CERN-TH. 4452/86
LAPP-TH-162

**MASSIVE LEPTON PAIR PRODUCTION REVISITED
WITH THE OPTIMIZED PERTURBATIVE QCD**

P. Aurenche
LAPP, Annecy-le-Vieux, France

P. Chiappetta,
CERN, Geneva, Switzerland

ABSTRACT

The existence of data on very massive Drell-Yan pairs produced by pion- or proton-induced reactions allows a comparison with the predictions of the optimized perturbative QCD. A good agreement is found.

CERN-TH. 4452/86
LAPP-TH-162
July 1986

1. INTRODUCTION

Since the pioneering idea [1] of Drell and Yan to explain lepton pair production, a large amount of theoretical work has been needed to understand correctly the series of experiments which became more and more precise. Let us recall that at the end of the last decade the experimentally measured cross-section [2] was more than twice as large as the naïve parton-model prediction, the discrepancy being called the K factor. At the same time the first QCD calculations at the next-to-leading order [3] gave a K factor of the order of 1.8 with $\Lambda = 500$ MeV and a factorization scale $M = Q$, where Q is the lepton pair mass. Such a huge first-order correction raises doubts about the convergence of the series until next orders have been computed. In fact it has been realized that the bulk of the first-order correction is of infrared origin which is calculable to all orders, giving rise to the soft-gluon exponentiation (SGE) increasing the first order by roughly 20% except at the boundary of the phase space where the effect is higher [4]. The recent NA10 data [5] for $d\sigma/d\sqrt{\tau} dx_F$ on π^- -W interactions have been found to be in agreement with SGE using their set of structure functions except for the last bin above the T ($Q \approx 11$ -12 GeV). Very recently it has been claimed [6] that this 'anomalous scaling violation' of $d\sigma/dQ^2 dy$ disappears with an appropriate choice of nuclear structure functions.

Since we have now a complete set of data [2,5,7,8] in pp, $p\bar{p}$, pN, and π^- -N collisions including high masses, our purpose is to revisit lepton pair production within the framework of the optimized perturbative QCD [9], which states that 'if an approximant depends on unphysical parameters, then their values should be chosen so as to minimize the sensitivity of the approximant to small variations in those parameters'. Here the unknown parameter is the factorization scale M which appears when collinear singularities are factorized into the bare distribution functions evolved to the scale M . In previous analyses the choice $M = Q$ is done, whereas in this scheme M is an arbitrary scale. The optimized procedure has been shown to be in good agreement with all available data on photoproduction and prompt photon reactions [10].

The paper is organized as follows. We give in Section 2 the expression for the cross-sections $d\sigma/dQ^2$, $d\sigma/dQ^2 dx_F$, and $d\sigma/dQ^2 dy$, and discuss the optimization procedure. Section 3 is devoted to numerical results for $Q^3(d\sigma/dQ)$ in pN and $\bar{p}N$ collision, dN/dQ in πN data, $d\sigma/dQ dy|_{y=0}$ in proton-induced reactions, and $d\sigma/d\sqrt{\tau} dx_F$ in π^- -W collisions. Our conclusions are given in Section 4.

Let us mention that we do not consider the transverse-momentum dependence of the cross-section whose theoretical re-evaluation has recently been done [11] and found to correctly describe data from ISR energies to W and Z production at the CERN $p\bar{p}$ Collider.

2. CROSS-SECTIONS IN THE OPTIMIZED PERTURBATON THEORY

When leaving the factorization scale M arbitrary, the Drell-Yan pair production cross-section for valence-dominated processes can be written as

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{9sQ^2} \sum_q e_q^2 \int_0^1 \frac{dt_1}{t_1} q_v(t_1, M) \int_0^1 \frac{dt_2}{t_2} q_v(t_2, M) \left[\delta(1-z) + \theta(1-z) \left(\frac{\alpha_s(M)}{\pi} 2 P_{qq}(z) \ln \frac{Q}{M} + \alpha_s(M) f_q'(z) \right) \right], \quad (1)$$

with $\tau = Q^2/S$ and $z = \tau/t_1 t_2$. The function $f_q'(z)$ is known from the perturbative calculation [3] and will be specified later. Under changes of the scale M , the variation of the Born term is obtained from the Altarelli-Parisi equations

$$\frac{d q_v(x, M)}{d(\ln M)} = \frac{\alpha_s(M)}{\pi} \int_x^1 \frac{dz}{z} P_{qq}(z) q_v\left(\frac{x}{z}, M\right). \quad (2)$$

