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STRING AND SUPERSTRING JACOBIANS*

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QUARTIC GHOST FIELD THEORIES IN TWO DIMENSIONS RELATED TO THE STRING AND SUPERSTRING JACOBIANS

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ABSTRACT

The functional measure for the string and superstring contains Jacobians which can be represented by integrals over ghost and antighosts with a bilinear action. The action is invariant under diffeomorphisms and Weyl transformations for the bosonic string and under local supersymmetry and super-Weyl transformations for the superstring. We extend these actions to include quartic ghost couplings while maintaining previous invariances. In the bosonic case, the new field theory is related to the massless Thirring model.

Recently,¹ we have considered forms of the string and superstring Jacobians for a general choice of gauge slice $g_{\mu\nu}(x)$ for the string and $e_{a\mu}(x)$, $\psi_\mu(x)$, $A(x)$ for the NSR superstring. The string Jacobian is represented as an integral over ghost $c_\rho(x)$ and traceless symmetric antighost $b^{\mu\nu}(x)$ fields with action

$$S = -i \int d^2x \sqrt{-g} b^{\mu\nu} D_\mu c_\nu \quad (1)$$

where D_μ is the covariant derivative for the world sheet metric $g_{\mu\nu}(x)$. The action (1) is obviously invariant under diffeomorphisms of $g_{\mu\nu}(x)$ combined with standard tensor transformation rules of $b^{\mu\nu}$ and c_ρ , and it is invariant under the Weyl transformations $\delta g_{\mu\nu} = 2\sigma g_{\mu\nu}$, $\delta c_\rho = 2\sigma c_\rho$, $\delta b^{\mu\nu} = -4\sigma b^{\mu\nu}$. For the superstring, a fermionic ghost $\lambda(x)$ and γ -traceless antighost $\beta^\mu(x)$ are added and the resulting representation of the Jacobian contains the action

$$S = \int d^2x \left\{ -i b^{\mu\nu} D_\mu c_\nu - i \bar{\beta}^\mu D_\mu \lambda + \bar{\psi}_\mu \left[\frac{3}{2} \beta^\nu D_\nu c^\mu - \frac{3}{2} \beta^\mu D_\nu c^\nu + D_\nu \beta^\mu c^\nu \right. \right. \\ \left. \left. - D_\nu \beta^\mu c^\nu - \frac{1}{2} b^{\mu\nu} \gamma_\nu \lambda \right] - (\bar{\psi}_\mu \gamma^\mu \psi) \left[\frac{3}{4} (\bar{\beta}^\mu \lambda) + \frac{1}{2} b^{\mu\nu} c_\nu \right] \right\} \quad (2)$$

It is invariant under supersymmetry and super-Weyl transformations given below.

In this note we study the addition of quartic ghost couplings to the actions (1) and (2) in such a way that all invariances are maintained. Although immediate contact with string theory is thus lost, we do find new conformal invariant field theories which may be of interest in themselves. In particular, the quartic ghost extension of (1) seems to

be closely related to the Thirring model,² and it is reasonable to believe that it should also be solvable³⁻⁵ and an example of a conformal field theory^{6,7} with anomalous dimensions.⁸

The general actions (1-2) are useful because the conserved currents, such as the stress tensor and supercurrent, can be computed¹ by direct variation which is more straightforward than conformal methods.^{6,7} The global ghost number and dual ghost number symmetries of (1) and (2) are of present interest and are given by the following variations with infinitesimal real parameters g and *g , respectively

$$\delta c_\mu = g c_\mu, \quad \delta b^{\mu\nu} = -g b^{\mu\nu}, \quad \delta \lambda = g \lambda, \quad \delta \beta^\mu = -g \beta^\mu \quad (3)$$

$$\delta c_\mu = ^*g \epsilon_\mu^\rho c_\rho, \quad \delta b^{\mu\nu} = ^*g \epsilon_\rho^\mu b^{\rho\nu}, \quad \delta \lambda = -^*g \gamma_5 \lambda, \quad \delta \beta^\mu = -^*g \gamma_5 \beta^\mu. \quad (4)$$

