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THE CONFORMAL EQUIVALENCE OF THE CANESCHI - SCHWIMMER - VENEZIANO AND WITTEN VERTICES

by

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ABSTRACT

We establish the conformal equivalence of the Caneschi - Schwimmer - Veneziano vertex and Witten's vertex. This proves that, for all levels, Witten's vertex yields the same couplings as the Caneschi - Schwimmer - Veneziano vertex for on-shell physical string states.

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I. INTRODUCTION

Free string field theory is now well understood [1-29]: The action is given by $S = \int \Psi^\dagger Q \Psi$ [1,3,11,12] where Q is the BRS operator [30,31]. Interactions have been introduced via a covariant generalization of the light-cone approach [32,33] and by E. Witten [1]. In the latter approach, the string is broken up into a right half and a left half with the midpoint separating the two halves. Two strings interact to produce a third string by pairwise joining halves [1]. The combined quadratic and trilinear terms constitute a Chern-Simons form in the space of string forms [1]. The vertex corresponding to the coupling of three arbitrary states has been computed in refs.[34-36]. For the first few levels, it has been shown [34,35] that the couplings agree with the open bosonic string results for physical on-shell states. Still lacking is a proof that couplings agree to all levels. The purpose of this note is to provide such a proof.

In ref.[35] a square root singular conformal mapping was used in which the authors argued that the singularity could be ignored. They inverted the mapping and substituted it into the half plane Neumann functions to obtain the couplings. Ref. [36] used a different approach. A six-string process was related to the three-string one. A non-singular conformal mapping was used to compute the six-string process. By analyzing the Goto-Naka conditions [37], it was straightforward to show [36] that the Witten vertex overlap conditions were satisfied and hence the six-string calculational method was justified. The method of A. Jevicki and D. Gross [34] seems to fall roughly between the above two methods. The two approaches give the same couplings through the fourth level although no direct proof exists for all levels. The different approaches may be useful for obtaining different results. For example, to construct the ghost vertex in terms of the original anticommuting ghosts, the six-string approach was used [38]. The results of ref.[38], when combined with those of refs.[34-36], mean that all couplings of physical and unphysical degrees of freedom are explicitly known for E. Witten's string field theory.

In this letter, we will show the conformal equivalence of a standard open bosonic vertex, the Caneschi-Schwimmer-Veneziano (CSV) vertex [39] and Witten's vertex. This means that there exists an operator, O , of the form $\exp[\sum_r (a_0^r (L_0^r - 1) + a_1^r L_1^r + a_2^r L_2^r + \dots)]$ (here r refers to the three different strings) such that $V^{CSV} O = V^{Witten}$. Thus E. Witten's way of joining strings yields the correct couplings for on-shell physical states for all levels. Our method of proof will rely on techniques developed by A. Neveu and P. West to show the equivalence of the light-cone and CSV vertices [33]. We were also aided in our search for the operator, O , by the paper of E. Cremmer, C. Thorn and A. Schwimmer [35].

Except for the space-time metric, we follow the conventions of ref.[40]. The letter r will be used to denote different strings, and sums over r go from 1 to 3.

II. THE CANESCHI - SCHWIMMER - VENEZIANO VERTEX

This section presents a complete mathematical description of the Caneschi - Schwimmer - Veneziano vertex [39]. We will use the form in which the vertex appears as a "bra" and hence the wave functions appear as "kets" (such a form is given in Eq.(3.2) of ref.[33]). Following ref. [33], let us review the overlap conditions for the CSV vertex. These overlap conditions will be used in the next section to establish the conformal equivalence of the CSV and the Witten vertices.

