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Superstrings from the $(1,0)$ Superspace)²

Non-Linear Sigma Model*

Roger Brooks

Center for Theoretical Physics
Laboratory for Nuclear Science
and Department of Physics
Massachusetts Institute of Technology
Cambridge, MA 02139 USA

ABSTRACT

We present the component action of the χ -supersymmetric $(2,1,0)$ supergravity non-linear sigma model coupled to a Wess-Zumino term. The action's matter superfields contain the superstring coordinates of $(10,1,0)$ superspace. Their equations of motion and Virasoro constraints are derived. Through suitable χ -supersymmetric gauge choices, the Neveu-Schwarz-Ramond and Green-Schwarz superstring theories are shown to appear separately.

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1. Introduction

To date, a complete understanding of how the world-sheet supergravity of the superstring [1] is coupled to supergravity in ten dimensions [2], has not been attained. With the advent of the heterotic string [3], two-dimensional $N=\frac{1}{2}$ or $(2,1,0)$ supersymmetry [4] (one right-handed world-sheet supersymmetry generator) has been receiving increased scrutiny [5,6]. A non-linear sigma model with a Wess-Zumino term possessing both bosonic and fermionic string coordinates with coupling to $(1,0)$ supergravity in ten dimensions, has already been given [7].

This theory is $(2,1,0)$ supersymmetric and the bosonic and fermionic superstring coordinates are taken to be the lowest components of $(2,1,0)$ Lorentz scalar superfields. These superfields reduce to a total of four matter fields, i.e. double the number present in the known superstring theories. In this paper we shall show that pairs of these fields may be removed through the use of the symmetries of the action.

In addition to being supersymmetric, general coordinate and superscale invariant, this theory is also χ -supersymmetric. The latter symmetry was first found [8] to be a property of the massless superparticle action. It was later used in the construction of the Green-Schwarz (GS) superstring [9]. In both of these cases it was used to remove unphysical fields. Its local parameter is a world-sheet vector and a space-time fermion.

We shall begin by giving, in section II, the component action for the full theory, this will include the coupling of the super Yang-Mills sector to the matter fields. The $(2,1,0)$ superspace will then be made conformally flat via the usual gauge choices and the $(10,1,0)$ superspace will be taken to be flat. In section III, the equations of motion along with the Virasoro constraints will be found. A set of solutions compatible with the $(2,1,0)$

supersymmetry transformation laws and particular χ -supersymmetric gauge choices will be shown to be equivalent to a chiral Neveu-Schwarz-Ramond (NSR) superstring [10]. A second set of solutions will reduce to the GS superstring under a different χ -gauge choice. Finally, the appendix contains an explanation of the notation used in the text.

11. (1,0)² Component Actions

The (1,0)² χ -supersymmetric action is [7]:

$$S_1 = -i \int d^3\sigma d\tau^- V^{-1} [e^{-\tilde{K}(\tilde{z})} \pi_+^{\underline{a}} \pi_{--}^{\underline{b}} \eta_{\underline{a}\underline{b}} + 2 \int_0^1 dy \hat{\pi}_+^{\underline{c}} \hat{\pi}_{--}^{\underline{d}} \hat{\pi}_y^{\underline{A}} G_{\underline{A}\underline{B}\underline{C}}(\tilde{z})], \quad (2.1)$$

where

$$\begin{aligned} \pi_{\underline{A}}^{\underline{B}} &\equiv V_{\underline{A}}^{\underline{M}} D_{\underline{M}} \tilde{Z}^{\underline{N}} E_{\underline{N}}^{\underline{B}}(\tilde{z}), \\ \hat{\tilde{Z}}^{\underline{M}} &\equiv \hat{\tilde{Z}}^{\underline{M}}(\tau, \sigma, \tau^+, y), \quad \hat{\tilde{Z}}^{\underline{M}}(y=1) = \tilde{Z}^{\underline{M}}, \quad \hat{\tilde{Z}}^{\underline{M}}(y=0) = 0, \\ \hat{\pi}_{\underline{A}}^{\underline{B}} &\equiv \pi_{\underline{A}}^{\underline{B}}(\hat{\tilde{z}}), \\ \hat{\pi}_y^{\underline{A}} &\equiv \frac{d\hat{\tilde{Z}}^{\underline{N}}}{dy} E_{\underline{N}}^{\underline{A}}(\hat{\tilde{z}}), \end{aligned} \quad (2.2)$$