It is easily verified that the term in $\ln(Q/M)$ in Eq. (1) compensates the variation of the Born term so that the changes of $d\sigma/dQ^2$ are of $o(\alpha_s^2)$ when M varies. This M dependence of the cross-section is not negligible however: only a calculation to all orders in α_s would yield M -independent results. It has been proposed [9] that M be chosen such that the $o(\alpha_s)$ cross-section [Eq. (1)] has the same property as the complete (uncalculated) series, namely

$$\frac{d(d\sigma/dQ^2)}{d(\ln M)} = 0 \quad (3)$$

This procedure enjoys several nice properties. In particular, the K -factor turns out to be very near 1: there are no ‘large’ corrections at the optimal point. Moreover, the scale M^{opt} , determined by Eq. (3), is smaller than Q ; this is related to the positivity of the correction [the term $f_q'(z)$ in Eq. (1)] when the calculation is performed with the standard scale Q [12]. This is illustrated in Fig. 1 where we give, for the non-singlet contribution $p\bar{p} - pp$ at $\sqrt{s} = 27.4$ GeV, the comparison between $\alpha_s(M^{\text{opt}})/\pi$ and $\alpha_s(Q)/\pi$; M^{opt} is bounded from below since it cannot be much smaller than the starting point Q_0 of the evolution. Thus the optimization method will not be effective in the small τ range. Before entering into the technical details of the approach [12], it should be recalled that, contrary to real photon reactions at large transverse momentum, the argument of α_s in Eq. (1) is not determined: only an $o(\alpha_s^2)$ calculation would allow the arbitrariness to be lifted. However, it is natural to use the variable which appears in the argument of α_s in the evolution equation, namely M .

The general expression for the cross-section $d\sigma/dQ^2$ including sea quarks and gluons is

$$\begin{aligned}
\frac{d\sigma}{dQ^2} = & \frac{4\pi\alpha^2}{9sQ^2} \sum_i e_i^2 \int \frac{dt_1}{t_1} \frac{dt_2}{t_2} \left\{ \left[q_i(t_1, M) \bar{q}_i(t_2, M) + (1 \leftrightarrow 2) \right] \right. \\
& \left[\delta(1-z) \left(1 + \frac{\alpha_s(M)}{\pi} C_F \left(\frac{1}{2} + \frac{2\pi^2}{3} + 3 \ln \frac{Q}{M} \right) \right) \right. \\
& \left. \left. + \theta(1-z) \frac{\alpha_s(M)}{\pi} C_F \left((1+z^2) \left(\frac{\ln(1-z)}{1-z} \right)_+ + \frac{3/2 + 2(1+z^2) \ln \frac{Q}{M}}{(1-z)_+} - 3 - 2z \right) \right] \right. \\
& \left. + \left[\left(q_i(t_1, M) + \bar{q}_i(t_2, M) \right) G(t_2, M) + (1 \leftrightarrow 2) \right] \right. \\
& \left. \left. \theta(1-z) \frac{\alpha_s(M)}{\pi} \frac{1}{4} \left[(z^2 + (1-z)^2) \ln \left(\frac{Q^2}{M^2} (1-z) \right) + \frac{9}{2} z^2 - 5z + \frac{3}{2} \right] \right] \right\} \quad (4)
\end{aligned}$$

The $\mathcal{O}(\alpha_s)$ terms are written in the non-universal convention of Altarelli et al., in Ref. (3) for the definition of quark distributions beyond the leading logarithm approximation (i.e. all the next-to-leading logarithmic corrections are absorbed in the definition of deep-inelastic structure functions). This is the correct convention to be used when only leading logarithmic parametrizations of quark densities are available.

