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MONTE CARLO RENORMALIZATION GROUP STUDY OF THE ABELIAN HIGGS MODEL

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ABSTRACT

Strong evidence supports the notion that pure ϕ^4 field theory is non-interacting, i.e., "trivial". Should such a situation persist when gauge fields are present, the idea that elementary scalars can break symmetries spontaneously is called into question. In principle, the question of whether a theory is trivial or not can be answered within the context of the Monte Carlo renormalization group (MCRG). Previously we applied the MCRG to the pure ϕ^4 , SU(2)-Higgs, and non-compact U(1)-Higgs models; here the four-dimensional compact U(1)-Higgs model is considered. All are lattice versions of limits of the SU(2) $\times$ U(1) standard model of the weak interaction without fermions.

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1. - PROLEGOMENA

The concept of particle mass generation by spontaneous symmetry breaking¹⁾ has become one of the most important [and least understood²⁾] ideas in field theory. Typically, this symmetry breaking is produced by the introduction of a scalar field. The new field supposedly interacts with itself in such a fashion (via a ϕ^4 coupling) as to generate a vacuum state which lacks a symmetry of the Lagrangian. Gauge particles propagating in the asymmetric vacuum then appear to be massive, while the underlying gauge theory remains renormalizable.

This Higgs-Kibble mechanism in fact forms an essential part of the standard model³⁾ of the weak interaction. Nevertheless, a large body⁴⁾⁻⁶⁾ of evidence has been presented which suggests that pure ϕ^4 field theory is "trivial", that is, non-interacting in the infinite cut-off limit in four dimensions. (Of course, if the theory is non-interacting, its interactions cannot break the symmetry of the vacuum.) Recently, studies^{7),8)} of the probable triviality of an SU(2) gauge field interacting with a fundamental representation scalar field have been made; a Monte Carlo renormalization group (MCRG) analysis of the non-compact U(1)-Higgs model has also been performed⁹⁾. No evidence for a non-trivial continuum limit was found in either model. In the latter case it is possible that the conclusions drawn were affected by the fact that the model studied was non-compact (little is known about the non-compact Abelian Higgs model). The present work therefore considers the interaction of a scalar field with a compact U(1) gauge theory.

All three of the models considered are lattice versions of limits of the SU(2) $\times$ U(1) standard model of the weak interaction (without fermions). If the standard model itself is trivial, it is fair to question the whole idea of symmetry breaking by elementary scalars. There is an alternative and much more exciting possibility: if the standard model is [like the O(N) gauge model discussed in Ref. 7)] non-trivial only for a select range of parameters, the Higgs mass can be bounded^{10),11),12)} or even calculated^{10),7)} from first principles.

It must be pointed out that it is not in general necessary for a theory to be asymptotically free in order for it to be non-trivial. It is only necessary that the running coupling constants approach a fixed-point value (corresponding to a zero of the beta function) as the momentum scale increases. This fixed-point value can even be infinite, corresponding to an appropriately-defined zero of the beta function at infinite coupling. What must be avoided is a "Landau ghost", where the running coupling becomes infinite at a finite momentum scale. Thus asymptotic freedom is sufficient to guarantee non-triviality (an asymptotically-

free theory has an attractive fixed point at zero coupling), but unnecessary. Whether or not the theory is asymptotically free, the domain of attraction (i.e., the set of values of the renormalized coupling constants for which the running couplings approach the fixed point) of the attractive "ultra-violet-stable" fixed point determines the allowed range of parameters for a non-trivial theory. If this domain of attraction is finite, an (upper) bound on the Higgs mass is a likely consequence¹⁰⁾⁻¹²⁾; if it is limited to a single point, the Higgs mass is calculable^{10),7),13)}. The implications of these ideas for real-space renormalization have been discussed by Hasenfratz⁴⁾ and ourselves^{6),7),9)}.

