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GEOMETRIC STRINGY GRAVITY

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A B S T R A C T

Following the indications that Einstein's gravity is corrected due to superstring effects, we propose a model for even dimensional gravity that has a very strong geometrical appeal. Its leading correction to Einstein's gravity is the Gauss-Bonnet invariant and for $D > 6$ it also contains all the additional Euler invariants. The model emphasizes the role of two-dimensional subspaces as the elementary building blocks of the higher dimensional space-time. It is the natural consistent spin-2 generalization of the Born-Infeld (spin-1) action.

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The recent developments in superstring theory¹⁾ which led to the construction of the heterotic string²⁾ point toward making heterotic superstring theories serious candidates for correctly describing quantum gravity.

There are a number of suggestions for the possible structure of the corrections to the classical Einstein action resulting from the slope expansion.

The first indication stems from the anomaly cancellation mechanism¹⁾. The requirement for anomaly freedom of $D = 10$ chiral supergravity³⁾ necessitates the addition of a gravitational Lorentz-Chern-Simons term (LCS). However, in order not to break the local supersymmetry of the initial $D = 10$ model this term has to be accompanied by its full supersymmetric completion, which is still unknown. Some partial results we are aware of up to now, indicate that the supersymmetric completion of the gravitational IOD-LCS term must contain the Gauss-Bonnet quadratic invariant⁴⁾. (There is, however, one calculation⁵⁾ providing a non-Gauss-Bonnet supersymmetric completion.) The fact that in 4D the full supersymmetric completion of the LCS term is known⁶⁾ lends support to the belief that such a term will also be present in the supersymmetric completion of the $d = 10$ theory. On the other hand, it was noted that when supersymmetrized the Gauss-Bonnet term requires the gravitational Chern-Simons term⁷⁾. Much more conclusive are the results obtained with the alternative dual version of $D = 10$ chiral supergravity⁸⁾ where the presence of the supersymmetrized Gauss-Bonnet term implies the presence of the Chern-Simons term required for anomaly cancellation⁹⁾.

Recent work on the low-energy limit of strings¹⁰⁾ gives results which are consistent with the Gauss-Bonnet term being the leading correction to Einstein's gravity provided unitarity is assumed. Further support to the need for a Gauss-Bonnet term stems from the Kaluza-Klein approach¹¹⁾ and from an effective action calculation¹²⁾ which confirmed that, starting from the heterotic string, the leading correction to Einstein's gravity is proportional to the precise combination of the square of the Riemann tensor appearing in the Gauss-Bonnet term.

All these arguments provide an independent support to the ghost freedom requirement of the string slope expansion, which was emphasized by Siegel, as mentioned in Ref. 13). This was later shown¹³⁾ to lead to the Gauss-Bonnet $\sim R^2$ combination as the leading ghost-free correction to classical Einstein gravity.

Thereafter Halpern and Zwiebach¹³⁾ conjectured that all the Euler densities may play a role in the low-energy limit of string theories. That cubic and higher order Euler actions will not spoil the unitarity of the corrected gravity theory was proved in a very elegant way by Zumino¹⁴⁾. This fact gives rise to the possibility of a large family of corrections to Einstein's gravity, containing higher powers of the Riemann tensor, for which the number of independent coefficients is in principle $D/2-2$ for even dimensional spaces (and $D/2-1$ for the odd dimensional ones). All these developments made it very attractive to pursue the qualitative study of the effective gravity theory characterized by Gauss-Bonnet terms in addition to the Einstein action¹⁵⁾. Several interesting features were discovered. It was shown, for instance, that the model has both flat and anti-de Sitter space-times as solutions, with the latter branch being unstable. Also the spherically symmetric solution was explicitly found to behave either like Schwarzschild with positive mass or as Schwarzschild-anti-de Sitter with negative mass.

