Faraday scaling and the BICEP2 observations

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As repeatedly speculated in the past, the linear polarization of the cosmic microwave background can be rotated via the Faraday effect. An economic explanation of the recent Bicep2 observations, not relying on long-wavelength tensor modes of the geometry, would stipulate that the detected B mode comes exclusively from a Faraday rotated E-mode polarization. We show hereunder that this interpretation is ruled out by the existing upper limits on the B-mode polarization obtained by independent experiments at observational frequencies much lower than the operating frequency of the Bicep2 experiment. We then derive the fraction of the observed B-mode polarization ascribable to the Faraday effect and suggest a dedicated experimental strategy for its detection.

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The Bicep2 Collaboration [1] recently measured the B-mode polarization for angular scales of the order of the degree and at a pivot frequency $\nu_p = 150$ GHz:

$$\mathcal{G}_{B\ell} = \frac{\ell(\ell+1)}{2\pi} C_\ell^{(BB)} = \mathcal{O}(10^{-2}) \mu\text{K}^2, \quad (1)$$

where $\mathcal{G}_{\ell B}$ denotes the angular power spectrum of the B-mode autocorrelation, $\ell \simeq \pi/\vartheta$ is the multipole moment and ϑ is the angular scale. If the whole effect is attributed to the primordial tensor modes, amplified by the pumping action of the gravitational field in a (spatially flat) Friedmann-Robertson-Walker geometry, the present frequency of the corresponding gravitons would be $\mathcal{O}(10^{-17})$ Hz. Before proceeding it is appropriate to mention that the observed signal could be contaminated by galactic dust emission or by the spatial variation of the polarized synchrotron emission [2]. More careful subtractions of the existing foregrounds may reduce the signal and ease the tension between the Bicep2 data and the satellite data leading, ultimately, to an inferred tensor to scalar ratio closer to 0.16 than to 0.2. We just mention this possibility as a plausible caveat that is, however, not central to this work.¹

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¹After this paper was circulating in the form of a preprint, there have been various debates on the BICEP2 data (see, e.g. [3] and references therein). At the moment the situation can be summarized by saying that the potential foreground contaminations from polarized dust are under active scrutiny. At the same time the only publicly available data on the foregrounds in the BICEP2 region are the WMAP data. The best evidence for the BICEP2 signal not being a foreground and the best evidence for the foregrounds being a possible contaminant both come from digitizing Power-Point presentations (see, in this context, the reference [33] quoted by the authors of Ref. [3]). The current debate is not conclusive but the present paper emphasizes the importance of multifrequency measurements that are important not only for the Faraday effect but also for proper foreground subtraction.

In what follows the sole assumptions will be instead that the tensor modes of the geometry are absent and that whole Bicep2 signal comes from the Faraday rotation of the polarization plane of the cosmic microwave background (CMB in what follows). If this is the case it will be shown that we are led into an interesting contradiction: direct upper limits on the B-mode polarization at frequencies $\nu \ll \nu_p$ are violated if the detected signal is ascribed to a purported Faraday effect. The B-mode polarization induced by the Faraday rotation scales with the frequency. Conversely the B mode induced by the tensor modes of the geometry is frequency independent.

In the recent past, direct upper limits on the B-mode autocorrelation $\mathcal{G}_{\ell B}$ have been presented, over different observational frequencies, by various collaborations aiming at a direct detection of the CMB polarization. The pivot frequencies of the corresponding experiments can be conventionally divided into two ranges denoted hereunder by ν_{low} and ν_{high} :

$$\begin{aligned} 26 \text{ GHz} &\leq \nu_{\text{low}} \leq 36 \text{ GHz}, \\ 100 \text{ GHz} &\leq \nu_{\text{high}} \leq 150 \text{ GHz}. \end{aligned} \quad (2)$$