It should also be pointed out that even if field theory is au fond a low-energy limit of another theory (e.g., a string theory) which becomes unified with other interactions at a much higher momentum scale (e.g., the Planck mass), the question of triviality is still relevant, for the field theory may well have a Landau ghost at "low" energies^{10),11)}.

Accordingly, in general it is necessary to consider the question of triviality in theories which are NOT asymptotically free in all couplings. A non-perturbative method must be applied to deal with this problem. We have used^{6),7),9)} the Monte Carlo renormalization group for this purpose. Specifically, a lattice version of the continuum model of interest is constructed, and the renormalization flows and fixed points under a comprehensive blocking transformation are studied. At any fixed points, the anomalous dimensions of the continuum theory (if one exists) associated with that point are calculated. If the scalar sector of the theory is non-trivial, i.e., interacting, the scalar field anomalous dimensions should be non-zero. Possible eventual phenomenological results of such an analysis include^{10),11),12)} upper bounds on the Higgs mass [as in the case of the $O(N)$ gauge model⁷⁾] or even calculable Higgs masses¹⁰⁾ [as in the case of eigenvalue theories¹³⁾]. After a brief discussion of the compact Abelian Higgs model, we determine its flow diagram, fixed points and critical exponents, and discuss our results in light of the above remarks.

2. - THE COMPACT ABELIAN HIGGS MODEL

In order to perform the study described above, it is necessary to formulate a lattice model of elementary scalars interacting via electromagnetism. Previously⁹⁾ we used a non-compact theory¹⁴⁾ of lattice electromagnetism for this purpose; here the analysis is applied to the more traditional compact theory¹⁵⁾⁻¹⁸⁾. The coupled $U(1)$ electrodynamics-Higgs theory has been well studied previously¹⁵⁾⁻²¹⁾.

One simplification used here is the fixing of the lattice Higgs field length to unity. Because the fixed- and variable-length models are in the same universality class, this constraint should not affect our results. Additionally, in both the pure scalar⁶⁾ and SU(2)-Higgs models^{7),8)}, evidence supports this idea (the scalar self-coupling appears to be irrelevant in an appropriate renormalization scheme). This fixed-length action is defined by¹⁹⁾ (2κ is sometimes called β_H):

$$S = \beta \sum_{\langle ijkl \rangle} (1 - \cos \theta_{ijkl}) + \\ + 2\kappa \sum_{\langle ij \rangle} (1 - \cos \Delta_{ij}) \quad ;$$

$$\Delta_{ij} \equiv \varphi_i - \varphi_j + \theta_{ij} \quad (1)$$

where β is related to the bare lattice gauge coupling g_0 by $\beta = g_0^{-2}$; κ is the "hopping parameter"; $\{\theta_{ij}\}$ and $\{\varphi_i\}$ are real dynamical variables associated with the gauge and scalar fields respectively and ranging from $-\pi$ to π , with each link between sites i and j associated with a θ_{ij} and each lattice site associated with a φ_i ; and $\theta_{ijkl} \equiv \theta_{ij} + \theta_{jk} - \theta_{kl} - \theta_{li}$ is the path-ordered sum of the four θ_{ij} defining an elementary plaquette $\langle ijkl \rangle$. The symbol $\langle ij \rangle$ refers to all nearest-neighbour pairs, summed once.

The theory governed by the fixed-length action Eq. (1) possesses¹⁵⁾⁻²¹⁾ three phases:

- 1) a confined phase, in which the free energy required to separate a pair of test charges increases without bound as the distance between them grows;
- 2) an electrodynamics or "Coulomb" phase, where interparticle forces between test charges obey Coulomb's law; and
- 3) a "Higgs" phase, where spontaneous symmetry breakdown occurs. The significance of these phases for the flow diagram is discussed below.

