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Removing Tachyons by Compactification

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Abstract

It is shown that tachyons in the recently discovered ten-dimensional non-supersymmetric heterotic string models can be removed by compactifying on torii in the presence of large Wilson lines.

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Recently a class of 10-dimensional non-supersymmetric heterotic string models have been constructed by correlating the boundary conditions of the right moving fermions (in the old formulation of the superstring) and that of the left moving fermions describing the gauge group^[1,2,3]. When these two boundary conditions are uncorrelated one gets the usual supersymmetric heterotic string^[4] but by correlating the boundary conditions in different ways, consistent with the requirement of modular invariance, one gets a class of non-supersymmetric models, and notably among them a tachyon-free model with gauge group $O(16) \times O(16)$. Other models of this type, such as $O(16) \times E_8$ and $SO(8) \times SO(24)$ have tachyons in ten dimensions. In this letter we address the question of compactifying these latter models on torii, and in particular whether it is possible to remove the tachyons by suitable compactification.

General toroidal compactification of supersymmetric heterotic string to d space-time dimensions was carried out in ref [5], where it was shown that modular invariance required that the left and right torii of dimensions $26-d$ and $10-d$ respectively should be parametrised by even hyperbolic self dual lattices $\Gamma_{26-d, 10-d}$ of $26-d$ positive signature and $10-d$ negative signature. Such lattices are known to be unique up to $SO(26-d, 10-d)$ transformation. It was further shown that the mass spectrum of the theory is invariant under the subgroup $SO(26-d) \times SO(10-d)$. This meant that consistent compactification involved $(26-d)(10-d)$ a number of parameters (equal to the dimension of the coset space $SO(26-d, 10-d)/SO(26-d) \times SO(10-d)$). It was further shown in ref [6], that these parameters could be interpreted as the vacuum expectation values of the massless fields: metric g_{IJ} , antisymmetric tensor B_{IJ} and the Wilson lines A_I^a where the index I runs over the $(10-d)$ -torus and a runs over the Cartan subalgebra of $E_8 \times E_8$ or $SO(32)$ gauge group. A remarkable feature of this compactification was that by giving suitable values to the Wilson lines, one could in fact enlarge the gauge group from $SO(32)$ to, for example, $SO(52-2d)$.

In ref [7], Nair, Shapere, Strominger and Wilczek carried out the toroidal compactification of the $O(16) \times O(16)$ model and obtained models in lower dimensions that were tachyon free and had finite cosmological constant. Just as in the construction of new heterotic string models of ref [1,2,3], where one modifies the usual G -parity projection operator to $-(-1)^F (-1)^{2P \cdot \delta}$, δ being a vector satisfying $2\delta \in$ lattice of $E_8 \times E_8$ or $spin(32)/Z_2$, in ref [7]

Nair et al considered the projection operator $-(-1)^F(-1)^{2(p \cdot \delta - \tilde{p} \cdot \tilde{\delta})}$, where p and $\tilde{p}$ are the momentum operators along the compact (26-d) left and (10-d) right directions and $(\delta, \tilde{\delta})$ is a vector satisfying $2(\delta, \tilde{\delta}) \in \Gamma_{26-d, 10-d}$. Since $\Gamma_{26-d, 10-d}$ is unique up to $SO(26-d, 10-d)$ transformations, one can start from a fixed lattice $\Gamma^0 \equiv \Gamma_{16} + (10-d)\sigma_1$, where σ_1 is a two dimensional even self dual lattice with signature $(+1, -1)$ and Γ_{16} is the lattice of $E_8 \times E_8$ or spin $(32)/Z_2$, and classify all $(\delta, \tilde{\delta})$ on this lattice. One can now get arbitrary consistent toroidal compactification by making suitable $SO(26-d, 10-d)$ transformation on this lattice as well as on the corresponding vector $(\delta, \tilde{\delta})$. In particular one can take $(\delta, \tilde{\delta})$ to be equal to $(\delta, 0)$ on Γ^0 , where δ is a vector such that $2\delta \in \Gamma_{16}$ (the other components of δ being set to zero) corresponding to different 10-dimensional models (eg for $O(16) \times O(16)$ model, $\delta = (V_{16}, V_{16})$ where V_{16} is an $SO(16)$ vector and Γ_{16} is chosen to be $E_8 \times E_8$). $SO(26-d, 10-d)$ transformations on these models would give compactifications of the corresponding 10-dimensional models. Since the value of $(\delta^2 - \tilde{\delta}^2)$ is an invariant, one would get different families of models characterised by this value. In ref [7], Nair et al studied the compactification of $O(16) \times O(16)$ model for which $\delta^2 - \tilde{\delta}^2 = 2$. The tachyonic models in 10-dimensions, such as $SO(16) \times E_8$ and $SO(8) \times SO(24)$ are of the class $\delta^2 - \tilde{\delta}^2 = 1$. In the followiwnng we will study the latter class of models. Instead of using the approach of $SO(26-d, 10-d)$ transformations we will mostly use the equivalent method of background fields discussed in [6], as the physical meaning of the parameters involved is manifest in this approach. We will see below that by giving suitable background fields (in particular Wilson lines) one can get tachyon free models in the family $\delta^2 - \tilde{\delta}^2 = 1$. As expected, in such models, the decompactification limit does not exist; one necessarily ends up with a tachyonic theory in 10-dimensions. We will also show that the converse is true, namely in the class $\delta^2 - \tilde{\delta}^2 = 2$, there is a region in the parameter space of background fields, for which the theory becomes tachyonic.