The Dasi (Degree Angular Scale Interferometer) [4] and the Cbi (Cosmic Background Imager) [5] experiments were both working in a range coinciding exactly with ν_{low} . Four other experiments have been conducted around ν_{high} and they are (i) Boomerang (Balloon Observations of Millimetric Extragalactic Radiation and Geophysics) working at 145 GHz with four pairs of polarization sensitive bolometers [6]; (ii) Maxipol (Millimeter Anisotropy experiment Imaging array) working at 140 GHz with 12 polarimeters [7]; (iii) Quad² working with 31 pairs of polarization sensitive bolometers: 12 at 100 GHz and 19

²An acronym or a contraction between the Quest (Q and U extragalactic sub-mm telescope) and the Dasi experiments.

at 150 GHz [8]; (iv) Bicep2 [1] and its precursor Bicep1 [9] work, respectively, at 150 GHz and 100 GHz.

There are finally three polarization sensitive experiments working in mixed or intermediate frequency ranges. They include: (a) the WMAP experiment [10] (Wilkinson Microwave Anisotropy Probe) [10] spanning five frequencies from 23 to 94 GHz; (b) the Capmap experiment (Cosmic Anisotropy Polarization Mapper) [11], with 12 receivers operating between 84 and 100 GHz and four receivers operating between 35 and 46 GHz; (c) the Quiet (Q/U imager experiment) [12] operating at 43 GHz (during the first season of the experiment) and at 95 GHz (during the second season of the experiment).

The dispersion relations governing the propagation of electromagnetic excitations in a cold, electrically neutral and magnetized plasma [13] determine the angular shift $\Delta\varphi$ of the polarization plane of the linearly polarized wave propagating, for instance, along the third Cartesian axis:

$$\Delta\varphi = \frac{\omega_{Be}}{2} \left(\frac{\omega_{pe}}{\omega} \right)^2 \Delta z, \quad (3)$$

showing that the rotation rate (i.e. $\Delta\varphi/\Delta z$) decreases as the inverse square of the angular frequency of the photons (i.e. $\omega = 2\pi\nu$) and it increases linearly with the magnetic field intensity, i.e. with the Larmor frequency of the electrons. The result (3) is approximate and it holds as long as the plasma and Larmor frequencies of the electrons (i.e. ω_{pe} and ω_{Be}) are both smaller than the angular frequency of the CMB photons.³

In the concordance cosmological lore the linear polarization is provided by the scalar adiabatic mode whose existence and properties have been firstly scrutinized by the WMAP experiment during the past decade [10] (see for instance [14] for an introduction to the polarization of the CMB). The WMAP experiment discovered the E-mode polarization whose existence has been later confirmed by different experiments (see e.g. [8]). The rotation rate of Eq. (3) determines a rotation in the Stokes parameters implying that the induced B-mode polarization can be computed in terms of the E-mode polarization generated by the scalar adiabatic mode and measured by WMAP experiment. The details of this process have been studied in the past [15,16] but are not essential here. The only crucial piece of information will be the Faraday scaling law stipulating that the B-mode polarization induced by the Faraday effect scales as the inverse of the fourth power of

the frequency (or, which is the same, as the fourth power of the wavelength). Given the signal at a certain pivot frequency ν_p (that we shall choose to be the Bicep2 frequency), the B-mode polarization induced by Faraday rotation at a different observational frequency ν can be obtained in terms of this simple scaling law (see e.g. [17] and third, fourth and fifth papers of Ref. [15]):

$$\mathcal{G}_{\ell B}(\nu) = \left(\frac{\nu_p}{\nu} \right)^4 \mathcal{G}_{\ell B}(\nu_p), \quad (4)$$

where $\nu_p = 150$ GHz is the pivot frequency that coincides with the Bicep2 frequency. The characteristic frequency dependence of Eq. (4) arises since the Faraday generated B mode is quadratic in the rotation rate. According to Eq. (4), for frequencies larger than ν_p the B mode is suppressed while for frequencies $\nu < \nu_p$ the angular power spectrum is enhanced. At a given frequency (be it for instance ν_p) the B-mode power spectrum induced by the Faraday rate first increases with the multipole ℓ , it has a maximum for $\ell \sim 1000$ and then it decreases again [15] (see also [16]). The maximum around $\ell = \mathcal{O}(10^3)$ is actually connected with the maximum of the E-mode autocorrelation occurring for $\ell \sim 1000$ [10].

