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HIGHER CURVATURE SUPERGRAVITY AND SUPERSTRINGS^{*)}

S. Ferrara
CERN,
1211 Geneva 23, Switzerland
and
Department of Physics,
University of California,
Los Angeles, CA 90024, USA

ABSTRACT

Recent attempts to construct higher curvature supergravity theories are reported and some implications concerning the effective low-energy action for superstring theories discussed.

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1. Introduction

Extensive investigation of quantum properties of gravity and supergravity theories [1] in the past decade has led to the conclusion that a consistent theory of quantum gravity is unlikely to be achieved in the framework of conventional point-field theories [2]. An intriguing and fascinating possibility is that gravity is described, at the fundamental level, by a string theory [3], namely a dynamical theory of one-dimensional extended objects. From the point of view of particle dynamics string theories describe the interactions of a finite set of massless states and an infinite set of massive states with masses which are multiples of the Planck mass. ‘Low-energy physics’ describes the interaction of the massless states upon integration over the massive modes.

Quantum consistency of a string theory requires, among other properties, world-sheet as well as space-time supersymmetry [3, 4]. For string propagation on a flat background this also implies $D = 10$ as the critical space-time dimension. With this philosophy in mind, supergravity theories can be regarded as the low-energy regime of superstring theories [3, 4]. However, the effective superstring action will originate higher terms in the curvatures of any order and these terms will have to respect all the symmetries of the underlying theory which are preserved upon integration over the massive modes, i.e. general coordinate invariance, gauge invariance, and local supersymmetry. Supersymmetry [1], as well known, is the most severe constraint, the reason being that it does not act linearly on the basic (massless) fields of the theory. Nevertheless, in theories linear in the Riemann curvature (Einstein supergravities) supersymmetry can be implemented rather easily, either using Noether techniques, superspace [5], or the group-manifold approach [6]. In fact all higher-dimensional Einstein-supergravity theories, which are known to exist up to $D = 11$ dimensions, have been constructed and their properties, such as symmetry breaking and compactification, are under control [7].

This state of affairs drastically changes when higher curvature terms or more generically higher derivative terms are added to the effective Lagrangian. The technical reason for this is the fact that the bosonic fields which appear in the fermion transformation rules no longer obey algebraic equations but rather differential equations [8]. There are different ways to understand this result and they will be described in Section 2. In Section 3 we will consider some of the implications of supersymmetry on recent results obtained by calculation of string scattering amplitudes and σ -model computations of higher-loops β functions.

In Section 4 we will describe geometrical theories of gravity with higher curvature terms and their possible supersymmetric extension.

2. Supersymmetric completion of higher curvature terms

The first higher-derivative term present in a D -dimensional supergravity theory is due to the Lorentz Chern-Simons three-form

$$\omega_{3L} = \text{tr} \left(d\omega \wedge \omega - \frac{2}{3} \omega \wedge \omega \wedge \omega \right) \quad (1)$$

introduced by Green and Schwarz [9] for the cancellation mechanism [10] of gauge and gravitational anomaly in $D = 10$ supergravity [10] coupled to Yang–Mills matter. This mechanism fixes the gauge group to be $E_8 \times E_8$ or $SO(32)$. The heterotic string theory [11] exists precisely for these two gauge groups while a third string theory (open string) only exists with $SO(32)$ [12]. The Chapline–Manton Lagrangian [10], because of the anomaly cancellation mechanism, must be modified with the substitution

$$H \rightarrow \hat{H} = dB + \omega_{3YM} - \omega_{3L} \quad (2)$$

where $H = dB + \omega_{3YM}$ is the field strength of the two-form B introduced in Ref. [10]. The above substitution implies that the original action is no longer supersymmetric so the question arises as to which terms one should add in order to restore supersymmetry. Supersymmetric completion, in general, requires two types of terms, purely bosonic ones and terms with fermions. The bosonic terms have an immediate interest for the practical reason that they may modify the allowed background configurations of the theory. Therefore, these terms are crucial in order to discuss compactification.