3. - DESCRIPTION OF THE PROCEDURE

How can the question of the triviality or non-triviality of a theory be answered in a sensible fashion? One way of approaching these questions is via the Monte Carlo renormalization group (MCRG). By dealing directly with the scaling properties of the various field and operators under consideration, it allows an economical (albeit numerical) formulation of the problem. Additionally, by restricting the analysis to a few carefully-chosen operators, the MCRG allows one to introduce accurate and systematically improvable approximations. More direct methods⁵⁾ of studying triviality do not allow this simplification.

The MCRG^{22),23),6),7)} is a combination of the ideas of Monte Carlo simulation²⁴⁾ with those of the real space renormalization group²⁵⁾. The system under consideration is divided up into "blocks" or collections of site variables, and a smaller set of block variables is defined by averaging in some fashion over these site variables. The new block system is governed by a block-renormalized action (defined by couplings $\{K'\}$) in the same fashion as the original site action (defined by couplings $\{K\}$) governed the site system. By studying the sequence of couplings $\{K\} \rightarrow \{K'\} \rightarrow \{K''\} \rightarrow \dots$ as repeated blockings are performed, the critical properties of the system can be determined. For a lattice gauge theory, these critical properties determine the continuum limit of the theory.

These ideas can be formulated in a more precise mathematical fashion as follows. Consider a system of site variables $\{\phi\}$ governed by a site action $S\{\phi\}$. A block-renormalized action $S'\{\phi\}$ is determined from the site action by use of the projection operator $P[\{\phi'\};\{\phi\}]$:

$$\exp[-S'\{\phi'\}] = \text{Tr}_{\{\phi\}} P[\{\phi'\};\{\phi\}] \exp[-S\{\phi\}] \quad (2)$$

where the trace (or functional integral) is only over the site variables $\{\phi\}$. The requirement that the renormalization group transformation preserve the partition function (vacuum-to-vacuum transition amplitude) of the system imposes the constraint

$$\text{Tr}_{\{\phi'\}} P[\{\phi'\};\{\phi\}] = 1 \quad (3)$$

The projection operator is otherwise entirely arbitrary. The generalization of Eqs. (2) and (3) to a case where both spin and gauge fields are present is obvious.

As there are two sets of fields (the $\{\theta_{ij}\}$ and the $\{\phi_i\}$), there are also two projection operators. It is convenient to choose these projection operators to be products of delta functions of the form

$$P[\{\varphi'\}; \{\varphi, \theta\}] = \prod_{\{\varphi'\}} \delta[\varphi'_i - f_i\{\varphi, \theta\}] \quad (4)$$

$$P[\{\theta'\}; \{\theta\}] = \prod_{\{\theta'\}} \delta[\theta'_{ij} - g_{ij}\{\theta\}] \quad (5)$$

The original (or "site") system is subdivided into blocks, each of which measures two lattice units on a side. The lattice site nearest the origin (all four coordinates are chosen positive throughout the lattice) is designated as the "block" lattice site. The phase f_i of the block scalar field is then taken to be the argument of the parallel transport of the sum of all scalar fields in the block. If more than one path of parallel transport exists, all of the shortest paths are averaged arithmetically. For example, if one blocks the two-dimensional system in Fig. 1a, then

$$f_a = \arg \left\{ e^{i\varphi_a} + e^{i(\theta_{ab} + \varphi_b)} + e^{i(\theta_{ac} + \varphi_c)} + \right. \\ \left. + \frac{1}{2} \left[e^{i(\theta_{ab} + \theta_{bd} + \varphi_d)} + e^{i(\theta_{ac} + \theta_{cd} + \varphi_d)} \right] \right\} \quad (6a)$$

The $\{g\}$ are taken as the arguments of the sum of all paths of the forms shown in Fig. 1b and their permutations. Again, for the two-dimensional example in Fig. 1a:

$$g_{ae} = \arg \left\{ e^{i(\theta_{ab} + \theta_{be})} + e^{i(\theta_{ac} + \theta_{cd} + \theta_{df} - \theta_{ef})} + \dots \right\} \quad (6b)$$

In order to determine the fixed points and critical exponents of a theory, it is necessary to be able to calculate the action for the blocked system. The original formulation²²⁾ of the MCRG did not permit the block action (and hence the block-renormalized couplings) to be extracted. Accordingly, we developed and repeatedly applied^{23), 6), 7)} the first practical general techniques for finding the block action. Our methods²³⁾ are applicable to systems with either continuous or discrete variables. [A totally different method which can be practical for discrete systems was developed and applied to the Ising model in Ref. 26).]