Let us therefore use our main assumption and posit that the entire signal observed by Bicep2 comes predominantly from a Faraday rotated E-mode polarization. This assumption requires that $\mathcal{G}_{\ell B}(\nu_p)$ appearing in Eq. (4) can be replaced by the angular power spectrum observed by Bicep2 since the correction coming from gravitational lensing is not essential for the angular scales considered hereunder. Consider then, for sake of concreteness, two specific ranges of multipoles scrutinized by the Bicep2 Collaboration,

$$\mathcal{G}_{\ell B}(\nu_p) = 3.37 \times 10^{-2} \mu\text{K}^2, \quad 196 \leq \ell \leq 230, \quad (5)$$

$$\mathcal{G}_{\ell B}(\nu_p) = 5.07 \times 10^{-2} \mu\text{K}^2, \quad 231 \leq \ell \leq 265. \quad (6)$$

It turns out that the ranges of Eqs. (5) and (6) overlap with the observations of other experiments producing upper limits on the B-mode polarization at lower frequencies compatible with ν_{low} . In particular the Dasi Collaboration provided upper limits for the B-mode polarization implying [4]

$$\mathcal{G}_{\ell B}(\nu_{\text{low}}) < 2.12 \mu\text{K}^2, \quad 28 \leq \ell < 245, \quad (7)$$

$$\mathcal{G}_{\ell B}(\nu_{\text{low}}) < 6.45 \mu\text{K}^2, \quad 246 \leq \ell < 420, \quad (8)$$

at 95% confidence level. The Faraday scaling law of Eq. (4) can now be used with the purpose of computing $\mathcal{G}_{\ell B}(\nu_{\text{low}})$ for the particular values of Eqs. (5)–(6). The result of this simple rescaling shall then be compared with Eqs. (7)–(8). Therefore, using Eqs. (5)–(6) into Eq. (4) the B-mode power spectrum at ν_{low} becomes

³During the formation of the CMB polarization $\omega_{Be} = 1.7 \times 10^{-2}$ Hz [for typical magnetic field strengths $\mathcal{O}(\text{nG})$, $\omega_{pe} = 0.285$ MHz for the fiducial set of WMAP9 parameters alone [10]]. For frequencies of the photons larger than few GHz the hierarchy between the scales of the problem is $\omega \gg \omega_{pe} \gg \omega_{Be}$. The plasma and Larmor frequencies of the ions do not play any role and are both smaller than the corresponding frequencies of the electrons.

$$\mathcal{G}_{\ell B}(\nu_{\text{low}}) = 3.37 \times 10^{-2} \left(\frac{\nu_p}{\nu_{\text{low}}} \right)^4 \mu\text{K}^2, \quad 196 \leq \ell \leq 230, \quad (9)$$

$$\mathcal{G}_{\ell B}(\nu_{\text{low}}) = 5.07 \times 10^{-2} \left(\frac{\nu_p}{\nu_{\text{low}}} \right)^4 \mu\text{K}^2, \quad 231 \leq \ell \leq 265. \quad (10)$$

Since we are discussing the bounds on a signal that decreases as ν gets larger, the most constraining frequencies compatible with the observational range are the lowest ones. Consequently if we estimate $\nu_{\text{low}} = 26$ GHz, Eqs. (9) and (10) imply, respectively,

$$\mathcal{G}_{\ell B}(\nu_{\text{low}}) = 37.33 \mu\text{K}^2, \quad 196 \leq \ell \leq 230, \quad (11)$$

$$\mathcal{G}_{\ell B}(\nu_{\text{low}}) = 56.16 \mu\text{K}^2, \quad 231 \leq \ell \leq 265. \quad (12)$$

Equations (11) and (12) violate the Dasi upper limits of Eqs. (7)–(8): instead of $\mathcal{O}(2)$ and $\mathcal{O}(6) \mu\text{K}^2$ we get, from Eqs. (11) and (12), $\mathcal{O}(40)$ and $\mathcal{O}(60) \mu\text{K}^2$ in an overlapping range of multipoles.

