COSMOLOGICAL BARYOGENESIS IN SUPERSTRING MODELS WITH STABLE PROTONS

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ABSTRACT

We discuss cosmological baryogenesis in phenomenological low-energy models inspired by the superstring which have an unobservably long baryon lifetime. The Affleck-Dine mechanism of baryogenesis in a cold ($\lesssim 10^6$ GeV) Universe is shown to be feasible, with a large baryon density being produced by the decays of large expectation values for squark and slepton fields after inflation. We catalogue the gauge-invariant quartic scalar operators in the low-energy effective action which could appear once supersymmetry is broken, show that the D-terms in the potential can vanish, and discuss the possibility that the F-terms have flat directions allowing large values for these scalar fields.

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Since the superstring\textsuperscript{1) is supposed to be the Theory of Everything, it should explain why the Universe is the way it is. This presumably means that superstring cosmology should provide an inflationary epoch\textsuperscript{2) and be able to generate a baryon asymmetry\textsuperscript{3). Since the details of the model of low-energy (\(E \ll m_p\)) four-dimensional physics emanating from the superstring are not yet clear, it may seem premature or foolhardy to tackle these questions. However, inflation and baryogenesis are significant challenges to low-energy models\textsuperscript{4) inspired by the superstring, which might only be met by a subset of models and could therefore guide us towards the correct one. It could even be that purely theoretical arguments alone do not uniquely favour any particular superstring-inspired low-energy model, and that ultimately the choice made in our part of the Universe can only be discovered by phenomenological and cosmological arguments, including successful inflation and baryogenesis.

It has been argued previously\textsuperscript{5) that inflation in superstring models should occur after the theory has compactified to four dimensions, but no elegant scenario for it has yet emerged. At first sight, the problem of baryogenesis also looks rather unpromising. Since it must occur after inflation, it can only occur at relatively low energies \(E \ll m_p\). The superpotential of the effective low-energy theory obtained from the superstring could \textit{a priori} contain combinations of renormalizable trilinear Yukawa terms leading to rapid proton decay\textsuperscript{4). The experimental longevity of the proton means that such combinations must be absent. Although the necessary zeroes in the Yukawa matrix could be due to discrete symmetries\textsuperscript{6)} or to topological properties\textsuperscript{7)} of the six-dimensional compactification space, no existence proof has yet been found. Assuming that it is indeed possible to avoid such catastrophic combinations of Yukawa couplings, the only interactions violating baryon number in perturbation theory are mediated by particles weighing \(O(m_p)\). In this case, proton decay becomes unobservably slow, and the decays of these heavy particles cannot contribute significantly to baryogenesis after an inflationary epoch\textsuperscript{4). So how did the baryon density of the observable Universe originate?

Affleck and Dine\textsuperscript{9)} have proposed a mechanism for baryogenesis which works at low energies in a cold \(\zeta 10^4\) GeV Universe. They notice that in many supersymmetric GUTs the effective scalar potential contains flat directions in the absence of supersymmetry breaking, which permit squark and slepton fields to have

\footnote{It has been suggested\textsuperscript{8)} that non-perturbative effects in electroweak theory might generate a useful baryon asymmetry, but we will not discuss this possibility here.}
large expectation values at the end of the inflationary epoch. The degeneracy of
the effective scalar potential along such directions is broken when supersymmetry
breaking is switched on. This enables the squark and slepton expectation values
to decay producing a large baryon asymmetry if C and CP are violated.

In this paper, we investigate the efficiency of this mechanism in low-energy
models inspired by the superstring. We catalogue the quartic, gauge-invariant
effective scalar operators which can be generated once supersymmetry is broken.
We show that the D-terms in the extended low-energy gauge group have flat direc-
tions allowing the squarks and sleptons in these quartic terms to have large
expectation values. We also argue that F-terms can have flat directions allowing
operator expectation values which can lead to a large baryon asymmetry at low
energies.