We first review the construction of non-supersymmetric heterotic string in 10-dimensions in a way that will be convenient for later purposes. In the bosonic formulation of heterotic string, we associate different 16-dimensional lattices describing the 16-torus, to the four different right-moving sectors.* Denote by Γ_1 and Γ_2 , the lattices in respectively Neveu-Schwarz even and odd G-parity sectors and by Γ_3 and Γ_4 the ones in

* These lattices are, of course, obtained from Γ_{16} by appropriately shifting by the vector δ mentioned above, but the details of this will not concern us here.

Ramond positive and negative chirality sectors. The mass and constraint formula in Ramond sector is

$$\tilde{N} = \frac{1}{2} p^2 + N - 1 = \frac{1}{8} m^2 , \quad (1)$$

where $\tilde{N}$ and N are the right and left moving oscillator numbers and are non-negative integers, and p are the momenta lying on Γ_3 and Γ_4 lattices for the two chiralities. Since modular invariance requires, in particular, left-right level matching condition^[8], Γ_3 and Γ_4 must be even lattices. It is clear from (1) that Ramond sector has no tachyons. The corresponding formula for the Neveu-Schwarz sectors is

$$\tilde{N} - \frac{1}{2} = \frac{1}{2} p^2 + N - 1 = \frac{1}{8} m^2 , \quad (2)$$

where $\tilde{N}$ now is half integral in the even G-parity sector and integral in odd G-parity sector, and p lies in Γ_1 and Γ_2 respectively in the two cases. From (2), it is clear that, even sector cannot have a tachyon, as minimum value of $\tilde{N} = \frac{1}{2}$. In the odd sector however, $\tilde{N}=0$, and hence there is a possibility for the existence of tachyon. Now, by level matching condition, Γ_2 must have vectors of odd integral length squares. If Γ_2 has vectors of length² equal to one then the theory has tachyon, whereas if minimum length square in Γ_2 is 3 or greater then there is no tachyon. For example in the tachyon free $O(16) \times O(16)$ model

$$\Gamma_2 = \{ (V_{16}, Sp'_{16}) , (Sp'_{16}, V_{16}) \} , \quad (3)$$

where V_{16} and Sp'_{16} are the vector and spinor' conjugacy classes of the two $SO(16)$'s respectively. It is clear that the minimum length² in Γ_2 is 3, and hence there is no tachyon.

Let us consider now an example with tachyon. Modular invariance imposes the following conditions on the lattices $\Gamma_1, \Gamma_2, \Gamma_3$ and Γ_4 . Let τ be the complex parameter defining the string world sheet torus. Under $\tau \rightarrow \tau+1$

$$\begin{aligned} Z(\Gamma_1) &\rightarrow Z(\Gamma_1) , \\ Z(\Gamma_2) &\rightarrow - Z(\Gamma_2) , \end{aligned} \quad (4)$$

$$Z(\Gamma_3) \rightarrow Z(\Gamma_3) \quad ,$$

$$Z(\Gamma_4) \rightarrow Z(\Gamma_4) \quad ,$$

and under $\tau \rightarrow -1/\tau$ (up to overall factors depending on $\text{Im}\tau$)