Our method^{23), 6)}, when applied to continuous systems⁶⁾, involved taking moments of the blocked variables, and was essentially equivalent to integration by parts. Some of the blocked variables were fixed. It was afterwards pointed out in Ref. 27) that in certain special cases (where the blocked action could be expanded in terms of known functions of a continuous variable) an improvement on this method can be made, and none of the block variables need to be fixed. This idea^{6), 27)} of integrating by parts can be expressed in a neat functional form²⁷⁾, which holds identically if the quantity in square brackets in Eqs. (7) either vanishes or is periodic at the limits of integration (note, however, that this trick will not work for discrete systems like the Ising model):

$$\int \mathcal{D}\theta' \mathcal{D}\varphi' \frac{\delta}{\delta \theta'_{ij}} \left[\mathcal{O}_1 \{ \theta' \} \exp(-S' \{ \theta', \varphi' \}) \right] = 0 \quad (7a)$$

$$\int \mathcal{D}\theta' \mathcal{D}\varphi' \frac{\delta}{\delta \varphi'_i} \left[\mathcal{O}_2 \{ \theta', \varphi' \} \exp(-S' \{ \theta', \varphi' \}) \right] = 0 \quad (7b)$$

for arbitrary periodic functionals $\mathcal{O}_1$ and $\mathcal{O}_2$. Specifically in our truncation scheme we employ [following the notation of Eq. (1)]:

$$\theta_1 \{ \theta' \} = \sum_{\langle ijkl \rangle} \sin \theta'_{ijkl} \quad (8a)$$

$$\theta_2 \{ \theta', \varphi' \} = \sum_{\langle ij \rangle} \sin \Delta'_{ij} \quad (8b)$$

[The notation $\langle ij \rangle$ in Eq. (8b) refers to all nearest-neighbour pairs, each summed once.] Equations (7) and (8) lead to a two-by-two set of linear equations for the block-renormalized couplings β' and κ' in terms of expectation values of local operators. (In practice, these expectation values are taken as an average over the entire lattice.) The results obtained using Eqs. (8) are compared below with those produced by a different set of operators; this comparison serves as one check on the validity of our truncation scheme. In principle, a larger set ($O_3, O_4, \dots$) of operators can be used to calculate an arbitrary number of block-renormalized couplings and refine our results to any given precision.

The truncation to two operators should be a good approximation, provided that the blocking transformation is sufficiently comprehensive. The assumption is essentially that the fixed points of the renormalization group transformation are near to the surface where β and κ are the only non-zero couplings. A second check of this assumption is thus whether or not critical exponents extracted from this two-parameter fit agree well with known results in appropriate limits of the model.

It is important also to mention that an implicit assumption has been made - that hyperscaling holds for the compact Abelian Higgs model. "Hyperscaling" is essentially the notion that there is one important (correlation) length scale in the system at criticality, i.e., that the critical scaling properties of the system depend only upon this one length. The assumption of hyperscaling is built into the present formulation of the MCRG, and therefore cannot be checked within the context of this calculation.