Let

$$z_r = 1/(1 - z_{r-1}) = (z_{r+1} - 1)/z_{r+1} \quad (r = 1, 2, 3), \quad (1)$$

where $z_4 \equiv z_1$ and $z_0 \equiv z_3$. Using three variables, z_1, z_2 and z_3 , is convenient even through Eq.(1) implies that there is only one independent variable. From Eq.(1)

$$\partial z_{r+1}/\partial z_r = z_{r+1}^2 = (1/(1 - z_r))^2 = ((z_{r-1} - 1)/z_{r-1})^2. \quad (2)$$

In the standard manner [40,41], define $\partial X^\mu(z) = \sum_n \alpha_n^\mu z^{-n-1}$. The overlap condition for the CSV vertex is [33]

$$0 = V^{CSV} [\sum_r A_r \partial X^\mu(z_r)] ,$$

where one of A_1, A_2 and A_3 is zero:

$$\text{if } A_r = 0 \text{ then } A_{r+1} = -z_{r-1}^{-2} A_{r-1}. \quad (3)$$

The reason for this restriction is that, in deriving Eq.(3), a geometric series arises which requires a convergence condition [33]. The convergence conditions are

$$|z_r| < 1. \quad (4)$$

These conditions (plotted in Figure 1 in the z_1 plane) become respectively in terms of z_1 , $|z_1| < 1$, $|z_1 - 1| > 1$ and $\text{Real}(z_1) > 1/2$. There are two points, common to the boundary of all three conditions, at $z_1 = 1/2 + \sqrt{3}/2 i$ and $z_1 = 1/2 - \sqrt{3}/2 i$. We denote these two complex numbers by Z_0 and Z_0^* . In contrast to the light-cone [32,33] and Witten [1] overlap conditions, there is presently no geometrical understanding of the CSV overlap conditions in Eq.(3). Nevertheless, the point Z_0 corresponds to the interaction point and the string positions "at infinity" are respectively at 0, ∞ and 1 in terms of the z_1 variable. The relations among the A_r in Eq.(3) are incompatible (there is one relative minus sign), but since they are never required to be simultaneously satisfied, there is no inconsistency.

Eq.(3) can be turned into an oscillator relation, by multiplying by $\int dz_1$ and choosing the paths of integration as those in Figure 2. Note that, on any given path, one of the A_r 's is zero and that the net result is that each term in Eq.(3) becomes a closed contour integral. After deforming the contours, one gets

$$0 = \text{VCSV} \left[\sum \oint_0 dz_r A_1 \partial X^\mu_r(z_r) \right] . \quad (5)$$

The relation between A_r 's and A_1 was precisely the Jacobian (see Eq.(2)) to change $dz_1 A_r$ into $dz_r A_1$. In Eq.(5) $\oint_0$ represents a contour around 0. Choosing, for example, $A_1 = -1/z_2 = (z_1 - 1) = z_3/(1 - z_3)$, generates Eq.(3.8) in ref.[33].

There is one other interesting result concerning the CSV vertex. The "string mappings", ξ_r , and the string points, Z^r , are given by

$$\begin{aligned} \xi^1(z) &= \ln((z-1)/z) , \quad Z^1 = 1 , \\ \xi^2(z) &= \ln(z) , \quad Z^2 = 0 , \\ \xi^3(z) &= \ln(1/(1-z)) , \quad Z^3 = \infty . \end{aligned} \quad (6)$$

When these are plugged in the formulas of refs.[42-45] (given in Eqs.(3-5) of ref.[36]), the CSV vertex in oscillator comes out.

The above represents a complete mathematical understanding of the CSV vertex. A physical interpretation is still lacking.

III. THE CONFORMAL MAPPING METHOD

A. Neveu and P. West have developed a powerful method for obtaining the conformal equivalence of two vertices without explicitly obtaining the exponential sum of Virasoro generators necessary to transform one vertex into another [33]. The idea is to find a conformal change of variables which maps the overlap conditions of one vertex into those of the other.

Start with Eq.(3) in a region where $A_3 = 0$ (i.e. $|z_1| < 1$ and $|z_1 - 1| > 1$ in Figure 1) and insert $1 = [C^3 C^2 C^1]^{-1} [C^3 C^2 C^1]$:

$$0 = \text{VCSV} [C^3 C^2 C^1]^{-1} [C^3 C^2 C^1] [A_1 \partial X^\mu_1(z_1) + A_2 \partial X^\mu_2(z_2)] , \quad (7)$$