We start with a brief reminder of the Affleck-Dine scenario for baryo-
genesis. They start from the observation that supersymmetric GUTs can have flat
directions whose degeneracy is only broken by supersymmetry-breaking effects
of order $\tilde{m}^2$ where $\tilde{m}$ is a soft susy-breaking mass of order $m_w$ and $\phi$ is a
generic scalar field. It has often been assumed the Universe after inflation has
all $\langle 0|\phi|0\rangle = 0(\tilde{m})$ and thermalizes to a reheating temperature
$T_R = 0(V_A^{1/4})$, where $V_A$ is the total energy density deposited by inflatons into light particles at the
end of the inflationary epoch. However, Affleck and Dine argue that this
need not be the case. In fact, De Sitter fluctuations during inflation when
$x \ll H_I \sim V_I^{1/4}/m_p$, where $H_I$ is the Hubble constant (field energy) during
inflation, could drive $\langle 0|\phi^2|0\rangle$ as large as $H_I^2 m_I^2 \sim m_I^2 / m_p^2$, implying
$m_I \sim m_I$. Analyses of density perturbations allow $V_I \lesssim 10^{-10} m_p^4$ corresponding to
$m_I \lesssim 10^{-5} m_p$, so $\langle 0|\phi^2|0\rangle$ could easily be as large as $m_p^2 / m_p$
during inflation. The inflationary epoch can be considered finished when the Hubble constant
$H_A \sim V_A^{1/2}/m_p \sim \Gamma_I$, where $\Gamma_I \sim m_I^2 / m_p^2$ is the inflaton decay rate. Inflaton decays
will produce a cold Universe if $V_A \lesssim \tilde{m}^2 \langle 0|\phi^2|0\rangle$, i.e., if $m_I^2 / m_p^2 \sim \Gamma_I \sim H_A \sim V_A^{1/2} / m_p
\lesssim \tilde{m}$. This condition is satisfied when $m_I \lesssim 4 \times 10^{-6} m_p$, which is almost all the
range $m_I \lesssim 10^{-5} m_p$ allowed by general analyses of density perturbations.
Actually, most specific models of inflation have $m_I \sim 10^{-8} m_p$, so the cold
Universe condition is comfortably satisfied. The Universe would then remain
cold as long as the field interaction rates.
\[ \Gamma \phi \sim \left( \frac{\alpha^2}{\pi} \right) \tilde{m}^3 \phi^2 < H \sim \tilde{m}^2 \phi / m_p \]  

(1)

where \( H \) is the Hubble expansion rate after inflation which is initially \( H_A \sim V_A^{1/2} / m_p \). The scalar fields \( \phi \) then evolve according to the zero-temperature classical equation

\[ \ddot{\phi} + 3H \dot{\phi} + \tilde{m}^2 \phi = V'(\phi) \]  

(2)

where we have separated off the quadratic and higher order terms in the supersymmetry-breaking effective potential. When \( t \ll \tilde{m}^{-1} \), the large damping term in (2) keeps the fields \( \phi \) almost static, and they begin to oscillate only when \( t \gtrsim \tilde{m}^{-1} \). These classical oscillations conserve baryon number as long as their amplitude is much smaller than \( m_\chi \), and one can associate with them a baryon number per scalar particle of order

\[ R \sim \frac{\text{Im} V_0}{\tilde{m}^2 \phi_0^2} \]  

(3)

where the vacuum expectation values \( V_0 \equiv \langle 0 | V | 0 \rangle \), \( \phi_0 \equiv \langle 0 | \phi | 0 \rangle \) are those specified at the end of inflation, and we have focused attention on an effective potential operator \( V \) which has non-zero baryon and lepton number (e.g., qqq\bar{\ell}) and assumed CP to be violated, so that the imaginary part \( \text{Im} V_0 \) of its initial expectation value is non-vanishing. Eventually, Eq. (2) drives \( \phi \) sufficiently small that the inequality (1) is no longer satisfied, which occurs when

\[ \phi \sim \left( \frac{\alpha^2}{\pi} \right)^{1/2} \phi_0 \tilde{m}^2 \]  

