕ_m and λ . All these fields are gauge singlets which interact with usual matter only gravitationally and hence form part of the "hidden sector" of the theory. The quark, lepton and Higgs fields are expected to arise from zero modes of the Yang-Mills sector, A^m ($m = 5-10$) and χ^α .

Several symmetries are expected to appear in the low-energy theory:

a) Peccei-Quinn Symmetries¹⁸⁻²²

The pseudoscalars $\theta(x)$ and $\eta(x)$ only have derivative couplings, since the field B_{MN} always appears through its field strength $H_{MNP} = \partial[M^B_{NP}] + \dots$. Thus there are two P-Q invariances under

$$\begin{aligned}\theta &\rightarrow \theta + c \\ \eta &\rightarrow \eta + c'\end{aligned}\tag{7}$$

This implies that the Kähler potential G , which determines the low-energy Lagrangian, depends on the dilaton superfields S and T only through the combinations $(S+S^*)$ and $(T+T^*)$. These symmetries are, however, broken by both space-time and string-world-sheet non-perturbative effects, as we will discuss below. Let us also remark that there could in fact be extra zero-mode light superfields originating in g_{MN} and B_{MN} apart from those considered above (e.g., moduli).

b) "S-scale Invariance"¹⁵

The $D = 10$ scale invariance in Eqs. (3) remains in four dimensions, since we assume that the compactification obeys the equations of motion and no tadpole for ϕ is produced. The form of this four-dimensional invariance is^{15,11,23}

$$\left. \begin{aligned} S &\rightarrow \lambda S \\ T, C_x &\rightarrow T, C_x \\ g_{\mu\nu} &\rightarrow \lambda g_{\mu\nu} \\ \psi_\mu &\rightarrow \lambda^{1/4} \psi_\mu \\ \chi &\rightarrow \lambda^{-3/4} \chi \end{aligned} \right\} \rightarrow e^{-1} \mathcal{L} \rightarrow \lambda^{-1} e^{-1} \mathcal{L}\tag{8}$$

where the fields C_x are the gauge non-singlet scalar fields. Of course, this is a classical scale invariance and any loop effect will break this symmetry. Notice that the T and C_x fields are left untouched by this symmetry, whereas S is not; that is why I call it S-scale invariance here. Notice also that the classical scalar potential of the truncated theory, since it obviously implies no space-time contractions, has necessarily to be of the general form¹¹

$$V(S, T, C_x) = \frac{1}{(S+S^*)} V(T+T^*; C_x, C_x^*)\tag{9}$$

with $V(T+T^*; C_x)$ scale invariant, in order to scale like λ^{-1} . Recalling the general $N = 1$, $d = 4$ supergravity expression for the scalar potential

(proportional to e^G), one obtains that at the classical level the S -dependent part of the Kähler potential G_s should be of the form

$$G_s = -\log(s + s^*) \quad (10)$$

The existence of this classical scale invariance dictates²³ also the form of the gauge kinetic function $f(S, T, C_x)$. This must be an analytic function of its arguments and induces the following bosonic terms in the Lagrangian

$$\text{Re} f_{ab} F_{\mu\nu}^a F_b^{\mu\nu} \quad ; \quad \text{Im} f_{ab} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^b \quad (11)$$

In order that these terms scale like λ^{-1} , the function f must necessarily be linear in S . One then has at the classical level^{24,23}

$$f_{ab} = \delta_{ab} S \quad (12)$$

One could, in principle, multiply f by an analytic product of scale-invariant fields like T and S , but one can easily see that the analyticity constraint plus the other T -scale invariance we define below forbids that possibility. Notice that the result (12) is the same for both observable and hidden sectors, i.e., for the whole of the unbroken $E_8 \times E_8$ generators.

c) "T-Scale Invariance"^{11,23}

The existence of this extra invariance is related to the fact that in the simplified tree-level analysis we are assuming that the compactification scale, which is related to $\text{Re}T$, is not determined. It can be intuitively understood as follows¹¹. In the compactification procedure, wherever there is an internal space contraction of indices, one gets a $(T+T^*)^{-1}$ factor from the internal metric g_{mn} ($m, n = 5-10$). The matter scalar fields C_x come from the A_m 's ($m = 5-10$), and hence whenever you have a field C_x in your Lagrangian, you expect a $(T+T^*)^{-\frac{1}{2}}$ factor from the internal contraction. Thus there should be a scale invariance under¹¹

$$\left. \begin{array}{l} T \rightarrow (X)^2 T \\ C_x \rightarrow \lambda' C_x \\ S \rightarrow S \end{array} \right\} \quad \lambda \rightarrow \lambda \quad (13)$$

This is also only a classical scale invariance since, as I will show below, there are loop terms which spoil it. This additional symmetry also restricts significantly the form of the tree-level Kähler potential. If the Kähler potential G is to contain the standard piece $\log |W|^2$ (W = superpotential), the only way to make it scale invariant^{11,23} is to add a term

$$-3 \log (T+T^* + \alpha_i |C_i|^2) + \log |W|^2 \quad (14)$$

where the factor 3 implies that the superpotential is trilinear and the α_i are just constants which may be different for different $SU(3) \times SU(2) \times U(1)$ representations (or families). Thus, from general symmetry considerations, one expects a tree-level Kähler potential for the low-energy $N = 1$, $d = 4$ theory of the general form^{15,23,11}

$$G_0 = -\log (S+S^*) - 3 \log (T+T^* + \alpha_i |C_i|^2) + \log |W|^2 + F((T+T^*)/|C_x|^2) \quad (15)$$

where we have added²³ an arbitrary function F of the scale-invariant combination $(T+T^*)/|C_x|^2$. The second term in (15) is very similar to the one appearing in the so-called "no-scale models"²⁵, but there are a couple of important differences [apart from the other terms in (15)]. First, an $SU(n)$ symmetry (n = number of chiral superfields) is not expected. Second, the term in Eq. (15) is just a tree-level term and it is known that the radiative corrections²⁶⁻²⁹ and non-perturbative effects³⁰ spoil its structure (since it has its origin in a classical invariance). This is to be contrasted with the very assumptions of the "no-scale" idea²⁵, in which the second term in Eq. (15) is assumed to represent the exact Kähler potential (including radiative gravitational corrections). Let us comment that the result in Eq. (15) can also be obtained (in a less general form) by performing a supersymmetry truncation of the $D = 10$ supergravity + Yang-Mills Lagrangian, either from the bosonic¹⁵ or the fermionic²¹ sectors.

There are many sources of corrections for the above low-energy interactions. First, there are higher derivatives and/or higher power in the inverse compactification scale terms coming from higher terms in the string world-sheet σ -model. This includes all tree-level string exchanges amongst massless external fields. Amongst these terms are

$$R^2 ; R^4 ; (R^2 - F^2)^2 ; H^4 \dots \quad (16)$$

couplings which have been shown to appear in string scattering amplitudes³¹. From the four-dimensional point of view, terms of the form¹¹

$$\frac{\left| \frac{\partial W}{\partial c_x} \frac{\partial W}{\partial c_y} d_{xy} \right|^2}{(S+S^*)(T+T^*)^4} ; \frac{\left| \frac{\partial W}{\partial c_x} \right|^4}{(S+S^*)(T+T^*)^4} ; \frac{|W|^4}{(S+S^*)(T+T^*)^6} ; \dots \quad (17)$$

respecting the symmetries described above will be induced in the scalar potential. Notice also that the Cremmer et al. formalism⁶ for $N = 1$, $d = 4$ supergravity only includes up to two derivatives and the $D = 10$ terms in Eq. (16) include more. Thus, a formalism including higher derivative terms in $d = 4$, $N = 1$ supergravity is also required for a complete description of the low-energy theory³².