The interplay between anomaly cancellation and superstring compactification (in the supergravity limit) is already seen from the supersymmetric completion of $\hat{H}$ terms in the modified Chapline–Manton action. Their completion requires a new bosonic term of the form

$$\frac{e}{4} e^{2\phi} R_{\mu\nu\rho\sigma}^2 \quad (3)$$

determined up to terms which vanish on the Einstein shell. Zwiebach [13] has pointed out that, in order to avoid spurious ghost states, (3) should be modified in to the Gauss–Bonnet combination

$$\frac{e}{4} e^{2\phi} (R_{\mu\nu\rho\sigma}^2 - 4 R_{\mu\nu}^2 + R^2) \quad (4)$$

Subsequently, Romans and Warner [14] pointed out that (4) is actually required by supersymmetry. A full supersymmetric completion of the $D = 4$ dimensionally reduced Chern–Simons term was achieved by Cecotti et al. [8]. The four-dimensional result is actually a rigorous proof that the Chern–Simons term in Eq. (2) requires a quadratic term in the (uncontracted) Riemann tensor.

However, there are some ambiguities both in the original argument [13] of ghost freedom which led to (4) as well as in the result of Ref. [14]. It has recently been clarified [15] that the expression given by Eq. (4) is ambiguous in the effective action since terms proportional to the Ricci tensor or scalar curvature can always be removed by a field redefinition of the metric. These terms do not contribute to (on-shell) S-matrix elements so (4) cannot be derived from a string calculation unless an off-shell string theory has been constructed. As far as the supersymmetric completion is concerned, the authors of Ref. [14] assumed the absence of fermionic ghosts, i.e. they did not modify the fermionic terms. Under that assumption the combination given by (4) is uniquely fixed since supersymmetry requires the same number of degrees of freedom both for physical and ghost states

[16]. However, a different supersymmetric completion of the $D=10$ Lorentz Chern-Simons modification was found in Ref. [17] with the quadratic combination given by (3) instead of (4). The two results are not in disagreement since both fermionic and bosonic ghost states were introduced in Ref. [17]. These results, if both correct, imply that there is a supersymmetric completion of curvature square terms which has not the Chern-Simons term as supersymmetric counterpart. This conjecture is in agreement with the $D=4$ dimensional reduction [8] and with a recent $D=10$ result obtained by Bergshoeff, Salam and Sezgin using superconformal tensor calculus [18]. In fact, in $D=4$ dimensions there are three independent chiral multiplets which are quadratic in the Riemann tensor. They are constructed in terms of the basic superfields of superspace supergravity [19, 5], namely two chiral superfields $W_{\alpha\beta\gamma}$ and R and a real superfield $E_{\alpha\dot{\alpha}}$ ($\alpha, \dot{\alpha} = 1, 2$ are two-component spinor indices). Their expression is [19, 8]

$$W_{\alpha\beta\gamma} W^{\alpha\beta\gamma} \quad (5)$$

$$(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8R) R R^* \quad (6)$$

$$(\bar{\mathcal{D}}\bar{\mathcal{D}} - 8R) E_{\alpha\dot{\alpha}} E^{\alpha\dot{\alpha}} \quad (7)$$

The dimensionally reduced Lorentz Chern-Simons term, which is related to the Hirzebruch signature $R^{ab} \wedge R^{ab}$, is only contained in (5). So there is a two-parameter family of supersymmetric completions of the Chern-Simons term. The multiplet in (5) also contains the square of the Weyl tensor which on-shell reduces to (3) or (4). If one requires the absence of ghost states [13] the Gauss-Bonnet combination is selected and then a unique combination of (5), (6), and (7) is fixed [8]. The resulting chiral multiplet has, as its last component, the two topological densities $R^{ab} \wedge R^{ab}$ and $R^{ab} \wedge R^{cd} \epsilon_{abcd}$ (Euler density) in addition to other terms required by supersymmetry. The multiplets given by (6) and (7) vanish on the super Einstein shell so their coefficient can be changed by superfield redefinitions. However, we could have equally obtained the result of Ref. [17] by choosing particular coefficients for the invariants given by (6) and (7). This shows that the two different $D=10$ supersymmetric completions are both consistent with the $D=4$ dimensional reduction. What is more important, they seem also to be consistent with the recent result of Ref. [18] where a one-parameter family of supersymmetric completion of the Lorentz Chern-Simons term was obtained. This result indicates that there is one quadratic invariant (rather than two as in $D=4$) which is not related to the Chern-Simons term and whose coefficient may be changed through supersymmetric field redefinition. Since the two independent invariants found in Ref. [18] are Gauss-Bonnet and the $(R_{\mu\nu}^{ab})^2$ invariant, the combination alluded to must be

$$4 R_{\mu\nu}^2 - R^2 \quad (8)$$

The above combination does not contain the Chern-Simons term as supersymmetric counterpart as it is also implied by its 4D reduction.