Note that $\epsilon_\rho^\mu b^{\rho\nu} = \epsilon^{\nu\rho} b_\rho^\mu$ and $\gamma_5 \beta^\mu = -\epsilon^\mu_\nu \beta^\nu$ because of the traceless properties. The symmetries (3-4) lead to conserved currents

$$G^\mu = b^{\mu\nu} c_\nu + \bar{\beta}^\mu \lambda + \frac{i}{2} \bar{\psi}_\nu (\beta^\mu c^\nu - \beta^\nu c^\mu) \quad (5)$$

$$^*G^\mu = \epsilon^\mu_\rho \bar{G}^\rho$$

$$\bar{G}^\mu = \left(b^{\mu\nu} c_\nu + \bar{\beta}^\mu \lambda + \frac{i}{2} \bar{\psi}^\mu \beta^\nu c_\nu \right) \quad (6)$$

Because of anti-commutativity, there is a unique quartic coupling for the bosonic ghost system. However, it is suggestive to write the extended action of (1) in terms of the ghost number current $J^\mu = b^{\mu\nu} c_\nu$ where it takes the form

$$S = \int d^2x \sqrt{-g} \left\{ -i b^{\mu\nu} D_\mu c_\nu + \frac{1}{2} g J^\mu J_\mu \right\} \quad (7)$$

To see that this is a close relative of the Thirring model, let us write the action on a flat world sheet in light-cone coordinates $x^\pm = \frac{1}{\sqrt{2}}(\tau \pm \sigma)$ where we obtain

$$S = \int dx^+ dx^- \left\{ -i b^{++} \partial_+ c_+ - i b^{--} \partial_- c_- + g J^+ J^- \right\} \quad (8)$$

with $J^\pm = b^{\pm\pm} c_\pm$. Let us compare this with the Thirring model action which involves a complex fermion field $\psi(x)$ and vector current $J^\mu = \bar{\psi} \gamma^\mu \psi$. We can write

$$\begin{aligned} S &= \int d^2x \left\{ i \bar{\psi} \not{\partial} \psi - \frac{1}{2} g J^\mu J_\mu \right\} \\ &= \int dx^+ dx^- \left\{ \sqrt{2} i (\psi^{*+} \partial_+ \psi^+ + \psi^{*-} \partial_- \psi^-) - g J^+ J^- \right\} \end{aligned} \quad (9)$$

where $\psi^\pm$ and $\psi^{*\pm}$ denote chirality components of ψ and its complex conjugate and $J^\pm = \sqrt{2} \psi^{*\pm} \psi^\pm$. Comparison of (8) and (9) suggests the rough identifications $c_\pm \sim \psi^\pm$ and $b^{\pm\pm} \sim \psi^{*\pm}$ which then give $J^\pm \sim J^\pm$. There are also differences between the

two models, namely different reality and Lorentz transformation properties, different anomalies, and different Hilbert space metric. Yet, the fact that the operator equations of motion are essentially identical suggests that both models should be solvable, and this question is now under study.

We now turn our attention to the supersymmetric generalization of (7). For this purpose we will need the local supersymmetry transformation rules under which (2) is invariant,¹ namely

$$\begin{aligned} \delta_Q e_\mu^a &= -i \bar{\epsilon} \gamma^a \psi_\mu \\ \delta_Q \psi_\mu &= 2 \left[\hat{D}_\mu \epsilon + \frac{1}{2} i \gamma_\mu A \epsilon \right] \end{aligned} \quad (9)$$

$$\delta_Q A = \bar{\epsilon} \left[\frac{1}{2} i A \gamma \cdot \psi - \gamma_5 \frac{e^{\rho\lambda}}{e} \hat{D}_\rho \psi_\lambda \right]$$

$$\delta_Q c_a = \bar{\epsilon} \gamma_a \lambda$$

$$\delta_Q \lambda = \frac{i}{2} e_\mu^a [D_\mu c^a - \gamma_5 \epsilon^{ab} D_\mu c_b] \epsilon$$

$$\begin{aligned} & - \frac{i}{4} [(\psi \cdot \gamma \epsilon) \lambda + (\bar{\psi} \cdot \gamma \gamma_5 \epsilon) \gamma_5 \lambda] - \gamma \cdot c A \epsilon \\ & + \frac{1}{2} (\bar{\psi} \cdot \gamma \psi_a) \left(c^a - \frac{1}{2} \gamma^a \gamma \cdot c \right) \epsilon \end{aligned} \quad (10)$$

$$\begin{aligned} \delta_Q b^{ab} &= \frac{i}{2} \bar{\epsilon} \left[D^a \beta^b + D^b \beta^a - \eta^{ab} D \cdot \beta \right. \\ & + \frac{1}{2} \gamma_c (\psi^a b^{bc} + \psi^b b^{ac} - \eta^{ab} \psi_d b^{cd}) + 3 i \gamma^a \beta^b A \\ & \left. + \frac{3i}{4} (\bar{\psi} \cdot \gamma \psi_\mu) (c^{a\mu} \beta^b + c^{b\mu} \beta^a - \eta^{ab} \beta^\mu) \right] \end{aligned}$$