(4)

At this stage the Universe thermalizes at a temperature \( T \sim (\tilde{m} \phi_0)^{1/4} \sim 10^{4} \text{ GeV} \) and the resulting baryon-to-entropy ratio is

\[ \frac{n_B}{n_\gamma} \sim \frac{R \tilde{m}^3 \phi_0}{T^3} \sim \left[ \left( \frac{\alpha^2}{\pi} \right)^{1/4} \frac{\phi_0}{\tilde{m}} \right] \sim \frac{\phi_0}{m_\chi} \]  

(5)

where we have assumed

\[ \text{Im} V_0 \sim \frac{\tilde{m}^2 \phi_0^2}{m_\chi^2} \]  

(6)

as in the examples to be discussed later. If \( \phi_0 / m_\chi \) is sufficiently large, \( n_B / n_\gamma \) (5) could be rather large.
Before exploring the implementation of this Affleck-Dine\textsuperscript{9)} scenario in the context of superstring models, we would like to emphasize the naturalness of the boundary conditions they assume. We consider the toy model

\[ V(x, y) = \tilde{m}^2(x^2 + y^2) + x^2 y^2 + y^4 \]  

(7)

which has a flat direction \((x, 0)\) whose degeneracy is broken by the \(\tilde{m}^2 x^2\) term. Normally one assumes that

\[ \chi_0 \sim y_0 \sim \tilde{m} \]  

(8)

after inflation, whereas Affleck and Dine propose

\[ \chi_0 \sim m_x, \quad y_0 \sim \tilde{m} \]  

(9)

Their proposition (9) clearly has a much larger phase space \(\Delta x_0 \Delta y_0 \sim \tilde{m}_x\) than the conventional assumption (8) for which \(\Delta x_0 \Delta y_0 \sim \tilde{m}^2\). Moreover, thermal and quantum effects in de Sitter space\textsuperscript{11)} tend to drive \(\langle 0 | \phi^2 | 0 \rangle = 0(m_x^2)\) as was previously discussed. Therefore, the Affleck-Dine initial conditions (9) are at least as natural as the conventional ones (8), and possibly even more natural.

To implement the rest of their scenario, one must demonstrate the existence of D- and F-flat directions whose degeneracy is broken by suitable quartic scalar operators with non-zero baryon and lepton numbers. Accordingly, we first catalogue all such \(\text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y\) invariant quartic scalar operators. We group them into classes which are of the form \((\phi^*)^p (f)^{4-p}\); \(p = 0, 1, 2\). The \(p = 0\) operators are conventional superpotential F-terms \(W\) projected onto their scalar components. Such terms are expected in the effective low-energy theory obtained from a supergravity GUT: in the case of minimal kinetic terms\textsuperscript{13)}

\[ V(\phi) \geq \exp \left| \phi^2 \right| \left( \phi^* W' + W \right)^2 \geq \tilde{m} \phi W' + \left( \text{heavily} \right) \]  

(10)

No quartic operators with \(p = 1\) are generated in supergravity GUTs with minimal kinetic terms, but they can be generated if the kinetic terms are non-trivial. Finally, operators with \(p = 2\) can be generated as a result of a mismatch between the conventional D-terms associated with massive gauge degrees of freedom \(V\) and exchanges of the massive scalars \(S\) associated with them in a massive \(N = 1\) supermultiplet. These two terms would cancel in a supersymmetric world, but supersymmetry breaking arranges \(\left| m_V^2 - m_S^2 \right| \sim \tilde{m}^2\) so that one is left with a residual
contribution
\[ V(\phi) \propto \frac{m_X^2}{m_C^2} (\bar{\phi} \phi)^2 \]  
(11)

Since these \( p = 2 \) quartic operators are the only ones which are guaranteed to appear in a supersymmetric GUT, we will concentrate on them in the rest of this paper.