As an additional fact we should mention the remarkable spin-1 result derived from the effective superstring action approach¹⁶⁾. The effective action for an Abelian vector field coupled to the open bosonic string in $D = 26$ coincides in the tree approximation of the string theory with the Born-Infeld action¹⁷⁾ $I_{BI} \sim \int d^Dx \sqrt{-\det(\eta_{rs} + g^{\frac{1}{2}} f_{rs})}$ which is known to have a supersymmetric extension in $D = 4$ ¹⁸⁾. The interesting property of Born-Infeld type actions is their non-linear form which admits non-trivial vortex solutions¹⁹⁾. Fradkin and Tseytlin pointed out that there may be some kind of bootstrapping, and the effective action derived from a fundamental superstring theory has stringlike $\sim$ solitonic classical solutions.

We present here a modified gravity model, which is only meaningful for even dimensional space-times, $D = 2d$. In odd dimensions, its stringy geometrical content, as will be seen later, disappears. The need for even dimensions has, in fact, a physical motivation; only even dimensional theories are flexible enough to accommodate chirality. Another property of this model is that it emphasizes the light-front structure of relativistic physics. The light-front breaking of the D -dimensional Lorentz invariance, requires two dimensions for the associated light-front variables, $x^\pm \equiv 2^{-\frac{1}{2}}(x^0 \mp x^{D-1})$. If in addition the covariant structure of D -dimensions is taken into account, all possible two-dimensional subspaces should be treated on the same footing. Thus we are led to consider space-time as being constructed out of all possible ensembles of two-dimensional subspaces.

Furthermore, since the effective pointlike action originates from a more fundamental heterotic superstring theory, which has two-dimensional objects (strings) built in, it seems that the two-dimensional substructure of the D-dimensional space-time should play a special role. Along these lines, Veneziano²⁰⁾ has pointed out recently that if indeed a dual superstring theory is a theory of everything (TOE), all phenomena must be understood in terms of quantum processes involving two-dimensional surfaces (he also emphasized that only two fundamental dimensionful constants and no dimensionless parameters should be present: the velocity of light c and the fundamental length $\lambda = \sqrt{\alpha'}$ needed for the quantization of the Nambu-Goto action²¹⁾).

It seems that in string physics geometry, is even more needed than in Einstein's gravity. Therefore, we use here the intrinsically geometric notation of Refs. 14) and 15) in order to stress the geometrical meaning of the pointlike fields entering into the model. These are the vielbeins $e^a \equiv dx^r e_r^a$ and the connections $\omega_{ab} \equiv dx^r \omega_{rab} = -\omega_{ba}$ one forms^{*)}.

The torsion T^a and the Riemann tensor two-forms are given by

$$T^a \equiv \frac{1}{2} dx^r dx^s T_{rs}^a = de^a + \omega^a_b e^b \equiv De^a = \frac{1}{2} dx^r dx^s (D_r e_s^a - D_s e_r^a) \quad (1a)$$

$$R_{ab} \equiv d\omega_{ab} + \omega_{ac} \omega^c_b = \frac{1}{2} dx^m dx^n R_{mnab}. \quad (1b)$$

Two Bianchi identities hold:

$$DT^a \equiv dT^a + \omega^a_b T^b \equiv R^a_b e^b, \quad (2a)$$

$$DR_{ab} \equiv dR_{ab} + \omega_a^c R_{cb} + \omega_b^c R_{ac} \equiv \frac{1}{3!} dx^{mnp} D_m R_{npab} \quad (2b)$$

as a direct consequence of the fundamental property of Cartan's exterior differential operator d : $d^2 = 0$. As is now becoming standard, we omit the wedge symbol for the external product of forms as it does not lead to ambiguities. We assume a Riemannian geometry, i.e., $T^a = 0$, a possible torsion is expected to be generated either by fermions or by spin-1 gauge matter.

*) Lorentz flat indices are indicated with the first letters of the alphabet, while world indices are denoted with letters from the middle of the alphabet. The metric η_{ab} has a signature D-2.

As was pointed out before we believe that two-dimensional subspaces play a special role in pointlike string-inspired effective actions. The idea that tangent two-planes should be considered as the fundamental dynamical variables for describing the bosonic string was implemented recently²²⁾ where the appropriate action was constructed.