$$\delta_Q \beta^a = b^{ab} \gamma_b \epsilon$$

where $\hat{D}_\mu$ is the spinor covariant derivative with torsion:

$$\begin{aligned} \hat{D}_\mu &= \partial_\mu + \frac{1}{2} \hat{\omega}_\mu \gamma_5 \\ \hat{\omega}_\mu &= \omega_\mu(\epsilon) - \frac{i}{2} \frac{e^{\mu\nu}}{e} (\bar{\psi}_\nu \gamma \cdot \psi) \end{aligned} \quad (11)$$

$$\omega_\mu(\epsilon) = -e^{-1} e_\mu^a e^{\rho\lambda} \partial_\rho e_{\lambda a}$$

There is also super-Weyl invariance,¹ with transformation rules

$$\begin{aligned} \delta_\sigma e_\mu^a &= \sigma e_\mu^a, & \delta_\sigma \psi_\mu &= \frac{1}{2} \sigma \psi_\mu, & \delta_\sigma A &= -\sigma A \\ \delta_\sigma e_\mu^a &= 0, & \delta_\sigma \psi_\mu &= i \gamma_\mu \eta, & \delta_\sigma A &= \frac{i}{2} \bar{\eta} \gamma \cdot \psi \\ \delta_u e_\mu^a &= 0, & \delta_u \psi_\mu &= 0, & \delta_u A &= u \end{aligned} \quad (12)$$

$$\begin{aligned}
\delta_c c^a &= \sigma c^a, & \delta_c \lambda &= \frac{1}{2} \sigma \lambda, & \delta_c b^{ab} &= -2\sigma b^{ab}, & \delta_c \beta^a &= -\frac{3}{2} \sigma \beta^a \\
\delta_s c^a &= 0, & \delta_s \lambda &= -\gamma \cdot c \eta, & \delta_s b^{ab} &= -\frac{3}{2} (\bar{\eta} \gamma^a \beta^b), & \delta_s \beta^a &= 0 \\
\delta_u (c_a, b_a, \lambda, \beta_\mu) &= 0
\end{aligned} \tag{13}$$

We begin the process of developing a supersymmetric extension of (7) by restricting (2) to a super-flat background $e_\mu^a = \delta_\mu^a$, $\psi_\mu = 0 = A$, and adding a quadratic current interaction, so that we have

$$S = \int d^2x \left\{ -i \bar{b}^{ab} \partial_a c_b - i \bar{\beta}^a \partial_a \lambda + \frac{1}{2} q \dot{G}^a \dot{G}_a \right\} \tag{14}$$

with $\dot{G}^a = b^{ab} c_b + \bar{\beta}^a \lambda$. We require that this action be invariant under the superflat restriction of (10) with constant ϵ , plus possible additions to the transformation rules (10) proportional to q . One key to a successful construction is to realize that, because of simple identities, the variation of the current can be written in a 1-parameter family of ways, namely

$$\begin{aligned}
\delta \dot{G}_a &= \frac{i}{2} \bar{\epsilon} \left\{ (1-y) [\partial_a (\beta \cdot c) + \partial^b (\beta_a c_b - \beta_b c_a)] \right. \\
&\quad \left. + y [2\partial_a (\beta \cdot c) - (\partial_a c_b + \partial_b c_a - \eta_{ab} \partial \cdot c) \beta^b - (\eta_{ab} + \gamma_5 \epsilon_{ab}) \partial \cdot \beta c^b] \right\}
\end{aligned} \tag{15}$$

