CERN-TH.4487/86

A BOSONIC REPRESENTATION OF THE $N = 2$ SUPERCONFORMAL ALGEBRA

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ABSTRACT

Motivated by the recent discovery by Waterson of a bosonic representation for the $N = 2$ superconformal algebra with a specific value of the central charge $\hat{c} = 2/3$, we present here a general bosonic realization of this algebra for an arbitrary value of the central charge. The analysis is based on a technique of bosonized ghosts, along the lines of Friedan, Martinec and Shenker.

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One of the most intriguing aspects of supersymmetry in two space-time dimensions is the possibility of representing the full supersymmetry algebra solely in terms of either bosonic or fermionic fields, by means of bosonization¹⁾. Several examples of both Abelian and non-Abelian kinds are already known^{1),2)}.

In an interesting recent paper, it has been shown by Waterson³⁾ that in two dimensions the complete $N = 2$ superconformal algebra can be realized in terms of just one bosonic field for one specific value of the central charge, $\hat{c} = 2/3$. This is a truly surprising result in view of the fact that a naïve counting of degrees of freedom would imply that at least three bosonic fields should be involved in the general case: two from the original Bose fields, and one from the bosonized fermion.

Our aim with the present paper is to present in detail one particular bosonic representation of the full $N = 2$ superconformal algebra for a general value of the central charge. Thus, although our representation will not, for certain specific values of the central charge, be the most economical one, it has the virtue of being completely general.

The somewhat unusual starting point of our construction is the observation made by Friedan, Martinec and Shenker⁴⁾ that the $N = 1$ superconformal algebra in fact contains a hidden $N = 2$ algebra when expressed in terms of first quantized ghost fields. Using a generalized Sugawara construction for the super stress-energy tensor, this will lead us directly to a bosonic representation of the $N = 2$ superconformal algebra.

In order to introduce the ghost field structure, we start with the BRS charge of the Neveu-Schwarz model ($n, m \in \mathbb{Z}$, $\alpha \in \mathbb{Z} + \frac{1}{2}$),

$$Q_{NS} = \sum_{m=-\infty}^{\infty} L_{-m}^{\text{matter}} c_m + \sum_{\alpha=-\infty}^{\infty} G_{-\alpha}^{\text{matter}} e_{\alpha} - \frac{1}{2} \sum_{m,n=-\infty}^{\infty} (m-n) :c_{-m} c_{-n} \bar{c}_{m+n}: \\ - \sum_{n,\alpha=-\infty}^{\infty} (\frac{n}{2} - \alpha) :c_{-n} e_{-\alpha} \bar{e}_{n+\alpha}: - \sum_{\alpha,\beta=-\infty}^{\infty} :e_{-\alpha} e_{-\beta} \bar{c}_{\alpha+\beta}: - \frac{1}{2} c_0 \quad (1)$$

(A completely similar analysis can be carried through for the Ramond sector.)

Here c and $\bar{c}$ are anticommuting ghosts with the standard $\{c_m, \bar{c}_n\} = \delta_{m+n}$ algebra in normal modes, while e and $\bar{e}$ are commuting ghosts, $[e_\alpha, \bar{e}_\beta] = \delta_{\alpha+\beta}$. All commutators between commuting and anticommuting ghost fields vanish.

From Eq. (1) one obtains the BRS invariant Virasoro and Neveu-Schwarz generators [see, e.g., Ref. 5)],

$$\mathcal{L}_n = \{Q_{NS}, \bar{c}_n\}, \quad \mathcal{G}_\alpha = [Q_{NS}, \bar{e}_\alpha] \quad (2)$$

which one can split into matter and ghost field contributions: $\mathcal{L}_n = L_n^{\text{matter}} + L_n$, $\mathcal{G}_\alpha = G_\alpha^{\text{matter}} + G_\alpha$. In the following we will need only the ghost field contributions.

