Submitted to
Nuclear Physics

MPI-PAE/PTh 29/86

Electroweak Corrections on the Toponium Resonance

B. GRZADKOWSKI*, J. H. KÜHN

Max-Planck-Institut für Physik und Astrophysik

– Werner-Heisenberg-Institut für Physik –

P.O.Box 40 12 12, Munich (Fed. Rep. Germany)

P. KRAWCZYK**

Institute of Theoretical Physics

Warsaw University

Warsaw (Poland)

and

R. G. STUART***

CERN

Geneva (Switzerland)

ABSTRACT

The full one loop electroweak radiative corrections to the longitudinal polarization asymmetry on the toponium resonance are evaluated. The techniques of the calculation are discussed in some detail. It is shown that to the order of interest the results are unaffected by QCD corrections. The dependence of the result on toponium and Higgs masses is studied. It is demonstrated that, after appropriate modifications, this calculation applies equally well to the forward backward asymmetry in $e^+e^- \rightarrow (t\bar{t}) \rightarrow \mu^+\mu^-$, and to asymmetries in the single quark decay.

* *On leave of absence from Institute of Theoretical Physics, Warsaw University, Warsaw (Poland)*

** *Work supported in part by Polish Ministry of Science, High Education and Technology, Problem MR.I.7.*

*** *Address after 1. September 1986: Max-Planck-Institut für Physik und Astrophysik, Munich*

1. Introduction

The Glashow-Salam-Weinberg model has been impressively confirmed in low energy experiments and through the recent discovery of the intermediate bosons. This has lent strong support to the idea that it will not only play the role of an effective theory, but also that the quantum corrections are an important ingredient. These should and will be tested in further experiments, in particular at the electron positron colliders presently under construction. The planned precision measurements will not only serve to test the theory but might eventually also allow to deduce indirectly information on heavy degrees of freedom like the Higgs structure of the theory, additional generations, etc.

A number of calculations of radiative corrections have been published to date [1]. These are limited to processes with bosons and light fermions only. In this paper we shall present a calculation of one loop corrections to asymmetries on a toponium resonance. In general reactions with top quarks in the final state are subject to uncertainties from hadronic interactions. Although QCD radiative corrections to asymmetries have been calculated [2] and the verification of the weak isospin assignment of top quarks seems possible, the remaining uncertainty is presumably too large to test electro-weak radiative corrections. The situation is more favorable if one studies asymmetries on the toponium resonance. In this paper we will show that uncertainties from QCD corrections are irrelevant to the order under consideration. Furthermore one expects to run on a toponium resonance for an appreciable length of time and one has various observable asymmetries with different systematic errors which, however, all measure the same theoretical quantity. Lastly one may hope for substantial reductions in the spread of the centre of mass energy [3] in electron positron collisions which would enhance the resonance and could help to improve the measurements substantially.

The following paper is organized as follows: In section 2 we briefly recapitulate the asymmetry measurements on toponium together with the lowest order results. In section 3 we present the basic formalism to calculate the one loop corrections. We specify our renormalization scheme and introduce the bound state formalism for nonrelativistic quarkonia. Since this is the first calculation of electroweak radiative corrections which involves a bound state of heavy quarks a more

detailed discussion of technical aspects seems appropriate. In section 4 we discuss qualitative aspects of our result and the absence of QCD corrections up to the order of interest. We present our numerical results and outline the experimental prospects. Section 5 deals with the extension of our work to other asymmetries. Section 6 summarizes our findings. Much of the tedious calculation is relegated to the appendix. A brief summary of our results has been presented in [4, 5].

2. Asymmetry measurements on toponium

The importance of weak interactions for toponium production and decay has been emphasized previously [6]. In particular the interference between neutral and electromagnetic current contribution leads, for polarized beams, to asymmetries in the resonance cross section

$$\alpha_{RL} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L} \quad (2.1)$$

which are different from those in the continuum.