$$Z(\Gamma_1)+Z(\Gamma_2) \rightarrow Z(\Gamma_1)+Z(\Gamma_2) \quad ,$$

$$Z(\Gamma_1)-Z(\Gamma_2) \leftrightarrow Z(\Gamma_3)+Z(\Gamma_4) \quad ,$$

$$Z(\Gamma_3)-Z(\Gamma_4) \rightarrow Z(\Gamma_3)-Z(\Gamma_4) \quad , \quad (5)$$

where $Z(\Gamma_i)$ are the partition functions

$$Z(\Gamma_i) = \sum_{p \in \Gamma_i} e^{i\pi\tau p^2} \quad .$$

These conditions can be easily obtained by examining the transformation properties of Neveu-Schwarz and Ramond sectors of the right moving fermionic string. For definiteness we consider here $SO(8) \times SO(24)$ heterotic string theory. The four lattices are

$$\Gamma_1 = \{(Sc_8, Sc_{24}) \quad , \quad (Sp_8, Sp_{24})\} \quad ,$$

$$\Gamma_2 = \{(Sc_8, Sp_{24}) \quad , \quad (Sp_8, Sc_{24})\} \quad ,$$

$$\Gamma_3 = \{(V_8, V_{24}) \quad , \quad (Sp'_8, Sp'_{24})\} \quad ,$$

$$\Gamma_4 = \{(V_8, Sp'_{24}) \quad , \quad (Sp'_8, V_{24})\} \quad , \quad (6)$$

where Sc , V , Sp and Sp' with subscripts 8 and 24 denote respectively scalar, vector, spinor and spinor' conjugacy classes of $SO(8)$ and $SO(24)$. It is easy to see that these lattices satisfy conditions (4) and (5). Now Γ_2 contains a vector $(\frac{1}{2} \frac{1}{2} \frac{1}{2} \frac{1}{2}, 00 \dots 0)$ whose length² is one. In fact the number of such vectors is equal to the dimension of the fundamental spinor representation of $SO(8)$ namely 8. The states corresponding to these vectors (with $\tilde{N}=N=0$) have negative m^2 and hence are tachyons.

Now let us compactify these theories on a (10-d)-torus given by a (10-d)-dimensional lattice Γ , in the absence of background antisymmetric tensor B_{IJ} and Wilson lines. Mass and constraint formula (1) and (2) now are modified to respectively:

Ramond sector:

$$\frac{1}{2} \left(\frac{1}{2} k-L \right)^2 + \tilde{N} = \frac{1}{2} \left(\frac{1}{2} k+L \right)^2 + \frac{1}{2} p^2 + N-1 = \frac{1}{8} m^2, \quad (1')$$

Neveu-Schwarz sector:

$$\frac{1}{2} \left(\frac{1}{2} k-L \right)^2 + \tilde{N} - \frac{1}{2} = \frac{1}{2} \left(\frac{1}{2} k+L \right)^2 + \frac{1}{2} p^2 + N-1 = \frac{1}{8} m^2, \quad (2')$$

where L is a winding vector lying in the lattice Γ and k is the corresponding momentum lying in the dual lattice Γ^* . Again Ramond sector and the even G-parity Neveu-Schwarz sector do not have tachyons, and the odd G-parity Neveu-Schwarz sector has a tachyon provided the corresponding 10-dimensional theory has a tachyon. The latter fact is easily seen by setting $L=k=0$ in (2'). If the 10-dimensional theory is modular invariant, so is the d-dimensional theory. Any other modular invariant d-dimensional model can be obtained by making "Lorentz" transformation $SO(26-d, 10-d)$ on the lattice $(W_L, p; W_R)$, where $W_L = \frac{1}{2} k+L$, $W_R = \frac{1}{2} k-L$ and $p \in \Gamma_1$ in the four different sectors^[5]. Let us focus on odd G-parity N-S sector, as tachyons can only occur in this sector. Furthermore, in order to have a tachyon we must set $N=\tilde{N}=0$. Then eq (2') becomes

$$\frac{1}{2} W_R^2 - \frac{1}{2} = \frac{1}{2} W_L^2 + \frac{1}{2} p^2 - 1 = \frac{1}{8} m^2.$$