Another comment on field redefinitions is in order. In the dimensional reduction of the Chern–Simons term it was found that the scalar potential of the resulting 4D theory [20] was modified owing to the presence of higher curvature terms. It was also found [8] that no new minima of the scalar potential were introduced by these terms. However, this result is no longer true if we change the coefficients of the R^2 and $R_{\mu\nu}^2$ terms because they would also affect the resulting scalar potential. This is not surprising because R^2 and $R_{\mu\nu}^2$ terms can change the spectrum of the theory so they can as well introduce spurious vacuum solutions (extrema of the potential) away from the original vacuum.

Up to now we have discussed the supersymmetric completion of the Lorentz Chern–Simons term to lowest order in α when α is defined by

$$\hat{H} = H - \alpha \omega_{3L} \quad (9)$$

Moreover, the discussion has been confined to those terms which do not vanish for $H=0$. The established result is that the Lorentz Chern–Simons three-form requires, by supersymmetry, a quadratic term in the (uncontracted) Riemann curvature whose coefficient is uniquely fixed. Quadratic terms which vanish on the Einstein-shell have, on the other hand, an ambiguous coefficient for two reasons: they can be removed by field redefinitions and they are not the supersymmetric partners of the Lorentz three-form modification.

The same result has also been obtained [21] using Noether techniques in the six-form formulation of $D=10$ supergravity [22] and using the group-manifold approach [23]. An alternative derivation [24], which is not directly related to supersymmetry, but gives the same result, is obtained in the framework of the Kaluza–Klein approach to the heterotic string [25], starting from a 506-dimensional space-time and upon compactification on the group-manifold $SO(32)$ or $E_8 \times E_8$.

It is worth mentioning that the presence of the term given by (3) in the anomaly-free $D=10$ Lagrangian is of crucial importance for compactification to $D=4$ dimensions. In fact, because of this term a previous no-go theorem [26] is overcome and consistent compactifications to $D=4$ preserving $N=1$ supersymmetry are allowed [27].

Exact supersymmetry (to all orders in α) for higher curvature terms is much harder to achieve. However, in $D=4$ a full answer was obtained because an off-shell formulation of $N=1$ 4D supergravity exists [28]. Higher-derivative supergravity coupled to matter has been obtained in the minimal auxiliary field formulation [8]. The presence of higher curvature terms has the immediate consequence that the auxiliary fields [28] A_μ , S , P of $N=1$ supergravity do not fulfil algebraic but rather differential equations. In the case of the Gauss–Bonnet combination the equations are first order and no spurious ghost states are introduced according to general expectations. Since the auxiliary fields A_μ , S , and P appear linearly in the fermion transformations they can be suitably identified (to lowest order) with some combination of ω , H , and the Yang–Mills field strength F of the corresponding higher-dimensional theory. This reasoning is correct provided one assumes that the minimal formulation of $N=1$ supergravity covers the most general form of the 4D supersymmetric Lagrangian, even in the presence of higher curvature terms. This is not immediate

since the equivalence of different auxiliary field formulations can only be established by suitable Legendre transformations [29].

Let us now consider, for definiteness, $D = 10$ supergravity, dimensionally reduced to $D = 4$ and truncated to an $N = 1$ supergravity theory as first obtained by Witten [20]. In the absence of higher curvature terms and dropping the Yang-Mills sector (pure supergravity) the equations of motion of the auxiliary fields are

$$S = P = 0$$

$$\frac{2}{3} \Phi A_a - \frac{i}{3} [\Phi^s \partial_a (s - s^*) + \Phi^t \partial_t (t - t^*)] = 0 \quad (10)$$

Here Φ is related to the Kahler potential of the $N = 1$ Standard Supergravity Lagrangian [30], and s and t are complex scalar fields, defined in Ref. [20], whose real parts are a combination of the dilation ϕ and the compactification mode σ .

In the presence of the Gauss-Bonnet combination the A_a field equation becomes

$$\begin{aligned} \frac{2}{3} \Phi A_a - \frac{i}{3} [\Phi^s \partial_s (s - s^*) + \Phi^t \partial_t (t - t^*)] = \\ = \alpha \partial_b (s - s^*) (3i) (R_{ab} - \frac{1}{6} g_{ab} R) + i \alpha \partial_a (s - s^*) A_b^2 \\ + \alpha \partial_b (s - s^*) (-\tilde{F}_{ab} + 2i A_a A_b) - 2\alpha \partial_a \partial_b (s + s^*) A_b \end{aligned} \quad (11)$$

which is a first-order non-linear differential equation for A_a and can be solved by iteration as a power series in α . The S and P field equations remain $S = P = 0$ to all orders in α if the Yang-Mills part is neglected. The 4D result seems to imply that the bosonic fields which appear in the gravitino transformation law in the $D = 10$ theory are no longer those defined by Green and Schwarz [9] and used by Candelas et al. [27]; rather, they will be a solution of first-order non-linear differential equations.