Explicitly, we have with $L_n = L_n^c + L_n^e$:

$$\begin{aligned} L_n^c &= \sum_{k=-\infty}^{\infty} [n-k] : \bar{c}_{n+k} c_{-k} : = n j_n^c + J_n^c \\ L_n^e &= \sum_{\alpha=-\infty}^{\infty} \left[\frac{n}{2} - \alpha \right] : e_{-\alpha} \bar{e}_{n+\alpha} : = -\frac{n}{2} j_n^e + J_n^e \end{aligned} \quad (3)$$

with the following commutation relations^{*)}

$$\begin{aligned} [j_n^e, j_m^e] &= -n \delta_{m+n} & [j_n^c, j_m^c] &= n \delta_{m+n} \\ [j_n^e, J_m^e] &= n j_{m+n}^e - \frac{n^2}{2} \delta_{m+n} & [j_n^c, J_m^c] &= n j_{m+n}^c - \frac{n^2}{2} \delta_{m+n} \\ [J_n^e, J_m^e] &= (n-m) J_{m+n}^e + \frac{n}{6} (n^2 + \frac{1}{2}) \delta_{m+n} \\ [J_n^c, J_m^c] &= (n-m) J_{m+n}^c - \frac{n}{6} (n^2 - 1) \delta_{m+n} \end{aligned} \quad (4)$$

^{*)} We choose to define our normal ordering for the zero modes of the ghosts c and $\bar{c}$ by $:\bar{c}_0 c_0: = \frac{1}{2}(\bar{c}_0 c_0 - c_0 \bar{c}_0)$.

Generalizing to the case of arbitrary conformal dimensions of the ghost fields, we introduce

$$L_n = -n(h'-1)j_n^e + J_n^e + n(h-1)j_n^c + J_n^c \quad (5)$$

corresponding to conformal weights^{4),5)}

$$\begin{aligned} w[e_\alpha] &= h' , & w[\bar{e}_\alpha] &= 1-h' \\ w[c_n] &= 1-h , & w[\bar{c}_n] &= h \end{aligned} \quad (6)$$

The standard BRS dimensions are recovered for $h = 2$, $h' = 3/2$.

It is straightforward to verify that the generator (5) indeed satisfies a standard Virasoro algebra with central element

$$A_n = (h'^2 - h' + \frac{1}{6}) + (-h^2 + h - \frac{1}{6}) \quad (7)$$

where so far h' and h are unrestricted.

We now apply the same general scheme to the Neveu-Schwarz operators. From (1) and (2) it follows that

$$G_\alpha = \alpha \sum_{n=-\infty}^{\infty} :c_{-n} \bar{e}_{n+\alpha}: - \frac{1}{2} \sum_{n=-\infty}^{\infty} n :c_{-n} \bar{e}_{n+\alpha}: - 2 \sum_{\beta=-\infty}^{\infty} :e_{-\beta} \bar{c}_{\alpha+\beta}: \quad (8)$$

which we write as

$$G_\alpha = \alpha V_\alpha^1 - \frac{1}{2} U_\alpha^1 - 2 V_\alpha^2 \quad (9)$$

in an obvious notation.

This G_α satisfies the standard Neveu-Schwarz algebra with $h = 2$ and $h' = 3/2$. On top of this, $V_\alpha^{1,2}$ and U_α^1 of Eq. (9) together with (4) give a set of closed (anti)commutation relations, a "super Kac-Moody algebra":

$$\begin{aligned}
[j_n^c, j_m^c] &= n \delta_{m+n} \\
[j_n^e, j_m^e] &= -m \delta_{m+n} \\
[V_\alpha^1, j_n^c] &= [V_\alpha^1, j_n^e] = V_{n+\alpha}^1 \\
[V_\alpha^2, j_n^c] &= [V_\alpha^2, j_n^e] = -V_{n+\alpha}^2 \\
\{V_\alpha^1, V_\beta^2\} &= -j_{\alpha+\beta}^c + j_{\alpha+\beta}^e + \alpha \delta_{\alpha+\beta}
\end{aligned} \tag{10}$$