Moving towards smaller angular scales but always remaining around ν_{low} , the constraints on a purported Faraday rotation signal may become even stronger once the direct measurements of $\mathcal{G}_{\ell B}$ will be available for larger multipoles (i.e. $\ell \sim 10^3$). Indeed, over these angular scales, Cbi [5] obtained an upper limit at 95% confidence level implying $\mathcal{G}_{\ell B}(\nu_{\text{low}}) < 3.76 \mu\text{K}^2$. This bound may be even more constraining than the Dasi limits since it corresponds to larger ℓ where we do know, on a theoretical ground, that the potential Faraday rotation signal will certainly be larger, i.e. $\mathcal{G}_{\ell B} \gg \mathcal{O}(10^{-2}) \mu\text{K}^2$. Hence the obtainable limit is potentially stronger but, at the moment, observationally weaker since direct measurements of the B-mode power spectrum are still lacking for these multipoles.

It is useful to reinstate, at this point, that the B-mode autocorrelations peak at a multipole which is of the same order of the maximum of the E-mode autocorrelation which is rotated via the angular power spectrum of the Faraday term. This value is mildly affected by the magnetized initial conditions (see sixth reference in [15]). Using a semi-analytical expression for the envelope of the B-mode autocorrelation it is possible to argue that [15] $\ell_{\text{max}} \simeq (\ell_d/2) \sqrt{(n_s + n_B + 2)/\alpha_1}$ where ℓ_d (estimated between 910 and 1000) is a dissipative scale while n_s and n_B are, respectively, the scalar and magnetic spectral indices; α_1 is a numerical constant of order 1. Conversely, the profile of the Faraday induced B mode at the peak is more strongly affected by the magnetic field spectrum; similarly the B-mode angular power spectrum can be narrowed or broadened depending on the properties of the magnetic field profile. This is at least the result of the numerical calculations recently discussed in [18]. At the moment,

however, the Faraday rotated E-mode polarization cannot be excluded as a potential candidate for the explanation of the current BICEP2 data *just by looking at the maximum of the B-mode correlation*: first there are no data points for multipoles order 1000, second the broadness of the induced angular power spectrum depends on various properties of the magnetic field. This is the reason why we must use what we have now hoping that more data at smaller angular scales will be available in the near future.

In spite of the last statement, we can be rather optimistic since the present investigation does not assume any theoretical prior besides the Faraday scaling of the induced B-mode polarization. If and when more precise measurements of the B-mode polarization will be conducted at smaller angular scales there will be interesting possibilities for further improvements of the derived bounds by cross examining the constraints on the B-mode polarization at lower frequencies (but higher multipoles).

Let us conclude this digression by corroborating the previous discussion with a simplified estimate recently tested via direct numerical calculations [18]:

$$\mathcal{G}_{B\ell} \simeq 4.9 \times 10^{-4} \times \left(\frac{\vec{B} \cdot \hat{n}}{\text{nG}} \right)^2 \left(\frac{150 \text{ GHz}}{\nu} \right)^4 \mu\text{K}^2. \quad (13)$$

In Eq. (13) it has been assumed that $\mathcal{G}_{E\ell}$ is well estimated by the measured E-mode autocorrelation at its maximum, i.e. $\mathcal{G}_{E\ell} \simeq 50 \mu\text{K}^2$ for $\ell_{\text{max}} \simeq 1000$. The actual value of the maximum of $\mathcal{G}_{E\ell}$ is slightly smaller than $50 \mu\text{K}^2$ [i.e. $\mathcal{O}(43) \mu\text{K}^2$]. Direct numerical calculations show that Eq. (13) is accurate within an order of magnitude in the region of the maximum of the E-mode autocorrelation and for blue or violet magnetic spectral indices. It becomes less accurate when these two conditions are not fulfilled [18].