and $\tilde{Z}^{\underline{M}} \equiv (\tilde{X}^{\underline{M}}(\partial))$ are the superfield superstring coordinates. The $\tilde{X}$ and ∂ superfields are expanded in terms of the (2,1,0) right-handed fermionic coordinate, $\tilde{S}^+$, as

$$\tilde{X}^{\underline{M}}(\tau, \sigma, \tau^+) \equiv X^{\underline{M}}(\tau, \sigma) + \tilde{S}^+ \beta_{\underline{A}}^{\underline{M}}(\tau, \sigma) \quad (2.3)$$

and

$$\partial^{\underline{M}}(\tau, \sigma, \tau^+) \equiv \theta^{\underline{M}}(\tau, \sigma) + \tilde{S}^+ S_{\underline{A}}^{\underline{M}}(\tau, \sigma) \quad (2.4)$$

The local χ -supersymmetric transformation laws under which the action (2.1) is invariant are [7]:

$$\delta_{\chi} \tilde{Z}^{\underline{M}} = i \frac{1}{2} \kappa_{++\alpha} (\sigma_{\underline{B}})^{\alpha\beta} \pi_{--}^{\underline{B}} E_{\underline{A}}^{\underline{M}}(\tilde{z}),$$

$$\delta_{\chi} \nabla_+ = \kappa_{++\alpha} \Lambda_+^{\alpha} \nabla_{--} - [\nabla_{--} (\kappa_{++\alpha} \Lambda_+^{\alpha})] M,$$

$$\begin{aligned} \delta_{\chi} \nabla_{++} &= -i [\nabla_+ (\kappa_{++\alpha} \Lambda_+^{\alpha})] \nabla_{--} + i \frac{1}{2} [\nabla_{--} (\kappa_{++\alpha} \Lambda_+^{\alpha})] \nabla_+ \\ &\quad + [\tilde{Z} (\kappa_{++\alpha} \Lambda_+^{\alpha}) \tilde{S}^+ + i (\nabla_+ \nabla_{--} (\kappa_{++\alpha} \Lambda_+^{\alpha}))] M, \end{aligned}$$

$$\delta_{\chi} \nabla_{--} = 0,$$

$$\Lambda_+^{\alpha} \equiv \pi_+^{\alpha} - i \frac{1}{2\sqrt{2}} (\sigma_{\underline{B}})^{\alpha\beta} \chi_{\underline{\beta}}(\tilde{z}) \pi_+^{\underline{B}}, \quad (2.5)$$

where $\tilde{S}^+$ is the (2,1,0) fermionic supergravity field strength. The (2,1,0) supergravity multiplet is given by the zweibein, $V_{\underline{a}}^{\underline{M}}$, and the gravitino, $\rho_{\underline{a}}^+$. Its rigid supersymmetry covariant derivatives are

$$D_{\underline{A}} \equiv (\partial_{\underline{A}}, D_+) = (\partial_{\pm\pm}, \frac{\partial}{\partial \tilde{S}^+} + i \tilde{S}^+ \partial_{++}), \quad (2.6)$$

where the two-dimensional light-cone coordinates are $\sigma^{\pm\pm} \equiv \pm \frac{1}{\sqrt{2}} (\tau \pm \sigma)$, with metric signature $(-, +)$. The (2,1,0) supergravity covariant derivatives are $\nabla_{\underline{A}} \equiv V_{\underline{A}}^{\underline{M}} D_{\underline{M}} + \omega_{\underline{A}}^{\underline{M}}$, with Lorentz spin-connection, $\omega_{\underline{A}}$.

The (10,1,0) supergravity superfields are [2] the dilaton, $\tilde{\Phi}(\tilde{z})$, zweibein, $E_{\underline{A}}^{\underline{M}}(\tilde{z})$, and $G_{\underline{A}\underline{B}}(\tilde{z})$, the field strength of the two-form, $\underline{B}_{\underline{A}\underline{B}}(\tilde{z})$. With this, the second (Wess-Zumino) term may be written as

$$S_{WZ} = -i 2 \int d^2\sigma d\tau^- V^{-1} \pi_+^{\underline{A}} \pi_{--}^{\underline{B}} B_{\underline{A}\underline{B}}(\tilde{z}) \quad (2.7)$$