First-order differential equations for the bosonic fields appearing in fermion variations cannot be obtained in $D = 10$ unless one goes beyond the Noether procedure.

There are actually two ways which lead toward the same conclusion. The first is the 'group-manifold approach' [6], which intrinsically uses a first-order formalism for gravity. In the presence of higher-curvature terms the torsion equation becomes a first-order differential equation [23]. The second way is the one suggested by string amplitudes and σ -model calculations in the heterotic string which indicate a possible role of the H field as a torsion contribution to the spin connection [31, 25]. The interpretation of H as a torsion term is also suggested by $D = 10$ superspace analysis [32]. Let us recall the fact that if H is interpreted as a torsion term a self-consistency problem [31] is posed since H contains in its definition the spin connection through ω_{3L} [see (4)].

Therefore the H -Bianchi identity becomes [31, 32]

$$dH = F \wedge F - R \wedge R (\omega) \quad (12)$$

where ω contains H .

This can be regarded as a differential equation for H which can be solved by iteration since, to lowest order in α , H is given by the Green–Schwarz ansatz (2). The relation of Eq. (10) to supersymmetry and its similarity to Eq. (9) was first discussed in Ref. [8].

3. Implication of supersymmetry on superstring effective actions

Several recent results on σ -model analysis of β -functions [33] up to fourth loops [34, 35], as well as direct calculation of three- and four-point graviton amplitudes in string theories [13, 36–39], give the curvature terms up to the fourth power in the curvatures which appear in the effective action for the massless modes of different string theories. In particular, recent results on four-loop β -functions have shown that Kahler Ricci-flat spaces are not solutions of the full fledged string equations [34–36]. It was later shown that for each Kahler Ricci-flat solution there is an exact solution of the string equations (to all orders in the σ -model loop expansion) with the same topological properties as the Ricci-flat solution [40]. We would like to comment on these results and to discuss to what extent symmetry arguments, based on supersymmetry invariance of the graviton scattering amplitudes, give information on the higher curvature terms.

Arguments based on space–time supersymmetry can only be applied to the heterotic and type II superstrings but not to bosonic strings.

Let us first consider the results on the three-point graviton amplitude. As is well known [41, 13], this amplitude has a leading term which is quartic in the momenta in the bosonic and heterotic theory but vanishes for type II strings [25, 33]. Moreover, the three-graviton on-shell vertex has a term of sixth power in the momenta in the bosonic string [13], while such a term is absent in the heterotic string [37–39]. All these results can be simply understood by the use of space–time supersymmetry of the S-matrix. In the heterotic string the $R^2_{\mu\nu\rho\sigma}$ term originates the contribution with four-momenta to the on-shell graviton vertex (this contribution is absent in $D=4$ because of the Gauss–Bonnet theorem). This term is related by supersymmetry to the Chern–Simons three-form, which is present because of anomaly cancellation. On the other hand, in type II superstrings the anomaly cancellation is automatic [42], without the Green–Schwarz mechanism, and no Lorentz Chern–Simons term is present. By supersymmetry this implies that no $R^2_{\mu\nu\rho\sigma}$ is possible in type II superstrings in agreement with σ -model calculations [33] and Kaluza–Klein techniques of σ -models on group manifolds [24, 25].

Let us now consider the terms of sixth power in the momenta. They can be originated by a cubic term in the Riemann curvature. However, some of these terms are forbidden by supersymmetry [43, 19] since they are related to the absence of the two-loop counterterm of point-field supergravity theories. The forbidden counterterm is

$$R_{\mu\nu}{}^{\rho\sigma} R_{\rho\sigma}{}^{\tau\lambda} R_{\tau\lambda}{}^{\mu\nu} \quad (13)$$

More generally, arguments based on 4D supersymmetry can only rule out those terms which do not vanish upon dimensional reduction. Therefore we conclude that such a term is possible (and in fact is present) in the bosonic string but is absent in superstrings [23]. At this point, we observe that the

absence of a term of sixth order in the momenta in the three-point graviton on-shell amplitude does not exclude all R^3 terms as concluded in Ref. [39]. Indeed a geometrical term of the form

$$R^{a_1 a_2} \wedge R^{a_3 a_4} \wedge R^{a_5 a_6} \wedge V^{a_7} \wedge \dots \wedge V^{a_{10}} \epsilon_{a_1 \dots a_{10}} \quad (14)$$