The tachyon, before the "Lorentz" transformation, corresponded to the vector $W_R = W_L = 0$ and $p^2 = 1$, but by making a suitable "Lorentz" transformation this vector can be transformed into $(\tilde{W}_L, \tilde{p}; \tilde{W}_R)$ where $\tilde{W}_R^2 > 1$ and this state would no longer be tachyonic. Of course one must check that in this process some other non-tachyonic state is not transformed into a tachyonic state. Conversely one can start from the $O(16) \times O(16)$ non-tachyonic theory, which has a vector $W_L = 0$, $W_R^2 = 2$, $p^2 = 3$ corresponding to a massive state; and "Lorentz" transform to a vector $W_L = 0$, $W_R = 0$ and $p^2 = 1$ (note that $W_L^2 + p^2 - W_R^2$ is an invariant under "Lorentz" transformation), which would give rise

to a tachyon. Thus we see that the existence or non-existence of tachyon depends on the details of the compactification.

Let us again consider $SO(8) \times SO(24)$ model in some detail. Let us consider compactifying to 9-dimensions on a circle given by a one dimensional lattice

$$\Gamma = \{Rn ; n \in \mathbb{Z}\} , \quad R > 0 . \quad (7)$$

The momenta k lie on the dual lattice Γ^* ,

$$k \in \Gamma^* = \left\{ \frac{1}{R} m ; m \in \mathbb{Z} \right\} . \quad (8)$$

Let us further switch on Wilson lines A^a given by

$$A = \frac{1}{R} (Sp'_8 , V_{24}) , \quad (9)$$

where Sp'_8 is a fundamental spinor weight of $SO(8)$ and V_{24} is a fundamental vector weight of $SO(24)$. It is known^[6] that in the presence of Wilson line, k and p are shifted in the following way

$$\begin{aligned} k \rightarrow k' &= k - p \cdot A - \frac{1}{2} A^2 L = k - p \cdot A - \frac{1}{2} A^2 Rn , \\ p \rightarrow p' &= p + A \cdot L = p + RnA , \end{aligned} \quad (10)$$

where $p \in \Gamma_2$ given in eq (6).

We want to choose an R so that there is no tachyon ie. in the odd G -parity N.S. sector (with $N=\tilde{N}=0$)

$$\begin{aligned} \frac{1}{4} m^2 &= \frac{1}{2} \left(\frac{1}{2} k' + L \right)^2 + \frac{1}{2} \left(\frac{1}{2} k' - L \right)^2 + \frac{1}{2} p'^2 - \frac{3}{2} \\ &= \frac{1}{4} k'^2 + L^2 + \frac{1}{2} p'^2 - \frac{3}{2} > 0 . \end{aligned} \quad (11)$$

There are two cases

1) $n=\text{even}$:

Since $A \cdot L = n(Sp'_8 , V_{24}) \in (Sc_8 , Sc_{14})$ the shifted p' again lies in Γ_2 .

Thus, $\text{Min}(p'^2)=1$ corresponding to $(\text{Sp}_8, \text{Sc}_{24}) \in \Gamma_2$. Now

$$\frac{1}{2} A^2 L = \frac{n}{2} \frac{1}{R} 2 = \frac{n}{R} ,$$

and

$$P.A = \frac{1}{R} \cdot \frac{2r+1}{2} \text{ for some } r \in \mathbb{Z} .$$

$$\text{Thus } k' = \frac{1}{R} (m - n - \frac{2r+1}{2}) ,$$

and hence

$$\text{Min}(k'^2) = \frac{1}{4R^2} . \quad (12)$$

From eq (11) we see that

$$\frac{1}{4} \text{Min}(m^2) = \frac{1}{16R^2} + \frac{1}{2} - \frac{3}{2} > 0 ,$$

$$\text{provided } R^2 < \frac{1}{16} . \quad (13)$$

2) n=odd:

In this case again k' is as before, hence $\text{Min}(k'^2)$ is again given by (12), p' in this case, however is

$$p' = p + n(\text{Sp}'_8, V_{24}) \in \tilde{\Gamma}_2 ,$$

$$\text{where } \tilde{\Gamma}_2 = \{ (V_8, V_{24}) , (\text{Sp}'_8, \text{Sp}'_{24}) \} .$$

Thus

$$\text{Min}(p'^2) = 2 .$$

Substituting these limits and $\text{Min}(L^2) = R^2$, in eq (11)

$$\frac{1}{4} \text{Min}(m^2) = \frac{1}{16R^2} + R^2 + 1 - \frac{3}{2} > 0 ,$$

for $R^2 < \frac{1}{16}$.

