Sweep-Sweep Correlations of Polyakov Loops for SU(3) Lattice Gauge Theory on 32^4 Lattices

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SWEEP-SWEEP CORRELATIONS OF POLYAKOV LOOPS FOR SU(3) LATTICE
GAUGE THEORY ON 32^4 LATTICES

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Abstract

We calculate the sweep-to-sweep correlations of Polyakov-loops at β -values of 6.3, 6.6 and 6.9 on a 32^4 lattice for SU(3) gauge theory. The distance versus correlation relations are determined. Confinement of adjoint sources is verified.

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Introduction

Quantitative predictions of physical quantities on the basis of lattice QCD require measurements in a β -regime where scaling holds. If the scaling behaviour follows the asymptotic two-loop β -function prediction dimensionful quantities can be predicted in terms of Λ_L . In principle Λ_L is known as its relation to $\Lambda_{\overline{MS}}$ is known [1], but the experimental determinations of $\Lambda_{\overline{MS}}$ are not very accurate. The same holds true if the scaling behaviour is universal and the β -function is measured. Even if the β -function is not measured, at least dimensionless ratios can still be predicted, if scaling is universal. Initial results of the ABCDE collaboration [2] had seemed to indicate that the β -function is almost asymptotic from $\beta = 6.0$ upwards, but recent measurements at higher β -values [3] indicate otherwise. In order to probe this region without substantial finite size effects it is necessary to use large lattices. Measurements on 16^4 lattices [4] have shown some finite size effects at $\beta \approx 6.4$. Therefore, in order to perform measurements at β -values of $\beta = 6.3, 6.6$ and possibly 6.9 we decided to generate configurations on a 32^4 lattice (note that the $\Delta\beta$ corresponding to a scale-change of 2 according to the asymptotic 2-loop β -function at $\beta = 6.3$ is $\Delta\beta = .612$).

The starting configuration was obtained from 16 copies of a 16^4 lattice which had been thermalized at $\beta = 5.8$ for 2000 sweeps [5]. The subsequent history is summarized in table 1. The configurations at $\beta = 6.6$ and 6.9 were thermalized starting from thermalized configurations at the previous β -value. Configurations were stored every 50 sweeps at $\beta = 6.3$ and 6.6 , and every 75 sweeps at $\beta = 6.9$. The programs used were an adaption of programs developed in Wuppertal [6], using 8-bit coding for the $SU(3)$ -matrices [7]. The performance of these programs on this size of lattice with a n-hit metropolis is:

$$34.5 + 2.4 n \quad \mu\text{sec/link update} \quad . \quad (1)$$

The 8-bit coding saves a lot of memory at the cost of unpacking and packing

the matrices every time. Also it is a great advantage for storing the configurations on tape. At $\beta = 5.8$ the only effect is a slight shift in the effective β , by $\delta\beta = .006$. By applying this small shift all measured Wilson loops up to a circumference of 22 could be mapped on the Wilson loops measured with the full SU(3) group [7]. At higher β 's the $\delta\beta$ begins to depend slightly on the size of the Wilson loop, though it is still small. This makes it necessary to do some sweeps with higher precision (full group or 16 bits per real element of the first two columns of the matrices) before measuring Wilson loops [8].

Another issue is the sweep-to-sweep correlations of the configurations, and this is the main subject of this paper. With increasing β the time scale for correlations is increasing too. One way of estimating this timescale is to measure the topological charge of the configurations and with present methods this becomes feasible. Here however we report only on sweep-to-sweep correlations between Polyakov loops [9]. The correlations between Wilson loops in subsequent sweeps are longer in range the longer their perimeter is, but measuring the correlations becomes harder as they disappear into the noise quickly. The Polyakov loops (Wilson loops closed by periodicity of the lattice) on a 32^4 lattice contain as many links as an 8×8 Wilson loop. The first result is that the average modulus of Polyakov loops is 1 within errors in the whole range $\beta = 6.3-6.9$,

$$\langle PP^* \rangle = .9997 \pm .0024 \quad (\beta = 6.3) , \quad (2a)$$

$$= .9998 \pm .0018 \quad (\beta = 6.6) , \quad (2b)$$

$$= 1.0001 \pm .0018 \quad (\beta = 6.9) , \quad (2c)$$

which implies confinement of adjoint sources [13] as shown below. Here P denotes the Polyakov loop

$$P_\mu = \text{tr } p_\mu(x) , \quad p_\mu(x) = \prod_{k=0}^{N-1} U(x + ka_\mu \pmod{N}, \mu) , \quad (3)$$