We now turn to the expression for the cross-section [3] $d\sigma/dQ^2 dx_F$, where x_F is the Feynman variable of the Drell-Yan pair. Introducing the variables x_1 and x_2 such that

$$x_1, x_2 = Q^2/s, \quad x_F = x_1 - x_2$$

and decomposing the cross-section into its annihilation and Compton pieces, we have

$$\frac{d^2 \sigma^A}{dQ^2 dx_F} = \frac{4\pi\alpha^2}{g_s Q^2} \sum_i e_i^2 \int_{x_1}^1 dt_1 \int_{x_2}^1 dt_2 \left[q_i(t_1, M) \bar{q}_i(t_2, M) + \bar{q}_i(t_1, M) q_i(t_2, M) \right]$$

$$\left\{ \frac{\delta(t_1 - x_1) \delta(t_2 - x_2)}{x_1 + x_2} \left[1 + \frac{\alpha_s(M)}{2\pi} C_F \left(1 + \frac{5\pi^2}{3} + \frac{3}{2} \ln \frac{(1-x_1)(1-x_2)}{x_1 x_2} \right. \right. \right.$$

$$\left. \left. + 2 \ln \frac{(1-x_1)}{x_1} \ln \frac{(1-x_2)}{x_2} + 2 \ln \frac{Q}{M} \left(3 + 2 \ln \frac{(1-x_1)(1-x_2)}{x_1 x_2} \right) \right) \right] \right\}$$

$$+ \frac{\delta(t_2 - x_2)}{x_1 + x_2} \frac{\alpha_s(M)}{2\pi} C_F \left[\frac{t_1^2 + x_1^2}{t_1^2 (t_1 - x_1)} \ln \left(\frac{Q^2 (x_1 + x_2)(1-x_2)}{M^2 (t_1 + x_2) x_2} \right) + \frac{3}{2} \frac{1}{(t_1 - x_1)} - \frac{2}{t_1} - \frac{3x_1}{t_1^2} \right] + 1 \leftrightarrow 2$$

$$+ \frac{\alpha_s(M)}{2\pi} \frac{C_F}{t_1 t_2} \left\{ \frac{(\tau^2 + (t_1, t_2)^2)}{(t_1 + x_2)(t_2 + x_1)((t_1 - x_1)(t_2 - x_2))_{x_1, x_2}} \left(\frac{1/t_1 + 1/t_2}{t_1 + t_2} - \frac{2}{t_1 + t_2} \right) \right\} \quad (5)$$

$$\frac{d^2 \sigma^c}{dQ^2 dx_F} = \frac{4\pi\alpha^2}{g_s Q^2} \sum_i e_i^2 \int_{x_1}^1 dt_1 \int_{x_2}^1 dt_2 \left\{ \left[G(t_1, M) (q_i(t_2, M) + \bar{q}_i(t_2, M)) \right] \right.$$

$$\left. - \frac{\alpha_s(M)}{2\pi} \frac{1}{2} \left[\frac{\delta(t_2 - x_2)}{(x_1 + x_2) t_1^3} \left((x_1^2 + (t_1 - x_1)^2) \ln \left(\frac{Q^2 (x_1 + x_2)(1-x_2)}{M^2 (t_1 + x_2) x_2} \right) - 6x_1(t_1 - x_1) + t_1^2 \right) \right. \right.$$

$$\left. + \frac{1}{t_1^2 t_2^2} \left(\frac{(\tau^2 + (t_1, t_2 - z)^2)}{(t_1, (t_2 + x_1)) (t_2 - x_2)_{x_2}} - \frac{t_1(t_2 + x_1)(t_2 - x_2) + 2\tau(t_1 + t_2)}{(t_1 + t_2)^2} \right) \right]$$

$$+ 1 \leftrightarrow 2 \left. \right\} \quad (6)$$

The distributions $1/(t-x)_x \dots$ are defined by

$$\int_x^1 dt \frac{f(t)}{(t-x)_x} = \int_x^1 dt \frac{f(t) - f(x)}{t-x}$$

$$\int_{x_1}^1 dt_1 \int_{x_2}^1 dt_2 \frac{f(t_1, t_2)}{((t-x_1)(t-x_2))_{x_1, x_2}} = \int_{x_1}^1 dt_1 \int_{x_2}^1 dt_2 \frac{(f(t_1, t_2) - f(t_1, x_2) - f(x_1, t_2) + f(x_1, x_2))}{(t_1 - x_1)(t_2 - x_2)}$$

The expressions for $d^2\sigma/dQ^2 dy$ are given in the Appendix.