For the critical value $R^2 = \frac{1}{16}$, we get massless 9-dimensional scalar states from case (1) (with $n=0$) transforming as fundamental spinor under $SO(8)$ and singlet under $SO(24)$, but for $R^2 < \frac{1}{16}$ all the states are massive. When $R^2 < \frac{1}{16}$, we can make infinitesimal changes in the values of R as well as Wilson line A , and all the states will be still massive. Thus in the 17-dimensional $(=(26-d)(10-d))$ parameter space of compactification $\text{Min}(m^2) > 0$, would describe open regions bounded by 16-dimensional surfaces defined by the points corresponding to $\text{Min}(m^2)=0$ (in the above example $R^2 = \frac{1}{16}$).

To get a tachyon-free theory in 4-dimensions, one can now simply compactify the remaining 5-dimensions on a 5-torus with, for example, no Wilson lines along these directions.

Now let us turn to the converse of the above example, namely let us start from the tachyon-free $O(16) \times O(16)$ model in 10-dimensions and compactify to 9-dimensions on the circle given by eq (7). In the odd-G-parity N-S sector, $p \in \Gamma_2$ where Γ_2 is given in eq (3), and has $\text{Min}(p^2) = 3$, signifying the absence of tachyon. Now we switch on Wilson line A given by

$$A = \frac{1}{R} (Sp'_{16}, V_{16}) ,$$

where Sp'_{16} and V_{16} are the fundamental weights of spinor' and vector representation of $SO(16)$. Let us consider the winding vector $L=Rn$ with n odd. Then

$$p' = p + A \cdot L = p + n(Sp'_{16}, V_{16}) \in \tilde{\Gamma}_2 ,$$

where $\tilde{\Gamma}_2 = \{(Sp_{16}, Sp_{16}), (Sc_{16}, Sc_{16})\}$, and $k' = k - P \cdot A - \frac{1}{2} A^2 L = \frac{(2r+1)}{2R}$ for some $r \in \mathbb{Z}$.

Thus

$$\frac{1}{4} \text{Min}(m^2) = \frac{1}{16R^2} + R^2 - \frac{3}{2} . \quad (14)$$

This shows that if we choose R such that

$$\frac{\sqrt{2}-1}{2} < R < \frac{\sqrt{2}+1}{2} , \quad (15)$$

we would get tachyons, corresponding to $p'=0$, $L = \pm R$ and $k' = \frac{\pm 1}{2R}$. Eq (14) for $\text{Min}(m^2)$ again shows a sort of duality $R \leftrightarrow \frac{1}{2R}$ as in the discussion of the cosmological constant in ref [7]. In particular, the fact that for large R one gets tachyon free theory, shows that decompactification limit exists and that the 10-dimensional theory is tachyon free as expected. On the other hand, eq (13) shows that tachyon free condition sets an upper bound on R ($\leq \frac{1}{4}$ for the Wilson line considered here), signalling that there is no decompactification limit - one necessarily ends up with a tachyonic theory in 10-dimensions. Stated another way, theories of this type (ie tachyonic in 10-dimensions) would dynamically compactify to lower dimensions. Even though we have shown the existence of an upper bound for R for a specific form of the Wilson line, this is easily seen to be a general feature. Indeed for any form of Wilson line, keeping the holonomy fixed (ie fixing $A \cdot R$), we see that k' in eq (10) scales as $1/R$. For zero winding number, the absence of tachyons requires that $k'^2 \geq 4$, which for sufficiently large R is certainly not satisfied. In fact, this argument also shows that the upper bound for R for arbitrary holonomy is less than or equal to the one found in the example considered here (namely $\frac{1}{4}$). This is so because, for zero winding number, the largest value of

$$\text{Min}(k'^2) = \frac{1}{R^2} \text{Min}(n - P \cdot A)^2 ; \quad n \in \mathbb{Z} ,$$

is achieved for half integral $P \cdot A$ and in this case $\text{Min}(k'^2)$ is equal to

$$\frac{1}{4R^2} .$$

This argument can be generalised for lower dimensional

compactification as follows. If the theory is compactified on a (10-d)-torus such that the corresponding momenta lie on a hypercubic lattice of characteristic lengths $1/R_i$ ($i=1, \dots, 10-d$), then a necessary condition for the absence of tachyons is

$$\sum_{i=1}^{10-d} \frac{1}{R_i^2} \geq 16 .$$

This again shows that not all the radii R_i can go to infinity simultaneously.