$N = 32$ is the lattice size, and a_μ a unit vector in direction μ . The adjoint sources come in through the relation

$$PP^* = |\text{tr}_{\text{fundamental}} p_\mu(x)|^2 = \text{tr}_{\text{adjoint}}(p_\mu(x)) + 1, \quad (4)$$

so the result (2) shows that

$$e^{-(E_{\text{adjoint}} - E_{\text{vac}})/T} \sim \langle \text{tr}_{\text{adjoint}} p \rangle \approx 0. \quad (5)$$

Thus the excess free energy of the adjoint sources is infinite, and they are confined.

We were able to follow sweep-to-sweep correlations between Polyakov loops up to a separation Δ between sweeps of 3, as summarized in table 2. The last three columns give the ratios of the absolute value of the correlation

$$c_\Delta = \langle P_x(s) P_x^*(s + \Delta) \rangle, \quad (6)$$

where s stands for sweep, and the standard deviation obtained by splitting the data into 31 bunches. The total number of loop correlations measured were 4.8 million at $\beta = 6.3$, and 14.3 million at $\beta = 6.6$ and $\beta = 6.9$. Because the average modulus of the Polyakov loops is one we can fit the data with two exponentials,

$$c_\Delta = ae^{-b\Delta} + (1-a)e^{-c\Delta}. \quad (7)$$

The results of the fit are reproduced in table 3, where also the number of sweeps necessary to get the correlation below some value is calculated. As expected we see that the lowest mass (in lattice units) relevant for the correlations decreases with increasing β , though the effect is not dramatic. Note that for measurements on the same physical scale as with 8×8 loops at $\beta = 6.3$, one needs 12×12 loops at $\beta = 6.6$ and 16×16 loops at 6.9 . Rough estimates for the values of the average Wilson loops of these sizes are 10^{-7} (6.3), 3×10^{-9} (6.6) and 10^{-10} (6.9). From table 3 we conclude that in order to get uncorrelated measurements of these loops a separation of 100 sweeps of the configurations will suffice. This does not exclude that global effects like remnants of metastability or like topological properties may be sustained on longer scales, but these will require separate study.

Acknowledgements

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Table 1

β	thermalising sweeps	generating sweeps	comment
6.3	1400	1350	ETA 1400 [10] UMRCC 1350 [11]
6.6	500	1430	UMRCC 1930 [11]
6.9	600	1575	SCI 1500 [12] ETA 450 [10] UMRCC 225 [11]

Number of sweeps at various β 's and where they were performed.

Table 2

β	$\Delta=1$	$ c_\Delta $		$ c_\Delta /(\text{standard deviation})$		
		2	3	$\Delta=1$	2	3
6.3	1.0057	.0155	.0032	31.9	6.2	1.3
	31	25	25			
6.6	1.2334	.0224	.0054	83.7	19.2	3.8
	15	12	14			
6.9	1.4250	.0289	.0071	100.0	21.7	5.3
	14	13	13			

Sweep-to-sweep correlations of Polyakov loops.

Table 3

$\beta =$	6.3	6.6	6.9
a	.797	.661	.781
b	2.76	2.58	2.55
c	1.40	1.30	1.26
$\Delta(c_{\Delta} < 10^{-8})$	12	13	13
$\Delta(c_{\Delta} < 10^{-16})$	25	28	28
$\Delta(c_{\Delta} < 10^{-24})$	38	42	43

Fitted values for eq (6) and sweep separations needed to reduce the correlations below the indicated level.