For unpolarized beams one may study the forward backward asymmetry of muon pairs. It is given by

$$\alpha_{FB} = \frac{4 N_F - N_B}{3 N_F + N_B} \quad (2.2)$$

For unpolarized beams the forward backward asymmetry for leptons from the weak decay of a single top quark can be used also, since this quantity serves to measure the polarization of the produced toponium state. The expectation value of the spin component parallel to the electron beam is then given by

$$\langle S_z \rangle = \frac{1}{f} \frac{N_F - N_B}{N_F + N_B} \quad (2.3)$$

where f denotes the analysing power of leptons from single quark decays and in a simple model is given by $3/4$ [7]. For toponium sufficiently far away from the Z and for sufficiently large beam energy spread the interference between resonance and continuum amplitude can be neglected. In lowest order polarization and forward backward asymmetries of muon pairs are simply related [8] by

$$\alpha_{FB} = \alpha_{RL}^2 \quad (2.4)$$

Also the polarization of toponium can be expressed through α_{RL} [7] as

$$\langle S_z \rangle = \alpha_{RL} \quad (2.5)$$

In this paper we do not want to touch upon the questions of interference between resonance and continuum which has been studied in ref [6] and of the model dependence of the depolarization factor f which appears in equ.2.3. The subsequent results are therefore immediately applicable for polarization asymmetries with final states which are characteristic for toponium decays and do not appear in the continuum, like the three gluon or the single quark decay modes. Apart from the peculiar situation of near degeneracy between toponium and Z the latter has a sizeable branching ratio and a characteristic experimental signature [7, 9]. We should emphasize, however, that our results can be immediately extended to the asymmetries described above, as discussed in more detail in section 5.

The amplitude for the production of toponium in electron positron collisions can be written in terms of reduced helicity amplitudes $h_{R,L}$ as

$$A = ie^2 f_v \bar{v} \not{E} (h_R \gamma_R + h_L \gamma_L) u \quad (2.6)$$

E and M denote the toponium polarization and mass. f_v is defined by the matrix element of the heavy quark vector current and can be calculated in terms of the boundstate wave function. In the nonrelativistic boundstate model it is given by the wave function at the origin:

$$f_v = \frac{R(0)}{\sqrt{\pi M^3}} \quad (2.7)$$

For final states which do not interfere with the continuum the polarization asymmetry is given through

$$\alpha_{RL} = \frac{h_R^2 - h_L^2}{h_R^2 + h_L^2} \quad (2.8)$$

This, and equ.2.6 holds true also with one loop corrections included. The mass of the electron will be neglected throughout. The prediction for the helicity amplitudes will be based on α , and the masses of the intermediate bosons - however, we only use such mass combinations which correctly (i. e. up to one loop order) reproduce the measured muon life time.

In lowest order the helicity amplitudes can be expressed in terms of M_Z and the electroweak couplings as [6]

$$h_{R,L} = Q^e Q^t + \beta_{R,L}^e \frac{\beta_R^t + \beta_L^t}{2} \frac{M^2}{M^2 - M_Z^2}$$

$$\beta_R^f = \frac{-s^2 Q^f}{sc}; \quad \beta_L^f = \frac{t_3^f - s^2 Q^f}{sc} \quad (2.9)$$

$$s \equiv \sin^2 \theta_W; \quad c \equiv \cos^2 \theta_W \equiv M_W^2 / M_Z^2$$

where Q and I_3 denote fermion charge and weak isospin. The asymmetry is then evidently independent of f_ν .

Note that it is only the vector party of the neutral current coupling which contributes at the toponium vertex. This is one of the reasons for the strong sensitivity of α_{RL} to the Weinberg angle. It is also evident that the asymmetry is independent of the quarkonium wave function in lowest order.

3. One loop corrections

To evaluate the one loop corrections we use the "on shell renormalization scheme" employed already in [10, 11, 12]. The poles of the Z and W propagators are fixed at the masses of the intermediate bosons. This requirement, together with the fine structure constant, the Higgs and the fermion masses fixes the normalization conditions. Since, furthermore, the muon lifetime has been measured with great precision, it is possible to express the W mass and $\sin^2 \theta_W$ in terms of the remaining quantities and we shall use only masses consistent with this requirement.

The radiative corrections are as follows:

- 1) photonic electron vertex corrections plus initial state radiation,
- 2) weak electron vertex corrections,
- 3) corrections to the A and Z propagators,
- 4) A - Z mixing,
- 5) corrections to the toponium A and Z vertices,
- 6) box diagrams.

The purely photonic corrections are irrelevant for the polarization asymmetry on a narrow resonance as discussed below. Corrections 2-4 have been published

previously and we simply list the results from [11] in a more compact form in appendix B.