In the product $q \dot{G}^a \delta \dot{G}_a$ the terms proportional to $(1-y)$ can be integrated back to act on the current which satisfies $\partial_a \dot{G}^a = 0$ and $\epsilon^{ab} \partial_a \dot{G}_b = 0$ on shell as a consequence of the symmetries (3-4). This means that $\partial_a \dot{G}^a$ and $\epsilon^{ab} \partial_a \dot{G}_b$ are proportional to field equations and we can define order q additions to (10) to cancel the order $q(1-y)$ terms in $q \dot{G}^a \delta \dot{G}_a$. In the terms proportional to y in (15), the term $2\partial_a (\beta \cdot c)$ is integrated back as above, while the remaining terms contain the field equation operators for c_a and β^a . The contribution of these terms to $q \dot{G}^a \delta \dot{G}_a$ can then be cancelled by new order q terms in $\delta b^{\mu\nu}$ and $\delta \lambda$. The order q modifications of the transformation rules (10) turn out to have a very simple interpretation in terms of the invariances (3-4). The modifications consist of a sum of three terms: 1) a ghost number gauge transformation (3) with g replaced by the field dependent gauge parameter $\frac{1}{2} q(1+y) \bar{\epsilon} \beta \cdot c$; 2) a dual ghost number gauge transformation (4) with parameter $-\frac{1}{2} q(1-y) \bar{\epsilon} \gamma_5 \beta \cdot c$; 3) plus

$$\begin{aligned}
\Delta b^{ab} &= -\frac{1}{2} q y \bar{\epsilon} (\dot{G}^a \beta^b + \dot{G}^b \beta^a - \eta^{ab} \dot{G} \cdot \beta) \\
\Delta \lambda &= \frac{1}{2} q y (\dot{G} \cdot c - \gamma_5 \epsilon^{ab} \dot{G}_a c_b) \epsilon \\
\Delta c_a &= \Delta \beta^a = 0
\end{aligned} \tag{16}$$

which can be interpreted as a ghost number covariantization of the derivatives in (10), namely $\partial_a c_b \rightarrow \partial_a c_b - i q y \dot{G}_a c_b$ and $\partial_a \beta_b \rightarrow \partial_a \beta_b + i q y \dot{G}_a c_b$. These order q variations must in turn be used in the $\frac{1}{2} q \dot{G}^a \dot{G}_a$ terms of (14) where they induce order q^2

terms in $\delta S = q \dot{G}^a \Delta \dot{G}_a$. We find quite simply that the order q contribution to $\Delta \dot{G}_a$ vanishes because the current is itself gauge invariant under the transformations 1) and 2) above and the variation due to (16) vanishes after a very short calculation. Thus, we have found that (14) is globally supersymmetric with a 1-parameter family of ghost transformation rules.

We now show how these results may be extended to a locally supersymmetric and super-Weyl invariant field theory of the ghost fields. Eq. (15) suggests that we consider the quantity

$$K_a^{(y)} = b_{ab} c^b + \bar{\beta}_a \lambda + \frac{1-y}{4} i \bar{\psi}^a (\beta_a c_b - \beta_b c_a) + \frac{1+y}{4} i \bar{\psi}^a \beta \cdot c \tag{17}$$

such that $K_a^{(+1)} = \dot{G}_a$ and $K_a^{(-1)} = G_a$ as in (5-6). The $\delta \bar{\psi}_\nu = 2\bar{\epsilon} \cdot \bar{D}_\nu$ variations of $K^{(y)}$ now combine with the covariant $\bar{\epsilon} D(\beta c)$ terms of (15) to give total derivatives $D(\epsilon \beta c)$. In fact, the full variation $\delta K_a^{(y)}$, computed using (9-10) to all orders in ψ_μ , can be written quite simply in terms of the full field equation operator for c_a and β^b obtained from (2) and the full currents G^a and $\tilde{G}^a$ of (5-6). This variation is

$$\begin{aligned}
\delta K_a^{(y)} &= \frac{i}{2} \left[(1+y) D_a (\bar{\epsilon} \beta \cdot c) - (1-y) \epsilon_{ab} D^b (\bar{\epsilon} \gamma_5 \beta \cdot c) \right] \\
&\quad - \frac{i}{2} y \left[\bar{\epsilon} \beta^b \mathcal{L}_{ab}^{(c)} + \bar{\epsilon} (\eta_{ab} + \gamma_5 \epsilon_{ab}) \mathcal{L}^{(\beta)} c^b \right] \\
&\quad + \frac{i}{2} \left[(1+y) (\bar{\epsilon} \gamma_5 \psi_a) \tilde{G}^b - (1-y) ((\bar{\epsilon} \gamma_a \psi_b) - \eta_{ab} (\bar{\epsilon} \gamma \cdot \psi)) G^b \right]
\end{aligned} \tag{18}$$