The terms in Eqs. (16) and (17) are tree level and that explains why the classical invariances described above still apply. Loop effects both in the effective low-energy Lagrangian²⁶⁻²⁸ and also including the effect of the string excitations²⁹ violate those symmetries. The same is true for non-perturbative effects, both in Minkowski space²⁹ and on the string world-sheet³⁰. We will consider these effects below. Let us start with the related problem of residual $N = 1$, $d = 4$ supersymmetry breaking.

The most appealing way to break the residual supersymmetry seems to be gaugino condensation^{33,24} in the hidden sector of the theory. It is well known that in the presence of non-minimal gauge kinetic terms³⁴, gaugino condensation $\langle \bar{\chi}\chi \rangle = \Lambda^3 \neq 0$ may break supersymmetry and give a mass to the gravitino $m_{3/2} \sim \Lambda^3/M_P^2$. Also in this scheme one may understand the smallness of the gravitino mass (and hence the weak scale) since $\Lambda \sim e_{M_P}^{-1/(2g^2 b_0)}$ can be exponentially small. It is certainly

interesting that this structure is given for free in the $E_8 \times E_8$ heterotic string. Since, in a simple compactification scheme, the vev of S [and hence the value of the coupling constant $g = (\text{Re } S)^{-1/2}$] is initially undetermined, instead of the creation of a condensate one can talk of an induced non-perturbative superpotential²⁴:

$$W_S \sim (e^{-1/2g^2 b_0} M_P)^3 \sim e^{-3\frac{S}{2b_0}} M_P^3 \quad (18)$$

This term obviously violates the classical S -invariance (it is a non-perturbative effect) and also the Peccei-Quinn symmetry associated with the S field, since (18) induces non-derivative couplings for the pseudoscalar $\theta = \text{Im } S$. This implies that one cannot use θ as an axion to solve the strong-CP problem. Furthermore, the existence of the interaction (18) destabilizes the vacuum since it gives a contribution to the scalar potential

$$V_S \sim \frac{1}{(S+S^*)(T+T^*)^3} |W_S|^2 \quad (19)$$

which induces a cosmological constant (except for the unphysical limits S and/or $T \rightarrow \infty$). There is, however, an interesting solution to this problem²⁴. An alternative to the gauge condensation breaking of supersymmetry is the existence of a vev for the antisymmetric field H_{ijk} in extra dimensions^{33,24,35}

$$\langle H \rangle = \langle H_{ijk} \epsilon^{ijk} \rangle = C \neq 0 \quad (20)$$

which, from the four-dimensional point of view, would look like a constant superpotential. This gives a mass to the gravitino $m_{3/2} \sim C/M_P^2$ and also induces a cosmological constant. The interesting point is that when one considers both SUSY-breaking mechanisms simultaneously, their contribution to the scalar potential comes in the form of a perfect square²⁴:

$$V_S \sim \frac{1}{(S+S^*)(T+T^*)^3} |\langle H \rangle + W_S + W|^2 \quad (21)$$

so that, upon minimization, the S field adjusts itself in such a way that the cosmological constant vanishes. Still, one can check that supersymmetry is broken ($F_T \neq 0$ although $F_S = 0$). This mechanism is very appealing, but one would have to show that it still works when further terms (e.g., H^4 $D = 10$ terms) are included, and also that it is stable under radiative corrections. One must admit that it would be a miracle if that were the case, but at least this provides us with a possible scenario in which SUSY-breaking and zero cosmological constant (at a certain level) are compatible. It could turn out that the cancellation of the zero-energies between $\langle H \rangle$ and $\langle \bar{\chi}\chi \rangle$ could be more general than the simple arguments given here.

As we stated above, a vev for H induces a gravitino mass $m_{3/2} \sim |C|/M_P^2$. Then, if we want to relate the $m_{3/2}$ mass to the weak scale M_W , one needs to choose $|C| \ll M_P$ by hand and we lose the opportunity offered by the gaugino condensation mechanism of determining $m_{3/2}$ in terms of M_P . Thus we have a nice mechanism for cancelling the cosmological constant, but we can no longer determine the $m_{3/2}$ scale dynamically. I think that a different interpretation of the cancelling cc mechanism is more interesting^{11,21,36}. It may well be that the values of $\langle S \rangle$ and $\langle T \rangle$ were determined upon compactification (e.g., by some string effects). In such a case, gaugino condensation will form at a definite mass scale $\Lambda \sim e_{M_P}^{-1/(2g^2 b_0)}$ (e.g., of order 10^{13} GeV). Then the $\langle H \rangle$ vev adjusts itself to cancel the condensate contribution to the vacuum energy. There is nothing wrong with this field having a dynamical rôle, since H_{ijk} contains a piece $\omega_{ijk}^G - \omega_{ijk}^L$ which could take a non-vanishing value $\sim C \epsilon_{ijk}$. This piece vanishes if one identifies gauge and spin connections⁵, but this identification should appear as a result of the dynamics of the system, not as an arbitrary input. Thus a small miscancellation²¹ $\omega_{ijk}^G - \omega_{ijk}^L \sim C \epsilon_{ijk}$ could appear if one considers simultaneously the dynamics of the compactification and the low-energy dynamics. In this situation the $m_{3/2}$ scale would be dynamically determined by the gaugino condensation scale and not by an arbitrary input parameter C .

The total tree-level (plus condensation effects) scalar potential is essentially given by V_S + the usual globally supersymmetric scalar potential [with appropriate powers of $(S+S^*)$ and $(T+T^*)$ in order to have a scale-invariant result]. However, although SUSY has been broken in the

hidden sector ($F_T \neq 0$), the observable world remains supersymmetric at this level

$$M = m = A = 0 \quad (22)$$

where M and m are the soft SUSY-breaking gaugino and scalar masses, and A parametrizes the soft trilinear scalar couplings. The parameters M and A vanish since they are proportional to $(\langle H \rangle + W_s)$ which vanishes upon minimization of the scalar potential Eq. (21). The value of m vanishes because of the scale-invariant structure of Eq. (15). Of course, this is only true at the tree level and one expects SUSY breaking soft terms to appear radiatively. It turns out that the required soft terms are not so easy to generate, at least if one just calculates radiative corrections starting with the low-energy truncated Lagrangian of Refs. 15 and 24. If one starts from $A = M = 0$, the effective theory has an R-symmetry which forbids non-vanishing A or M to be generated. It is sometimes stated in the literature that the usual observable sector interactions may generate radiatively some modification to the gauge kinetic function f_{ab} involving non-singlet superfields C_x . This is not possible, since radiative corrections are unable to give rise to an analytic contribution to f_{ab} . With a trilinear superpotential, as is the case here, any radiative graph will contain as many outgoing as incoming chiral superfields C_x . The appearance of such a radiative contribution to f_{ab} is also forbidden by the scale invariance in Eq. (13).

Masses for observable scalars are not generated²⁶⁻²⁸ at one loop (at least in the truncated Lagrangian), but do have contributions at two loops²⁷. However, this source for SUSY breaking would in general be problematic³⁶ since it gives rise to squark and slepton masses

$$m_{\tilde{q}, \tilde{e}} \sim h_{q, e} \frac{m_{3/2}}{(4\pi)^2} \quad (23)$$

and hence the soft scalar masses will not be universal but proportional to each fermion partner's mass. This would be a disaster for the suppression of flavour-changing neutral currents (FCNC). Thus, although one obtains SUSY-breaking terms from the truncated Lagrangian, the phenomenological prospects do not seem very promising.