The central part of this paper consists in the evaluation of the toponium $-A$ and $-Z$ vertex corrections (Fig. 1a-k) and of the box diagrams (Fig. 1l-n). Corrections from Φ_Z , Higgs and photon exchange (Fig. 1h-k) contribute with equal strength at the A and Z vertex, are thus irrelevant for the asymmetry and could be dropped, together with the appropriate contributions from the counterterms. We nevertheless give the full weak corrections for completeness, apart from the photon exchange contribution (Fig. 1d), which can be found in the literature [13]. The amplitude which contains a Higgs $-Z$ coupling (Fig. 1k) is finite by itself and, among others, will be responsible for the Higgs mass dependence of the final answer. Scalar exchange in the s channel does not contribute in the limit of vanishing electron mass.

The evaluation of box diagrams is tedious but straightforward. We note that all these amplitudes are separately infrared and ultraviolet finite.

Since we are interested in the production of a vector bound state we have to combine the quark and antiquark wave functions with the appropriate spins and angular momentum into a 3S_1 configuration. For the decay this amounts effectively to the substitution [14]

$$\bar{v}O u \Rightarrow \frac{f_v M}{4} \text{Tr}\{O(+\not{P} + M)\not{E}\} \quad (3.1)$$

and similarly for the production

$$\bar{u}O v \Rightarrow \frac{f_v M}{4} \text{Tr}\{O(-\not{P} + M)\not{E}^*\} \quad (3.2)$$

O stands for an arbitrary combination of gamma matrices and may depend on momenta and Lorentz indices. Quark spinors will thus appear only in intermediate steps but not in the final amplitudes.

As we discuss in more detail below it is possible to reduce the loop integrands from tensors to scalars. After the integration the results for the helicity amplitudes will depend on masses only and not on any external momentum. Furthermore, the vertex correction can be expressed as a sum of two point functions and the box diagram as a sum of three point functions.

We now proceed with the detailed discussion of the vertex corrections depicted in Fig. 1a-i.

We use dimensional regularization and employ the n -dimensional γ -algebra, with

$$\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu} \quad (3.3)$$

and employ an anticommuting γ_5 . All terms proportional to $1/(n-4)$ will cancel against the appropriate counter terms and the result is therefore independent of the definition of $\text{Tr}(1)$ as long as it approaches 4 in the limit $n \rightarrow 4$.

We perform the trace and split off an overall factor $ie\frac{\alpha}{4\pi}f_vM^2$. The dimensionally regularized corrections to the A and Z toponium couplings are then given by