where the field equation operators are

$$\begin{aligned}
\mathcal{L}_{\mu\nu}^{(c)} &= D_\mu c_\nu + D_\nu c_\mu - g_{\mu\nu} D \cdot c - \frac{1}{4} (\bar{\psi}_\mu \gamma_\nu + \bar{\psi}_\nu \gamma_\mu - g_{\mu\nu} \bar{\psi} \cdot \gamma) \lambda \\
&\quad + \frac{i}{4} \bar{\psi} \cdot \gamma (\psi_\mu c_\nu + \psi_\nu c_\mu - g_{\mu\nu} c \cdot \psi) \\
\mathcal{L}^{(\beta)} &= D_\mu \beta^\mu + \frac{1}{2} \gamma_\mu \psi_\nu b^{\mu\nu} - \frac{3}{4} \bar{\psi} \cdot \gamma \psi_\mu \beta^\mu
\end{aligned} \tag{19}$$

Furious Fierz rearrangements and identities such as

$$b^{ac} c^d - b^{cd} c^a = \eta^{cd} b^{ab} c_b - \eta^{ac} b^{db} c_b \tag{20}$$

play a role in deriving (18).

It is now fairly clear that the quartic ghost coupling in the locally supersymmetric theory must be chosen so that the term $D_a (\bar{\epsilon} \beta \cdot c)$ of (18) can be integrated back onto the current G^a , while the $\epsilon_{ab} D^b \bar{\epsilon} \gamma_5 \beta \cdot c$ can be integrated back on $\tilde{G}^a$. This is arranged by taking

$$S_q = \frac{1}{2} q \int d^2x \epsilon G^a \tilde{G}_a \tag{21}$$

The total variation of S_q can be computed with the help of (18), which gives

$$\delta S_q = \frac{-i}{2} q \int d^2x \, e \left[(\bar{\epsilon}\beta \cdot c) D_a G^a + \bar{\epsilon}\gamma_5 \beta \cdot c \, \epsilon^a b D_a \tilde{G}_b + \frac{1}{2} \left[\bar{\epsilon}\beta^b \mathcal{L}_{ab}^{(c)} + \bar{\epsilon}(\eta_{ab} + \epsilon_{ab}\gamma_5) \mathcal{L}^{(b)} c \right] (G^a - \tilde{G}^a) \right] \quad (22)$$

Note that the $\bar{\epsilon}\gamma\psi G$ and $\bar{\epsilon}\gamma\psi \tilde{G}$ terms of (18) cancel partially with each other and partly with the variation $\delta\epsilon = -i\bar{\epsilon}\bar{c}\gamma \cdot \psi$ of (21). The $D_a G^a$ and $\epsilon^{ab} D_a \tilde{G}_b$ terms in (22) involve the full field equation operators for all four ghost fields. Thus, these terms are cancelled by the same field-dependent gauge transformations described above (16), with $y = 1$ (resp. $y = -1$) in the ghost number (resp. dual ghost number) parameters. The remaining terms in (22) involving $\mathcal{L}_{ab}^{(c)}$ and $\mathcal{L}^{(b)}$ are cancelled by additional terms $\Delta\delta^{ab}$ and $\Delta\lambda$ similar to (16) but with $y\tilde{G}_a$ replaced by $\frac{1}{2}(G_a - \tilde{G}_a)$ which appears because δK_a effectively contributes to (22) as $\delta K_a^{(+)} + \delta K_a^{(-)}$. We see that the parametric freedom of the global theory is now fixed. It is easy to see that the $O(q^2)$ variations cancel as in the global theory (14), and invariance under super-Weyl transformations (12-13) can also be verified without difficulty.

Now that the theory is known, it can be reformulated using auxiliary gauge fields A_μ and B_μ for ghost number and dual ghost number transformations. In this form there is a much simpler proof of the local supersymmetry properties. We introduce the potentials A_μ and B_μ , with gauge transformation laws $\delta A_\mu(x) = -\partial_\mu g(x)$ and $\delta B_\mu = -\partial_\mu^* g(x)$, in the covariant derivatives of ghost fields, i.e. we define

$$\begin{aligned} \nabla_\mu c_\nu &= D_\mu c_\nu + A_\mu c_\nu + B_\mu \epsilon_\nu{}^\rho c_\rho \\ \nabla_\mu \lambda &= D_\mu \lambda + A_\mu \lambda - B_\mu \gamma_5 \lambda \\ \nabla_\mu b^{\rho\sigma} &= D_\mu b^{\rho\sigma} - A_\mu b^{\rho\sigma} + B_\mu \epsilon^\rho{}_\nu b^{\nu\sigma} \\ \nabla_\mu \beta_\nu &= D_\mu \beta_\nu - A_\mu \beta_\nu - B_\mu \gamma_5 \beta_\nu \end{aligned} \quad (23)$$

