CERN-TH. 4462/86
N TH 86-7

**ANGULAR DISTRIBUTION OF LEPTON PAIRS
IN DRELL-YAN-LIKE PROCESSES**

P. Chiappetta,
CERN, Geneva, Switzerland

M. Le Bellac
CERN, Geneva, Switzerland, and
Laboratoire de Physique théorique^{*)}, Nice, France

ABSTRACT

We compute the angular distribution of lepton pairs at fixed transverse momentum using soft-gluon resummation. Theoretical predictions are compared with the recent NA10 experimental data. We also give predictions in the case of Z^0 production.

^{*)} Unité associée au CNRS (France).

Lepton pair production is one of the classic tests of quantum chromodynamics (QCD), and for this reason has been extensively studied experimentally and theoretically during the past 10 years [1]. One can find in the literature detailed descriptions of the cross-sections $d\sigma/dQ^2$, $d\sigma/dQ^2 dy$, $d\sigma/dQ^2 dy dq_T^2$, where Q , y and q_T are the mass, rapidity, and transverse momentum of the lepton pair — or the virtual photon. Some attention has also been given to the angular distribution of the leptons [2–6], but, until recently, no detailed comparison with data could be made. The recent results of the high-statistics NA10 experiment [7], using a π^- beam of 194 GeV/c (and 286 GeV/c under analysis) led us to reconsider the QCD calculation of this angular distribution. Indeed all the previous theoretical calculations at fixed q_T in the low-energy domain took into account only the so-called ‘hard component’ of the cross-section, in order to avoid the well-known singularity at $q_T = 0$. The theoretical curves presented in Ref. (7) rely on these calculations, which seem to give an acceptable fit to the data. Unfortunately, as we shall see later, this agreement is quite misleading.

The calculation of Ref. [6], which was done five years ago, took into account the singular part of the cross-section; however, as it was not entirely clear at that time how to treat the soft-gluon problem, the authors preferred to integrate over q_T . Nevertheless, they were able to show that the deviation with respect to the $(1 + \cos^2 \theta)$ distribution should be very small, much smaller than that predicted from the consideration of the hard component alone. Moreover, soft-gluon resummation has been shown [8] to restore the naïve Drell–Yan prediction.

As the experimental data exhibit a rather strong dependence on q_T , we decided to re-analyse the problem completely, by making use of the important progress which has recently been made in soft-gluon resummation at low q_T [9, 10]. The outcome of this analysis can be stated briefly: we find that the deviations from the $(1 + \cos^2 \theta)$ law are less than 5% in the range $0 \leq q_T \leq 3$ GeV/c, and therefore smaller than the experimental data [7]. Our results confirm that the authors of Ref. 6 were basically correct, although they were not able to give any information on the q_T dependence.

In Section 1 we recall briefly the theoretical analysis [9, 10] and describe our numerical results in the case of lepton pair production via virtual photons. In Section 2 we extend an analysis to the case of Z^0 production, and show that a measurement of the angular distribution of the lepton pairs from Z^0 decay could be — when enough statistics are collected — an interesting check of QCD, more reliable than ordinary lepton pair production owing to the much higher energy.

1. ANGULAR DISTRIBUTION IN LEPTON PAIR PRODUCTION

Let us first recall the theoretical formalism for including soft-gluon effects when one wishes to calculate the q_T dependence of the differential cross-section. We follow the formalism of Altarelli et al. [9], which is equivalent, for practical purposes, to that of Collins and Soper [10], and is somewhat simpler to follow. Schematically, in the case of $q\bar{q}$ annihilation, the perturbative q_T distribution at order α_s is of the form

$$\frac{d\sigma}{dq_T^2} = \sigma_0 (1 + A) S(q_T^2) + B \left(\frac{\ln Q^2/q_T^2}{q_T^2} \right)_+ + \frac{C}{(q_T^2)_+} + Y(q_T^2) \quad (1)$$

where σ_0 is the Drell-Yan cross-section, and A, B, C, and Y are higher-order corrections. The term $Y(q_T^2)$ is regular at $q_T = 0$. The singular terms B and C are exponentiated after Fourier transformation in impact parameter space, leading to the following form of the differential cross-section:

$$\frac{d\sigma}{dq_T^2} = Y(q_T^2) + \int \frac{d^2b}{4\pi} e^{i\vec{q}_T \cdot \vec{b}} \sigma_0 (1 + A) \exp[S(b)] \quad (2)$$

where the Sudakov form-factor $S(b)$ is given by

$$S(b) = \int_0^{(q_T^2)_{\max}} \frac{dk^2}{k^2} (J_0(kb) - 1) \left(B \ln \frac{Q^2}{k^2} + C \right) \quad (3)$$

It is important to note that the exponentiation alters only the *shape* of the q_T dependence, but leaves unchanged the q_T -integrated cross-sections $d\sigma/dQ^2 dy$ or $d\sigma/dQ^2$. We have checked (Fig. 1) that the previous expressions give an acceptable fit to the q_T distributions extracted from Ref. (11), especially for low masses, whereas the hard component is in disagreement with data. Let us remark that we have not attempted to make a fit since we have directly taken the values of the parameters extracted from W and Z^0 production and have not included an intrinsic transverse momentum.