Now let us turn to the discussion of cosmological constant Λ . One loop contribution to the cosmological constant is given by

$$\Lambda = - T \int \frac{d^2 \tau}{\tau_2^2} \frac{1}{(\tau_2)^{\frac{d-2}{2}}} (p_B - p_F) ,$$

where p_B and p_F are the bosonic and Fermionic partition functions, T is a positive constant depending on string tension, and the integral is over the fundamental domain; $-\frac{1}{2} \leq \tau_1 < \frac{1}{2}$, $\tau_2 > 0$ and $|\tau| > 1$. In the region of the parameter space corresponding to tachyon-free models, one expects that Λ would be finite. In order to understand which particular background fields are stable, one must extremise Λ with respect to the background fields. Keeping R fixed let us first calculate the variation of Λ with respect to the holonomy $H^a (= A^a R)$. Now the partition functions involve terms of the type

$$p \sim \sum_L e^{-\frac{1}{2} \pi m^2 \tau_2} e^{2i\pi \tau_1 (L_0 - \tilde{L}_0)} ,$$

where the sum is over all the states. Under a variation of the parameters, $L_0 - \tilde{L}_0$ does not change, therefore we need only to look at the variation of m^2 . It can be seen from eqs (10,11) that

$$\delta m^2 \propto \frac{1}{R} (L - \frac{1}{2} k') \cdot p' \cdot \delta H .$$

Now if the lattice $\{p'\}$ is such that for every vector p' , there also exists another vector $(-p')$ for a fixed k' and L , then it is clear that $\delta \Lambda$ would be zero. Under what conditions can this happen? The lattice $\{p'\}$ is given by vectors of the type

$$p' = L_{16} + \delta + nH ,$$

where L_{16} is a vector in $E_8 \times E_8$ or spin $(32)/Z_2$ lattice Γ_{16} , δ is a Z_2 shift (ie $2\delta \in \Gamma_{16}$) and n is the winding number. If $(-p')$ is also a vector in the lattice, then there must exist $\tilde{L}_{16} \in \Gamma_{16}$ such that

$$-p' = \tilde{L}_{16} + \delta + nH \quad ,$$

for each n . This is possible only if H is also a Z_2 vector (ie. $2H \in \Gamma_{16}$). Furthermore if k' is to be fixed as $p' \rightarrow -p'$ we see from eq (10) that $2\delta.H$ must be integer. The holonomy H in the example of 9-dimensional compactification of $SO(8) \times SO(24)$ model considered here in fact satisfies all these conditions.

Let us next look at the variation of Λ with respect to the radius R . From eq (10) and (11) we obtain

$$\delta m^2 \propto (L^2 - \frac{1}{4} k'^2) \frac{\delta R}{R} \quad .$$

If for fixed p'^2 , for every vector (L, k') there exists another vector $(\tilde{L}, \tilde{k}')$ such that $\tilde{L} = \frac{1}{2} k'$ and $\tilde{k}' = 2L$, then again variation of Λ would be zero.

However, such a simple analysis is not possible in this case, as this would require, among other conditions, that $R = \frac{1}{\sqrt{2}}$ which gives a tachyonic theory. Nevertheless a few statements can be made. Firstly we note that if for a given holonomy H (satisfying the conditions of the previous paragraph), the maximum allowed radius for a tachyon-free theory is $R_{\max}$ then for $R > R_{\max}$, the cosmological constant Λ is negative infinity. As we decrease R below $R_{\max}$, Λ becomes finite and when $R \rightarrow 0^+$, one can show that Λ approaches zero as $R^{\frac{1}{d}}$, where d is the non-compact dimensions. It will be interesting to see if there is any local extremum of Λ for $0 < R < R_{\max}$, otherwise the theory would drive towards $R \rightarrow 0^+$ limit.

In conclusion, we have shown that by compactifying appropriately on a torus, one can get tachyon-free theory from the tachyonic 10-dimensional heterotic string models (even though we have explicitly worked out the example of $SO(8) \times SO(24)$ model, but the same result is true for all the models described in [2]). For toroidal compactification, it seems that the theory drives towards the singular limit $R \rightarrow 0^+$. It would be interesting to investigate orbifold compactification of these models, as they may be able to provide, besides the chiral fermions, a reasonable extremum for cosmological constant.

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