Generalizing also G_α to the case of arbitrary conformal weight for the ghost fields, we find as a necessary requirement that

$$h' = h - 1/2 \tag{11}$$

and a solution

$$G_\alpha(h) = \alpha(h-1)V_\alpha^1 - \frac{1}{2}U_\alpha^1 - 2V_\alpha^2 \tag{12}$$

For this choice of G_α we have

$$[L_n(h), G_\alpha(h)] = \left(\frac{n}{2} - \alpha\right) G_\alpha(h) \tag{13}$$

$$\{G_\alpha(h), G_\beta(h)\} = 2L_{\alpha+\beta}(h) + \frac{\hat{c}}{2}(\alpha^2 - \frac{1}{4})\delta_{\alpha+\beta} \tag{14}$$

with $\hat{c} = 6-8h$. The same $\hat{c}$ -term appears in the Virasoro part of the algebra, once we impose (11).

The important observation now is that we can actually uncover an $N = 2$ superconformal algebra from the $N = 1$ expressions by splitting G_α into two pieces:

$$\begin{aligned}
G_\alpha^+(h) &= \alpha(h-1)V_\alpha^1 - \frac{1}{2}U_\alpha^1 \\
G_\alpha^-(h) &= -2V_\alpha^2 \\
G_\alpha^1(h) &= G_\alpha^+(h) + G_\alpha^-(h) = G_\alpha(h) \\
G_\alpha^2(h) &= G_\alpha^+(h) - G_\alpha^-(h)
\end{aligned} \tag{15}$$

One readily verifies that, for example,

$$\{G_\alpha^1, G_\beta^2\} = (\alpha - \beta)[-2(h-1)j_{\alpha+\beta}^c + (2h-1)j_{\alpha+\beta}^e] \quad (16)$$

i.e., one is led to identify a Kac-Moody generator

$$T_n(h) = -2(h-1)j_n^c + (2h-1)j_n^e \quad (17)$$

On account of (4) this operator in fact satisfies

$$[T_n(h), T_m(h)] = \frac{\hat{c}}{2} n \delta_{m+n} \quad (18)$$

with precisely the correct central element, $\hat{c} = 6-8h$. Similarly, by straightforward algebra, one verifies all (anti)commutation relations of the full $N = 2$ superconformal algebra:

$$[L_m(h), L_n(h)] = (m-n)L_{m+n}(h) + \frac{\hat{c}}{8}(n^3-n)\delta_{m+n}$$

$$[L_m(h), G_\alpha^\pm(h)] = \left(\frac{m}{2} - \alpha\right) G_{m+\alpha}^\pm(h)$$

$$[L_m(h), T_n(h)] = -n T_{m+n}(h)$$

$$\{G_\alpha^+(h), G_\beta^+(h)\} = \{G_\alpha^-(h), G_\beta^-(h)\} = 0 \quad (19)$$

$$\{G_\alpha^+(h), G_\beta^-(h)\} = L_{\alpha+\beta}(h) + \frac{1}{2}(\alpha-\beta)T_{\alpha+\beta}(h) + \frac{\hat{c}}{4}(\alpha^2 - \frac{1}{4})\delta_{\alpha+\beta}$$

$$[T_m(h), G_\alpha^\pm(h)] = \pm G_{m+\alpha}^\pm(h)$$

$$[T_m(h), T_n(h)] = \frac{\hat{c}}{2} m \delta_{m+n}$$

We are now ready to perform the bosonization^{4),6)}. First note that the ghost number current, which satisfies $[j_n^c, j_m^c] = n\delta_{m+n}$, can be viewed as an oscillator basis for a corresponding Bose degree of freedom. We are hence led, in a standard Sugawara construction of the stress-energy tensor, to consider the following Bose representation of the Virasoro generator:

$$L_n^c(Q) = \frac{1}{2} \left(\sum_{k=-\infty}^{\infty} :j_{-k}^c j_{k+n}^c: - Q n j_n^c \right) \quad (20)$$

where we have added a term linear in the current in order to be able to reproduce

the appropriate central element⁷⁾. This term, which gives an extra explicit contribution to the stress-energy tensor, can be thought of as the result of an addition of background charge Q to the system which modifies the boundary conditions for the Bose fields⁸⁾. Requiring that the bosonized Virasoro operator (20) completely specifies the anomalous parts of the commutators $[L_n^c(Q), L_m^c(Q)]$ and $[L_n^c(Q), j_m^c]$ gives the unique identification $Q = 1-2h$.