Let us now turn to the angular distribution of the lepton pairs; let $\Omega = (\theta, \varphi)$ be the angles defining the direction of one of the leptons in the rest frame of the lepton pair; we shall use exclusively the Collins-Soper frame [12], which minimizes possible intrinsic transverse momentum effects whilst being by far the most convenient frame for theoretical calculations. We recall that in this frame the momentum $\vec{p}_1$ of the projectile and that ($\vec{p}_2$) of the target lie in the xz plane, the z axis being the bissectrix of the directions of $\vec{p}_1$ and $-\vec{p}_2$. The angle β between $\vec{p}_1$ and the z axis is given as a function of $\varrho = q_T/Q$ by

$$\tanh \beta = \varrho \quad (4)$$

The angular distribution defines three coefficients, A_0 , A_1 , and A_2 :

$$\begin{aligned} \frac{d\sigma}{dQ^2 dy dq_T^2 d\Omega} = \frac{3}{16\pi} \frac{d\sigma}{dQ^2 dy dq_T^2} & \left[(1 + \cos^2 \theta) + \frac{1}{2} A_0 (1 - 3 \cos^2 \theta) \right. \\ & \left. + A_1 \sin 2\theta \cos \varphi + \frac{1}{2} A_2 \sin^2 \theta \cos 2\varphi \right] \end{aligned} \quad (5)$$

which are related to the more conventional coefficients λ , μ , and ν by [7]

$$\lambda = \frac{2 - 3 A_0}{2 + A_0}, \quad \mu = \frac{2 A_1}{2 + A_0}, \quad \nu = \frac{2 A_2}{2 + A_0} \quad (6)$$

Let us now compute the partonic cross-section to first order in α_s ; it will read:

$$\frac{d\hat{\sigma}}{dQ^2 d\gamma dq_T^2 d\Omega} = \alpha_s N [|M|^2 (1 + \cos^2 \theta) + \frac{1}{2} \hat{A}_0 (1 - 3 \cos^2 \theta) + \hat{A}_1 \sin 2\theta \cos \varphi + \frac{1}{2} \hat{A}_2 \sin^2 \theta \cos 2\varphi] \quad (7)$$

where N is a normalization factor and $|M|^2$ the matrix element squared summed over all photon polarizations. Following Altarelli et al. [9], we introduce the notation

$$x_1^0 = \sqrt{\tau} e^Y, \quad x_2^0 = \sqrt{\tau} e^{-Y}, \quad z = \frac{Q^2}{s} \quad (8)$$

where s is the centre-of-mass energy squared of the hadronic process. It will also be convenient to define the partonic variables $z_i = x_i/x_i^0$, where x_i is the fractional parton momentum inside the hadron. These variables allow us to write, without any approximation, rather simple formulae for the coefficients $\hat{A}_i$ in Eq. (7). In the case of the subprocess $q + \bar{q} \rightarrow \gamma^* + g$, we obtain

$$|M|^2 = \frac{(1+e^2)(z_1^2 + z_2^2)}{z_1 z_2 e^2} \quad (9)$$

$$\hat{A}_0 = \hat{A}_2 = \frac{z_1^2 + z_2^2}{z_1 z_2} = \frac{e^2}{(1+e^2)} |M|^2 \quad (10)$$

$$\hat{A}_1 = \frac{1}{z_1 z_2 e} (z_2^2 - z_1^2) \quad (11)$$

In the case of the subprocess $g + q \rightarrow \gamma^* + q$ (gluon in the projectile taken as particle 1), the formulae are slightly more involved:

$$|M|^2 = \frac{(z_1 + z_2)}{(1 - z_2^2) z_1} \left[(z_1 z_2)^2 + (1 - z_1 z_2)^2 + \frac{z_1^2 (1 - z_2^2) (1 + z_2^2 + 2 z_1 z_2)}{(z_1 + z_2)^2} \right] \quad (12)$$

$$\hat{A}_0 = \hat{A}_2 = \frac{(1 - z_1^2) (z_2^2 + 2 z_1^2 + 3 z_1 z_2)}{z_1 (z_1 + z_2)} \quad (13)$$

$$\hat{A}_1 = \frac{1}{z_1} \sqrt{\frac{1-z_1^2}{1-z_2^2}} (z_2^2 - 2z_1^2) \quad (14)$$