The pertinent relations between the (10,1,0) supergravity superfields are [2,11]

$$G_{\alpha\beta\gamma} = i \frac{1}{2} e^{-\tilde{K}(\tilde{z})} (\sigma_{\underline{B}})^{\alpha\beta}$$

$$G_{ab\gamma} = \frac{1}{2\sqrt{2}} e^{-\Phi(z)} (\sigma_{ab})_{\gamma}^{\delta} \chi_{\delta}$$

$$\nabla_a \Phi = -\frac{1}{\sqrt{2}} \chi_a. \quad (2.8)$$

The projection of the action (1.1) onto components is performed by applying the density formula [6]

$$\int d^5 V^{-1} \mathcal{L} = V^{-1} (\nabla_+ - i \rho_{++}) \mathcal{L}, \quad (2.9)$$

where the determinants are $V \equiv \text{sdet}(V_A^M)$ and $v \equiv \text{det}(V_A^M)$. Upon defining $Z^a \equiv Z^M$ and $\gamma_+^a \equiv \gamma_+^M Z^a$, we have

$$\begin{aligned} S_1 = \int d^2 \sigma \, v^{-1} e^{-\Phi(z)} \{ & i \gamma_+^p \bar{\psi}_p \gamma_+^q \psi_q E_M^a d_{--} Z^M E_N^b \\ & + \mathcal{D}_{++}(v, \rho) Z^M E_M^a d_{--} Z^M E_N^b \\ & + (-)^M i \gamma_+^M \gamma_+^p \psi_p E_M^a E_N^b d_{--} Z^M E_N^b \\ & + i \gamma_+^M E_M^a \bar{\psi}_- \gamma_+^N \psi_N^b E_N^b \\ & + (-)^M i \gamma_+^M E_M^a d_{--} Z^M \gamma_+^p \psi_p E_N^b \} Z_{ab} \\ & + \int d^2 \sigma \int_0^1 dy \, v^{-1} \{ [\mathcal{D}_{++}(v, \rho) \hat{Z}^M \hat{E}_M^c d_{--} \hat{Z}^N \hat{E}_N^c \frac{d\hat{Z}^a}{dy} \hat{E}_a^A \\ & + (-)^M i \gamma_+^M \gamma_+^p \hat{\psi}_p \hat{E}_M^c \hat{E}_N^c d_{--} \hat{Z}^N \hat{E}_N^c \frac{d\hat{Z}^a}{dy} \hat{E}_a^A \\ & + (-)^M i \gamma_+^M \hat{E}_M^c \hat{E}_N^c \bar{\psi}_- \gamma_+^N \psi_N^b \frac{d\hat{Z}^a}{dy} \hat{E}_a^A \\ & + (-)^{[c+M]} i \gamma_+^M \hat{E}_M^c d_{--} \hat{Z}^N \gamma_+^p \psi_p \hat{E}_N^c \frac{d\hat{Z}^a}{dy} \hat{E}_a^A \\ & + (-)^{[c+M]} i \gamma_+^M \hat{E}_M^c d_{--} \hat{Z}^N \hat{E}_N^c \frac{d\hat{Z}^a}{dy} \hat{E}_a^A \\ & + (-)^{[c+M+Q]} i \gamma_+^M \hat{E}_M^c d_{--} \hat{Z}^N \hat{E}_N^c \frac{d\hat{Z}^a}{dy} \gamma_+^p \psi_p \hat{E}_a^A] G_{ABc} \\ & + (-)^{[c+M+Q]} i \gamma_+^M \hat{E}_M^c d_{--} \hat{Z}^N \hat{E}_N^c \frac{d\hat{Z}^a}{dy} \gamma_+^p \psi_p \hat{E}_a^A G_{ABc} \} \end{aligned} \quad (2.10)$$

in which $(-)^{\underline{A}}$ is defined to be +1 for vector indices, $\underline{A} = \underline{a}$, and -1 for spinor indices, $\underline{A} = \alpha$. The covariant derivative appearing in (2.10) is

$$\mathcal{D}_a(v, \rho) = v_a^m \partial_m + \omega_a(v, \rho) | M \quad (2.11)$$

where

$$\begin{aligned} \omega_{++} | &= C_{++}, -- \\ \omega_{--} | &= C_{++}, --^{++} + i 2 \rho_{++}^+ \rho_{--}^+ \end{aligned} \quad (2.12)$$

with the anholonomy coefficients C_{ab}^c defined through $[v_a, v_b] = C_{ab}^c v_c$. We have also defined

$$d_{--} Z^M \equiv \mathcal{D}_{--}(v, \rho) Z^M + \rho_{--}^+ \gamma_+^M \quad (2.13)$$

and

$$\tilde{d}_{--} \gamma_+^M \equiv \mathcal{D}_{--}(v, \rho) \gamma_+^M + i \rho_{--}^+ \mathcal{D}_{++}(v, \rho) Z^M \quad (2.14)$$

for ease in expressing the action.