4. - RESULTS

Before presenting the results of the calculation, a few technical details must be mentioned. The analysis was performed (except for the XY model limit described below) in the unitary gauge. Choosing the unitary gauge is essentially equivalent to setting all of the $\{\phi_i\}$ to zero. After the blocking, the $\{\phi'_i\}$ are also set to zero by performing a gauge transformation (which depends upon the blocked variables) on the $\{\theta'_{ij}\}$, a procedure described in Ref. 7). The site lattice configurations of $\{\theta'_{ij}\}$ were generated according to the algorithm of Metropolis et al.^{28),24)}. The values of β' and κ' were measured by the procedure described in Section 3 over 2,500 iterations (following 250 equilibration iterations during which no measurements were taken) on a 6^4 lattice. The machine used for the majority of the computations was the Livermore MFE Cray; for comparison, each point used in this calculation would have used roughly nine VAX hours.

The first result of the calculation is the flow diagram, displayed in Fig. 2. Note the presence of fixed points F, G, T and E, and three sinks: S_C , for the confined phase (the true confined phase is actually limited to the $\kappa = 0$ axis and $0 < \beta \lesssim 1.0$); S_H , for the Higgs phase, and S_E , for the electrodynamics or "Coulomb" phase. Each of these points is discussed below in turn.

When κ equals zero, the model reduces to a pure compact U(1) gauge theory. A plot of β' versus β is shown in Fig. 3. There is a fixed point at $\beta^* \cong 1.01$. A two-parameter least-squares fit to the seven points nearest to the transition (shown as a dashed line in the Figure) yields:

$$\left. \frac{\partial \beta'}{\partial \beta} \right|_{\beta^*} = 14.3 \quad (9a)$$

$$\left. \frac{\partial \beta'}{\partial \beta} \right|_{\beta^*} \equiv 2^y \quad (9b)$$

$$y = 3.8 \quad (9c)$$

[where the number 2 in Eq. (9b) is just the scale factor of the blocking transformation].

The value in Eq. (9c) is suspiciously close to the value ($y = 4$) expected²⁹⁾

for a first-order phase transition. Additionally, the width $\Delta\beta$ of the critical region is quite small, so errors in convergence should tend to reduce the measured value of y below its true value. Evidence in support of the theory that this transition is first order has also been presented previously¹⁸⁾. However, an MCRG calculation can never prove that a transition is first order, for y must equal four (i.e., the space-time dimensionality of the system) exactly in this event, and an MCRG calculation always has an associated error (here we estimate 10-15%). Thus we cannot entirely eliminate the possibility that this transition is second order with a critical exponent $\nu = 1/y \cong 0.3$, as found by different authors¹⁷⁾. In either case, as the fixed point F is attractive along the segment TF , the critical behaviour along this segment should be governed entirely by this fixed point.

The next fixed point to consider is the one marked "G" in Fig. 2. In this limit of infinite β , the theory reduces to an XY model. According to an expansion³⁰⁾ in inverse powers of the co-ordination number q (for a hypercubic lattice $q = 8$), this model has a phase transition at $2\kappa^* \cong 0.30$. If the theory is trivial, it should have a Gaussian exponent ($\nu = \frac{1}{2}$), corresponding³¹⁾ to the vanishing of the anomalous dimension γ_{ϕ^2}

$$\gamma_{\phi^2} = 2 - \bar{\nu}^{-1} = 0 \quad (10)$$

Because of the difficulties associated with simulating the model in this limit (the Metropolis et al. algorithm²⁸⁾ performs extremely poorly for large β), the XY model limit was simulated directly. This model is governed by an action

$$S = -2\kappa \sum_{n,\mu} \cos(\varphi_n - \varphi_{n+\mu}) \quad (11)$$

(n is a lattice site, μ is a direction). The renormalized coupling (in the truncation scheme applied above) can be determined by using the operator O_2 defined in Eq. (8b), which is gauge-equivalent to

$$O_2 \longrightarrow \sum_{n,\mu} \sin(\varphi'_n - \varphi'_{n+\mu}) \quad (12)$$

in this limit. The block-renormalized coupling κ' is calculated by varying with respect to $\varphi'_{n,\mu}$ [cf. Eq. (7b)] to obtain