For the bosonic ghosts, a little more care is required if one wants to follow a Sugawara procedure, essentially because we are now "bosonizing bosons". The anomaly term can now no longer be matched by a simple Chodos-Thorn method as in Eq. (20), but requires an additional term⁴⁾

$$L_n^e(Q') = -\frac{1}{2} \left(\sum_{k=-\infty}^{\infty} :j_{-k}^e j_{k+n}^e: + Q' n j_n^e \right) + L_n^{(-2)} \quad (21)$$

with

$$L_n^{(-2)} = - \sum_{k=-\infty}^{\infty} k : \bar{d}_{n+k} d_{-k} : , \quad \{ \bar{d}_n, d_m \} = \delta_{m+n} \quad (22)$$

This corresponds to an additional fermion system of $h = 1$ which can always be bosonized in the standard way. The anomaly-matching now requires $Q' = (1-2h') = 2(h-1)$. Note that the total central charge of the $N = 2$ algebra satisfies $\hat{c} = 2(Q-Q')$.

Identifying (20), (21) with (5) we see that

$$J_n^c = \frac{1}{2} \sum_{k=-\infty}^{\infty} :j_{-k}^c j_{k+n}^c: + \frac{1}{2} n j_n^c \quad (23)$$

$$J_n^e = -\frac{1}{2} \sum_{k=-\infty}^{\infty} :j_{-k}^e j_{k+n}^e: + L_n^{(-2)} - \frac{1}{2} n j_n^e$$

and we have hence expressed the whole Virasoro sector in terms of fundamental bosonic currents. The next step now consists in identifying the corresponding bosonic fields of the Neveu-Schwarz sector. This amounts essentially to a Mandelstam representation of fermions in terms of vertex operators.

To carry out this bosonization programme in detail, we use the (anti)commutation relations

$$\begin{aligned}
[V_\alpha^1, J_n^c] &= -U_{\alpha+n}^1 & [V_\alpha^2, J_n^c] &= -nV_{\alpha+n}^2 - U_{\alpha+n}^2 \\
[V_\alpha^1, J_n^e] &= \alpha V_{\alpha+n}^1 + U_{\alpha+n}^1 & [V_\alpha^2, J_n^e] &= (n+\alpha)V_{\alpha+n}^2 + U_{\alpha+n}^2 \\
[U_\alpha^1, j_n^c] &= nV_{\alpha+n}^1 + U_{\alpha+n}^1 & [U_\alpha^1, j_n^e] &= U_{\alpha+n}^1 \\
\{U_\alpha^1, V_\beta^2\} &= J_{\alpha+\beta}^c + J_{\alpha+\beta}^e + \beta j_{\alpha+\beta}^e - \frac{1}{2}(\alpha^2 - \frac{1}{4})\delta_{\alpha+\beta}
\end{aligned} \tag{24}$$

to express $U_\alpha^{1,2}$ in terms of the "neutral" Bose currents j_n^e, j_n^c and the two "charged" currents⁹⁾ $V_\alpha^{1,2}$ with the help of the Sugawara expressions (23). In Eq. (24) we have defined

$$U_\alpha^2 = \sum_{n=-\infty}^{\infty} n : \bar{c}_{-n} e_{n+\alpha} : \tag{25}$$

which together with Eqs. (8) and (9) immediately leads to (24).