Among the symmetries of the action given in equation (2.10), is superscale invariance. This follows because the gravitino ρ_{++}^+ , a component of the conformal compensator [6], is absent from the expression. Since the action was computed on both a curved world-sheet superspace and a curved space-time superspace, it is manifestly (10,1,0) locally supersymmetric and invariant under the (2,1,0) supergravity transformations

$$\begin{aligned} \delta_\epsilon^L Z^M &= -\epsilon^+ \gamma_+^M \\ \delta_\epsilon^L \gamma_+^M &= -i \epsilon^+ (\mathcal{D}_{++} Z^M + \rho_{++}^+ \gamma_+^M) \\ \delta_\epsilon^L v_a^m &= -i 2 \epsilon^+ \rho_a^+ v_{++}^m \\ \delta_\epsilon^L \rho_a^+ &= (v_a^m \partial_m + \frac{1}{2} \omega_a(v, \rho)) \epsilon^+. \end{aligned} \quad (2.15)$$

it is also general coordinate transformation invariant.

The action (2.10) couples (2,1,0) world-sheet supergravity to (10,1,0) space-time supergravity. In particular, it builds a curved space-time superspace out of the fields defined on the super world-sheet. Additionally, we may include a space-time super Yang-Mills potential, $A_{\alpha\beta\gamma}^a$, through the introduction [5a,6] of Weyl fermions minimally coupled to the super potential in the action

$$S_M = -\int d^2\sigma d^5V^{-1} \bar{\Psi}_{-}^{\dot{\alpha}} [\delta_{\dot{\alpha}\dot{\beta}}^{\alpha} \nabla_{+} + \Pi_{+}^B A_{\alpha\dot{\alpha}\dot{\beta}}(z)] \Psi_{-}^{\dot{\beta}}. \quad (2.16)$$

The Fradkin-Tseytlin [12] term

$$S_{FT} = \int d^2\sigma d^5V^{-1} \bar{\Sigma}^{\alpha} \bar{\Phi}(z) \quad (2.17)$$

may also be introduced. However, this last term breaks superscale invariance [6] and a Noether procedure for incorporating it into a X-supersymmetric action has not been found [7]. We will therefore not consider it.

The coupling of the super Yang-Mills potential to X^M , β_{+}^M , θ^{α} , and S_{+}^M (its coupling to the first two fields has been previously given [6,14]) is given by the component action

$$\begin{aligned} S_M = & \int d^2\sigma V^{-1} \{ i \bar{\eta}_{-}^{\dot{\alpha}} \bar{D}_{++}(V,0) \bar{Z}_{-}^N \eta_{-}^{\dot{\alpha}} + F^{\dot{\alpha}\dot{\beta}} F^{\dot{\gamma}\dot{\delta}} \\ & + i \bar{\eta}_{-}^{\dot{\alpha}} \bar{D}_{++}(V,0) \bar{Z}_{-}^N E_{N\dot{\alpha}\dot{\beta}}^B A_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} \eta_{-}^{\dot{\beta}} \\ & - i \bar{\eta}_{-}^{\dot{\alpha}} \bar{\psi}_{+}^{\dot{\beta}} \bar{E}_{+}^B E_{N\dot{\alpha}\dot{\beta}}^B A_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} F^{\dot{\gamma}\dot{\delta}} \\ & + \bar{\psi}_{+}^{\dot{\alpha}} \bar{\eta}_{-}^{\dot{\beta}} \bar{\psi}_{+}^{\dot{\gamma}} \bar{E}_{+}^B E_{N\dot{\alpha}\dot{\beta}}^B A_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} \eta_{-}^{\dot{\delta}} \\ & + \bar{\psi}_{+}^{\dot{\alpha}} \bar{E}_{+}^B \bar{\eta}_{-}^{\dot{\beta}} \bar{\psi}_{+}^{\dot{\gamma}} \bar{E}_{+}^B E_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} A_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}} \eta_{-}^{\dot{\delta}} \} \end{aligned} \quad (2.18)$$

where $\eta_{-}^{\dot{\alpha}} \equiv \bar{\Psi}_{-}^{\dot{\alpha}}$ and $iF^{\dot{\alpha}\dot{\beta}} \equiv \nabla_{+}^{\dot{\alpha}} \bar{\Psi}_{-}^{\dot{\beta}}$. This action is superscale invariant as a result of the gravitino terms in the pull-back of the potential onto the super world-sheet.