(where V^a is the vierbein one-form) does not vanish on shell but it does not contribute to the three-point amplitude because of antisymmetry. In fact, it is a general property of geometrical terms, with n curvatures, which can be written as differential forms, that their leading contribution is a total derivative so they can only contribute to the scattering amplitudes with more than n gravitons. The possible existence of a term as in (14) can only be inferred by a study of the four-point on-shell graviton scattering amplitude.

Let us now move to the four-point on-shell graviton scattering amplitude. S matrix calculations in superstring theories [36–39] seem to indicate that there is no term of sixth power in the momenta but there are terms of eighth power. This rules out the term given by (14) and implies the presence of a term quartic in the Riemann curvature (or one derivative and three Riemann curvatures).

However, an S-matrix calculation cannot determine the full $(R)^4$ term, even on shell, because of the possible existence of a geometrical term of the form

$$R^{a_1 a_2} \wedge R^{a_3 a_4} \wedge R^{a_5 a_6} \wedge R^{a_7 a_8} \wedge V^{a_9} \wedge V^{a_{10}} \epsilon_{a_1 \dots a_{10}} \quad (15)$$

which does not contribute to the four-point graviton scattering amplitude. The precise form of the $(R)^4$ term has been obtained using σ -model calculations for the fourth-loop order β -function [34]. Of course, these calculations determine the $(R)^4$ terms modulo terms which are ambiguous because of field redefinitions, i.e. terms which are proportional to the Ricci tensor or curvature scalar. The $(R)^4$ term, which does not vanish when reduced to $D=4$ dimensions, is related to the three-loop counterterm [44, 19] of point-field supergravity theories and it is therefore allowed by supersymmetry [23]. This term is unique [up to terms as given in (15) which vanish in $D=4$ dimensions] and is the analytic continuation of the square of the Bel–Robinson tensor [44, 19]. In terms of the superfield $W_{\alpha\beta\gamma}$ defined in Section 2, its expression is [19]

$$W_{\alpha\beta\gamma} W^{\alpha\beta\gamma} \bar{W}_{\hat{\alpha}\hat{\beta}\hat{\gamma}} \bar{W}^{\hat{\alpha}\hat{\beta}\hat{\gamma}} \quad (16)$$

The existence of the additional term given by (15) shows that claims based on S-matrix calculations are not complete. The four-point on-shell graviton scattering amplitude does not fix all $(R)^4$ terms while the σ -model calculation does (up to terms which vanish on the Einstein shell).

The coefficient of the term in (15) can only be fixed by computing the five-point scattering amplitude.

The existence of $(R)^4$ terms which contribute to the four-point on-shell graviton scattering amplitude shows that non-geometrical terms do appear in the effective action of superstrings, although they do not appear up to $(R)^3$ terms. This seems to indicate that the group-manifold

approach [6] to supergravity and superstring theories needs additional variables (forms) in order to describe all the higher-order terms in the effective action.

4. Geometrical approach to gravity and supergravity theories

In the study of the effective action of superstring theories it is an important point to understand how to carry out the supersymmetric completion of higher curvature terms. In 4D $N=1$ supergravity or superstrings compactified to $D=4$ preserving $N=1$ supersymmetry, this problem can be solved, for any higher curvature term, since an off-shell formulation of 4D supergravity coupled to matter exists [28]. However, in 10D superstrings other methods must be found (at least in the point-field limit) since an off-shell formulation of 10D supergravity Lagrangian is not known.

A convenient technique to attack the problem of formulating supersymmetric theories in higher dimensions is given by the so-called group-manifold approach [6], which makes extensive use of differential forms in superspace. This approach is intrinsically a first-order (Palatini) formulation of gravity in the sense that it uses the spin-connection and vierbein one-form as basic fields (with the addition of the gravitino one-form in the case of supergravity).