Each C^r represents $\exp(a_0^r(L_0^r - 1) + a_1^r L_1^r + \dots)$. When acting on VCSV , the oscillators in the L_n 's generate a transformation on the vertex which is algebraically difficult to compute. When acting to the right, they are equivalent to a conformal change of variables, $z_r \rightarrow w_r$, under which [40,41]

$$\partial X^\mu_r(z_r) \rightarrow \partial w_r / \partial z_r \partial X^\mu_r(w_r) . \quad (8)$$

The ω_r must have a convergent Taylor series in z_r in a region near $z_r = 0$. The attractive feature about this method is that one does not have to compute explicitly the effects of operators; one must simply find the right mapping. Finding the right change of variables is much easier than computing the effects of the L_n operators on the vertex. For the light-cone case, the mappings are:

$$z_r \rightarrow \omega_r = z_r (1 - z_r)^{(\alpha_i - 1/\alpha_i)} , \quad (9)$$

where α_i are the string lengths (and satisfy $\alpha_1 + \alpha_2 + \alpha_3 = 0$). ω_r is analytic in z_r in a region around $z_r = 0$. Using the fact that $z_1^{\alpha_1} (1 - z_1)^{\alpha_3} = z_2^{\alpha_2} (1 - z_2)^{\alpha_1} = z_3^{\alpha_3} (1 - z_3)^{\alpha_2}$ (use Eq.(1)), one finds that the three ω 's are related to one variable: $\omega_1 = \zeta^{(1/\alpha_1)}$, $\omega_2 = \zeta^{(1/\alpha_2)} \exp(-i\alpha_1/\alpha_2 \pi)$, $\omega_3 = -\zeta^{(1/\alpha_3)}$. Using these as arguments of the ∂X^μ_r , the light-cone overlap conditions between strings 1 and 2 are obtained [42-45]. The interaction points Z_0 and Z_0^* get mapped onto the light-cone interaction point (the point where they split). Notice that the mappings in Eq.(9) are similar (but not exactly the same) as the Mandelstam mapping from the half plane to the light-cone [37].

Now consider the Witten vertex. Since a half plane Mandelstam map has recently been found [35], one suspects that the conformal mapping needed to transform the CSV vertex into the Witten vertex should be a closely related mapping. Several clues help one guess the map. Since both vertices are cyclicly symmetric, the same conformal mapping functions should appear:

$$z_r \rightarrow w_r \equiv \Omega(z_r) . \quad (10)$$

Furthermore, one wants ω_1 to correspond to ζ and ω_2 to correspond to $-1/\zeta$ so that the overlap condition relates $X^1(\sigma)$ to $X^2(\pi - \sigma)$ ($\zeta = \exp(i\sigma)$). This implies that $\Omega(z_1) \Omega(z_2) = -1$. We require that $\Omega(z)$ be analytic near $z = 0$ and one knows that $\Omega(z)$ must begin as $3\sqrt{3}/4 z + \dots$. With insight from ref.[35], we found Ω :

$$\Omega(z) = -2/(3\sqrt{3}) [(1/2 - z)(2 - z)(1 + z) - (1 - z + z^2)^{3/2}] / z(1 - z) . \quad (11)$$

$\Omega(z)$ satisfies the above requirements. It is essentially the map of E. Cremmer, A. Schwimmer and C. Thorn (Eq.(5) of ref.[35]). The sign of the root $3/2$ factor is necessary in order that $\Omega(z)$ be analytic at $z=0$.

Applying these mappings to Eq.(8), we get

$$0 = V^{CSV} [C^3 C^2 C^1]^{-1} \{A_1 1/\zeta \partial\zeta/\partial z_1\} [\zeta \partial X^\mu_1(\zeta) - 1/\zeta \partial X^\mu_2(-1/\zeta)] , \quad (12)$$

Now divide Eq.(12) by $\{A_1 1/\zeta \partial\zeta/\partial z_1\}$ to obtain the overlap condition among strings 1 and 2 for E. Witten's way of joining strings [1]. The overlap conditions of strings 2 and 3 and strings 3 and 1 follow using Eq.(3) when $A_1 = 0$ and $A_2 = 0$. The only remaining question is "What is the range of ζ ?". Does Eq.(12) apply for

$\pi/2 < \sigma < 3\pi/2$ (i.e. ζ on the left half of the unit circle)? The overlap conditions apply in the region $|z_1| < 1$ and $|z_2| < 1$ displayed in Figure 1. The extrema of these regions are given at Z_0 and Z_0^* . The corresponding ζ values are $\zeta_0 = \Omega(Z_0) = i$ and $\zeta_0^* = \Omega(Z_0^*) = -i$ and hence the correct range is obtained. The operator, O , which implements the conformal equivalence is $[C^3 C^2 C^1]^{-1}$.