To simplify the calculations and in order to readily compare the resulting expressions to their counterparts in known superstring theories, we set $E_{\dot{\alpha}}^M \equiv \delta_{\dot{\alpha}}^M$ and $\bar{\Phi}(z) = 0$. The space-time manifold is now flat. We observe that the Wess-Zumino term remains through $\hat{G}_{\alpha\beta\gamma} = i_{\gamma}^{\alpha}(\hat{G}_{\beta\gamma})_{\alpha\beta}$. The full action $S = S_1 + S_M$ then becomes

$$\begin{aligned} S = & -i \int d^2\sigma d^5V^{-1} \bar{\nabla}_{+} \bar{Z}^B \nabla_{-} \bar{Z}^B \eta_{\alpha\beta} \\ & - i \int d^2\sigma d^5V^{-1} \bar{\nabla}_{+} \bar{Z}^B \nabla_{-} \bar{Z}^B \frac{d\hat{Z}^A}{dy} \hat{G}_{AB\dot{\alpha}\dot{\beta}} \\ & - \int d^2\sigma d^5V^{-1} \bar{\Psi}_{-}^{\dot{\alpha}} [\delta_{\dot{\alpha}\dot{\beta}}^{\alpha} + \nabla_{+} \bar{Z}^B A_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}}(z)] \Psi_{-}^{\dot{\beta}}. \end{aligned} \quad (2.19)$$

In the Wess-Zumino term, the integral over y is done using the super anti-symmetry of $\hat{G}_{\dot{\alpha}\dot{\beta}\dot{\gamma}\dot{\delta}}$. It reduces to

$$V^{-1} \int dy \nabla_{+} \bar{Z}^B \nabla_{-} \bar{Z}^B \frac{d\hat{Z}^A}{dy} \hat{G}_{AB\dot{\alpha}\dot{\beta}} \equiv \frac{1}{3} (A_{-} - B_{-}) \quad (2.20)$$

where

$$\begin{aligned} A_{-} = & \int dy \left\{ \frac{d}{dy} [V^{-1} \nabla_{+} \bar{Z}^B \nabla_{-} \bar{Z}^B \hat{Z}^A] \right. \\ & + (-)^{A\dot{B}} (-)^{N\dot{C}} D_M [V^{-1} (\nabla_{+}^N \nabla_{-}^M - (-)^{NM} \nabla_{+}^M \nabla_{-}^N)] * \\ & \left. * \hat{Z}^B D_N \bar{Z}^A \frac{d\hat{Z}^B}{dy} \hat{G}_{AB\dot{\alpha}\dot{\beta}} \right\} \hat{G}_{AB\dot{\alpha}\dot{\beta}} \end{aligned} \quad (2.21)$$

and

$$\begin{aligned} B_{-} = & \int dy (-)^{A\dot{B}} (-)^{N\dot{C}} [(-)^{MN} D_M (V^{-1} \nabla_{+}^N \nabla_{-}^M) - D_M (V^{-1} \nabla_{+}^M \nabla_{-}^N)] * \\ & * \hat{Z}^B D_N \bar{Z}^A \frac{d\hat{Z}^B}{dy} \hat{G}_{AB\dot{\alpha}\dot{\beta}} \end{aligned} \quad (2.22)$$

The B_- term vanishes because of the (2,1,0) supergravity constraints. In particular the constraints imply anholonomy coefficients, C_{AB}^C , such that $[C_{-+}^{++} + C_{-+}^{--} + 2C_{-+}^{+-}] = 0$, $[C_{++}^{++} - C_{++}^{+-}] = 0$, and $C_{-+}^{+-} = 0$. These are exactly the terms appearing in B_- . As for the A_- term, $\int dy dS^- = \int dy d\tau$ reduces the D_M factor to surface terms when integrated over the world-sheet. The Wess-Zumino term then becomes

$$S_{WZ} = -i \frac{2}{3} \int d^2\sigma dS^- V^{-1} \nabla_+ Z^A \nabla_- Z^B Z^C G_{ABC}. \quad (2.23)$$

This is trivially obtained from equation (2.19) after defining $\hat{Z}^M(\tau, \sigma, S^+) \equiv Y^M(\tau, \sigma, S^+)$.

