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DYNAMICS OF SIX-QUARK SYSTEMS IN THE SOLITON BAG MODEL

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ABSTRACT

The soliton bag model introduced by Friedberg and Lee is applied to S-wave nucleon-nucleon scattering. The dynamics of six quarks and their confining soliton field is treated in the generator co-ordinate method. We find an attraction of 120 MeV at a NN separation of 2.5 fm, but no attraction in the vicinity of the spherically symmetric shape of the system. This is in contradiction to approaches using naive adiabatic potentials.

Several attempts^{1),2)} have been made to describe the nucleon-nucleon interaction in a relativistic quark model. The starting point has always been the MIT bag model, where confinement is enforced via a static boundary condition. Albeit that model is not able to treat the dynamical problem of different confining field configurations. To improve the static approximation to confinement, we explicitly introduce a non-topological soliton field which takes into account the important gluon content of the baryonic wave function. Several proposals for such models have been made in the framework of a colour dielectric theory³⁾⁻⁵⁾. We use the Friedberg-Lee soliton model^{3),4)}. It describes quarks and gluons interacting with the scalar field σ via the following Lagrangian density

$$\mathcal{L} = \mathcal{L}_q + \mathcal{L}_\sigma + \mathcal{L}_{q\sigma} + \mathcal{L}_g \quad (1)$$

The first term $\mathcal{L}_q = \sum_f \bar{\psi}_f (i\gamma\partial - m_f)\psi_f$ describes the quarks with flavour f and current quark mass m_f ($m_u \approx m_d = 0$). The scalar field is parametrized by $\mathcal{L}_\sigma = \frac{1}{2}(\partial_\mu \sigma)^2 - U(\sigma)$, where the self-interaction $U(\sigma)$ has the form of a fourth order polynomial

$$U(\sigma) = a \left(\left(\frac{\sigma}{\sigma_v} \right)^2 - 1 \right) + b \left(\left(\frac{\sigma}{\sigma_v} \right)^3 - 1 \right) + c \left(\left(\frac{\sigma}{\sigma_v} \right)^4 - 1 \right) \quad (2)$$

with σ_v as vacuum expectation value of σ in the absence of quarks. We regard excitation quanta of the scalar σ -field as glueballs and have adjusted the parameters to let the mass $m_{GB} \approx 707$ MeV of the glueball agree with lattice estimates⁶⁾. The quarks interact with the scalar field via a Yukawa term $\mathcal{L}_{q\sigma} = -g\bar{\psi}\sigma\psi$ ($g = 13.06$) in such a way that the sum of $U(\sigma) + g\bar{\psi}\sigma\psi$ for $\bar{\psi}\psi \neq 0$ in the baryon develops a minimum near $\sigma = 0$. The change of the σ -field from $\sigma = \sigma_v$ outside of the hadron to $\sigma \approx 0$ inside creates the bag.

In this paper, we choose $a = 195 \text{ MeV fm}^{-3}$, $b = -390 \text{ MeV fm}^{-3}$, $c = 195 \text{ MeV fm}^{-3}$ and $\sigma_v = 77.5 \text{ MeV}$. [Note that the definition of a, b, c differs from Refs. 3) and 4).] The soliton also manifests itself in the gluon part of the Lagrangian

$$\mathcal{L}_g = -\frac{1}{4} \kappa(\sigma) \sum_{i=1}^8 F_{\mu\nu}^i F_{\mu\nu}^i - g_s \bar{\psi} \gamma_\mu \frac{1}{2} \lambda^i A_i^\mu \psi \quad (3)$$

where the conventional colour field tensor $F_{\mu\nu}$ is modified by a dielectric function $\kappa(\sigma)$ [$\equiv \kappa(\sigma)$ of Ref. 4)] which vanishes at $\sigma = \sigma_v$ forbidding coloured field

to leak out of the bag. Asymptotic freedom inside the hadron is assured if $\kappa(\sigma=0) = 1$. For the present paper we neglect all gluonic interactions and treat only $\mathcal{L}_{\text{eff}} = \mathcal{L}_q + \mathcal{L}_\sigma + \mathcal{L}_{q\sigma}$ as zeroth order approximation. With this Lagrangian, the single nucleon has the following properties: a mass of $\bar{m} \equiv \frac{1}{2}(m_N + m_\Delta) = 1300 \text{ MeV}/1087 \text{ MeV}$ and a r.m.s. charge radius for the proton $\langle r_p^2 \rangle^{\frac{1}{2}} = 0.96 \text{ fm}/0.83 \text{ fm}$ without and with c.m. corrections⁷⁾.