When the gluon belongs to the target, one should interchange the indices 1 and 2 and change the sign of $\hat{A}_1$. Were the matrix elements $|M|^2$ not singular at $q_T = 0$, the hadronic coefficients A_i would be simply obtained by integrating the $\hat{A}_i$'s with hadron structure functions and by dividing the result by the sum of the Drell-Yan term and the integral of $|M|^2$. However, $|M|^2$ is singular at $q_T = 0$, and we must of course give a prescription in order to deal with this singularity. On the other hand, the $\hat{A}_i$'s have at most a q_T^{-1} singularity, and are thus integrable. This must be the case, since the singularity is absorbed into a redefinition of the structure functions, and must be proportional to the Drell-Yan process: only the term proportional to $(1 + \cos^2 \theta)$ in Eq. (7) can be singular.

The singularity in $|M|^2$ will be handled in exactly the same way as in the calculation of the cross-section, namely by exponentiation of soft gluons in impact parameter space. The hadronic coefficients A_i will be obtained by integrating the $\hat{A}_i$'s with hadron structure functions, and by dividing the result by $d\sigma/dQ^2 dy dq_T^2$. We are thus able to predict the A_i 's for arbitrary values of q_T , whilst previous calculations [2-5] were restricted to q_T 'large enough' (and, as we shall see later on, this is never the case in practice), or to predictions integrated over q_T [6-8].

In our numerical calculations, we have chosen the first set of Duke-Owens structure functions [13] with $\Lambda = 200$ MeV. The results for λ , μ , and ν are plotted in Fig. 2 as a function of q_T , for $y = 0$ and $Q^2 = 25 \text{ GeV}^2$ at $p_{\text{lab}} = 194 \text{ GeV}/c$. We get λ close to one and μ and ν close to zero, a result which could have been anticipated from Ref. (6). If we compare the values obtained by using the hard component, we get very strong deviations when q_T increases, since λ decreases with q_T reaching 0.3 at 3.5 GeV, whereas ν increases up to 0.35 at 3.5 GeV. In Fig. 3 we give the values of three coefficients for $Q^2 = 64 \text{ GeV}^2$ and $y = 0$. The QCD corrections become smaller when the lepton-pair mass increases. We also show in Fig. 3 our results for $Q^2 = 25 \text{ GeV}^2$ and $y = 0.5$: the coefficients do not vary appreciably when we vary the rapidity. Since the NA10 data at fixed q_T are integrated over Q and positive Feynman x_F , we can only compare our QCD predictions to the general trend of the data: if μ in the whole q_T range and λ for $q_T > 1 \text{ GeV}$ are in agreement with the NA10 measurements, we do not reproduce the q_T behaviour of ν . Since the main part of the data lie in an x_F range not sensitive to higher twist contributions [14] the transverse momentum dependence of ν cannot be a higher twist effect [15].

It is now necessary to comment on the fits given in Ref. (7). These fits use only the hard component of the cross-section: more precisely, it uses the form of $|M|^2$ given in Eqs. (9) and (12) for $q_T \neq 0$. This allows simple predictions: for example, if the annihilation process dominates, one finds from Eq. (10), $A_2 = q^2/(1 + q^2)$, independently of the structure functions. Unfortunately, as can be seen from Fig. 1 the hard component fails to reproduce the differential q_T cross-section by an order of magnitude, even at $q_T = 3 \text{ GeV}/c$! In order to check the fits of Ref. (7), we give in Figs. 2 and 3 our predictions when we use the same assumptions, and find a good agreement with the data for μ and ν but a disagreement for λ at large q_T . It is clear, at least in our opinion, that this comparison has no theoretical significance.

We have not attempted to include any intrinsic transverse momentum in our calculation, as it is not clear how to do it in a consistent way; neither is it clear, since we want to test QCD, what we would learn from a fit involving some completely arbitrary parameters.