The same strategy discussed for $\nu = \mathcal{O}(\nu_{\text{low}})$ can be used to analyze the upper limits reported by other experiments operating at $\nu = \mathcal{O}(\nu_{\text{high}})$ or even at intermediate frequencies $\nu_{\text{low}} < \nu < \nu_{\text{high}}$. As it can be easily argued thanks to the Faraday scaling law of Eq. (4), the tensions between the upper limits and the Bicep2 measurement get progressively less pronounced as ν approaches ν_p . For instance between Bicep2 and Bicep1 (located at $\nu \sim 100$ GHz) the gain in frequency from ν_p will be given by a comparatively smaller factor, i.e. $(150/100)^4 = 5.06$. As a representative of the experiments working in a mixed frequency range, it is useful to mention the WMAP bound stipulating $\mathcal{G}_{\ell B}(\nu_V) < 0.25 \mu\text{K}^2$ for $50 \leq \ell \leq 100$ and for $\nu_V = 41$ GHz (the so-called V band) [10]. In an overlapping range of multipoles the Bicep2 data would imply $\mathcal{G}_{\ell B}(\nu_p) = (1.33 \pm 0.17) \times 10^{-2} \mu\text{K}^2$ for $56 \leq \ell \leq 90$. Again, using Eq. (4) we get that the Bicep2 signal rescaled at frequency ν_V becomes $2.38 \mu\text{K}^2$ so that the WMAP upper limit is violated. The WMAP Collaboration has also a stronger bound on $\mathcal{G}_{\ell B}$ for $2 \leq \ell \leq 7$ but it cannot be used for the present purposes

due to lack of measurements by the Bicep2 Collaboration in that specific range.

While this analysis was in progress some authors tried to argue that the observed B-mode polarization is the result of a primordial magnetic field [19]. In spite of some different earlier statement, the authors now conclude that this option is ruled out. The authors consider the B mode induced by the vector and tensor modes generated, in turn, by the magnetic field. In the present paper the E-mode autocorrelation is the one coming from the scalar sector and directly observed by the WMAP and by other polarization experiments. As soon as we switch on a predecoupling magnetic field, all the sources of B mode should be democratically considered and among these sources there is the Faraday rotation of the E-mode polarization. The Faraday contribution is present even if the magnetic fields are not included in the initial conditions of the Einstein-Boltzmann hierarchy. Since the Faraday contribution is there and it cannot be erased the problem is to gauge its importance. The present analysis shows that if the Faraday effect is the only source of B-mode polarization then its contribution must be at most 0.01 of the BICEP2 signal which implies a magnetic field which is at most of the order of 0.4 nG. If larger magnetic fields are postulated they will be in patent contradiction with the existing upper limits on the B-mode polarization at lower frequencies, as explained above in general terms.

The strategy discussed here cannot be applied in the case where the birefringent effect is obtained thanks to a pseudoscalar field (see, e.g. fifth paper in Ref. [15]). The reason is that this type of birefringence, recently confronted with the Bicep2 data [20], is frequency independent. According to some this is a relevant point and, therefore, it is appropriate to stress that when a magnetic field and a pseudoscalar coupling are simultaneously present Eq. (3) gets modified as [15]

$$\Delta\varphi = \frac{\omega}{2} \left[\frac{\omega_\sigma}{\omega} + \sqrt{\frac{1}{4} \left(\frac{\omega_\sigma}{\omega} \right)^2 + \epsilon_+(\omega)} - \sqrt{\frac{1}{4} \left(\frac{\omega_\sigma}{\omega} \right)^2 + \epsilon_-(\omega)} \right] \Delta z, \quad (14)$$

where $\epsilon_\pm(\omega) = \epsilon_1 \pm \epsilon_2$ are the components of the dielectric tensor related to the circularly polarized waves:

$$\epsilon_\pm(\omega) = 1 - \frac{\omega_{pi}^2}{\omega[\omega \mp \omega_{Bi}]} - \frac{\omega_{pe}^2}{\omega[\omega \pm \omega_{Be}]}. \quad (15)$$

The frequency⁴ $\omega_\sigma = \beta\sigma'/M$ measures the rate of variation of the polarization because of the presence of the

pseudoscalar background. Equations (14) and (15) imply that whenever $\omega \gg \omega_\sigma$ the explicit result of Eq. (3) is recovered since, as already mentioned, $\omega \gg \omega_{pe} \gg \omega_{Be}$ and the plasma and Larmor frequencies for the ions are both smaller than the corresponding frequencies of the electrons. In the opposite limit (i.e. $\omega \ll \omega_\sigma$) the shift in the polarization plane of the CMB will essentially be independent upon the channel of observation. As anticipated this is the case treated by Ref. [20] and, in this case, the argument discussed in this paper cannot be applied.