To calculate radiative corrections using just the low-energy truncated Lagrangian is probably unreliable since one is neglecting, for example, the effect of heavy string modes. However, quantum corrections involving heavy string modes can be important²⁹. Specific examples of such terms are the "Wess-Zumino" one-loop couplings^{2,3}

$$\frac{1}{18(2\pi)^5} \left[4(\omega^4 - \frac{1}{30}\omega^6) X_7 - 6 B X_8 \right] \quad (24)$$

where $X_8 = dX_7$ and

$$X_8 = \frac{1}{24} \text{Tr} F^4 - \frac{1}{7200} (\text{Tr} F^2)^2 - \frac{1}{240} \text{Tr} F^2 \text{Tr} R^2 + \frac{1}{8} \text{Tr} R^4 + \frac{1}{32} (\text{Tr} R^2)^2 \quad (25)$$

in the notation of Ref. 3. These one-loop terms have to be present in the low-energy $D = 10$ field theory if one is to understand the fact that anomalies are cancelled at the string level. When one considers a truncation of these couplings down to four dimensions, one obtains, for example, from the second term in Eq. (24) four-dimensional couplings of the form¹⁸

$$B^{\mu\nu} \text{Tr}(F^{\mu\nu} F^{\rho\sigma}) \langle \text{Tr} R^{\rho\sigma} R^{\tau\delta} \rangle \longrightarrow \sim \frac{1}{(2\pi)^5} \eta F \tilde{F} \quad (26)$$

These are axion-type couplings of the pseudoscalar field $\eta = \text{Im } T$ with the four-dimensional gauge particles. In the $E_8 \times E_8$ case, one can easily check that the coefficients of the observable E_6 and the hidden E_8 (or subgroups) axion couplings are equal and opposite^{21,22}

$$-\varepsilon \eta F_6 \tilde{F}_6 \quad ; \quad +\varepsilon \eta F_8 \tilde{F}_8 \quad , \quad \varepsilon \sim (2\pi)^{-5} \quad (27)$$

Since one assumes that the low-energy theory is supersymmetric, one concludes that the gauge kinetic function Eq. (12) gets a one-loop correction from these terms^{21,29}

$$\int_{ab}^8 = \delta_{ab} (S + \varepsilon T) \quad ; \quad \int_{ab}^6 = \delta_{ab} (S - \varepsilon T) \quad (28)$$

Notice that the couplings in Eq. (27) violate S-scale invariance (since they are one loop) and T-scale invariance (since they are proportional to a large vev $\langle R^2 \rangle \neq 0$). One does not expect Eq. (24) to be the only effective local one-loop terms induced by heavy string mode exchanges. Thus, in order to obtain the supersymmetric counterparts of the axion couplings at low energies, D = 10 couplings of the form²⁹

$$\varepsilon \sum^{MNPQRSTUUVW} \psi^{3/4} (\partial_M B_{NP}) \text{Tr}(\bar{\chi} \Gamma_{RS} \chi) \text{Tr}(F_{TU} F_{VW}) \quad (29)$$

are also required. By the same token, one also expects analogous gravitino couplings

$$\varepsilon \sum^{MNPQRSTUUVW} \psi^{3/4} (\partial_M B_{NP}) \text{Tr}(\bar{\Psi}_Q \Gamma_R \Psi_S) \text{Tr}(F_{TU} F_{VW}) \quad (30)$$

which, after truncation, induce four-dimensional terms²⁹

$$\frac{i \varepsilon}{(S + S^*)} (\partial_\mu \eta) \sum^{\mu\nu\rho\sigma} (\bar{\Psi}_\nu \gamma_\rho \Psi_\sigma) \quad (31)$$

Comparing this result with the corresponding general N = 1, d = 4 supergravity coupling $(G^T D_\mu T) \varepsilon^{\mu\nu\rho\sigma} (\bar{\psi}_\nu \gamma_\rho \psi_\sigma)$, one concludes that the tree-level Kähler potential G_0 gets a one-loop correction δG ²⁹:

$$G = G_0 + \delta G \quad ; \quad \delta G = -\alpha \varepsilon \frac{(T + T^*)}{(S + S^*)} \quad (32)$$

where α is a number O(1) and G_0 is the tree-level Kähler potential of Eq. (15). As indicated in that equation, one may have extra scale-invariant factors $\sim (|C|^2 / (T + T^*))$. To obtain the result in Eq. (32), it is in

fact enough to consider the transformation properties under the S and T scale invariances. The above modification of G_0 is the only one which leads to terms in the Lagrangian which transform like the axion couplings (i.e., $\eta \tilde{F}\tilde{F} \rightarrow \lambda' \lambda^{-2} \eta \tilde{F}\tilde{F}$).

Notice that the results in Eqs. (28) and (32) substantially modify the tree-level situation. First, the fact that the T-scale invariance is broken means that the $(-3 \log(T+T^*) + \log|W|^2)$ structure guaranteeing a positive definite scalar potential disappears. Secondly, if a gaugino condensation is generated in the hidden sector, Eq. (28) implies that a non-perturbative superpotential is also generated for the T-field

$$W_g \sim (e^{-1/2g^2 b_0} M_P)^3 \sim e^{-\frac{3S}{2b_0}} e^{-\frac{3\epsilon T}{2b_0}} M_P^3 \quad (33)$$

so that non-derivative interactions appear for $\eta = \text{Im } T$ and the second Peccei-Quinn symmetry is broken, as already happened to the one associated with $\theta = \text{Im } S$. In fact, it has recently been argued that this symmetry is also broken by string world-sheet instantons³⁰, and analogous superpotentials of the form $\exp(T)$ may also be generated by that mechanism. Of course, as in the case of the superpotential Eq. (18), the existence of W_g destabilizes the vacuum unless one finds a mechanism to cancel the cosmological constant.

One interesting observation²⁹ is that we may define a new field $S' = S + a\epsilon T$ and reabsorb the δG correction into the $-\log(S'+S'^*)$ term (up to ϵ^2). Then the gauge kinetic functions may be re-expressed as

$$f_{ab}^8 = \delta_{ab} (S' + \epsilon(1-a)T) ; \quad f_{ab}^6 = \delta_{ab} (S' - \epsilon(1+a)T) \quad (34)$$

where we recall that a is a number $O(1)$ which gives us the coefficient of the one-loop corrections to G . In terms of the redefined field, one then obtains for the scalar potential

$$V = e^K \left[\left| W + 3(s' + s'^*) \frac{W_g}{b_0} \right|^2 + \frac{t_c}{3} \left| \frac{\partial W}{\partial c_x} \right|^2 + D^2 \text{term} \right] + e^K 3 \varepsilon' \frac{t_c}{b_0} W_g^* (W_g + c) + \text{h.c.} \quad (35)$$

where $\varepsilon' \equiv \varepsilon(1-a)$, $2t_c = (T+T^* - 2|C_x|^2)$ and $\exp(K) = (s' + s'^*)^{-1} (t_c^{-3})/8$. The term in brackets is analogous to the tree-level result, but the term in ε' is not positive definite and spoils the mechanism for cancelling the cosmological constant that we explained above. On the other hand, soft SUSY-breaking terms are generated in the observable sector, $M, m \sim \varepsilon m_{3/2}$ and $A_m \sim \varepsilon m_{3/2}$, since the symmetries giving rise to the result in Eq. (22) are no longer present.