For example, in normal modes we have from $[V_\alpha^1, J_0^1] = -U_\alpha^1$:

$$U_\alpha^1 = -\sum_{k=1}^{\infty} V_{\alpha-k}^1 j_k^c - \sum_{k=1}^{\infty} j_{-k}^c V_{\alpha+k}^1 - \frac{1}{2} V_\alpha^1 j_0^c - \frac{1}{2} j_0^c V_\alpha^1 \tag{26}$$

or the equivalent expression in terms of the j_n^e current, which follows from $[V_\alpha^1, J_0^e] = \alpha V_\alpha^1 + U_\alpha^1$:

$$U_\alpha^1 = -\sum_{k=1}^{\infty} V_{\alpha-k}^1 j_k^e - \sum_{k=1}^{\infty} j_{-k}^e V_{\alpha+k}^1 - \frac{1}{2} V_\alpha^1 j_0^e - \frac{1}{2} j_0^e V_\alpha^1 - \alpha V_\alpha^1 + [V_\alpha^1, L_0^{(-2)}] \tag{27}$$

The fact that these two expressions are equal already tells us that it should be possible to express j_n^c, j_n^e, V_α^1 and the bosonized field from $L_n^{(-2)}$ in terms of just three underlying Bose fields, which we call $\phi^c(z, h)$, $\phi^e(z, h)$ and $\sigma(z)$. Due to the modified boundary conditions imposed by the background charge through h , the fields ϕ^c and ϕ^e will explicitly depend on h . It turns out that the auxiliary field σ is associated with the fermions of $L_n^{(-2)}$ and hence corresponds to a fixed value of h , $h = 1$.

Before giving the explicit Mandelstam form for $V_\alpha^{1,2}$, we note in passing that

since $G(z)$ is a primary conformal field of dimension $3/2$, it follows from Eq. (9) that also $w[V^2(z)] = 3/2$, while $w[V^1(z)] = \frac{1}{2}$. Their normal modes then follow from

$$V_\alpha^1 = (2\pi i)^{-1} \oint \frac{dz}{z} z^{\alpha + \frac{1}{2}} V^1(z) \quad (28)$$

$$V_\alpha^2 = (2\pi i)^{-1} \oint \frac{dz}{z} z^{\alpha + \frac{3}{2}} V^2(z) \quad (29)$$

and similarly for G_α . Bosonizing the ghosts^{4),6)}, we now find from Eq (8) the following vertex operator representation of these two charged currents:

$$V^1(z) = : e^{\phi^c(z,h) - \phi^e(z,h) + \sigma(z)} \partial_z \sigma(z) : \quad (30a)$$

$$V^2(z) = : e^{-\phi^c(z,h) + \phi^e(z,h) - \sigma(z)} : \quad (30b)$$

where

$$w[e^{\pm \phi^c(z,h)}] = \frac{1}{2}(1 \pm (1-2h)) \quad (31a)$$

$$w[e^{\pm \phi^e(z,h)}] = -\frac{1}{2}(1 \mp 2(1-h)) \quad (31a)$$

and $\sigma(z)$ is associated with anticommuting ghosts of $h = 1$, i.e., from Eq. (31a): $w[e^{\sigma(z)}] = 0$ and $w[e^{-\sigma(z)}] = 1$.

Miraculously, the particular combinations (30) precisely render $V^{1,2}(z)$ completely independent of h , and of the correct conformal weights $3/2$ and $\frac{1}{2}$. To check the independence of h , we note that only the zero modes of $\phi^c(z,h)$, $\phi^e(z,h)$ and $\sigma(z)$ are affected by a background charge. The extra terms read $\Delta\phi^c(z,h) = \phi^c(z,h) - \phi^c(z,0) = -\frac{1}{2}(1-2h)\ln z$, $\Delta\phi^e(z,h) = (1-h)\ln z$ and $\Delta\sigma(z) = \frac{1}{2}\ln z$, i.e., in total extra terms of $\pm \ln z$, independent of h .