We now consider the action

$$S_T = S_{\text{cov}} + 2q^{-1} \int d^2x \, \epsilon^{\mu\nu} A_\mu B_\nu \quad (24)$$

where S_{cov} is simply given by (2) with D_μ replaced uniformly by ∇_μ . A similar replacement is made in the variations (10). The solution of the auxiliary field Euler-Lagrange equations of (24) gives $A_\mu = \frac{1}{2} i q \tilde{G}_\mu$ and $B_\mu = \frac{1}{2} i q \epsilon_\mu{}^\nu G_\nu$. Upon substitution in (23) one finds that $S_T = S + S_q$ of (2) and (20) as discussed above. To show directly that S_T is locally supersymmetric, we note that, as a consequence of gauge invariance, nearly all variations of S_{cov} using (9) and the gauge covariantized version of (10) automatically vanish by the same calculations⁷ which established the invariance of (2) under (9-10). The only new terms involve the commutator $[\nabla_\mu, \nabla_\nu]$ which occurs in the term $i(\delta b^{\mu\nu} D_\mu c_\nu + \bar{\beta}^\mu D_\mu \delta \lambda)$ and yields the uncancelled variation

$$\begin{aligned} \delta S_{\text{cov}} &= \bar{\epsilon} \tilde{e} \beta^\mu ([D_\mu, D_\nu] c^\nu - R_{\mu\nu}{}^\rho{}_\sigma c^\sigma) \\ &= \epsilon (\partial_\mu A_\nu - \partial_\nu A_\mu) \bar{\epsilon} \beta^\mu c^\nu + \epsilon (\partial_\mu B_\nu - \partial_\nu B_\mu) \bar{\epsilon} \beta^\mu \epsilon_\rho{}^\nu c^\rho \\ &= -\epsilon^{\mu\nu} \partial_\mu A_\nu \bar{\epsilon} \gamma_5 \beta \cdot c - \epsilon^{\mu\nu} \partial_\mu B_\nu \bar{\epsilon} \beta \cdot c \end{aligned} \quad (25)$$

This term is cancelled by the variation of the second term in (24) under the field-dependent gauge transformation of A_μ and B_μ , namely

$$\delta A_\mu = -\frac{1}{2} q \partial_\mu (\bar{\epsilon}\beta \cdot c) \equiv -\partial_\mu P(x) \quad \delta B_\mu = \frac{1}{2} q \partial_\mu (\bar{\epsilon}\gamma_5 \beta \cdot c) \equiv -\partial_\mu Q(x) \quad (26)$$

The transformations (26) induce new order q variations of S_{cov} but these are neatly cancelled by making the corresponding field dependent gauge transformations on the ghost fields, specifically

$$\begin{aligned} \delta c_\mu &= P c_\mu + Q \epsilon_\mu{}^\nu c_\nu, & \delta b^{\mu\nu} &= -P b^{\mu\nu} + Q \epsilon^\mu{}_\rho b^{\rho\nu} \\ \delta \lambda &= P \lambda - Q \gamma_5 \lambda, & \delta \beta^\mu &= P \beta^\mu - Q \gamma_5 \beta^\mu \end{aligned} \quad (27)$$

Then the entire theory is given by the action (24) and transformation rules which consist of (9) for the supergravity fields, the covariantized version of (10) plus (27) for the ghosts and antighosts, and (26) for the gauge potentials. The additional gauge transformations (27) and the effects of A_μ and B_μ in δb^{ab} and $\delta \lambda$ coincide exactly with the $O(q)$ transformation rules previously introduced to cancel (23). The action (24) is Weyl invariant because A_μ and B_μ are dimensionless. The proof of super-Weyl invariance is the same as for (2).

The introduction of auxiliary gauge potentials should be useful in studying the quantum theory of the action (24) because the effective action of A_μ and B_μ is quite simple and should allow a simple computation of the Green's function of currents. There are also analogous formulations of the bosonic theory (7), such as that given by the action

$$S = \int d^2x \sqrt{-g} \left\{ -i b^{\mu\nu} (D_\mu + A_\mu) c_\nu + \frac{1}{2q} A_\mu A^\mu \right\} \quad (28)$$

Although we cannot foresee specific applications of the ghost field theories discussed here, we note that they were inspired by string theory and thus we can speculate that they might be useful in the future development of that subject.

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