We denote two-dimensional subspaces by their associated two-forms $e_{c_1 c_2}$

$$e_{c_1 c_2} \equiv e_{c_1} e_{c_2} = \frac{1}{2} dx^{\eta_1} dx^{\eta_2} (e_{\eta_1 c_1} e_{\eta_2 c_2} - e_{\eta_2 c_1} e_{\eta_1 c_2}) \quad (3)$$

whose product, contracted with the totally antisymmetric Levi-Civita tensor $\varepsilon^{c_1 \dots c_D}$, $\varepsilon^{012 \dots, D-1} = +1$, gives the D-dimensional invariant volume.

The geometrical stringy gravity action we suggest to consider is the following:

$$I = \frac{1}{D! \alpha'^{D/2}} \int \varepsilon^{c_2 c_3 c_4 \dots c_{D-1} c_D} (e_{c_1 c_2} + 4(n-1)\alpha' R_{c_1 c_2}) \dots (e_{c_{D-1} c_D} + 4(n-1)\alpha' R_{c_{D-1} c_D}) - \\ - \frac{1}{D! \alpha'^{D/2}} \int \varepsilon^{c_2 \dots c_{D-1} c_D} e_{c_1 c_2} \dots e_{c_{D-1} c_D} - \frac{1}{D! \alpha'^{D/2}} \int (4(n-1)\alpha')^{D/2} \varepsilon^{c_2 \dots c_{D-1} c_D} R_{c_1 c_2} \dots R_{c_{D-1} c_D} \quad (4)$$

where $D = 2+n$, $\lambda^2 = \alpha' = \alpha\kappa$, $\kappa \equiv 4(\int d\Omega_n)G$, $\int d\Omega_n$ is the area of the unit n-dimensional sphere and G is Newton's constant. It is the effective volume generated by the set of all the two-dimensional subspaces, where each such two-dimensional subspace is represented by its gravitationally augmented two-form

$$e_{c_1 c_2} \rightarrow e_{c_1 c_2} + 4(n-1)\alpha' R_{c_1 c_2} . \quad (5)$$

This seems to be the most geometrical two-dimensionally-minded expression one could make out of the whole set of the D-dimensional Euler invariants $I_0, I_1, \dots, I_p, \dots, I_d$ and which contains the Einstein action as its first contribution. These invariants were studied in Ref. 14) and shown to have the remarkable property of not introducing higher than second order derivatives in the field equations even though generally they are expected to generate such higher order derivatives. Here

$$I_p \equiv \frac{1}{(\alpha')^{d-p} (d-2p)!} \int \varepsilon^{c_2 \dots c_{2p-1} c_{2p} c_{2p+1} \dots c_D} R_{c_1 c_2} \dots R_{c_{2p-1} c_{2p}} e_{c_{2p+1} \dots c_D} . \quad (6)$$

As this propagating behaviour is a common feature of all the Euler invariants, other possible effective stringy actions may be constructed out of Einstein's action augmented by any linear combination of the whole set of these invariants. The conjecture that all these invariants may contribute to the effective string generated action appears in Ref. 13). Here we are proposing one specific action of this family, motivated by the two-dimensional stringy covariance mentioned before. Note that the action (4) is a generalized spin-2 Born-Infeld type action that is obtained by the substitution of the one-dimensional metric $\eta_{r_1 r_2}$ by the two-dimensional one $e_{c_1 c_2}$, and the spin-1 Maxwell field strength $f_{r_1 r_2}$ by the field strength of the gravity spin-two field $R_{c_1 c_2}$.

Expanding the action (4) leads to

$$I = \sum_{p=1}^d b^p \binom{d}{p} (d-2p)! \frac{1}{D!} I_p \quad (7)$$

where $b = 4(n-1)$ is determined by the requirement that the leading correction to Einstein's gravity is the string low energy Gauss-Bonnet term.