Further simplification is achieved by using the superconformal invariance of the action to make the gauge choices $V_a^m = e^{h(\tau, \sigma)} \delta_a^m$ and $\rho_a^+ = (\delta_a)^+ \lambda_\beta$.

With this, we have the final form of the action

$$\begin{aligned} S = & \int d^2\sigma \left[\partial_{++} X^a \partial_{--} X^b + i \beta_+^a \partial_{--} \beta_+^b \right] \eta_{ab} \\ & + \int d^2\sigma \left[i \partial_{++} \theta^\alpha \partial_{--} \theta^\beta X^\epsilon + \beta_+^\epsilon \partial_{--} \theta^\beta S_+^\alpha \right. \\ & \quad \left. - \frac{1}{2} S_+^\beta \partial_{--} X^\epsilon S_+^\alpha \right] (\sigma_\epsilon)_{\alpha\beta} \\ & + \int d^2\sigma \left[i \eta_-^\epsilon \partial_{++} \eta_-^\epsilon + F^\epsilon F^\epsilon \right. \\ & \quad + i \eta_-^\epsilon \partial_{++} X^b A_{b\epsilon\epsilon} \eta_-^\epsilon + i \eta_-^\epsilon \partial_{++} \theta^\beta A_{\beta\epsilon\epsilon} \eta_-^\epsilon \\ & \quad - i \eta_-^\epsilon \beta_+^\epsilon A_{b\epsilon\epsilon} F^\epsilon - i \eta_-^\epsilon S_+^\beta A_{\beta\epsilon\epsilon} F^\epsilon \\ & \quad + \beta_+^\epsilon \eta_-^\epsilon \beta_+^\epsilon A_{b\epsilon\epsilon} \eta_-^\epsilon + \beta_+^\epsilon \eta_-^\epsilon S_+^\beta A_{b\epsilon\epsilon} \eta_-^\epsilon \\ & \quad \left. + S_+^\beta \eta_-^\epsilon \beta_+^\epsilon A_{\beta\epsilon\epsilon} \eta_-^\epsilon + S_+^\beta \eta_-^\epsilon S_+^\beta A_{\beta\epsilon\epsilon} \eta_-^\epsilon \right] \quad (2.24) \end{aligned}$$

The fields X^a and β_+^a along with the first term in (2.24) compose the NSR [10] sector of the heterotic string [3]. The auxiliary field, F^ϵ (the

superpartner of the Yang-Mills η_-^ϵ), may be replaced in the action by its equation of motion. This results in the Yang-Mills sector of the heterotic string. The Wess-Zumino term couples the X^a and θ^α fields as in the GS superstring theory [9] with only one real, fermionic field transforming as the 16 spinor representation of $SO(1,9)$. Of course the θ^α superpartner, S_+^α , is a new field. In section III, we will see that the latter field may be κ -gauged away.

The action (2.24) is easily seen to be invariant under the (10,1,0) rigid supersymmetry transformations $\delta_\epsilon X^a = -i \epsilon^\alpha (\sigma^a)_{\alpha\beta} \theta^\beta$ and $\delta_\epsilon \theta^\alpha = \epsilon^\alpha$, where ϵ^α is now the (10,1,0) supersymmetry parameter.

III. Field and Constraint Equations

With no space-time super Yang-Mills coupling, the equations of motion derived from the action (2.24) are

$$\partial_{++} \partial_{--} X_a + i \frac{1}{2} \partial_{--} \theta^\alpha \partial_{++} \theta^\beta (\sigma_a)_{\alpha\beta} - \frac{1}{2} S_+^\beta \partial_{--} S_+^\alpha (\sigma_a)_{\alpha\beta} = 0 \quad (3.1)$$

$$\partial_{--} \beta_+^a - i \frac{1}{2} S_+^\beta \partial_{--} \theta^\alpha (\sigma_a)_{\alpha\beta} = 0, \quad (3.2)$$