It is of course quite possible that the cross-section we are looking at is particularly sensitive to non-perturbative effects since we are in a low-energy domain. Nevertheless, we found it interesting to point out that there are some QCD predictions that are in disagreement with data, in a range of Q^2 [$16 \text{ (GeV/c)}^2 \lesssim Q^2 \lesssim 100 \text{ (GeV/c)}^2$], which is typical in the determination of structure functions. In Fig. 4 we give our predictions for λ , μ , and ν for $Q^2 = 25 \text{ GeV}^2$ and 64 GeV^2 at $y = 0$ and $p_{\text{lab}} = 286 \text{ GeV/c}$, an energy which has recently been investigated by the NA10 Collaboration and whose data are under analysis.

2. ANGULAR DISTRIBUTIONS FOR Z^0 PRODUCTION

Let us now turn to the angular distribution of lepton pairs in the case of Z^0 production. The effects of soft-gluon emission via quark form factors in the DLLA approximation have been studied in Ref. 16 for $p\bar{p}$ collisions. We write the coupling of Z^0 's to leptons and quarks as

$$\bar{\nu}(l') (a_l \gamma_\mu + b_l \gamma_\mu \gamma_5) \nu(l) \quad (\text{leptons}) \quad (15)$$

$$\bar{\nu}(p') (a \gamma_\mu + b \gamma_\mu \gamma_5) \nu(p) \quad (\text{quarks}) \quad (16)$$

with the following values of (a_l, b_l) and (a, b) :

$$a_l = \frac{-1 + 4 \sin^2 \theta_w}{2 \sin 2 \theta_w}, \quad b_l = \frac{1}{2 \sin 2 \theta_w} \quad (17)$$

$$a = \frac{1 - 8/3 \sin^2 \theta_w}{2 \sin 2 \theta_w}, \quad b = \frac{-1}{2 \sin 2 \theta_w} \quad (u, c \text{ quarks}) \quad (18)$$

$$a = \frac{-1 + 4/3 \sin^2 \theta_w}{2 \sin 2 \theta_w}, \quad b = \frac{1}{2 \sin 2 \theta_w} \quad (d, s \text{ quarks}) \quad (19)$$

In the Born approximation, the angular distribution $N(\Omega)$ is given by the following formula:

$$N(\Omega) = 1 + \cos^2 \theta + 8 \alpha \alpha_l \cos \theta \quad (20)$$

with

$$\alpha = \frac{\text{Re } ab^*}{|a|^2 + |b|^2} \quad , \quad \alpha_e = \frac{\text{Re } a_e b_e^*}{|a_e|^2 + |b_e|^2} \quad (21)$$

The new feature is of course the parity-violating term in Eq. (20). The corrections of order α_s will have exactly the same form as in Section 1 for the parity-conserving process, and they will modify the parity-violating one in the following way:

$$\cos \theta \rightarrow (1 - A_3) \cos \theta + A_4 \sin \theta \cos \varphi \quad (22)$$

The formulae for the partonic coefficients $\hat{A}_3$ and $\hat{A}_4$ are analogous to those given previously; in the case of the subprocess $q + \bar{q} \rightarrow Z^0 + g$ we find

$$\hat{A}_3 = \frac{(1 + z_1 z_2)(z_1^2 + z_2^2)}{z_1 z_2 (1 + z_1)(1 + z_2)} \quad (23)$$

$$\hat{A}_4 = \frac{(1 + z_1 z_2)(z_1 + z_2)(z_2^2 - z_1^2)}{z_1 z_2 (1 - z_1^2)(1 - z_2^2)} e \quad (24)$$

whilst for the subprocess $g + q \rightarrow Z^0 + q$ (gluon in the projectile) we find

$$\begin{aligned} \hat{A}_3 = \frac{1}{z_1(1+z_2)} & [2z_1^3 + 4z_1^2 - z_1 - 1 + (1-z_2)(-2z_1^3 - 6z_1^2 - z_1 + 1) + z_1(1-z_2^2) \\ & \cdot (1+2z_1)] - z_2(1+z_2^2 + 2z_1 z_2) / (z_1 + z_2) \end{aligned} \quad (25)$$

$$\hat{A}_4 = z_2(z_1 + z_2) \left[\frac{1}{z_1} + \frac{1}{(1-z_2^2)} \left(\frac{(1-z_1^2)(z_2 + 2z_1)}{z_1(z_1 + z_2)} - \frac{1}{z_2} \right) \right] e \quad (26)$$

When the gluon is in the target, one has to interchange the indices 1 and 2 and to change the sign of $\hat{A}_4$; let us recall that the calculation has been done in the Collins-Soper frame. In Fig. 5 we give predictions for λ , ν , and the parity-violating coefficients C and S (obtained for $\hat{A}_3$ and $\hat{A}_4$ after folding with the structure functions) for two center-of-mass energies $\sqrt{s} = 630 \text{ GeV}$ and $\sqrt{s} = 2 \text{ TeV}$. If the coefficient S remains very small, the QCD corrections reach 15% for C at $\sqrt{s} = 2 \text{ TeV}$ and $q_T = 40 \text{ GeV}$.