Recall the ϵ -identities

$$\epsilon^{a_1 \dots a_p b_1 \dots b_{D-p}} \epsilon_{a'_1 \dots a'_p b_1 \dots b_{D-p}} = -(D-p)! \delta^{a_1 \dots a_p}_{a'_1 \dots a'_p}, \quad (8a)$$

$$e_{m_1 m_2 \dots m_p c_{p+1} \dots c_D} \epsilon^{m_1 m_2 \dots m_p m_{p+1} \dots m_D} = e \epsilon_{b_1 \dots b_p c_{p+1} \dots c_D} e^{b_{m_1}} \dots e^{b_{m_p}}, \quad (8b)$$

where $e \equiv \det e_r^a$ and e_a^s is the inverse vielbein $e_r^a e_a^s = \delta_r^s$. These identities are useful for translating expressions written in terms of differential forms to the more traditional component formalism. The first Euler invariant appearing in (7) is Einstein's action and the second is the Gauss-Bonnet term, which in $D > 4$ is not an exact form. In fact,

$$I_1 = -\frac{1}{(\alpha')^{D-1}} \langle e R^{c_1 c_2}_{c_1 c_2} \rangle, \quad (9a)$$

$$I_2 = -\frac{1}{(\alpha')^{D-2}} \langle e [R^{c_1 c_2}_{c_3 c_4} R^{c_3 c_4}_{c_1 c_2} - 4 R^{c_1 c_2}_{c_2 c_1} + R^2] \rangle, \quad (9b)$$

$$I_3 = -\frac{1}{(d')^{d-3}} \langle e [R^{a_1 a_2}_{b_1 b_2} R^{b_1 b_2}_{c_1 c_2} R^{c_1 c_2}_{a_1 a_2} - 24 R^{a_1 a_2}_{b_1 b_2} R^{b_1 b_2}_{a_1 a_3} R^{b_2 a_3}_{a_2 b_3} - \\ - 32 R^{a_1 a_2}_{b_1 b_2} R^{b_1 b_2}_{a_1 a_2} R^c_d + 24 R^{a_1 a_2}_{b_1 b_2} R^{b_1}_{a_1} R^{b_2}_{a_2} + 3 R R^{a_1 a_2}_{b_1 b_2} R^{b_1 b_2}_{a_1 a_2} (9c) - \\ - 4 R R^a_b R^b_a + R^3] \rangle$$

where $R^{a_1 a_2}_{b_1 b_2} \equiv R_{m_1 m_2 b_1 b_2} e^{a_1 m_1 a_2 m_2}$, $R^a_b \equiv R^{ac}_{bc}$, $R \equiv R^a_a$. An interesting property is that in the absence of torsion, I_d is an exact form¹⁴⁾. The first non-trivial Gauss-Bonnet term appears in $D = 6$. Indeed the action (4) reduces to the Gauss-Bonnet model of gravity which was introduced in Ref. 15). It is expected that six-dimensional field theoretical models may have some importance in heterotic string theories as implied by Ref. 23). For $D > 6$, the action (4) deviates from that of Ref. 15) by higher order corrections in α' .

The field equations can be derived directly from (4) by varying the vielbein only. In the language of forms they read

$$\begin{aligned} & * \left\{ e^{c_2 \dots c_{d-2}} e_{c_2} (e_{c_3 c_4} + b \alpha' R_{c_3 c_4}) \dots (e_{c_{d-1} c_d} + b \alpha' R_{c_{d-1} c_d}) \right\} - \\ & - \left\{ e^{c_2 \dots c_{d-2}} e_{c_2} e_{c_3 c_4} \dots e_{c_{d-1} c_d} \right\} = 0 \end{aligned} \quad (10a)$$

where the star $*$ means the dual form in D -dimensions. In the component formalism, (10a) is equivalent to the following expression