$$\begin{aligned} & \partial_{++} X^a \partial_{--} \theta^\alpha (\sigma_a)_{\alpha\beta} - \partial_{--} X^a \partial_{++} \theta^\alpha (\sigma_a)_{\alpha\beta} + i \beta_+^a \partial_{--} S_+^\alpha (\sigma_a)_{\alpha\beta} \\ & + i S_+^\alpha \partial_{--} \beta_+^\alpha (\sigma_a)_{\alpha\beta} = 0, \end{aligned} \quad (3.3)$$

$$\beta_+^a \partial_{--} \theta^\alpha (\sigma_a)_{\alpha\beta} - \partial_{--} X^a S_+^\alpha (\sigma_a)_{\alpha\beta} = 0, \quad (3.4)$$

$$\partial_{--} \eta_-^\epsilon = 0, \quad (3.5)$$

along with the trivial result $F^\epsilon = 0$. The Virasoro constraints may be found by varying equation (2.10) (coupled to a flat space-time only) with respect to the gravitino, ρ_{-+}^+ , and the zweibein, V_a^m . The same results may also be

obtained from the component reduction of the super stress-energy tensor. Either way we have

$$\beta_+^a \partial_{++} \chi_a - \frac{1}{2} \beta_+^a S_+^\beta S_+^\alpha (\sigma_a)_\alpha{}^\beta = 0, \quad (3.6)$$

as the fermionic equation. The bosonic equations are

$$\partial_{--} \chi^a \partial_{--} \chi_a + i \eta_{-}^{\hat{x}} \partial_{--} \eta_{-}^{\hat{x}} = 0 \quad (3.7)$$

and

$$i \beta_+^a \partial_{--} \beta_{+a} + [\beta_+^a \partial_{--} \theta^\beta S_+^\alpha - \frac{1}{2} S_+^\beta \partial_{--} \chi^a S_+^\alpha] (\sigma_a)_\alpha{}^\beta = 0 \quad (3.8)$$

arising from δV_{4+}^F , along with

$$\begin{aligned} & \partial_{++} \chi^a \partial_{++} \chi_a + i \beta_+^a \partial_{++} \beta_{+a} \\ & + [\beta_+^a \partial_{++} \theta^\beta S_+^\alpha - \frac{1}{2} S_+^\beta \partial_{++} \chi^a S_+^\alpha] (\sigma_a)_\alpha{}^\beta = 0 \end{aligned} \quad (3.9)$$

from δV_{4-}^F . Note that (3.6) and (3.9) are superpartners.

Counterparts of equations (3.6), (3.7) and (3.9) are known to exist in the heterotic string (and hence the NSR superstring) with $\eta_{-}^{\hat{x}} = 0$. Recalling equation (3.2), we see that (3.8) is present solely because of the Mess-Zumino term. The cubic terms in (3.6 - 3.9) are obviously present due to the Mess-Zumino action. This in turn couples to a flat space-time superspace through the presence of the superpartner, S_+^α , of the Grassmann coordinate, θ^α .

Although the X-supersymmetric coupling of a super world-sheet to a super space-time introduces the non-linearities in (3.1 - 3.9), we will soon see that this combination leads to a two-dimensional superfield X-supersymmetry parameter which facilitates the removal of S_+^α from equations (3.1 - 3.9).

Before continuing, we note that the super Yang-Mills term is not X-supersymmetric under the transformations (2.5). In fact even the Dirac

contribution is not invariant. This lack of invariance is primarily due to the restrictions on $\delta_X \nabla_+$ imposed by the X-invariance of the two terms in (2.1). In particular, we see this through the absence in $\delta_X \nabla_+$ of a factor proportional to ∇_+ . Full invariance is present only in the (2,0,0) theory. The latter is true after modification of the $\delta_X \nabla_a$ [13].