To obtain the numerical results, we proceed as follows. In order to avoid the problem of the mixing between quarks and gluons we shall determine the optimal scale at each Q (and x_F) value for the valence-dominated part of the cross-section, i.e. for the reactions of type $(\pi^- N \rightarrow \pi^+ N)$ or $(\bar{p}N \rightarrow pN)$. The full cross-section (including sea quark and gluon contributions) is then evaluated at the previously determined optimal scale. The results are in fact the same as those which would be obtained by working directly with $\pi^- N$ or $\bar{p}N$: minor differences appear only at the lowest τ values.

It has often been suggested [4] that part of the π^2 term appearing in Eqs. (4) or (5) should be exponentiated. For instance, in Eq. (4) one should replace the term $\{1 + [\alpha_s(M)/\pi]c_F(2\pi^2/3)\}$ by an overall factor $\exp [\alpha_s(M)/2\pi]c_F\pi^2$, leaving only $[\alpha_s(M)/\pi]c_F(\pi^2/6)$ in the next-to-leading logarithm part. The exponentiation will diminish the first-order correction, increasing therefore the optimized factorization mass. Consequently the procedure will be valid for smaller lepton pair masses: down to $\sqrt{\tau} = 0.35$ at $\sqrt{s} = 20$ GeV and $\sqrt{\tau} = 0.24$ at $\sqrt{s} = 63$ GeV. We will see that in the domain where the optimization using the first-order formula or the exponentiated one is valid the discrepancy is very small, smaller than the experimental error bars.

In our comparison with data, we give results based on the parton density parametrization of Duke and Owens [13], set 1 ($\Lambda = 0.2$ GeV). The reasons are twofold: Drell-Yan pair production has been used for the determination of the proton structure functions, and a parametrization of the pion structure functions has been worked out.

3. NUMERICAL RESULTS

We first consider cross-sections integrated over the longitudinal variable. In Fig. 2 we compare our predictions for $Q^3 (d\sigma/dQ)$ with the NA3 data for the reactions $\bar{p}N$ at 150 GeV/c and pN at 200 GeV/c. As can be seen, a very good agreement is obtained. The results derived from the exponentiated formula (solid line) or from the first-order one (dashed line) are very similar except for the narrower τ range in the latter case. Figure 3 shows a comparison of the predictions of the optimized perturbative QCD with the NA3 data on $\pi^\pm N$ scattering at $p_{lab} = 200$ GeV/c. The agreement down to $Q = 7$ GeV between theory and experiment is also very good. At this point it is appropriate to compare our results with the standard next-to-leading logarithmic calculations. If we

denote by $\bar{K}$ the ratio of the optimized predictions to the standard next-to-leading logarithmic ones, the factor $\bar{K} \approx 1.4$ for $\pi^\pm N$ at $p_{\text{lab}} = 200$ GeV/c in the range $7 \text{ GeV} < Q < 12 \text{ GeV}$. This factor $\bar{K}$ decreases with energy: it is $\bar{K} \approx 1.3$ at ISR energies and $\bar{K} \approx 1.1$ at collider energies in the range $\tau > 0.03$ where the optimization is possible. It is interesting to note in this respect that the recent calculations [11] of W/Z production cross-sections at $\sqrt{s} = 630$ GeV ($\tau = 0.021$) tend to be somewhat lower than the data, although within experimental error bars, and such an enhancement factor would be welcomed (unfortunately the kinematical domain for W/Z production at these energies is slightly outside the range of our numerical optimization procedure). We find that the scale defined by $M^2 \approx 0.1 (1-\tau)^{1/2} Q^2$ is a good approximation to the optimal scale in the domain $\tau < 0.5$ for all the energies considered above.