$$2\kappa' = \frac{\langle \sum_{n,\mu} (\cos \Delta'_{n,\mu} + \cos \Delta'_{n-\mu,\mu}) \rangle}{\langle \sum_n \left(\sum_{\mu} (\sin \Delta'_{n,\mu} + \sin \Delta'_{n-\mu,\mu}) \right)^2 \rangle} ;$$

$$\Delta'_{n,\mu} \equiv \varphi'_n - \varphi'_{n+\mu} \quad (13a)$$

The blocking scheme is of course just the limit of that used for the full model [compare Eq. (6a)]:

$$f_n = \arg \left\{ \sum_{\text{block}} e^{i\varphi_a} \right\} \quad (13b)$$

Note that this entire procedure is just the limit of both the model and Eqs. (7) as β increases without bound.

The result of this calculation is shown in Fig. 4. A fixed point is clearly in evidence; a 12-point least-squares fit to $\kappa'(\kappa)$ at this fixed point yields

$$2\kappa^* \cong 0.28$$

$$\left. \frac{\partial \kappa'}{\partial \kappa} \right|_{\kappa^*} \cong 3.93$$

$$= 2^y$$

$$y \cong 1.97$$

$$\nu = 1/y \cong 0.51$$

$$\gamma_{\varphi_2} \cong 3 \cdot 10^{-2} \quad (14)$$

This is in accord with the expectation ($\nu = \frac{1}{2}$) of triviality, as well as with a result ($\nu = 0.56$) reported²¹⁾ for the variable-magnitude model at $\lambda = 3$. Because this fixed point is attractive in the β direction, this second-order behaviour and critical exponent should be valid all along the line segment TG. Note also that the value of the fixed point is in good agreement with the expected³⁰⁾ value $2\kappa^* \cong 0.30$, suggesting that the true fixed point (in the infinite parameter

space) does indeed lie close to the plane where β and κ are the only non-zero parameters.

Next consider the tricritical point T. A simultaneous double least-squares fit was performed on 26 points in this region; the result is a fixed point located at approximately (1.0, 0.18) with exponents

$$\begin{aligned} y_1 &= 3.7, \\ y_2 &= 2.16 \end{aligned} \tag{15}$$

Note that (given the expected 10%-15% errors) the value of y_2 is near to the Gaussian value $y = 2$. Within these same limits, the value for y_1 is close to the value ($y = 4$) indicated²⁹⁾ for a first-order phase transition.

The exponent for the fixed point E was determined by a least-squares fit to 18 points:

$$\text{leading eigenvalue} \quad \lambda_E = 3.0 \approx 2^y \tag{16a}$$

$$y = 1.6 \tag{16b}$$

$$v = 1/y = 0.6 \tag{16c}$$

at $\beta^* = 0.41$, $\kappa^* = 0.65$. As discussed below, there may be another relevant operator associated with this fixed point, so this number should perhaps not be taken too seriously. Interestingly enough, however, it is within 20% of the Gaussian value $v = \frac{1}{2}$, $y = 2$.

Finally it remains to discuss the sinks S_C and S_H . According to an old result of Fradkin and Shenker²⁰⁾, based upon work of Osterwalder and Seiler¹⁵⁾, the Higgs and confinement phases are analytically connected. Specifically, there is a domain of finite width connecting the two phases in which no phase transitions occur. Naively, this exact result seems to belie the existence of separate sinks for the two phases. However, in a more general parameter space [where more operators are included in Eqs. (7) and (8)], the two "sinks" S_C and S_H may

actually flow to a common sink. This scenario would suggest that another operator is relevant for the fixed point E.

A second possibility is that an improved blocking transformation might ameliorate this difficulty. Consider the half-line from the point E to $\kappa = \infty$, $\beta = -\infty$, which serves as a separatrix between the flows going to the sinks S_H and S_C . (This half-line is roughly described by the equation $\kappa = 1.1-1.6\beta$). If another fixed point, repulsive along the direction tangent to this half-line and lying between the point E and the axis $\beta = 0$ appeared in an attempted scheme, the above problem would be solved. Indeed, such a flow diagram would imply a terminating line of phase transitions, in accord with MC simulations¹⁹⁾. Thus it is likely that our renormalization procedure has simply missed such a fixed point.