As the original \( E_6 \) GUT symmetry in superstring models is broken by the Hosotani mechanism\(^{14}\) rather than the Higgs mechanism, one might wonder whether the low-energy effective theory necessarily includes relics of D-terms such as (11). We believe it does for the following reason. The Hosotani mechanism gives vector boson masses \( m_X \) of order \( m_C^2/m_p^3 \), where \( m_C \) is the compactification scale defined by the inverse of the radius of the curled-up dimensions. If \( m_C \) were much less than \( m_p \), then \( m_X \) would be \( \ll m_C \). In this case, the effective theory at energies between \( m_X \) and \( m_C \) would necessarily look like a gauge theory spontaneously broken by the Higgs mechanism. Moreover, the massive vector particles would necessary belong to massive \( N = 1 \) supermultiplets along with scalar particles. Therefore, the partial cancellation between D-terms and scalar exchanges which give (11) in conventional supersymmetric GUTs would also apply in this case. In practice, we do not expect that \( m_C \ll m_p \), but since (11) exists for all smaller values of \( m_C \), we expect it also to be present when \( m_C \sim m_p \) as we anticipate.

The Table shows our catalogue of quartic scalar operators with \( p = 0, 1 \) and 2. Each operator in the Table should be understood as representing all operators containing fields from different generations with the same gauge quantum numbers. The first two columns exhibit operators which appear in the minimal supersymmetric Standard Model, while the last two exhibit additional operators appearing in a Calabi-Yau compactification yielding generations with 27 matter fields each: \((Q, L, u^c, d^c, \lambda^c; H, \bar{H}; D, D^c; \nu^c; N)\). For these operators to be permitted in the low-energy limit of the superstring, they must be invariant under the residual subgroup of \( E_6 \) left unbroken by the Hosotani mechanism. We expect this to be of the form \( SU(3) \times SU(2)_L \times U(1)_Y \times U(1) \)', where the specification of the addition \( U(1)' \) hypercharges is model-dependent. There is a unique \( U(1)' \) which can be produced by the Hosotani breaking of \( E_6 \) directly to a rank-5 subgroup\(^{14,15}\), and this is the one favoured in some phenomenological analyses\(^{16}\). However, if one postulates Hosotani breaking to a rank-6 subgroup followed by Higgs breaking to a rank-5 subgroup, one can arrive at a different low energy \( U(1)' \). Integrating out the heavy degrees of freedom which acquire masses through this Higgs breaking of
rank 6 to rank 5 could also yield $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)'$ quartic operators. Shown alongside each of the operators involving just Standard Model fields in the Table is the algebraic relation between the $U(1)'$ hypercharges $Y_{10}$ of the $10$'s of conventional $SU(5)$ (i.e., $Q, u^c, e^c$) and $Y_5$ of the $5$'s of conventional $SU(5)$ (i.e., $d^c, L$) which is needed if the total $U(1)'$ charge of the quartic operator is to vanish. The algebraic relation between $Y_{10}$ and $Y_5$ which is needed to make the total $U(1)'$ hypercharge of the two $p = 0$ operators vanish is that provided by the extra $U(1)$ in $SO(10)/SU(3)_c \times SU(2)_L \times U(1)_Y$. However, Hosotani breaking$^{14,15}$ of $E_6$ does not suggest that this $U(1)'$ survives down to low energies. Nor does Hosotani breaking lead us to expect $Y_{10} = 3Y_5$ as is required for the $p = 1$ Standard Model operator to be $U(1)'$ invariant. However, the two $p = 2$ Standard Model operators are automatically invariant under any possible $U(1)'$. Therefore, they are candidates for Affleck-Dine operators in any superstring-inspired model.