We now turn to more exclusive quantities and compare our results for $d^2\sigma/dQ^2 dy|_{y=0}$ with the CFS data (Ito et al. [2]) on pPt at $\sqrt{s} = 27.4$ GeV and ISR data [7] on pp at $\sqrt{s} = 62.3$ GeV (Fig. 4). As before, a good agreement between theory and experiment is achieved. One notes the small difference between the results obtained from the exponentiated formula (solid line) and the non-exponentiated one (dashed line) as was also the case for $d\sigma/dQ^2$. In Fig. 5 we compare the NA10 data on $d^2\sigma/d\sqrt{\tau} dx_F$ in $\pi^- - W$ reactions at $p_{\text{lab}} = 194$ GeV/c for several $\sqrt{\tau}$ bins. These data, obtained in a high-statistics experiment, allow a detailed comparison of various theoretical predictions. We consider only bins with $\sqrt{\tau} > 0.36$ where the optimized procedure applies. The first-order calculation (with $M = Q$) does not fit the data except for the last bin ($0.6 < \sqrt{\tau} < 0.66$). A good agreement is obtained with the exponentiated optimized formula with $\Lambda = 100 \text{ MeV}^*$ except for the last bin where the predictions fall above the data: we therefore encounter the anomalous scaling behaviour. Since the reaction we consider involves heavy nuclei, the last bin, which corresponds to large x_2 values, may be sensitive to the EMC effect and to Fermi motion. Moreover, since in the NA10 procedure [14] to extract the cross-sections the Fermi motion has been taken into account, the structure functions we use have to be corrected for Fermi motion. In the Duke-Owens parametrization the proton and pion structure functions are extracted from data involving different nuclei, some of them not having been Fermi-corrected. In our study we consider the model of Ref. [15], which predicts a relatively large decrease of the structure functions with x and M . As can be seen from inspection of Fig. 6 (solid line: $\Lambda = 200 \text{ MeV}$; dashed line: $\Lambda = 100 \text{ MeV}$) this improves the agreement with the data for negative and small positive x_F values and especially for the last bin. Since the Fermi corrections are relatively model-dependent we believe that the predictions of Fig. 5 without Fermi motion and those of Fig. 6 including Fermi corrections reflect the maximal uncertainties owing to our lack of knowledge of nuclear structure functions. It should be noticed that the systematic normalization uncertainty of data of Ref. [5] is about 7%. Further uncertainties (such as the EMC effect) can also modify the predictions as discussed in Refs. [6] and [16].

*) In the previously considered data (Figs. 1 to 4) the predictions are insensitive to a change of the value of Λ from 200 MeV to 100 MeV.

4. CONCLUSIONS

We have applied the optimized perturbative theory to massive lepton pair production. A good agreement has been found in a large lepton pair mass domain for various reactions including those at small p_{lab} ($p_{\text{lab}} \approx 150 \text{ GeV}/c$). We note that the optimized procedure allows the range of applicability of perturbation theory to be extended down to low p_{lab} values where the usual perturbative results fall below the data. Let us emphasize that the same method has been successfully applied to reactions involving real photons, adding further confidence to the optimized perturbative QCD. The level of theoretical understanding of perturbative QCD is such that an improvement of the knowledge of the nuclear structure functions becomes necessary.

Acknowledgements

We gratefully acknowledge discussions with R. Baier, E. Berger, M. Fontannaz, D. Schiff and M. Winter.