$$aE_\mu = \int \frac{d^n q}{i\pi^2} J_\mu \quad (3.4)$$

$$\begin{aligned} J_\mu(W, W) &= \left(\frac{1}{c/s}\right) \frac{t_3^t}{s^2} \frac{q^2 E_\mu + (n-2)q_\mu q E}{q^2((q - \frac{P}{2})^2 - M_W^2)((q + \frac{P}{2})^2 - M_W^2)} \\ J_\mu(W, \Phi_W) &\sim \frac{q_\mu}{q^2((q - \frac{P}{2})^2 - M_W^2)((q + \frac{P}{2})^2 - M_W^2)} \\ J_\mu(\Phi_W, \Phi_W) &= \left(\frac{1}{\frac{c^2-s^2}{2sc}}\right) \frac{t_3^t}{s^2} \frac{M^2}{4M_W^2} \frac{q E q_\mu}{q^2((q - \frac{P}{2})^2 - M_W^2)((q + \frac{P}{2})^2 - M_W^2)} \\ J_\mu(Z, H) &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{\beta_R^t + \beta_L^t}{4s^2 c^2} \frac{(-\frac{M^2}{2} + qP) E_\mu}{(q^2 - \frac{M^2}{4})((q + \frac{P}{2})^2 - M_H^2)((q - \frac{P}{2})^2 - M_Z^2)} \\ J_\mu(W) + J_\mu(\Phi_W) &= \begin{pmatrix} -Q_t + 2t_3^t \\ -\beta_L^t + 2t_3^t \frac{c}{s} \end{pmatrix} \frac{1}{2s^2} \left(\frac{n-2}{2} + \frac{M^2}{8M_W^2}\right) \frac{(\frac{M^2}{4} - q^2) E_\mu + 2q E q_\mu}{(q^2 - M_W^2)(q + \frac{P}{2})^2 (q - \frac{P}{2})^2} \\ J_\mu(Z) &= \begin{pmatrix} -Q_t(\beta_R^{t^2} + \beta_L^{t^2}) \\ -(\beta_R^{t^3} + \beta_L^{t^3}) \end{pmatrix} \left(\frac{n-2}{2}\right) \frac{2q_\mu q E - q^2 E_\mu}{(q^2 - M_Z^2)(q^2 + qP)(q^2 - qP)} \\ &\quad + \begin{pmatrix} -Q_t^t \\ -\frac{\beta_R^t + \beta_L^t}{2} \end{pmatrix} \frac{\beta_R^{t^2} + \beta_L^{t^2}}{2} \left(\frac{n-2}{2}\right) \frac{M^2 E_\mu}{(q^2 - M_Z^2)(q^2 + qP)(q^2 - qP)} \\ J_\mu(\Phi_Z) &= \begin{pmatrix} -Q_t^t \\ -\frac{\beta_R^t + \beta_L^t}{2} \end{pmatrix} \frac{1}{s^2} \frac{M^2}{16M_W^2} \frac{2q_\mu q E - q^2 E_\mu}{(q^2 - M_Z^2)(q^2 + qP)(q^2 - qP)} \\ J_\mu(H) &= \begin{pmatrix} -Q_t^t \\ -\frac{\beta_R^t + \beta_L^t}{2} \end{pmatrix} \frac{1}{s^2} \frac{M^2}{16M_W^2} \frac{2q_\mu q E - q^2 E_\mu + M^2 E_\mu}{(q^2 - M_H^2)(q^2 + qP)(q^2 - qP)} \end{aligned}$$

The amplitudes are listed in the same order as in Fig. 1 and the upper and lower entries refer to A and Z couplings respectively.

Terms proportional to EP have been dropped since toponium is on shell, terms proportional to P_μ since it is connected to the conserved lepton current.

The final answer depends on the scalar helicity amplitudes only. It should therefore be possible to reduce also the factor $(Eq)q_\mu$ in the integrand to scalar form. This can be achieved since after integration $q_\mu q_\nu$ will lead to terms proportional to $g_{\mu\nu}$ and $P_\mu P_\nu$. Only the former contribute and these can be obtained if we substitute

$$Eq q_\mu \rightarrow E_\mu (g_{\alpha\beta} - \frac{P_\alpha P_\beta}{P^2}) q^\alpha q^\beta / (n-1) \quad (3.5)$$

The integrands can now be decomposed into a sum of two point functions, since the propagators depend on q^2 , qP and masses only. The first integrand in equ.3.4 for example can be reexpressed as

$$\begin{aligned} & \frac{q^2 E_\mu + (n-2)qEq_\mu}{q^2((q - \frac{P}{2})^2 - M_W^2)((q + \frac{P}{2})^2 - M_W^2)} \\ \rightarrow & E_\mu \frac{q^2 + \frac{n-2}{n-1}(q^2 - (qP)^2/P^2)}{q^2((q - \frac{P}{2})^2 - M_W^2)((q + \frac{P}{2})^2 - M_W^2)} \\ \rightarrow & E_\mu \left[\frac{1 + \frac{n-2}{n-1}}{((q + \frac{P}{2})^2 - M_W^2)((q - \frac{P}{2})^2 - M_W^2)} + \frac{\frac{n-2}{n-1}qP/P^2}{q^2((q + \frac{P}{2})^2 - M_W^2)} \right] \end{aligned} \quad (3.6)$$

and similarly for the other terms.