An interesting situation²⁹ would occur if the "a" coefficient of the correction δG were $a = 1$ ($\varepsilon' = 0$). In that situation, the second term in Eq. (35) would vanish and the scalar potential would be completely analogous to the tree-level result, giving rise to SUSY breaking with zero cosmological constant ($F_{S'} = 0$, $F_T \neq 0$). There is, however, an important difference now. Due to the one-loop correction for f^8 and f^6 , although the minimization conditions imply vanishing "hidden" gaugino mass $M^8 = 0$, the observable gaugino mass will be in general non-vanishing:

$$M_8 = \int_{S'}^8 F_{S'} = 0$$

$$M_6 = \int_{S'}^6 F_{S'} + \int_T^6 F_T = -2(1+a) F_T \sim \varepsilon m_{3/2} \quad (36)$$

Thus, in the case $a = 1$ ($\varepsilon' = 0$), we would have soft terms²⁹

$$M_6 \sim \varepsilon m_{3/2} \quad ; \quad m = A = 0 \quad , \quad \varepsilon \sim (2\pi)^{-5} \quad (37)$$

and there would be no problem in transmitting SUSY breaking to the observable sector. Gaugino masses would induce the supersymmetry breaking.

Since for phenomenological reasons we want $M \lesssim 1$ TeV one necessarily has

$$m_{3/2} \lesssim 10^7 \text{ GeV} \quad (38)$$

This is certainly an appealing possibility, but there is no obvious reason why "a" should be equal to one. Furthermore, other (or higher) loop effects may probably modify the mechanism which cancels the cosmological constant.

What is the conclusion of this long excursion through SUSY-breaking-dilaton-axion physics? It seems that both loop and non-perturbative effects substantially alter the results of a first naive tree-level truncation of the theory. The scalar potentials one obtains for the dilatons $\text{Re } S$ and $\text{Re } T$ lead to unphysical limits ($S, T \rightarrow 0$ or ∞). This is probably an indication that the vevs for the dilatons (and axions) are not determined by low-energy physics but by full string dynamics, and it does not make much sense to use S and T as undetermined dynamical variables at low energies. On the optimistic side, one may think of several aspects of the low-energy analysis which may survive. Gaugino condensation in the hidden sector seems a rather appealing mechanism for breaking the residual supersymmetry and obtaining a hierarchically small gravitino mass $m_{3/2} \sim \langle \bar{\chi}\chi \rangle / m_P^2$ which would presumably be related to the weak scale. In any case, there are loop effects which will transmit the supersymmetry breaking to the observable sector so that the result of Eq. (38), $m_{3/2} \lesssim 10^7$ GeV, seems unavoidable if we want to maintain the gauge hierarchy. The cancellation of the gaugino condensation cosmological constant by the vev of H_{ijk} could also be a more general mechanism. If that were the case, one would expect vanishing hidden gaugino mass M_g and A-parameter, since both are proportional to $(\langle H \rangle - \langle \bar{\chi}_g \chi_g \rangle)$. In this case, the existence of radiative corrections which make $f^8 \neq f^6$ [i.e., Eq. (28)] is important, since it allows for non-vanishing observable gaugino masses M^6 to transmit supersymmetry breaking to the quark-lepton-Higgs world.

SUPERSTRING-INSPIRED LOW-ENERGY MODELS

We have explored in the previous chapter mostly the singlet "hidden" sector of the theory. Since the conclusions were not so positive, one may be led to the conclusion that nothing can be said about the low-energy limit of the $E_8 \times E_8$ string after compactification. This is not correct. There are some general features expected for the low-energy "observable" theory which are more or less independent of the details of the compactification³⁷⁻⁴⁰. Thus, for example, if we compactify on a "Calabi-Yau" manifold embedding the gauge connection into the spin connection, we know that we obtain an E_6 (or some subgroup) model with several families of $\underline{27}$'s (or some subset of states). Also, some pairs of $(\underline{27} + \overline{\underline{27}})$ could be present to start with³⁷. The rank of the group can be lowered further. If the compact manifold has a certain non-Abelian discrete symmetry³⁷, one can break E_6 down to $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'''$, where $U(1)'''$ is a specific $U(1)$ to be described below. This is not the only possible low-energy group, since one can also obtain other rank = 5 or 4 examples³⁶⁻⁴⁶. This may be done by using the chiral fields in the $\underline{27} + \overline{\underline{27}}$ representation which models usually have. A vev for them can lower the rank from 6 down to 5 or 4. This symmetry breaking may occur at an intermediate scale (e.g., 10^{10} - 10^{13} GeV) by radiative corrections. The final unbroken gauge group can then be just the standard $SU(3) \times SU(2) \times U(1)$ model (see, e.g., Ref. 17).

There are other possible compactification schemes which may lead to a variety of low-energy models. If one goes beyond the usual recipe of identifying spin- and gauge connections, one may find manifolds¹⁹ which break the original E_8 directly down to an $SO(10)$ or $SU(5)$ subgroup. These may be further broken (e.g., through the Wilson-loop mechanism³⁷) down to some rank 5 subgroup or the standard model. Low-energy $SO(10)$ or $SU(5)$ subgroups may also be obtained¹⁶, compactifying on certain classes of "orbifolds". The orbifolds are obtained by modding out some six-torus by a discrete subgroup of $SU(3)$ (so that supersymmetry is preserved). They are in general "singular manifolds" with a discrete holonomy group, which may often be obtained as singular limits of some Calabi-Yau manifolds. If the original torus has some Wilson lines, or if (as in the manifolds mentioned above) one generalizes the usual procedure of identifying spin and gauge connections, one may break E_8 directly down to $SU(5)$. The Wilson-loop mechanism may then give rise again to the standard model.

The opposite possibility exists, i.e., one may get rank > 6 low-energy gauge groups. For example, if the "twisting" group of our orbifold¹⁶ is an Abelian discrete subgroup (as, for example, in the Z-orbifold), one obtains low-energy rank = 8 groups (subgroups from E_8). If, furthermore, one considers the new class of tori compactifications⁴⁷ (or some orbifold versions of these) introduced by Narain, the rank of the low-energy gauge theory may be even larger. However, the existence of a large low-energy gauge group is not phenomenologically welcome, so we will restrict ourselves from now on to low-energy groups contained in E_6 , which is the simplest possibility.

Since it seems that one may obtain low-energy groups with rank 4, 5 and 6, it is useful to discuss what extra gauge interactions (if any) one may have at low energies. We are going to confine ourselves to models involving only (at most) extra $U(1)$'s, since non-Abelian generalizations of these lead in general to bad predictions for $\sin^2\theta_W$ and M_x ^{39,40}. Extra $U(1)$ interactions are conveniently analyzed in terms of the diagonal generators of the $SU(3)_C \times SU(3)_L \times SU(3)_R$ subgroup of E_6 . Apart from the Cartan subalgebra of the $SU(3)_C \times SU(2)_L$ group, one has the extra diagonal generators

$$T_L = \begin{pmatrix} 1 & 1 & -2 \end{pmatrix}_L ; T_R = \frac{1}{2} \begin{pmatrix} -2 & 1 & 1 \end{pmatrix}_R ; T_N = \frac{1}{2} \begin{pmatrix} 0 & 1 & -1 \end{pmatrix}_R$$

(39)

The quantum numbers of the fields in a 27 of E_6 with respect to these generators are shown in the Table.