Since $\omega_\sigma \propto \sigma'$ the contribution to the rate can be, in principle, either positive or negative. This will affect the sign of the cross correlation between the temperature and the B-mode (i.e. the TB correlation) and the cross correlation between the E-mode and the B-mode (i.e. the EB correlation). If the birefringence effect violates parity (as it can in principle happen), the measurements on the TB and EB correlations can be used to constrain the rotation angle as done, for instance, by the WMAP Collaboration in the five-year data release. If the birefringent effect does not violate parity the TB and EB power spectra vanish. The only possibility to constrain the effect will be through the small-scale measurements of the B-mode polarization.

All in all, the Bicep2 measurements together with the other bounds on the polarization observables demand that the fraction of B-mode polarization potentially coming from a purported Faraday rotation of the E mode can be, at most, $\mathcal{O}(10^{-4}) \mu K^2$ for angular scales of the order of the degree. This result suggests the possibility of a mixing between the tensor B mode and the Faraday rotated E mode that will be independently investigated. The derived bound can be sharpened either by considering smaller angular scales or by improving the observational bounds at low frequencies. The Planck satellite did not publish yet any polarization data and the available analyses borrow the polarization observables from the WMAP data. The Planck instrument is composed by a low frequency instrument (LFI) (with frequencies between 30 and 70 GHz) while the remaining six frequencies are located between 100 and 857 GHz and form the high frequency instrument (HFI). It is unclear from the published calibrations of the LFI and of the HFI [22] if the sensitivity to the B-mode polarization will be sufficient for a detection. Barring for specific experimental considerations, it will be extremely important if the three low frequency channels of the LFI could obtain explicit bounds on the B-mode polarization not only for angular scales of the order of the degree but also for subdegree scales possibly improving the Dasi and Cbi bounds. By going at smaller angular scales (i.e. higher multipoles) the B-mode signal is theoretically rather well understood both analytically and numerically. It will therefore be possible to infer rather robust bounds on the magnetic field strength independent on any other theoretical consideration.

⁴Note that β is a numerical constant and M is typical scale weighting the pseudoscalar coupling in the action. In the case of the axion M is related to the Peccei-Quinn breaking scale [21].

Summarizing the gist of the argument we can say that if the B-mode autocorrelation arises as a Faraday rotated E mode, then, the B-mode angular power spectrum scales with the frequency. If we identify the measured B-mode power spectrum at $\nu_p = 150$ GHz with the Faraday rotated E mode, the corresponding signal will be larger for frequencies lower than ν_p . By rescaling the result from ν_p to ν_{low} we are in condition of checking the existing upper limits on the B-mode polarization at lower frequencies. Since the observational upper limits are violated, the main (and sole) assumption of the analysis is fallacious and the observed B-mode polarization cannot come predominantly from a rotated E-mode polarization. In the light of the current bounds at different frequencies and different

angular scales we can say that the fraction of B-mode polarization potential attributable to a Faraday effect is at most $\mathcal{O}(10^{-4}) \mu\text{K}^2$ for $\nu = \mathcal{O}(150)$ GHz and degree angular scales. Further analyses of the Faraday scaling of CMB polarization observables, in particular at low frequency, could open a new observational window for a closer scrutiny of the Faraday effect of the CMB and of large-scale magnetism prior to photon decoupling. In this respect, specific observational strategies have been suggested and we hope they will be soon considered by the forthcoming polarization experiments also in connection with the ongoing debate on multifrequency foreground measurements (and subtractions).

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