5. - EFFECTS OF TRUNCATION

Systematic errors enter the above analysis due to the effects of truncation, that is, because only two operators were used in Eqs. (8), only two (effective) couplings β' and κ' can be extracted. A more complete analysis would employ more operators, but would lose much of the conceptual clarity of the study presented here, for the flow diagram would then occupy a space of many more (e.g., 10-100) dimensions. Thus we believe that it is useful to adopt such a truncation scheme for the first renormalization-group analysis of this model.

In principle this truncation is an uncontrolled approximation, for there is a priori no small expansion parameter. However, we did repeat the analysis with a different pair of operators as a check on our results. The new equations determining the block couplings were produced by using Eq. (7a) but replacing Eq. (7b) with

$$\int \mathcal{D}\theta' \mathcal{D}\varphi' \frac{\delta}{\delta \theta'_{ij}} \left[\mathcal{O}_2 \{ \theta', \varphi' \} \exp \left(-S' \{ \theta', \varphi' \} \right) \right] \quad (7b')$$

The flow diagram was qualitatively and quantitatively virtually identical to Fig. 2. The exponent for the fixed point F is identical to that for the previous scheme (the two procedures are equivalent when κ vanishes). The exponents for the fixed point T (located at $\beta^* = 1.0$, $\kappa^* = 0.19$) were found by a least-squares fit to 27 points to be $y_1 = 3.4$, $y_2 = 2.0$, and for the fixed point E (located at

$\beta^* = 0.31$, $\kappa^* = 0.71$) a fit to 12 points yields $\gamma = 1.4$. The exponent associated with the fixed point G could not be determined due to statistical errors [the Metropolis et al. algorithm²⁸⁾ performs poorly in the large β limit]; however, a fixed point at $2\kappa^* \approx 0.3$ was in evidence. Thus the results obtained here with two different truncation schemes are at least consistent within the expected (10-15%) errors.

6. - CONCLUSIONS AND DISCUSSION

We have presented the results of the first MCRG study of the compact Abelian Higgs model, i.e., a fixed-length scalar field in the fundamental representation interacting with a compact U(1) gauge field. Within the context of a two-parameter truncation scheme (employed for convenience and clarity) we have identified the flow structure and fixed points of the theory within a particular blocking scheme and calculated the corresponding critical exponents. We have also established the machinery by which this calculation may be improved systematically.

Our results suggest the interpretation that the Higgs-Coulomb phase transition (the line segment TG in Fig. 2) is second order and is governed by the fixed point G. The critical exponent for this fixed point is consistent with the Gaussian value $\nu = \frac{1}{2}$, i.e., with the statement that the corresponding two-component ϕ^4 theory is trivial. Note that this result and the irrelevance of the gauge coupling $g_0^2 = \beta^{-1}$ along the segment TG imply the triviality of any continuum limit along this phase transition (within the accuracy of our procedure).

The above analysis was, of course, performed within the context of the fixed-magnitude model, where the bare lattice scalar self-coupling is infinite. If the universality hypothesis holds for this model, this self-coupling should be irrelevant for this fixed point (in the sense of the renormalization group). Experience with both pure scalar⁶⁾ and SU(2)-Higgs^{7),8)} models suggests that this should be the case, but we have not verified this point for the Abelian Higgs model.

Numerical analyses of the variable-magnitude model have been performed by other authors^{19),21)} for several values of the scalar self-coupling. For small λ , this Higgs-Coulomb phase transition was seen to be first-order [by Munehisa and later by others¹⁹⁾], a result which is consistent with the Coleman-Weinberg calculation³²⁾ [a good review can be found in Ref. 33)]. More recent studies²¹⁾,

however, suggest that the phase transition becomes second-order in the fixed-length limit, with a critical exponent $\nu \cong 0.56$ for $\lambda = 3$, consistent both with our results and with triviality.