This table also shows our notation³⁶ for the quark, lepton and Higgs superfields. Let us recall that a 27 of E_6 contains, apart from a standard family, two coloured triplets $D, \bar{D}$, a right-handed neutrino ν_L^C and some other neutral object N . There is a set of Higgses $H + \bar{H}$ per 27 generation. One can write for the usual hypercharge generator

$$Y = \sqrt{\frac{3}{5}} \left(\frac{2}{3} T_R - \frac{1}{6} T_L \right)$$

(40)

	Q_L	U_L^c	e_L^c	d_L^c	L	H	$\bar{D}$	$\bar{H}$	D	ν_L^c	N
$T_L (\times \frac{1}{2\sqrt{3}})$	-1	0	-2	0	1	1	0	1	2	-2	-2
$L_R (\times \frac{1}{\sqrt{3}})$	0	-1	1	1/2	-1/2	-1/2	1/2	1	0	-1/2	-1/2
T_N	0	0	0	1/2	-1/2	-1/2	-1/2	0	0	-1/2	1/2
$Y (\times \sqrt{\frac{3}{5}})$	1/6	-2/3	1	1/3	-1/2	-1/2	1/3	1/2	-1/3	0	0
$Y' (\times \frac{1}{\sqrt{40}})$	-1	-1	-1	-2	-2	3	3	2	2	0	-5
$Y'' (\times \frac{1}{\sqrt{40}})$	-1	-1	-1	3	3	-2	-2	2	2	-5	0
$Y''' (\times \frac{1}{\sqrt{15}})$	-1	-1	-1	1/2	1/2	1/2	1/2	2	2	-5/2	-5/2
$Y'_I (\times \frac{1}{4\sqrt{6}})$	-2	-2	-2	4	4	-2	-2	4	4	-8	-2
$Y''_I (\times \frac{1}{4\sqrt{6}})$	-2	-2	-2	-2	4	4	4	4	4	-2	-8

Any linear combination of T_L , T_R and T_N orthogonal to Y is, in principle, a candidate for extra $U(1)$ and some candidates are shown in the Table.

The superpotential of the low-energy $N = 1$ supergravity model may, in general, contain any term present in the $(27)^3 E_6$ coupling, although the Yukawa couplings will not usually obey any E_6 relationship, since the Wilson-loop breaking alters those³⁷. Under these conditions, the most general superpotential originating in the $(27)^3$ coupling will be

$$\begin{aligned}
 W = \sum_{\text{families}} & h_L H L e_L^c + h_d H d_L^c + h_\nu \bar{H} \nu_L^c + \\
 & + h_\nu \bar{H} L \nu_L^c + h'_\nu D d_L^c \nu_L^c + \\
 & + \lambda_2 N H \bar{H} + \lambda_3 N D \bar{D} + \\
 & + \lambda_L L Q \bar{D} + \lambda'_L e_L^c u_L^c D + \\
 & + \lambda_B Q u D + \lambda'_B u_L^c d_L^c \bar{D}
 \end{aligned}
 \tag{41}$$

It is well known that if all the couplings in (41) are present, several phenomenological disasters may occur: (a) the $D, \bar{D}$ fields mediate fast proton decay unless $\lambda_B = \lambda'_B = 0$ or $\lambda_L = \lambda'_L = 0$ (or those fields are heavier than $\sim 10^{10}$ GeV). These couplings cannot all vanish simultaneously since then D and $\bar{D}$ would be absolutely stable, causing cosmological trouble; (b) there are unacceptable Dirac neutrino masses unless $h_\nu = 0$ (or the ν_L^c 's have Majorana masses bigger than ~ 10 TeV); (c) unless essentially only one of the three sets of Higgses $H, \bar{H}$ couples to quarks and leptons, tree-level FCNC's will appear (or the extra two Higgses are heavy enough). All these problems become less severe the smaller the rank of the low-energy group is. For a rank = 6 model, the only possibility is to assume that some symmetries exist which forbid the dangerous couplings, since the gauge symmetries do not allow us to give mass to any of the dangerous particles. In rank = 5 models, one may get rid of some of the unwanted particles ($D, \bar{D}, H, \bar{H}$ or ν_L^c 's), so that one only needs to forbid some of the couplings. Finally, in rank 4 models (i.e., the standard model gauge group) the possibility exists that no dangerous particles at all remain in the low-energy spectrum. Still, one has to be sure that Weinberg-Salam (WS) doublets remain light. We now consider these three cases in turn.

i) Two Extra Z^0 's

In this case the relevant gauge group is

$$G_6 = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)^2
 \tag{42}$$

and all the 27 states in a fundamental of E_6 remain light, since they are

all chiral under the G_6 group. Thus one has to assume there is some symmetry forbidding dangerous couplings. Radiative corrections may break G_6 down to the standard model inducing non-vanishing vevs $\langle H \rangle \neq 0$, $\langle \bar{H} \rangle \neq 0$, $\langle \tilde{\nu}_L^c \rangle \neq 0$ and $\langle N \rangle \neq 0$. However, several diseases will in general appear. A vev for $\tilde{\nu}_L^c$ will give rise to $d_L^c - \bar{D}$ and L-H mixing through the h_ν and h'_ν couplings. This would ruin the GIM mechanism. Also, one gets induced vevs for the left-handed $\tilde{\nu}_L$ giving rise to lepton number violation. All these problems, along with the fact that a good number of symmetries forbidding certain Yukawa couplings are needed, make this model rather contrived.

ii) One Extra Z°

The low-energy gauge group is in this case

$$G_5 = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1) \quad (43)$$

The extra $U(1)$ may be any of the linear combinations of T_L , T_R and T_N which commute with the hypercharge. The more general form for it will be³⁶

$$Y_{\text{extra}} = (\text{constant}) (T_L + T_R + d T_N) \quad (44)$$

Thus there is a one-parameter (d) family of possible extra $U(1)$'s. There are, however, three cases of special physical interest which correspond to the hypercharges Y' , Y'' and Y''' in the Table.

Y' hypercharge. It is defined as the one under which the ν_L^c field is inert, $Y'(\nu_L^c) = 0$, and is given by³⁶

$$Y' = \frac{1}{2\sqrt{10}} (T_L + T_R - 5 T_N) \quad (45)$$

with the same normalization as the hypercharge. The interest of this $U(1)$ is that a large Majorana mass term $\nu_L^c \nu_L^c$ is not forbidden by this symmetry, and hence this leaves the door open to a solution of the neutrino mass problem mentioned above^{11,36}. Some of the mechanisms discussed

in Ref. 21 may be the source of these ν_L^c masses. A model like this may be obtained from a rank = 6 model if a field N_R with the quantum numbers of a ν_L^c gets a vev at a large scale. Alternatively, it may be obtained after Wilson-loop breaking of an $SO(10)$ model with fermion content $3 \times (\underline{16} + \underline{10})$. However, the embedding of a standard family is unusual in that $\underline{16} = (Q_L, U_L^c, e_L^c; H, \bar{D}, N)$, $\underline{10} = (L, d_L^c; \bar{H}, D)$. Thus, N does not behave like a right-handed neutrino (it is harmless) and L, d_L^c are inside a $\underline{10}$ and not inside a $\underline{16}$. This extra Z' model has been studied in detail in Ref. 36.

Y'' hypercharge. It is defined as the one under which the N field is inert, $Y''(N) = 0$, and is given by

$$Y'' = \frac{1}{2\sqrt{10}} (T_L + T_R + 5 T_N) \quad (46)$$

In fact, this is obtained from Y' by changing $T_N \rightarrow -T_N$ and just corresponds to $U(1)'$ after redefining $(d_L^c, L, \nu_L^c) \leftrightarrow (\bar{D}, H, N)$. $U(1)''$ is the $U(1)$ inside $SO(10)$ which commutes with $SU(5)$ and the quark-lepton embedding is the standard one. A model with a Y'' hypercharge is only physically distinguishable from a Y' model if you destroy the symmetry $T_N \rightarrow -T_N$ in the low-energy spectrum. This is what is done in the models with an intermediate symmetry-breaking scale ($\langle N \rangle \sim 10^{10} - 10^{14}$ GeV) of Refs. 21, 39 and 45. In those models, a Higgs field N breaks the $U(1)$ orthogonal to Y and Y'' and at the same time gives masses to the unwanted $D, \bar{D}, H$ and $\bar{H}$ fields. In this way we may get rid of the fast proton decay problem. The low-energy superpotential is then just

$$W'' = \sum_{\text{families}} h_L H L e_L^c + h_d H Q d_L^c + h_u \bar{H} Q u_L^c + h_\nu \bar{H} L \nu_L^c + \lambda_2 N H \bar{H} \quad (47)$$

Unfortunately³⁶, with such a superpotential it is hard to obtain the desired pattern of symmetry breaking $SU(2) \times U(1)^2 \rightarrow U(1)_{\text{e.m.}}$. In order to forbid Dirac ν -masses, we have to set $h_\nu = 0$, and then it is hard to understand how $\tilde{\nu}_L^c$ may acquire a negative (mass)² and radiatively break

$U(1)''$. Furthermore, one can see that the soft coupling $m A(\widetilde{H} \widetilde{L} \widetilde{\nu}_L^c)$ will induce a vev for the left-handed sneutrino $\widetilde{\nu}_L$, leading to lepton number violation. All these problems make this $U(1)$ interaction rather unattractive.