$$\begin{aligned} & \binom{d-1}{1} (d-3)! \bar{2}^1 \delta_{b_1 b_2} R^{b_1 b_2}_{c_1 c_2} + \binom{d-1}{2} (d-5)! (b \alpha')^2 \bar{2}^2 \delta_{b_1 b_2 b_3 b_4} R^{b_1 b_2}_{c_1 c_2} R^{b_3 b_4}_{c_3 c_4} + \\ & + \dots + \binom{d-1}{d-1} (b \alpha')^{d-2} \bar{2}^{d-1} \delta_{b_1 b_2 \dots b_{d-3} b_{d-2}} R^{b_1 b_2}_{c_1 c_2} \dots R^{b_{d-3} b_{d-2}}_{c_{d-3} c_{d-2}} = 0. \end{aligned} \quad (10b)$$

Note that the first term is just the Einstein tensor written in vielbein form, and the second is the Gauss-Bonnet contribution. Thus, the first two leading order contributions for small α' coincide with the Gauss-Bonnet model of Ref. 15), as was mentioned before.

We would like to find now the maximally symmetric spaces which solve these equations. Inserting the maximally symmetric ansatz

$$R^{b_1 b_2}_{c_1 c_2} = \lambda \delta_{c_1 c_2}^{b_1 b_2} \quad (11a)$$

into Eq. (10b), it is found that λ is any one of the real roots of

$$p_{d-1}(b\lambda\alpha') \equiv (1+b\lambda\alpha')^{d-1} - 1 \quad (11b)$$

provided $D = 2d$.

Note that for $d = 2k$ only Minkowski space with $\lambda = 0$ is a solution of (11b), whereas for $d = 2k+1$ there is in addition the solution $b\lambda\alpha' = -2$ describing anti-de Sitter space-time. In particular, for $D = 6, 10$ and 26 we have these two solutions.

In order to study the stability of these backgrounds, we note that in the semiclassical approximation of the corresponding quantum theory, an expansion around the classical solutions is performed. Keeping only quadratic terms in the quantum fluctuations (or those linear in the field equations) in the action one can study the stability of the backgrounds. We observe the polynomial structure in the Riemann tensor of action (4) and field Eqs. (10b). Then, the methods of Boulware and Deser¹⁵⁾ given for the Gauss-Bonnet model can also be applied here. It is enough to calculate the values of the characteristic polynomial (11b) on its real roots. Thus, for $\lambda = 0$ (Minkowski space) $\delta^2 I \sim p'_{d-1}(0) = d-1 > 0$ which makes this background stable, having only the gravitational excitations. However, for $b\lambda\alpha' = -2$ (anti-de Sitter space), $\delta^2 I \sim p'_{d-1}(-2) = -2k < 0$, which makes it unstable, excitations around it behave as those of ghost cosmological gravity, as pointed out in Ref. 15). Note that here we obtain this qualitative behaviour only for spaces of dimension $d = 4k+2$; for spaces of dimension $D = 4k$ only the Minkowski background appears (for odd D , our model is not defined).

The action we presented in this work has an extended type of "self-duality" symmetry

$$e_{\alpha} c_2 \rightarrow b\alpha' R_{\alpha} c_2 \quad ; \quad b\alpha' R_{\alpha} c_2 \rightarrow e_{\alpha} c_2$$

as can be seen directly from (4). This in addition to its geometrical significance as emphasized by the special role played by the two-dimensional subspaces, makes it a particularly attractive extension to Einstein's gravity when higher order string corrections are included.

In conclusion, we have introduced a gravity action which is inspired in two ways by string theory. It contains all the Euler invariants which introduce stringy-gravitational interactions without affecting the kinetic content of a spin-2 theory. The particular way in which these invariants contribute to the

action was determined by emphasizing the two-dimensional substructure of the even dimensional space, which is certainly an idea motivated by string theory. The action constructed in this way is a natural generalization to spin-2 of the spin-1 Born-Infeld theory, which was found as the effective pointlike action in the "tree" approximation of string theory. For all dimensions $D = 4k+2$ the action presents a uniform qualitative behaviour containing the stable Minkowski background and the unstable anti-de Sitter cosmological branch.

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