We now finally construct the full Neveu-Schwarz operators $G^\pm(z)$. There are three independent contributions [see, e.g., Eq. (15)]. One is $V^2(z)$, whose bosonic representation has already been given above. Next consider the term which in a mode expansion reads αV_α^1 . Since $G^\pm(z)$ are of conformal weight $3/2$, we find

$$\begin{aligned} \sum_{\alpha=-\infty}^{\infty} z^{-\alpha-\frac{3}{2}} \alpha V_{\alpha}^1 &= [-z \partial_z - \frac{3}{2}] z^{-1} \sum_{\alpha=-\infty}^{\infty} z^{-\alpha-\frac{1}{2}} V_{\alpha}^1 \\ &= -\frac{1}{2} V^1(z)/z - \partial_z V^1(z) \end{aligned} \quad (32)$$

The term proportional to $V^1(z)/z$ corresponds to modified boundary conditions for the Bose fields, and can conveniently be represented by Eq. (20a) with $\phi^c(z, h) \rightarrow \phi^c(z, h-1)$. The only remaining term for $G^{\pm}(z)$ is now $-\frac{1}{2} U_{\alpha}^1$. Using a similar procedure as above we find, from the definition (8):

$$\begin{aligned} -\frac{1}{2} \sum_{\alpha=-\infty}^{\infty} z^{-\alpha-\frac{3}{2}} U_{\alpha}^1 &= -\frac{1}{2} \sum_{n=-\infty}^{\infty} \sum_{\alpha=-\infty}^{\infty} n : c_{-n} \bar{e}_{n+\alpha} : z^{-\alpha-\frac{3}{2}} \\ &= -\frac{1}{2} \sum_{n=-\infty}^{\infty} n z^n : c_{-n} V^1(z) : \\ &= -\frac{1}{2} \sum_{n=-\infty}^{\infty} [z \partial_z z^n] : c_{-n} V^1(z) : \\ &= +\frac{1}{2} V^1(z)/z + : [\partial_z \phi^c(z, h)] V^1(z) : \end{aligned} \quad (33)$$

so that in total we have

$$G^-(z) = -2 : e^{-\phi^c(z, h) + \phi^e(z, h) - \sigma(z)} : \quad (34a)$$

$$\begin{aligned}
 G^+(\bar{z}) = & (1 - \frac{1}{2}h) : e^{\phi^c(z,h-1) - \phi^e(z,h) + \sigma(z)} \partial_{\bar{z}} \sigma(z) : \\
 & + (1-h) : \partial_{\bar{z}} \{ e^{\phi^c(z,h) - \phi^e(z,h) + \sigma(z)} \partial_{\bar{z}} \sigma(z) \} : \\
 & - \frac{1}{2} : \partial_{\bar{z}} \phi^c(z,h) e^{\phi^c(z,h) - \phi^e(z,h) + \sigma(z)} \partial_{\bar{z}} \sigma(z) :
 \end{aligned} \tag{34b}$$

The $N = 2$ Neveu-Schwarz generators have thus been expressed in terms of three free Bose fields. One of these fields, $\phi^e(z,h)$, has negative metric.

With this, the full $N = 2$ superconformal algebra has been put in a purely bosonic representation. The Virasoro generators have already been presented in their Sugawara forms, or equivalently, in terms of derivatives of free Bose fields $\partial_z \phi^c(z,h)$ and $\partial_z \phi^e(z,h)$. From Eq. (17), we see that the same holds for the Kac-Moody generators $T_n(h)$.

We have so far not touched upon the conjugation properties of the original ghost-antighost system, where by a standard conjugation we understand $c^\dagger = c_{-n}$, etc. Under this conjugation the currents transform as

$$\begin{aligned}
 j_n^c{}^\dagger &= -j_{-n}^c, & j_n^e{}^\dagger &= -j_{-n}^e \\
 V_\alpha^1{}^\dagger &= -V_{-\alpha}^1, & V_\alpha^2{}^\dagger &= +V_{-\alpha}^2
 \end{aligned} \tag{35}$$