3. CONCLUSIONS

In view of the problems encountered in lepton pair production via virtual photons in a low-energy domain, it would be interesting to check the QCD predictions for angular distributions at energies where non-perturbative effects (such as intrinsic transverse momentum) are presumably unimportant. We are of course aware that it may take some time before the necessary number of Z^0 's is collected, but Z^0 production seems particularly well suited to such a study.

Acknowledgements

We are very grateful to G. Martinelli for providing us with his computer program, and also for discussions. We also wish to thank P. Darriulat, A. Romana and H. Suter for useful discussions.

REFERENCES

- [1] For a review see, for example, G. Altarelli, Phys. Rep. **81** (1982) 1.
- [2] J.C. Collins, Phys. Rev. Lett. **42** (1979) 291.
- [3] J. Cleymans and M. Kuroda, Nucl. Phys. **B155** (1979) 480; Phys. Lett. **80B** (1979) 385.
- [4] J. Lindfors, Phys. Scri. **20** (1979) 19.
- [5] R.L. Thews, Phys. Rev. Lett. **43** (1979) 987, 1968 (E); Phys. Lett. **100B** (1981) 339.
- [6] T. Grandou, J.L. Meunier and G. Plaut, Phys. Lett. **101B** (1981) 267.
- [7] S. Falciano et al., (NA10 Collaboration), preprint CERN-EP/86-35 (1986).
- [8] P. Chiappetta and T. Grandou, Z. Phys. **C26** (1985) 345.
- [9] G. Altarelli, R.K. Ellis, M. Greco and G. Martinelli, Nucl. Phys. **B246** (1984) 12.
- [10] J.C. Collins, D.E. Soper and G. Sterman, report CERN-TH. 3923 (1984), unpublished.
- [11] P. Bordalo et al., Paper No. 30, submitted to Session P13, Int. Conf. on High-Energy Physics, Bari, 1985.
- [12] J.C. Collins and D.E. Soper, Phys. Rev. **D16** (1977) 2219, and references therein.
- [13] D.W. Duke and J.F. Owens, Phys. Rev. **D30** (1984) 49 and **D30** (1984) 943.
- [14] E.L. Berger and S.J. Brodsky, Phys. Rev. Lett. **42** (1979) 940.
E.L. Berger, Z. Phys. **C4** (1980) 289.
- [15] H. Suter, private communication.
- [16] M. Chaichian, M. Hayashi and K. Yamagishi, Phys. Rev. **D26** (1982) 3078.

Figure captions

- Fig. 1 :** The differential cross-section ($1/q_T$) ($d\sigma/dq_T dQ dy$) at $p_{\text{lab}} = 194 \text{ GeV}/c$ for $4.5 < Q < 5.5 \text{ GeV}$ and $7.5 < Q < 8.5 \text{ GeV}$ for $0 < y < 0.2$. Full curves: prediction from Ref. (9). Dashed curves: hard component only.
- Fig. 2 :** The coefficients λ , μ , and ν as a function of q_T at $p_{\text{lab}} = 194 \text{ GeV}/c$ for $Q = 5 \text{ GeV}$ and $y = 0$. Full curve: our prediction. Dashed curve: hard component. Data (integrated over Q and x_F) are from ref. 7.
- Fig. 3 :** The coefficients λ , μ , and ν as a function of q_T at $p_{\text{lab}} = 194 \text{ GeV}/c$. Full curve: our prediction for $Q = 8 \text{ GeV}$ and $y = 0$. Dashed curve: hard component for $Q = 8 \text{ GeV}$ and $y = 0$. Dash-dotted curve: our prediction for $Q = 5 \text{ GeV}$ and $y = 0.5$.
- Fig. 4 :** The coefficients λ , μ , and ν as a function of q_T at $p_{\text{lab}} = 286 \text{ GeV}/c$. Full curve: our prediction for $Q = 5 \text{ GeV}$ and $y = 0$. Dashed curve: our prediction for $Q = 8 \text{ GeV}$ and $y = 0$.
- Fig. 5 :** The coefficients λ , ν , C , and S as a function of q_T for Z^0 production in $p\bar{p}$ collisions. Full curve: $\sqrt{s} = 630 \text{ GeV}$. Dashed curve: $\sqrt{s} = 2 \text{ TeV}$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5