A phase transition which changes its order in this fashion [as predicted to occur in the Abelian Higgs model in Ref. 34)] is associated with tricritical behaviour, a phenomenon which has also been conjectured¹⁸⁾ to occur in the compact pure U(1) theory. Such a change in the order of the Higgs transition may also occur in the non-compact Abelian Higgs model^{9),14)} and in a lattice superconductor^{35),36)}.

In the ($\kappa = 0$) limit of pure compact QED, our results provide evidence for the deconfinement phase transition being of first order, consistent with previous work of other authors¹⁸⁾. Nevertheless, as discussed above, one cannot, on the basis of an MCRG calculation alone, exclude the possibility that the phase transition is of second order with a critical exponent $\nu \cong 0.3$, in accord with even earlier work¹⁷⁾ on the model. A tricritical point, labelled "T" in Fig. 2, was also observed, with one of the exponents (γ_2) near to the Gaussian value $\gamma = 2$.

According to early numerical work¹⁹⁾, the compact Abelian Higgs model has a line of phase transitions extending into the interior of the phase diagram which terminates at a critical point. The location of this line and its endpoint are extremely difficult to determine by numerical simulation because of statistical fluctuations. In our (truncated) calculation, this line did not in fact terminate at such a critical point. This negative result probably indicates that our truncation scheme was naive enough to miss a fixed point, as discussed above. This line is, however, entirely within the Higgs phase of the theory, so a continuum limit with Coulombic forces between charged particles is probably impossible along it. Indeed, as such a terminating line does not appear in the non-compact theory⁹⁾, it may well be an artifact of the lattice model and irrelevant for a continuum limit of this model, should one exist.

Some support for the idea that no continuum limit exists along this line also comes from mean-field calculations. Although a mean-field study³⁷⁾ of the Z_2 -Higgs model finds indications of such a limit at a corresponding critical point (not surprisingly with mean-field exponents), corresponding analyses³⁸⁾ of the compact U(1)-Higgs theory find this line to be first-order. These analyses are flawed, however - if the gauge is not fixed, the line does not terminate in a critical point for positive β ; if the gauge is fixed, the Higgs-Coulomb transition disappears.

In conclusion, therefore, we have performed the first MCRG analysis of the four-dimensional compact Abelian Higgs model. Within the framework of a two-operator truncation scheme, the flow diagram, fixed points and critical exponents were identified. The general formalism for systematically improving the calculation was elucidated. Several important assumptions were made (hyperscaling and universality), and errors arising from truncation and volume effects were discussed. Further studies on this model should certainly be made. As the results obtained here have no obvious generalization to more complicated theories [for example, a new fixed point might appear if more scalar generations are included³⁵⁾], we are presently performing an analysis of the full $SU(2) \times U(1)$ standard model³⁾ minus fermions.

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FIGURE CAPTIONS

- Fig. 1a : Two-dimensional projection of the blocking scheme. Block boundaries are shown as dashed lines, block points as heavy dots; site points as dots.
- Fig. 1b : Schema of paths whose sum constitutes a block link. The ellipsis (...) refers to obvious symmetry transformations.
- Fig. 2 : Flow diagram of the four-dimensional compact Abelian Higgs model.
- Fig. 3 : Plot of β' versus β approximating the fixed point F. A solid line $\beta' = \beta$ is shown to indicate the fixed point β^* ; a dashed line shows the least-squares fit used to extract the critical exponent.
- Fig. 4 : Plot of κ' versus κ approximating the fixed point G. A solid line $\kappa' = \kappa$ is used to indicate the fixed point κ^* ; a dashed line shows the least-squares fit used to extract the critical exponent.

Fig. 1a

Fig. 1b

Fig. 2

Fig. 3

Fig. 4