In order to calculate NN-scattering we use a variational method which is known as generator co-ordinate method (GCM)⁸⁾. The two nucleon trial wave function is taken as a superposition of basis state $|R\rangle$ characterized by a collective parameters R

$$|\psi^{\text{trial}}\rangle = \int dR |R\rangle \Phi(R) \quad (4)$$

* *carries* The weight function $\Phi(R)$ carries the information about the process and can be determined after a variation

$$\frac{\delta}{\delta \Phi^*(R)} \langle \psi^{\text{trial}} | H - E | \psi^{\text{trial}} \rangle = 0 \quad (5)$$

which yields a Hill-Wheeler equation for $\Phi(R)$ ⁸⁾. In order to avoid the complications of the variational method for the scattering system, we will approximate the Hill-Wheeler integration equation (5) by a Schrödinger equation with an effective interaction potential. We construct the basis states $|R\rangle$ from an external scalar field $\phi_R(\vec{r})$ which depends on the collective parameter R . For R we employ the centre-to-centre distance of two sphere configurations. The external potential $\phi_R(\vec{r})$ is assumed to have the following shape:

$$\psi_R(\vec{r}) = g\sigma_v \left(1 - 1.7 \int d^3\tau' T_R(\tau') \frac{\Gamma^2}{4\pi} \frac{e^{-\Gamma|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \right) \quad (6)$$

with $T_R(r)$ as the union of two spheres of radius a at a distance R :

$$T_R(\vec{r}) = \begin{cases} \theta(a - |\vec{r} - \frac{R}{2}\vec{e}_z|) & z \geq 0 \\ \theta(a - |\vec{r} + \frac{R}{2}\vec{e}_z|) & z < 0 \end{cases} \quad (7)$$

The folding function in (6) and the constant 1.7 are determined in such a way that the self-consistent solution $\sigma(r)$ of two free nucleons is reproduced at asymptotic R , i.e.,

$$\psi_{R \rightarrow \infty}(r) = g \sigma(\vec{r} - \frac{R}{2} \vec{e}_z) + g \sigma(\vec{r} + \frac{R}{2} \vec{e}_z) \quad (8)$$

In order to select a definite path in configuration space, we impose the constraint that the volume of the 6q cavity is fixed at the value of two nucleon volumes, i.e.,

$$\int T_R(\vec{r}) d^3r = 2 \cdot \frac{4}{3} \pi a_0^3, \quad (9)$$

where $a_0 = 1.21$ fm gives the radius of the nucleonic bag. This determines the size a of the two overlapping spheres in Eq. (7). We do not consider a as an independent collective parameter. Certainly the choice of the generating potential ϕ_R is of crucial importance to the calculation. We think the above parametrization of $\phi_R(\vec{r})$ is a very natural one combining numerical convenience with the asymptotic constraints. The potential $\phi_R(\vec{r})$ serves to create a basis of single particle quark states $\psi_R^k(\vec{r})$. These are determined in the equation of motion

$$(\vec{\alpha} \vec{p} + \beta \psi_R(\vec{r})) \psi_R^k(\vec{r}) = \epsilon_R^k \psi_R^k(\vec{r}) \quad (10)$$