Y''' hypercharge. It is just the $U(1)$ generator^{37,41,46} orthogonal to Y and T_N

$$Y''' = \frac{1}{\sqrt{15}} (T_L + T_R) \quad (48)$$

The $U(1)'''$ quantum numbers of the different particles are shown in the Table. This model can be obtained directly from E_6 through Wilson-loop breaking. All the particles in a 27 are chiral under this symmetry, so that the complete 27 states must remain light. Thus, one has to rely on the possible existence of symmetries to forbid the dangerous couplings in the superpotential. Particularly, one has to set the couplings $h_\nu = 0$ to avoid Dirac neutrino masses^{41,46}. The right-handed neutrinos are then massless, and this could cause a serious cosmological problem, since six massless neutrinos are probably too much for consistent nucleosynthesis⁴⁸.

Apart from these three physically interesting $U(1)$'s, there is a family of other possible $U(1)$'s as one varies the parameter "d" in Eq. (44). They may be obtained, e.g., by giving vevs both to N_R and N in a rank = 6 model. Then, depending on the size of the vevs, one gets an unbroken $U(1)$:

$$Y_{extra} = \cos \alpha Y' + \sin \alpha Y'', \quad t_{y\alpha} = N/N_R \quad (49)$$

The $U(1)'$, $U(1)''$ and $U(1)'''$ above correspond to the values $\alpha = 0, \pi/2, \pi/4$. The new possibilities in (49) have no special advantage over the Y''' case, so we will not consider them any longer here.

From the above discussion, it seems clear that the least contrived extra- $U(1)$ models are those based on the $U(1)'$ and $U(1)'''$ hypercharges. The first one especially, $U(1)'$, has the advantage of being the only one

consistent with a heavy right-handed neutrino, and hence may avoid problems with neutrino masses. On the other hand, for the $U(1)'''$ hypercharge, one has to forbid the h_ν neutrino couplings, and then the right-handed neutrinos are massless. This leads to six massless neutrinos which, as we remarked above, could be incompatible with nucleosynthesis bounds. This possibly makes the $U(1)'$ case more interesting. However, one can make a rather general analysis of the process of symmetry breaking

$$SU(2)_L \times U(1)_Y \times U(1) \longrightarrow U(1)_{e.m.} \quad (50)$$

for an arbitrary extra- $U(1)$ model. This is explained in some detail in Ref. 36. The relevant pieces of the low-energy superpotential regarding the process of symmetry breaking are

$$W_Y = h_t \bar{H} Q u_L^c + \lambda_2 N H \bar{H} + \lambda_3 N D \bar{D} \quad (51)$$

where the first term corresponds to the top quark. To simplify the computations, we also assume that only one of the three N -fields couples strongly enough to H , $\bar{H}$, D , $\bar{D}$. Then the scalar potential (along the neutral directions) which is relevant for the symmetry breaking is

$$\begin{aligned} V = & \frac{g_1^2}{2} (X_{\bar{H}} |\bar{H}|^2 + X_H |H|^2 + X_N |N|^2)^2 + \frac{(g_1^2 + g_2^2)}{8} (|H|^2 - |\bar{H}|^2)^2 + \\ & + \lambda_2^2 (|N|^2 |H|^2 + |N|^2 |\bar{H}|^2 + |H|^2 |\bar{H}|^2) + m_N^2 |N|^2 + m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 + \\ & + m A_2 \lambda_2 (N H \bar{H}) + h.c. \end{aligned} \quad (52)$$

where $X_{\bar{H}}$, X_H and X_N are the extra- $U(1)$ charges of the corresponding fields (see the Table). We have included in the potential the usual soft SUSY-breaking terms, i.e., scalar mass terms and trilinear scalar couplings.

All the parameters in the scalar potential are assumed to be taken at the biggest of the two symmetry-breaking scales, i.e., at $Q = M'_{Z'}$. One also expects $M'_{Z'}$ not to be very much larger than $M_{Z'}$. (e.g., $M'_{Z'} \lesssim 500-600$ GeV), otherwise we would need to do some unnatural fine tuning in order to avoid radiative corrections giving an unwanted large contribution to $M_{Z'}$. We have then to compute how the parameters m_H^2 , $m_{\bar{H}}^2$, m_N^2 , λ_2 and A_2 are renormalized in going from the compactification scale down to low energies³⁶.

Some intuition into how the double symmetry breaking (50) in this type of model occurs can be obtained from a discussion of the form of the scalar potential (52). Since we obviously want to obtain that $M'_{Z^0} \gg M_{Z^0}$ (because of neutral current constraints), the symmetry breaking should appear as follows. The parameter m_N^2 becomes negative not much above the weak scale, and a vev for N is induced. Upon renormalization one gets $m_{\bar{H}}^2 \ll m_H^2$ (although $m_{\bar{H}}^2$ does not need to be negative) and the extra- $U(1)$ D^2 terms give some (usually negative) contribution to the H and $\bar{H}$ effective masses. This, along with the trilinear term, induces vevs for H and $\bar{H}$. The vev for $\bar{H}$ is usually bigger than that for H because, as we remarked above, renormalization effects yield $m_{\bar{H}}^2 \ll m_H^2$ (the contribution of the D^2 term is not so important in this respect). It is important to remark that the form of the renormalization group equations for the soft SUSY-breaking terms is such that one may obtain this small hierarchical ($M'_{Z^0} \gg M_{Z^0}$) symmetry-breaking for wide ranges of the parameters. This has been checked for specific models in Refs. 36 and 46. In particular, a model based on the $U(1)'$ hypercharge was recently studied numerically³⁶. One obtains consistent $SU(2) \times U(1)^2 \rightarrow U(1)_{e.m.}$ symmetry breaking for wide ranges of the parameters $\lambda_2, \lambda_3, A, m_t, m$ and M . An interesting point to remark is that the suggested boundary conditions in Eq. (37) (i.e., $m/M \ll 1, |A| \ll 1$) are consistent with the desired pattern of symmetry breaking, but only for top-quark masses $m_t \lesssim 70$ GeV. The lightest supersymmetric particle (LSP) is usually a Higgsino or a "singlino" $\tilde{N}$ (unless the top quark is very heavy). As in all extra- $U(1)$ models, there are plenty of new particles ($D, \bar{D}, \tilde{H}^\pm, H^\pm$, etc.), which could be detected at present accelerators. For a discussion of the typical spectra in these models, see Ref. 36.

Although the models with an extra $U(1)$ are technically viable, they do, however, look rather artificial. To avoid all phenomenological problems, extra- $U(1)$ models require a number of lucky coincidences to happen:

- i) some Yukawa couplings are absent in order to avoid fast proton decay;
- ii) other B-violating couplings are present in order to avoid the absolute stability of the D -fields;
- iii) the extra $SU(2)_L$ doublets in the models have negligible couplings to quarks in order to avoid FCNC's;
- iv) $\nu_L - \nu_R - \bar{H}$ Yukawas are absent in order to avoid unobserved neutrino masses [this is, in principle, solved for the $U(1)'$ hypercharge].