REFERENCES

- [1] S. Drell and T. Yan, Phys. Rev. Lett. **25** (1970) 316.
- [2] J. Badier et al., Phys. Lett. **89B** (1979) 145.
A.S. Ito et al., Phys. Rev. **D23** (1981) 604.
- [3] G. Altarelli, R.K. Ellis and G. Martinelli, Nucl. Phys. **B143** (1978) 521, **B146** (1978) 544, and **B147** (1979) 461.
B. Humpert and W.L. Van Neerven, Phys. Lett. **85B** (1979) 293.
J. Kubar-André and F.E. Paige, Phys. Rev. **D19** (1979) 221.
J. Kubar, M. Le Bellac, J.L. Meunier and G. Plaut, Nucl. Phys. **B175** (1980) 251.
- [4] G. Parisi, Phys. Lett. **90B** (1980) 295.
G. Curci and M. Greco, Phys. Lett. **92B** (1980) 175 and **102B** (1981) 280.
P. Chiappetta, T. Grandou, M. Le Bellac and J.L. Meunier, Nucl. Phys. **B207** (1982) 251.
- [5] B. Betev et al., Z. Phys. **C28** (1985) 9 and 15.
M. Winter, Talk given at the 21st Rencontre de Moriond, Les Arcs, 1986, to appear in the Proceedings.
- [6] Y. Gabellini, J.L. Meunier and G. Plaut, Z. Phys. **C29** (1985) 439.
- [7] C. Kourkouvelis et al., Phys. Lett. **91B** (1980) 475.
A.L.S. Angelis et al., Phys. Lett. **147B** (1984) 472.
- [8] J. Badier et al., Phys. Lett. **96B** (1980) 422; Z. Phys. **C18** (1983) 281.
- [9] P.M. Stevenson, Phys. Rev. **D23** (1981) 2916; Nucl. Phys. **B203** (1982) 472.
H.D. Politzer, Nucl. Phys. **B194** (1982) 493.
- [10] P. Aurenche, R. Baier, A. Douiri, M. Fontannaz and D. Schiff, Phys. Lett. **135B** (1984) 164, **140B** (1984) 87 and work in progress;
R. Baier, Bielefeld preprint BI-TP 85/29 (1985);
M. Fontannaz, Orsay preprint LPTHE 85/41 (1985);
P. Aurenche, Annecy preprint LAPP-TH-159 (1986);
D. Schiff, Talk given at the 17th Symposium on Multiparticle Dynamics, Seewinkel, Austria, June 1986, to appear in the Proceedings.
- [11] G. Altarelli, R.K. Ellis, M. Greco and G. Martinelli, Nucl. Phys. **B246** (1984) 12.
G. Altarelli, R.K. Ellis and G. Martinelli, Phys. Lett. **151B** (1985) 457.
- [12] P. Aurenche, R. Baier, M. Fontannaz and D. Schiff, in preparation.
- [13] D.W. Duke and J.F. Owens, Phys. Rev. **D50** (1984) 49 and **D30** (1984) 943.
- [14] M. Winter, private communication.
- [15] A. Bodek and J.L. Ritchie, Phys. Rev. **D23** (1981) 1070 and **D24** (1981) 1400.
M. Winter, Thèse d'Etat, CRN Strasbourg (1985).
- [16] E.L. Berger, Nucl. Phys. **B267** (1986) 231.

APPENDIX

We give in this Appendix the general expression for the cross-section $d\sigma/dQ^2 dy$.

For the annihilation piece, one gets

$$\begin{aligned} \frac{d^2\sigma^A}{dQ^2 dy} &= \frac{4\pi\alpha^2}{g_s Q^2} \sum_i e_i^2 \int_{x_1}^1 dt_1 \int_{x_2}^1 dt_2 \left[q_i(t_1, M) \bar{q}_i(t_2, M) + \bar{q}_i(t_1, M) q_i(t_2, M) \right] \\ &\left\{ \delta(t_1 - x_1) \delta(t_2 - x_2) \left[1 + \frac{d_s(M)}{2\pi} c_F \left(1 + \frac{5\pi^2}{3} + \frac{3}{2} \ln \frac{(1-x_1)(1-x_2)}{x_1 x_2} \right. \right. \right. \\ &\quad \left. \left. \left. + 2 \ln \frac{(1-x_1)}{x_1} \ln \frac{(1-x_2)}{x_2} + 2 \ln \frac{Q}{M} \left(3 + 2 \ln \frac{(1-x_1)(1-x_2)}{x_1 x_2} \right) \right) \right] \right. \\ &\quad \left. + \delta(t_2 - x_2) \frac{d_s(M)}{2\pi} c_F \left[\frac{t_1^2 + x_1^2}{t_1^2 (t_1 - x_1) x_1} \ln \left(\frac{Q^2}{M^2} \frac{2x_1(1-x_2)}{x_2(t_1 + x_1)} \right) + \frac{3/2}{(t_1 - x_1) x_1} - \frac{2}{t_1} - \frac{3x_1}{t_1^2} \right] + 1 \leftrightarrow 2 \right. \\ &\quad \left. + \frac{d_s(M)}{\pi} \frac{c_F}{t_1 t_2} \left[\frac{(\tau^2 + (t_1, t_2)^2)}{(t_1 + x_1)(t_2 + x_2)} \frac{(\tau/t_1, t_2 + 1)}{((t_1 - x_1)(t_2 - x_2))_{x_1, x_2}} - \frac{2\tau (\tau + t_1, t_2)}{(t_1 x_2 + t_2 x_1)^2} \right] \right\} \end{aligned}$$