We will now provide an algorithm for computing the exponential series of Virasoro generators which take the CSV vertex into the Witten vertex. We will explicitly carry this out for the first three L_n 's and check that the mapping does the job. The first step is to work out the Taylor series for $\Omega(z)$:

$$\Omega(z) = 3\sqrt{3}/4 z (1 + 1/2 z + 9/16 z^2 + \dots) \quad (13)$$

For positive n , a Virasoro generator L_n generates a conformal change on a function, $f(z)$, of $f(z) \rightarrow L_n f(z)$ where $L_n f(z) = z^{n+1} \partial f / \partial z$. It is convenient to get rid of

the $3\sqrt{3}/4$ factor by multiplying V^{Witten} by $\exp[-\ln(3\sqrt{3}/4) \sum_r (L_0^r - 1)]$. The second step is to find a set of exponentials of Virasoro generators which transform z into $\Omega(z)$. It is easy to check that

$$(1 + 1/2 z + 9/16 z^2 + \dots) = \exp(5/16 \sum_r L_2^r) \exp(1/2 \sum_r L_1^r) z \quad (14)$$

The final step is to apply the inverse (see. Eq.(12)) of these transformations to V^{CSV} :

$$\begin{aligned} V^{\text{CSV}} \exp(-1/2 \sum_r L_1^r) \exp(-5/16 \sum_r L_2^r) (1 + \dots) \\ = V^{\text{Witten}} \exp[-\ln(3\sqrt{3}/4) \sum_r (L_0^r - 1)] \end{aligned} \quad (15)$$

Since the Virasoro generators, L_n , for $n \geq 3$ affect $n \geq 3$ levels, Eq.(15) should map all couplings of V^{CSV} into V^{Witten} through the second level. This is the case for all 3·13 couplings (the 3 indicates couplings obtained by cycling the indices) as explicit computation demonstrates. The above method can be carried out to arbitrarily high order to obtain explicitly the exponential sum of Virasoro generators which map V^{CSV} into V^{Witten} .

This completes the demonstration that the Witten vertex is conformally equivalent to the Caneschi - Schwimmer - Veneziano.

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FIGURE CAPTIONS

Figure 1. Regions of Convergence for Eq.(3).

The slanted lines, dotted region, and horizontal lines indicate respectively the regions where A_1 , A_2 and A_3 may be non-zero. The plane is broken up into six regions: three regions where only one A is non-zero and three regions where two A 's are non-zero. The latter correspond to where two strings can overlap: A_1 and A_2 non-zero corresponds to $|z_1| < 1$ and $|z_1 - 1| > 1$; A_2 and A_3 non-zero corresponds to $|z_1 - 1| > 1$ and $\text{Real}(z_1) > 1/2$; A_3 and A_1 non-zero corresponds to $\text{Real}(z_1) > 1/2$ and $|z_1| < 1$. The boundaries of the six regions have two points in common: one at $Z_0 = 1/2 + \sqrt{3}/2 i$ and one at $Z_0^* = 1/2 - \sqrt{3}/2 i$.

Figure 2. Integration Curves for Eq.(5).

With the arrows as drawn, the integral for string 1 (along curves $A_2 = 0$ and $A_3 = 0$) and string 2 (along curves $A_3 = 0$ and $A_1 = 0$) convert into closed contours. The integral for string 3 (along curves $A_1 = 0$ and $A_2 = 0$ for which one arrow is in the wrong direction) also converts into a closed contour when the incompatibility of Eq.(3) is taken into account.

$|z_1| < 1$

KEY: $|z_2| < 1$

$|z_3| < 1$

Figure 1

Figure 2