There are, in fact, further conditions coming from the fact that the new particles of the first two generations should be heavy enough also to suppress other sources of FCNC. In this situation, one must admit that the models with an extra $U(1)$ are rather contrived. This makes more desirable the obtention of compactification schemes leading directly to the standard model.

iii) No Extra Z .³⁶

As we discussed above, one can have compactification schemes which lead to the standard model gauge group $SU(3) \times SU(2) \times U(1)$ and no additional gauge bosons. This is probably the most interesting possibility, since in this case there is no need to have at low energies any of the extra new particles (D , $\bar{D}$, $H^\pm$, ν_R 's, etc.) which lead to problems in the models with an extra $U(1)$. The minimal model with gauge group $SU(3) \times SU(2) \times U(1)$ should contain, apart from the standard quark-lepton superfields, the Higgs doublets H and $\bar{H}$ and, possibly, a singlet N . In the absence of a singlet N° , the resulting model would be precisely the "minimal low-energy supergravity model" studied in the last few years^{9,49-51}. A mass coupling of the form $\epsilon H\bar{H}$ has then to be present in the superpotential to avoid the appearance of an axion and to induce a $\langle H \rangle \neq 0$. Although there are no obvious sources for a term $\epsilon H\bar{H}$ at the tree level, it could appear through radiative corrections⁴⁹. Then one expects ϵ to be a small parameter, as would be the corresponding soft term in the scalar potential $\mu_3^2 \sim \epsilon m$. This leads in turn to a small value for $\langle H \rangle \ll \langle \bar{H} \rangle$ and to a very light "chargino" in the spectrum^{9,49} with mass $m_{\tilde{W}} \sim \sqrt{v}/M$. If ϵ is very small [e.g., $\epsilon \sim (\alpha/\pi)m$], this light chargino may even be incompatible with experiment. As we have seen above, the presence of a singlet N° coupling to $H\bar{H}$ is a very natural feature in superstring-inspired models and it is likely to appear also in the minimal $SU(3) \times SU(2) \times U(1)$ case. In this more general case, $\langle H \rangle$ can be as large as $\langle \bar{H} \rangle$ and there is no danger associated with a possible light chargino. The relevant Higgs superpotential in the general case is

$$W_H = \lambda_2 N H \bar{H} \quad (53)$$

where we define $N \equiv N^\circ + \epsilon/\lambda_2$. The scalar potential (along the neutral direction) in this model is

$$V = \frac{(g_1^2 + g_2^2)}{8} (|H|^2 - |\bar{H}|^2)^2 + \lambda_2^2 |H|^2 |\bar{H}|^2 + \lambda_2^2 |N|^2 (|H|^2 + |\bar{H}|^2) + \\ + m_N^2 |N|^2 + m_H^2 |H|^2 + m_{\bar{H}}^2 |\bar{H}|^2 + m A_2 \lambda_2 (N H \bar{H}) + h.c. \quad (54)$$

Notice that Eq. (54) implicitly assumes that the soft bilinear coupling $B \approx A_2$. We assume this in order to simplify the computations and also because the term missing in Eq. (54) $[(B-A_2)\epsilon H \bar{H}]$ is expected to be small for ϵ small. It also vanishes in the interesting case in which gaugino masses are the only source of supersymmetry breaking ($A = B = 0$ and A_2, B renormalize in a very similar way). Notice, however, that a non-vanishing ϵ is required in order to give a mass to a would-be Goldstone boson which appears when N° gets a vev.

The scalar potential above looks similar to that of "minimal low-energy supergravity" (MLES), but it has several important differences due to the existence of the $\lambda_2 N H \bar{H}$ coupling. In particular, it has no flat direction ($\langle H \rangle = \langle \bar{H} \rangle$) for $m_H^2 = m_{\bar{H}}^2 = \mu_3^2$ as happened in the MLES case. This was important in that case because one could have radiative $SU(2) \times U(1)$ breaking with a low t-quark mass⁴⁹⁻⁵¹, since a large difference between m_H^2 and $m_{\bar{H}}^2$ was not needed. In the present case, one needs a negative $m_{\bar{H}}^2$ to be generated since there is no flat direction, and an extra positive term in the potential ($\lambda_2^2 |H|^2 |\bar{H}|^2$) has to be overwhelmed in order to obtain a stable minimum. This has the important consequence that a relatively heavy top quark is required to trigger $SU(2) \times U(1)$ breaking, as happened in the first version of the MLES model⁹.

Solving the renormalization group equations for $m_N^2, m_H^2, m_{\bar{H}}^2, A_2$ and λ_2 , one can analyze numerically the constraints in the parameters (essentially A, m, M, m_t and λ_2) obtained by imposing the appropriate $SU(2) \times U(1)$ breaking. One only finds symmetry breaking for a top-quark mass³⁶

$$m_t \gtrsim 70 \text{ GeV}$$

just because of the absence of an approximate flat direction $H = \bar{H}$ in the potential. Radiative corrections generate symmetry breaking through a

negative $m_{\tilde{H}}^2$ which in turn requires $|\langle \tilde{H} \rangle| > |\langle H \rangle|$. Also in this case the suggested boundary conditions in Eq. (37) (i.e., $m/|M| \ll 1$, $|A| \ll 1$) are consistent with the desired minimum of the potential. However, this is only the case for $m_t \lesssim 100$ GeV (particularly for $m_t = 70-80$ GeV).

The typical SUSY spectra in this "minimal stringy model" is in general lighter³⁶ than in the extra-U(1) models, since in the latter the overall SUSY-breaking scale is $\sim M'_Z$. For example, the charged sleptons may be relatively light (e.g., 25-60 GeV). The sneutrino $\tilde{\nu}_L$ is often the lightest supersymmetric particle and the decays $W^\pm \rightarrow \tilde{e}\tilde{\nu}$ and $Z^\circ \rightarrow \tilde{\chi}^+\tilde{\chi}^-$, $\tilde{\nu}\tilde{\nu}$ are then possible. Sometimes the LSP is a Higgsino-singlino state ($\tilde{H}^\circ - \tilde{N}$), but it is rarely the photino. However, unlike the extra-Z^o case, in this minimal model there is not a clear signature from the stringy origin of the low-energy Lagrangian (except for, to a small extent, the singlet N). Stringiness could be difficult to reveal.

To conclude, let us remark that the heterotic $E_8 \times E_8$ superstring offers us the possibility of embedding the "low-energy supergravity models" developed in the last few years inside a complete unification scheme which includes gravity. It contains a possible built-in mechanism for residual supersymmetry breaking (gaugino condensation in the "hidden sector"). Although the physics of the "hidden sector" and supersymmetry breaking is not yet clarified, one expects in the "observable" sector some truncated version of an E_6 GUT. The gauge structure of the low-energy Lagrangian may have two, one or no extra Z^o's. Whereas the case of two extra U(1)'s does not seem viable, models with an extra Z^o seem to be consistent with the required symmetry breaking structure. However, they have a variety of phenomenological problems which can only be avoided if one assumes that some Yukawa couplings are absent but others are present. From the low-energy point of view, the absence of any extra Z^o would be rather more simple. This makes specially important the study of compactification schemes leading directly to the standard model gauge structure $SU(3) \times SU(2) \times U(1)$.

ACKNOWLEDGEMENTS

I acknowledge discussions with F. Del Aguila, J. Mas, C. Muñoz and H.P. Nilles.