The energies of the lowest positive and negative parity solutions $|S\rangle$ and $|A\rangle$ with eigenvalues ϵ_S and ϵ_A show the binding of the positive parity state at short distances as well as the convergence of both levels for $R \rightarrow \infty$. In order to construct six quark wave function with good symmetry we use right and left combinations of the parity eigenstates

$$\begin{aligned} |R\rangle \\ |L\rangle \end{aligned} = \begin{pmatrix} |S\rangle \pm \mu_R |A\rangle \end{pmatrix} \frac{1}{\sqrt{1 + \mu_R^2}} \quad (11)$$

such that $\mu_R \rightarrow 1$ and $\langle R|L\rangle \rightarrow 0$ for $R \rightarrow \infty$. Hence for asymptotic R we approach the situation of two orthogonal nucleons. Inside the interaction region the minimum of the energy is obtained for $\mu = 0$ which at zero separation $R = 0$ leads to the

symmetric $[s_1^6, 2]$ and mixed symmetry configuration $[s_1^4, 2, p_3^2, 2]$, as described by Harvey⁹⁾.

Since presently we do not include the gluon part $\mathcal{L}_G$ of the Lagrangian Eq. (3) no dynamical mixing of the two configurations is possible. Without any two-body one gluon exchange force, the mixing coefficient μ changes asymptotically from $\mu = 0$ to $\mu = 1$ without influence on the dynamics.

The solution of the Dirac equation and the symmetry considerations above yield the six quark scalar density $\langle \bar{\psi}\psi \rangle$ which is used as input to the soliton field equation. We only used ϕ_R to generate the quark basis for our trial wave function. The resulting σ -field does not crucially deviate from the input ϕ_R/g potential. The starting potential ϕ_R can be regarded as an implicit constraint on the multitude of solutions. Our procedure is similar to shell model calculations for deformed nuclei in the Nilsson model¹⁰⁾ where the mean field potential generates the nuclear orbits. Numerically, the σ -equation

$$-\vec{\nabla}^2 \sigma(\vec{r}) + \frac{\partial}{\partial \sigma} \mathcal{U}(\sigma) = -g \langle \bar{\psi}\psi \rangle. \quad (12)$$

is solved with a relaxation method on a two-dimensional grid¹¹⁾.

In order to evaluate overlaps of soliton configurations of different R , the σ wave function (state vector) is constructed as a coherent state¹²⁾. Then overlaps $\langle R|R' \rangle$ and $\langle R|H|R' \rangle$ which are needed in

$$\langle \psi^{\text{trial}} | H-E | \psi^{\text{trial}} \rangle = \iint dR dR' \bar{\Phi}^*(R) \langle R | H-E | R' \rangle \Phi(R) \quad (13)$$

can be calculated. The Hamiltonian H ¹³⁾ following from the Lagrangian Eq. (1) is given by¹³⁾

$$H = n_S \varepsilon_S + n_A \varepsilon_A + n_S \int d^3r \bar{\psi}_S (g\sigma - \psi) \psi_S + n_A \int d^3r \bar{\psi}_A (g\sigma - \psi) \psi_A \quad (14)$$

$$+ \int d^3r \left\{ \frac{1}{2} |\vec{\nabla}\sigma|^2 + \frac{1}{2} \pi_\sigma^2 + \mathcal{U}(\sigma) \right\},$$

where π_σ is the conjugate momentum of the soliton field σ .

In order to derive a Schrödinger equation instead of the Hill-Wheeler equation, we introduce new co-ordinates $\vec{\beta} = (\vec{R} + \vec{R}')/2$ and $\vec{\delta} = \vec{R} - \vec{R}'$ and expand

the weight function $\Phi^*(R)$ and $\Phi(R')$ around β up to second order in δ . After integrating over δ we obtain zeroth and second moments of the norm and energy kernels

$$\begin{aligned} N_k(\beta) &= \int d\delta \delta^k \left\langle \beta + \frac{\delta}{2} \mid \beta - \frac{\delta}{2} \right\rangle \\ H_k(\beta) &= \int d\delta \delta^k \left\langle \beta + \frac{\delta}{2} \mid H \mid \beta - \frac{\delta}{2} \right\rangle \end{aligned} \quad (15)$$

with $k = 0$ or 2 . We rewrite Eq. (14) as

$$-\frac{1}{\tilde{N}_0} \frac{d}{d\beta} \frac{\tilde{N}_0}{2H(\beta)} \frac{d}{d\beta} \Phi + \frac{\tilde{H}_0}{\tilde{N}_0} \Phi = E \Phi \quad (16)$$

with an effective mass

$$M^{-1}(\beta) = \frac{N_2}{\tilde{N}_0} (E - H_2/N_2) \quad (17)$$

and the abbreviations

$$\begin{pmatrix} \tilde{H}_0 \\ \tilde{N}_0 \end{pmatrix} = \begin{pmatrix} H_0 \\ N_0 \end{pmatrix} + \frac{1}{8} \frac{d^2}{d\beta^2} \begin{pmatrix} H_2 \\ N_2 \end{pmatrix} \quad (18)$$