It is widely known that the Einstein–Hilbert action can be written in pure geometrical terms as

$$R^{a_1 a_2} \wedge V^{a_3} \wedge \dots \wedge V^{a_D} \varepsilon_{a_1 \dots a_D} \quad (17)$$

The field equation for the spin-connection, derived from (17) is

$$T^a = R^a = \mathcal{D}V^a = dV^a - \omega^{ab} \wedge V^b = 0 \quad (18)$$

when T^a is the torsion two-form. This implies that ω_μ^{ab} is the Riemannian spin-connection. Expression (17) has a natural generalization to higher curvature terms of the form

$$R^{a_1 a_2} \wedge R^{a_3 a_4} \wedge \dots \wedge R^{a_{p-1} a_p} \wedge V^{a_{p+1}} \wedge \dots \wedge V^{a_D} \varepsilon_{a_1 \dots a_D} \quad (19)$$

where $p=1, \dots, D$. For D even the term with $p=D$ is a topological invariant (Euler density), while for D odd one can write an extra term with the Lorentz Chern–Simons three-form

$$T_n (R \wedge R \wedge \dots \wedge R \wedge \omega_{3L}) \quad (20)$$

with $(D-3)/2$ curvatures. This term does not contribute to the Einstein equations although it is not a total derivative.

Lagrangians including higher curvature terms of the form given by (19) have the special property that the torsion equation is still solved by Eq. (18) since the torsion equation is homogeneous in T^a . Theories of this type have recently been considered by Zumino [45] and others [46, 47] in view of their possible relevance to string theories. Indeed the Chern–Simons three-form just requires, as discussed in Section 2, the Gauss–Bonnet combination, which corresponds to the $p=4$ quadratic

geometrical term in (19). In Section 3 we have seen that the coefficient of the trilinear geometrical invariant vanishes in superstring theories if one uses the results of the existing literature. However, the $(R)^4$ term, as discussed in Section 3 is not geometrical, so this approach seems to fail when applied to the effective action of superstrings. However, from the point of view of supergravity theories, it is quite interesting to explore the possibility of a supersymmetric completion of higher curvature terms which are geometrical.

An investigation of the same problem in lower-dimensional supergravity has also been carried out [48]. The group-manifold approach seems to select geometrical invariants as simplest candidates for a supersymmetric completion [23]. The interesting fact is that the presence of higher curvature terms gives rise to a non-polynomial theory in the curvatures when bosonic matter fields are coupled to the spin connection.

The simplest case is the presence of a $D-4$ form B : it can be naturally coupled to the gravitational sector with two terms

$$\alpha T^a \wedge V_a \wedge dB \quad (21)$$

$$\beta dB \wedge \text{tr} \left(d\omega \wedge \omega - \frac{2}{3} \omega \wedge \omega \wedge \omega \right) \quad (22)$$

Expression (21) gives rise to a kinetic term for the B field [49], while (22) is the Chern-Simons term required by anomaly cancellation in the six-form formulation of $D=10$ supergravity [22].

If the Gauss-Bonnet term is added (with coefficient γ) the full torsion equation becomes

$$\begin{aligned} & \Sigma_{abm} c_{1\dots C_{D-3}} T^m \wedge V^a_{1\dots} V^{C_{D-3}} + \alpha V_a \wedge V_b \wedge dB \\ & + \beta R_{ab} \wedge dB + \gamma \Sigma_{abcdm} c_{1\dots C_{D-5}} R^{cd} \wedge T^m \wedge V^a_{1\dots} V^{C_{D-5}} = 0 \end{aligned} \quad (23)$$

A dilaton field can also be naturally incorporated. The modified Einstein term

$$e^\phi R^{a_1 a_2} \wedge V^{a_3} \wedge \dots \wedge V^{a_D} \epsilon_{a_1 \dots a_D} \quad (24)$$

leads to a ϕ kinetic term because the torsion equation acquires an extra term

$$\delta T^a \propto V^a \wedge d\phi$$

For $\beta = \gamma = 0$ the Lagrangian given by adding (24) to (21) (with an additional Weyl factor $e^{-\phi}$) reproduces the complete bosonic section of $D=10$ supergravity in the six-form formulation. An interpretation of the dilaton and the two-form B (dual to the six-form) as torsion contribution in string theories was given long ago by Scherk and Schwarz [50].

For $\beta, \gamma \neq 0$ the bosonic Lagrangian becomes non-polynomial owing to the torsion equation. The torsion satisfies a first-order non-linear differential equation that can be solved by iteration in

exact analogy to Eq. (12). In fact, it can be shown that the B equation of motion, up to terms in the Riemann curvature and up to higher-order terms, does reproduce Eq. (12). These are indications that the supersymmetric completion of the class of bosonic Lagrangians considered in this section can be achieved by extending the group-manifold approach to supergravity theories [23].

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