The Compton piece reads:

$$\frac{d^2\sigma^c}{dQ^2 dy} = \frac{4\pi\alpha^2}{9sQ^2} \sum_i e_i^2 \int_{x_1}^1 dt_1 \int_{x_2}^1 dt_2 \left\{ G(t_1, M) \left(q_i(t_2, M) + \bar{q}_i(t_2, M) \right) \right.$$

$$\frac{d_s(M)}{2\pi} \frac{1}{2} \left[\frac{\delta(t_2 - x_2)}{t_1^3} \left((x_1^2 + (t_1 - x_1)^2) \ln \left(\frac{Q^2}{M^2} \frac{2x_1(1-x_2)}{(t_1+x_1)x_2} \right) - 6x_1(t_1-x_1)+t_1^2 \right) \right.$$

$$+ \frac{2(\tau + t_1 t_2)}{t_1^2 t_2^2 (t_1 x_2 + t_2 x_1)} \left(\frac{(\tau^2 + (t_1 t_2 - \tau)^2) x_2}{t_1 (t_2 + x_2) (t_2 - x_2) x_2} \right.$$

$$\left. \left. + \frac{\tau(t_1 t_2^2 x_1 + \tau(t_1 x_2 + 2t_2 x_1))}{(t_1 x_2 + t_2 x_1)^2} \right) \right] + 1 \leftrightarrow 2 \Bigg\},$$

with x_1, x_2 such that

$$x_1, x_2 = \tau = \frac{Q^2}{s}, \quad x_1 = \sqrt{\tau} e^y, \quad x_2 = \sqrt{\tau} e^{-y}.$$

Figure captions

- Fig. 1 :** $\alpha_s(M)/\pi$ for $M = M^{\text{opt}}$ (the optimal scale) and $M = Q$.
- Fig. 2 :** Comparison of the optimized perturbative QCD with the NA3 data [8] for $Q^3 (d\sigma/dQ)$ on $\bar{p}N$ at $p_{\text{lab}} = 150$ GeV/c and pN at $p_{\text{lab}} = 200$ GeV/c. Solid line: exponentiated case. Dashed line: non-exponentiated case.
- Fig. 3 :** Same as Fig. 2 for NA3 data [2] for dN/dQ in $\pi^\pm N$ collisions at $p_{\text{lab}} = 200$ GeV/c.
- Fig. 4 :** Same as Fig. 2 for $d^2\sigma/dQdy|_{y=0}$. The data at 27.4 GeV are from Ito et al., [2] and those at 62.3 GeV from Angelis et al. [7].
- Fig. 5 :** Comparison of theoretical models (Fermi motion uncorrected) with NA10 data [5] for $d^2\sigma/d\sqrt{\tau} dx_F$.
Full line (resp. dashed line): exponentiated optimized procedure with $\Lambda = 200$ MeV (resp. with $\Lambda = 100$ MeV).
Dash-dotted line: next-to-leading-order prediction.
- Fig. 6 :** Comparison of theoretical models, including Fermi motion, with NA10 data [5] for $d^2\sigma/d\sqrt{\tau} dx_F$. Full line (resp. dashed line): exponentiated optimized procedure with $\Lambda = 200$ MeV (resp. with $\Lambda = 100$ MeV).

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5 a)

Fig. 5 b)

Fig. 5 c)

Fig. 5 d)

Fig. 6 a)

Fig. 6 b)

Fig. 6 c)

Fig. 6 d)