In Fig. 1 we show the ratio of the dynamical mass $M(\beta)$ over the reduced mass $\mu = \frac{1}{2}m_N$ ($m_N = 1300$ MeV). At small distances the ratio becomes very small, asymptotically it converges towards 1 aside from numerical inaccuracies. The local adiabatic potential V_{loc} without effective mass corrections can be read off as $V_{loc} = \tilde{H}_0/\tilde{N}_0 - 2\bar{m}$ in Eq. (16). It is shown in Fig. 2. At zero distance, this adiabatic potential is large and attractive, a sign of the energy which is gained when one combines six-quarks in one bag of slightly larger radius. Taking the MIT bag model we estimate the ratio $E_{6q}/2E_{3q} = 6^{3/4}/2 \cdot 3^{3/4} = 0.84$ or $\tilde{H}_0/\tilde{N}_0 - 2\bar{m} \approx -400$ MeV at $R = 0$ in agreement with the value of the adiabatic potential. The extension of the local potential is determined by the size of the nucleon. Since we used rather extended ("soft") soliton bags, the potential drops to half of its value only at 2.2 fm. As we will see this long range still obtains in the more refined dynamical treatment.

To determine the dynamical potential we transform the β -dependent effective mass term $1/(2M(\beta))$ into a constant coefficient $1/(2\mu)$ by two alternative methods¹⁴⁾. One of these methods¹⁵⁾ gives the following effective potential $V_{\text{eff}}(\beta)$ which is to be used in the final Schrödinger equation

$$\left(-\frac{1}{2\mu} \frac{d^2}{d\beta^2} + V_{\text{eff}}(\beta) \right) f(\beta) = E_{\text{rel}} f(\beta) \quad (19)$$

The potential $V_{\text{eff}}(\beta)$ for NN scattering at the threshold ($E^{\text{rel}} = 0$) is shown in Fig. 2. Note that V_{eff} is independent of the NN spin-isospin state since our effective Lagrangian contains only scalar-isoscalar interactions. Because of the small value of $M(\beta)/\mu$ at small distances (β) the deep attraction of -400 MeV in V_{loc} has disappeared. The final results differ significantly from the local adiabatic potential. $V_{\text{eff}}(\beta)$ shows a minimum of -120 MeV at $\beta \approx 2.5$ fm. Asymptotically it approaches zero rather slowly. In fact comparing to the empirical NN-potential we see some difficulty to make the soliton bag model consistent with phenomenology. The Reid potential¹⁷⁾ has a minimum much further inside than the result of the soliton model. We conjecture that this result points towards a valence quark bag with a radius smaller than the one we used. But before this question can be settled we have to include more of the relevant physics in the soliton model like the gluon terms $\mathcal{L}_G$ and the treatment of the c.m. energy. Presently, the soliton bag model shows promising features to generate intermediate range attraction from σ (= glueball) exchange between nucleons. This presents a complementary picture to the conventional description of the attraction in the NN interaction in terms of 2π exchange.

If there is a strong glueball (0^+) $\rightarrow 2\pi$ coupling as indicated in recent theoretical work¹⁶⁾, this gluon picture may not be so different from the conventional one for the intermediate range. At short distances it connects well to the quark gluon description, which will to be investigated next in the soliton bag model.

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FIGURE CAPTIONS

Fig. 1 : Ratio of effective mass $M(\beta)$ to reduced mass μ as a function of distance β .

Fig. 2 : The adiabatic potential V_{loc} is shown together with the effective potential V_{eff} .

Fig. 1

Fig. 2