while both J 's transform as $J_n^\dagger = J_{-n}$. This conjugation obviously respects ghost number, i.e., eigenvalues of j_0^c and j_0^e . Another interesting method of conjugation (which we will denote by a tilde) has been suggested by Gervais¹⁰⁾. Here ghosts are interchanged with antighosts according to $\tilde{c}_n = \bar{c}_{-n}$, etc., i.e., it changes ghost number from 1 to -1, or vice versa. It is amusing to first perform this conjugation and then follow it with the usual one. Under these combined operations we find

$$\begin{aligned}
 j_n^c &\rightarrow -j_n^c, & j_n^e &\rightarrow -j_n^e, & V_\alpha^1 &\rightarrow V_\alpha^2, & U_\alpha^1 &\rightarrow U_\alpha^2 \\
 J_n^c &\rightarrow J_n^c - n j_n^c, & J_n^e &\rightarrow J_n^e + n j_n^e, & V_\alpha^2 &\rightarrow -V_\alpha^1, & U_\alpha^2 &\rightarrow -U_\alpha^1
 \end{aligned} \tag{36}$$

These operations do not change the algebra of the currents, and also the

full $N = 2$ superconformal algebra remains invariant. Since $L_m^c(h) \rightarrow m(-h) \frac{c}{jm} + J_m^c$ [and similarly for $L_m^e(h)$], it corresponds to $h \rightarrow 1-h$. This immediately shows that the central charge of the Virasoro algebra [see Eq. (7)] is invariant. One simple way to understand this result is to observe that $h \rightarrow h-1$ corresponds to $Q \rightarrow -Q$. Comparing this with the ghost current anomaly⁴⁾ (No. zero modes of $\bar{c}$) - (No. zero modes of c) = const. $\times Q$, we see that this is exactly as required. In Bose language, it takes a particularly simple form since, e.g., $c = :\exp(\phi^c(z,h)):$ $\rightarrow \bar{c} = :\exp(\phi^c(z,1-h)):$, and similarly for the other fields.

Let us finally point out that the purely bosonic representation presented here can be used to give an independent derivation of the highest weight eigenvalues for given central elements $\hat{c}$ of the $N = 2$ superconformal algebra^{11),12)}. The highest weight vectors are eigenstates of L_0 [see, e.g., Ref. 13)],

$$L_0|h\rangle = h|h\rangle \quad (37)$$

and $L_n|h\rangle = 0$, $n \neq 0$. The eigenvalues then essentially follow from the intercept:

$$\langle h | \{ \frac{1}{2}(j_0^c)^2 - \frac{1}{2}(j_0^e)^2 + \frac{1}{2}(j_0^\sigma)^2 + \frac{1}{8}(1 + \frac{1}{2}\hat{c}) \} | h \rangle$$

[our $\hat{c} = 2c_2$ (or $2\tilde{c}$) in the notations of Refs. 11) and 12)]. However, since the $\phi^e(z,h)$ field comes with negative metric, the representation will only be unitary when restricted to a positive definite Fock space in terms of the free Bose oscillators. This restriction, together with the requirement that $G^\pm(z)$ remain analytic³⁾, should uniquely determine the $\hat{c}$, h values allowed by unitarity. We suspect that this technicality can be avoided by expressing the general $N = 2$ algebra in terms of just two (positive-metric) free Bose fields, through the addition of twists.

ACKNOWLEDGEMENTS

H.A. would like to thank M. Jacob for hospitality at the Theory Division of CERN, and acknowledges many stimulating discussions with A.H. Zimmerman on related subjects. P.H.D. wishes to thank Y. Frishman and the theory group at the Weizmann Institute for the kind hospitality extended to him while this work was in its early stage.

NOTE ADDED

After the completion of this work, we become aware of a recent paper by P. Di Vecchia, J.L. Petersen, M. Yu and H.B. Zheng, Phys. Lett. 174B, 280 (1986), in which the $N = 2$ superconformal algebra has been expressed in terms of free fermions, thereby giving an elegant rederivation of the highest weights and central charges for unitary representation of the algebra^{11),12)}.

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