Shown alongside the additional quartic operators involving superstring fields are analogous conditions for the vanishing of the total $U(1)'$ hypercharge of the operator. These are arrived at assuming that $U(1)'$ is contained in $E_6$ and hence is some linear combination of the $U(1)$ in $E_6/ SO(10)$ and that in $SO(10)/SU(3)_c \times SU(2)_L \times U(1)_Y$. This assumption fixes the $U(1)'$ hypercharges of the $H, \tilde{H}, D$ and $D^c$ fields in terms of $Y_5$ and $Y_{10}$. None of the $p = 0$ or $p = 1$ operators is invariant under the specifically Hosotani $U(1)'$: $2Y_5 + Y_{10} = 0$ which has some phenomenological advocates$^{16}$. However, five of the $p = 2$ operators are invariant under any $U(1)'$, and three more are invariant under this Hosotani $U(1)'$. Therefore, we have several more candidates for Affleck-Dine operators which involve the additional superstring fields.

To check if any of these $p = 2$ operators are indeed suitable for baryo-synthesis, we must check that they can have non-zero vacuum expectation values in directions where the conventional renormalizable $D$- and $F$-terms vanish. Affleck and Dine$^9$ already showed that when (all the subscripts are colour indices)

$$
\begin{align*}
\langle 0 | U_{3}^c | 0 \rangle &= \alpha, \\
\langle 0 | U_1 | 0 \rangle &= -\alpha, \\
\langle 0 | S_{e}^c | 0 \rangle &= \alpha
\end{align*}
$$

(12)

all the $D$- and $F$-terms of the Standard Model would vanish, while a $p = 2$ operator of the class $L QU^{c+k} d^c k$ has a non-zero vacuum expectation value. It is easy to
check that the combination (12) of vacuum expectation values also gives zero D-terms for an extra $U(1)'$ if

$$\gamma_{10} + 2\gamma_5 = 0$$

This is just the extra $U(1)'$ expected if the Calabi-Yau manifold has a non-Abelian discrete symmetry leading to a rank-5 group in four dimensions\(^6,15,16\).

We expect that other combinations of field values can be found which yield vanishing D-terms for $SU(3)_C \times SU(2)_L \times U(1)_Y$ and other $U(1)'$ subgroups of $E_6$, as well as non-vanishing expectation values for other operators in the Table, but we have not investigated this problem systematically. Since for any choice of $U(1)'$ there are 13 conditions for the vanishing of D-terms, and 81 field expectation values which can be chosen independently in attempts to satisfy them, we expect many solutions in general. We are content with (12) and (13) as an existence proof.

The expectation values (12) also give vanishing F-terms in the minimal Standard Model with just one pair of Higgs doublets $(H, \bar{H})^9)$. However, this conclusion would not hold in general for superstring models with three sets of Higgs doublets $(H, \bar{H})$ whose couplings to $Q$ and $Q^c$ fields are not simultaneously diagonalizable in generation space. Vanishing of the F-terms imposes non-trivial constraints on the Yukawa couplings of a superstring model, but there is no principle to prevent these constraints from being satisfied. Indeed, they are satisfied in at least one\(^15\) superstring-inspired model, if only for the simple reason that it contains only one pair of light Higgs doublets $(H, \bar{H})$.

We conclude that there is no obvious obstacle to implementing the Affleck-Dine scenario\(^9,10\) for baryogenesis in a superstring model. The quartic operators they require appear naturally in the theory, can be invariant under an extended low-energy gauge group $SU(3)_C \times SU(2)_L \times U(1)^2$, and can have non-zero expectation values while still having zero D-terms for each of the $SU(3)_C \times SU(2)_L \times U(1)^2$ gauge generators. The F-terms may also vanish if the Yukawa couplings obey certain constraints. These should perhaps be regarded as desirable criteria to be satisfied by a phenomenologically acceptable superstring-inspired model. Cosmological baryogenesis following inflation with little reheating in superstring models with a long-lived proton is by no means a lost cause!
ACKNOWLEDGEMENTS

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Allowed quartic scalar operators: \((\phi^*)^p \phi^4-p\)

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<th>Superstring Model</th>
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<td>operator</td>
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