REFERENCES

1. J.H. Schwarz, Physics Reports 89:223 (1982);
M.B. Green, Surveys in High Energy Phys. 3:127 (1982).
2. M.B. Green and J.H. Schwarz, Phys. Lett. 149B:117 (1984); ibid. 151B:21 (1985).
3. M.B. Green, J.H. Schwarz and P. West, Nucl. Phys. B254:327 (1985).
4. D. Gross, J. Harvey, E. Martinec and R. Rohm, Phys. Rev. Lett. 55: 502 (1985); Nucl. Phys. B256:253 (1985).
5. P. Candelas, G. Horowitz, A. Strominger and E. Witten, Nucl. Phys. B258:46 (1985).
6. E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, Nucl. Phys. B212:413 (1983).
7. For reviews, see, e.g.,
H.P. Nilles, Physics Reports 110:1 (1984);
L.E. Ibáñez, Proceedings of the 12th International Winter Meeting on Fundamental Physics, J.E.N. Madrid, ed., (1984);
L. Hall, Proceedings of the Winter School in Theoretical Physics, Mahabaleshwar, India (1984), Springer Verlag Lecture Notes in Physics 208:197 (1984).
8. L.E. Ibáñez, Phys. Lett. 118B:73 (1982);
H.P. Nilles, Phys. Lett. 115B:193 (1982); Nucl. Phys. B217:366 (1983);
P. Nath, R. Arnowitt and A.H. Chamseddine, Phys. Rev. Lett. 49:970 (1982);
R. Barbieri, S. Ferrara and C.A. Savoy, Phys. Lett. 119B:343 (1982).
9. L.E. Ibáñez, Nucl. Phys. B218:54 (1983);
L.E. Ibáñez and C. Lopez, Phys. Lett. 126B:54 (1983);
L. Alvarez-Gaumé, J. Polchinski and M. Wise, Nucl. Phys. B221:495 (1983);
J. Ellis, J. Hagelin, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 125B:275 (1983).
10. L.E. Ibáñez and G.G. Ross, Phys. Lett. 110B:215 (1982);
L. Alvarez-Gaumé, M. Claudson and M. Wise, Nucl. Phys. B207:16 (1982);
B. Ovrut and C. Nappi, Phys. Lett. B113:65 (1982).
11. L.E. Ibáñez, Phenomenology from superstrings, CERN preprint TH.4308/85 (1985), to appear in the proceedings of the 1st Torino Meeting on Superunification and Extra Dimensions, World Scientific, Singapore.
12. H.P. Nilles, CERN preprint TH.4444/86 (1986).
13. G. Segrè, Low-energy physics from superstrings, University of Pennsylvania preprint (1985);
J. Ellis, CERN preprints TH.4255/85 (1985); TH.4439/86 (1986).
14. A. Chamseddine, Nucl. Phys. B185:403 (1981);
F. Berghoeff, M. de Roo, B. de Witt and P. van Nieuwenhuizen, Nucl. Phys. B195:97 (1982);
G. Chapline and N. Manton, Phys. Lett. 120B:105 (1983).
15. E. Witten, Phys. Lett. 155B:151 (1985).
16. L. Dixon, J. Harvey, C. Vafa and E. Witten, Nucl. Phys. B261:651 (1985), and Princeton preprint (1986).
17. R. Greene, K. Kirklin, P. Miron and G. Ross, A three-generation superstring model, parts I and II, Oxford University preprints (1986).
18. E. Witten, Phys. Lett. 153B:243 (1985).
19. E. Witten, New issues in manifolds of SU(3) holonomy, Princeton preprint (1985).

20. X. Wen and E. Witten, Phys. Lett. 166B:397 (1986);
M. Dine and N. Seiberg, String theory and the strong CP problem, IAS-Princeton preprint (1986).
21. J.-P. Derendinger, L.E. Ibáñez and H.P. Nilles, Nucl. Phys. B267:365 (1986).
22. K. Choi and J. Kim, Phys. Lett. 165B:71 (1985).
23. C. Burgess, A. Font and F. Quevedo, Texas preprint UTTG-31-85 (1985).
24. M. Dine, R. Rohm, N. Seiberg and E. Witten, Phys. Lett. 156B:55 (1985).
25. E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, Phys. Lett. 133B:61 (1983);
J. Ellis, A. Lahanas, D.V. Nanopoulos and K. Tamvakis, Phys. Lett. 134B:429 (1984);
J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. B241:406 (1984); B274:373 (1984).
26. J. Breit, B. Ovrut and G. Segrè, Phys. Lett. 162B:303 (1985);
P. Binétruy and M.K. Gaillard, Phys. Lett. 168B:347 (1986).
27. Y.J. Ahn, J. Breit and G. Segrè, Pennsylvania preprint (1985).
28. M. Quiros, CERN preprint TH.4363/86 (1986).
29. L.E. Ibáñez and H.P. Nilles, Phys. Lett. 169B:354 (1986).
30. M. Dine, N. Seiberg, X. Wen and E. Witten, Non-perturbative effects on the string world sheet, Princeton preprint (1986);
J. Ellis, C. Gomez, D.V. Nanopoulos and M. Quiros, CERN preprint TH.4395/86 (1986).
31. D. Gross and E. Witten, Superstring modifications of Einstein's equations, Princeton preprint (1986);
Y. Kikuchi, C. Marzban and Y. Ng, North Carolina University preprint IFP-272-UNC (1986).
32. S. Cecotti, S. Ferrara, L. Girardello and M. Porrati, CERN preprint TH.4253/85 (1985);
S. Cecotti, S. Ferrara, L. Girardello, A. Pasquinucci and M. Porrati UCLA preprint 85-TEP 24 (1985).
33. J.-P. Derendinger, L.E. Ibáñez and H.P. Nilles, Phys. Lett. 155B:65 (1985).
34. S. Ferrara, L. Girardello and H.P. Nilles, Phys. Lett. 125B:457 (1983).
35. R. Nepomechie, Y. Wu and A. Zee, Washington preprint 40048-02 P5 (1985).
36. L.E. Ibáñez and J. Mas, CERN preprint TH.4426/86 (1986).
37. E. Witten, Nucl. Phys. B258:75 (1985).
38. J. Breit, B. Ovrut and G. Segrè, Phys. Lett. 158B:33 (1985).
39. M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi and N. Seiberg, Nucl. Phys. B259:549 (1985).
40. S. Cecotti, J.-P. Derendinger, S. Ferrara, L. Girardello and M. Roncadelli, Phys. Lett. 156B:318 (1985).
41. E. Cohen, J. Ellis, K. Enqvist and D.V. Nanopoulos, Phys. Lett. 165B:76 (1985).
42. S.M. Barr, Phys. Rev. Lett. 55:2778 (1985).
43. M. Drees, N. Falck and M. Glück, Dortmund preprint 85/25 (1985).
44. P. Binétruy, S. Dawson, I. Hinchliffe and M. Sher, LBL preprint 203173 (1985).
45. F. Del Aguila, G. Blair, M. Daniel and G.G. Ross, Nucl. Phys. B272:413 (1986).
46. J. Ellis, K. Enqvist, D.V. Nanopoulos and F. Zwirner, CERN preprint TH.4323/85 (1985).
47. K.S. Narain, Rutherford preprint RAL-85-097 (1985).
48. J. Ellis, K. Enqvist, D.V. Nanopoulos and S. Sarkar, Phys. Lett. 167B:457 (1986);
G. Steigman, K. Olive, D. Schramm and M. Turner, Minnesota preprint UMN-TH-562 (1986).

49. L.E. Ibáñez and C. Lopez, Nucl. Phys. B233:511 (1984);
L.E. Ibáñez, C. Lopez and C. Muñoz, Nucl. Phys. B256:218 (1985).
50. S. Jones and G. Ross, Phys. Lett. 155B:69 (1984).
51. C. Kounnas, A. Lahanas, D.V. Nanopoulos and M. Quiros, Nucl. Phys.
B236:438 (1984).