Henceforth, we will only consider the X-supersymmetric part of the action (2.24), i.e. we neglect $\eta_{-}^{\hat{x}}$. Writing out the X-supersymmetry transformation laws in components and on a flat space-time coupled to a conformally flat super world-sheet, we find

$$\delta_X \chi^a = 0, \quad \delta_X \beta_{+a} = 0, \quad (3.10)$$

$$\delta_X \theta^\alpha = i \frac{1}{2} \chi_{++\gamma} (\sigma_B^\alpha)^{\gamma\alpha} \partial_{--} \chi^B, \quad (3.11)$$

and

$$\delta_X S_+^\alpha = i \frac{1}{2} \chi_{++\gamma} + \gamma (\sigma_B^\alpha)^{\gamma\alpha} \partial_{--} \chi^B - i \frac{1}{2} \chi_{++\gamma} (\sigma_B^\alpha)^{\gamma\alpha} \partial_{--} \beta_{+a}^B, \quad (3.12)$$

where

$$\chi_{++\alpha}(\tau, \sigma, \zeta, \sigma^+) \equiv \chi_{++\alpha}(\zeta, \sigma) + \zeta^+ \chi_{++\alpha}(\tau, \sigma). \quad (3.13)$$

We may then choose a gauge such that $\theta^\alpha = S_+^\alpha = 0$. In this gauge the equations of motion become familiar,

$$\square \chi^a = 0 \quad \text{and} \quad \partial_{--} \beta_{+a} = 0. \quad (3.14)$$

The Virasoro conditions become

$$\beta_{+a}^a \partial_{++} \chi_a = 0, \quad (3.15)$$

$$\partial_{++} \chi^a \partial_{++} \chi_a + i \beta_{+a}^a \partial_{++} \beta_{+a} = 0, \quad (3.16)$$

and

$$\partial_{--} X^{\underline{a}} \partial_{--} X_{\underline{a}} = 0. \quad (3.17)$$

We then have the NSR contribution to the heterotic string [3], a chiral NSR superstring. This is not surprising since in this gauge, the Wess-Zumino term no longer contributes and the theory reduces to the non-linear sigma model term which is a chiral NSR theory.

On the other hand, if we choose $X_{++\gamma}$ such that $S_{+}^{\alpha} = 0$ but keep $\theta^{\alpha} = \theta^{\alpha}(\sigma^{++}, \sigma^{--})$, then equations (3.2) and (3.4) imply $\dot{\beta}_{+}^{\underline{a}} = 0$. The first equation, (3.1), may be simplified through the use of equation (3.11) to choose a X -gauge such that

$$\partial_{++} \theta^{\underline{a}} = 0 \quad (3.18)$$

and

$$\partial_{--} \theta^{\underline{a}} = 0, \quad (3.19)$$

where $\underline{a} = 1, \dots, 8$ and $\underline{a} = 9, \dots, 16$. This choice is possible if we require $\partial_{-} X_{++\gamma} = 0$. The bosonic coordinate field equation then becomes

$$\square X^{\underline{a}} = 0. \quad (3.20)$$

We then have the GS superstring theory. The constraint equations become

$$\partial_{-} X^{\underline{a}} \partial_{\sigma} X_{\underline{a}} = 0 \quad \text{and} \quad \partial_{-} X^{\underline{a}} \partial_{\tau} X_{\underline{a}} + \partial_{\sigma} X^{\underline{a}} \partial_{\sigma} X_{\underline{a}} = 0, \quad (3.21)$$

i.e. the bosonic string Virasoro conditions. There is also the double set of eight equations each,

$$\partial_{++} X^{\underline{a}} \partial_{--} \theta^{\underline{a}} (\sigma_{\underline{a}})_{\underline{\alpha}\underline{\beta}} = 0 \quad (3.22)$$

and

$$\partial_{--} X^{\underline{a}} \partial_{++} \theta^{\underline{a}} (\sigma_{\underline{a}})_{\underline{\alpha}\underline{\beta}} = 0. \quad (3.23)$$

The first set of gauge choices (NSR model), break space-time supersymmetry but maintain world-sheet supersymmetry. The second set of choices (GS model), break world-sheet supersymmetry but retain manifest space-time supersymmetry. As is already known [9], the Wess-Zumino term is absolutely required in the latter case.

We conclude by noting that the two sets of gauge choices lead to the simplest reductions of the theory. It is otherwise unclear how to solve equations (3.1 - 3.9). Though these sets of gauge choices naturally lead to the two known superstring theories, there may be an entire family of gauge choices associated with the parameters $X_{++\gamma}$ and $X_{++\underline{a}}$, which might lead to interesting results.

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Appendix

Underlined indices label space-time quantities, those not underlined are super world-sheet indices. The exceptions to this rule are space-time spinor indices (λ, μ etc.); these need not be distinguished from their super world-sheet counterparts.

The slash, $|$, is used to denote $S(\tau, \sigma, \underline{\tau}^{\dagger} = 0)$, for some arbitrary superfield S .

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