We then obtain for the toponium- A and - Z vertex corrections

$$\begin{aligned} a(W, W) &= \left(\frac{1}{c/s} \right) \frac{2t_3^t}{s^2} \frac{1}{12} \left\{ -1 + 10B_0(P, M_W, M_W) + 2B_1\left(\frac{P}{2}, 0, M_W\right) \right\} \\ a(W, \Phi_W) &= 0 \\ a(\Phi_W, \Phi_W) &= \left(\frac{1}{\frac{c^2-s^2}{2sc}} \right) \frac{2t_3^t}{s^2} \frac{M^2}{48M_W^2} \left\{ 1 + 2B_0(P, M_W, M_W) + B_1\left(\frac{P}{2}, 0, M_W\right) \right\} \\ a(Z, H) &= \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{\beta_R^t + \beta_L^t}{4s^2 c^2} \frac{1}{M_Z^2 + M_H^2 - M^2} \left\{ (-M_Z^2 + M_H^2 - M^2)B_0(P, M_H, M_Z) \right. \\ & \quad \left. + M_Z^2 B_0\left(\frac{P}{2}, \frac{M}{2}, M_Z\right) + (-M_H^2 + M^2)B_0\left(\frac{P}{2}, \frac{M}{2}, M_H\right) \right\} \\ a(W) + a(\Phi_W) &= \begin{pmatrix} -Q^t + 2t_3^t \\ -\beta_L^t + 2t_3^t \frac{c}{s} \end{pmatrix} \frac{1}{6s^2} \left\{ \frac{5}{2} + \frac{M^2}{8M_W^2} \right\} \end{aligned}$$

$$\begin{aligned}
& + (1 + \frac{M^2}{8M_W^2})(2\frac{M^2 - 4M_W^2}{M^2 + 4M_W^2}B_0(\frac{P}{2}, 0, M_W) \\
& - \frac{4M^2}{M^2 + 4M_W^2}B_0(P, 0, 0) - B_1(\frac{P}{2}, 0, M_W)) \} \\
a(Z) = & \left(\frac{-Q^t(\beta_R^{t^2} + \beta_L^{t^2})}{-(\beta_R^{t^3} + \beta_L^{t^3})} \right) \frac{1}{3} \left\{ \frac{5}{2} - B_0(\frac{P}{2}, M_Z, M/2) + B_1(\frac{P}{2}, M_Z, \frac{M}{2}) \right\} \\
& + \left(\frac{-Q^t}{-\frac{\beta_R^t + \beta_L^t}{2}} \right) \frac{\beta_R^{t^2} + \beta_L^{t^2}}{2} \frac{M^2}{M_Z^2} (B_0(\frac{P}{2}, M_Z, \frac{M}{2}) - B_0(P, \frac{M}{2}, \frac{M}{2})) \\
a(\Phi_Z) = & \left(\frac{-Q^t}{-\frac{\beta_R^t + \beta_L^t}{2}} \right) \frac{1}{s^2} \frac{M^2}{48M_W^2} \left\{ 1 - B_0(\frac{P}{2}, M_Z, \frac{M}{2}) + B_1(\frac{P}{2}, M_Z, \frac{M}{2}) \right\} \\
a(H) = & \left(\frac{-Q^t}{-\frac{\beta_R^t + \beta_L^t}{2}} \right) \frac{1}{s^2} \frac{M^2}{48M_W^2} \left\{ 1 + (3\frac{M^2}{M_H^2} - 1)B_0(\frac{P}{2}, M_H, \frac{M}{2}) \right. \\
& \left. + B_1(\frac{P}{2}, M_H, \frac{M}{2}) - 3\frac{M^2}{M_H^2}B_0(\frac{P}{2}, \frac{M}{2}, \frac{M}{2}) \right\}
\end{aligned} \tag{3.7}$$

The functions B are defined in the standard manner [15] and a collection of formulas is listed in the appendix. The divergencies cancel against those from the counter terms which are derived from the corrections to the top quark propagator shown in Fig. 2 plus charge and field renormalizations. They lead to the following counterterms:

$$\begin{aligned}
a_c^A &= ief_v M^2 (Q^t(\delta Z_R + \delta Z_L)/2 + t_3^t \delta k) \\
a_c^Z &= ief_v M^2 ((\beta_R^t \delta Z_R + \beta_L^t \delta Z_L)/2 + \frac{c}{s} t_3^t \delta k) \\
\delta Z_{R,L} &= A_{R,L} + m^2 (A'_R + A'_L - 2C') \\
\delta k &= -\frac{\alpha}{4\pi s^2} B_0(0, M_W, M_W) \\
A_R &= \frac{\alpha}{4\pi s^2} \left\{ s^2 \beta_R^{t^2} (1 + 2B_1(p, m, M_Z)) \right. \\
& \quad \left. + \frac{m^2}{4M_W^2} (B_1(p, m, M_Z) + B_1(p, m, M_H) + 2B_1(p, m_b, M_W)) \right\} \\
A_L &= \frac{\alpha}{4\pi s^2} \left\{ s^2 \beta_L^{t^2} (1 + 2B_1(p, m, M_Z)) + \frac{1}{2} + B_1(p, m_b, M_W) \right. \\
& \quad \left. + \frac{m^2}{4M_W^2} (B_1(p, m, M_Z) + B_1(p, m, M_H)) + \frac{m_b^2}{2M_W^2} B_1(p, m_b, M_W) \right\} \\
C &= \frac{\alpha}{4\pi s^2} \left\{ 2s^2 \beta_R^t \beta_L^t (1 - 2B_1(p, m, M_Z)) \right.
\end{aligned} \tag{3.8}$$

$$+ \frac{m^2}{4M_W^2} (B_0(p, m, M_H) - B_0(p, m, M_Z)) - \frac{m_b m}{2M_W^2} B_0(p, m_b, M_W) \}$$

$$m = M/2; \quad m_b = 0; \quad p \equiv P/2; \quad A' \equiv \frac{dA}{dp^2} (p^2 = m^2).$$

The sum of vertex corrections and contributions from the counterterms is finite. In the mass region above M_W a top quark may decay into a W plus a bottom quark, and $B(m, 0, M_W)$ develops an imaginary part. However, this does not interfere with the Born term and thus does not contribute to order α .

The box diagrams can be tackled in a rather straightforward way since they are evidently ultraviolet finite. It is, however, noteworthy that all contributions from box diagrams, in particular also those from $Z\gamma$ exchange are also infrared finite, essentially owing to the fact that toponium is a neutral state, which implies that the Z -photon-toponium coupling vanishes for soft photons. This result is also expected from the infrared structure of the vertex corrections. The only source of infrared divergencies are photonic corrections to the electron vertex, which are compensated separately by initial state bremsstrahlung. Infrared divergencies at the toponium vertex from QED are absorbed in the toponium wave function. Since the rate from the reaction $e^+e^- \rightarrow Z \rightarrow {}^3S_1\gamma$ (possible through the axial coupling of the Z) is infrared finite and even strongly damped for $E_\gamma \rightarrow 0$, the $Z\gamma$ box amplitude has to be separately finite.

Using equ.3.2 the coupling between the nonrelativistic boundstate and two gauge bosons is readily evaluated. After combining with the lepton lines one finds for the full amplitude expressions of the form

$$\int \frac{d^4q}{i\pi^2} \frac{\bar{v}_e T^{\alpha\beta} E_\alpha \gamma_\beta u_e}{N_1 N_2 N_3 N_4} \quad (3.9)$$

$T^{\alpha\beta}$ denotes a tensor and may depend quadratically on internal and external momenta, N_i stand for the relevant propagators. Note that only one permutation contributes to h^{WW} , two to h^{ZZ} and four to $h_{Z\gamma}$. To get rid of the lepton spinors and to reduce again the loop integral to the evaluation of a scalar, it is necessary to transform the term proportional to $q\gamma E(q+l)$ in the above expression to a

simpler form. After integration $q_\alpha(q+l)_\beta$ will lead to a tensor of the form

$$ag_{\alpha\beta} + bP_\alpha P_\beta + cl_\alpha l_\beta + dP_\alpha l_\beta + el_\alpha P_\beta \quad (3.10)$$

Combined with $E_\alpha \gamma_\beta$ only the first term contributes in our case, since

$$\bar{v} \not{P} u = \bar{v} \not{P} \gamma_5 u = \bar{v} \not{l} u = \bar{v} \not{l} \gamma_5 u = 0. \quad (3.11)$$

It can be isolated if we contract with

$$(g_{\alpha\beta} - \frac{P_\alpha P_\beta}{P^2} - \frac{l_\alpha l_\beta}{l^2})/2 \quad (3.12)$$

For the contribution from box diagrams to the helicity amplitudes we find

$$\begin{aligned} h_{R;L}^{WW} &= \frac{\alpha}{4\pi} (0; \frac{-1}{s^4}) \frac{1}{2} F^{WW} \\ h_{R;L}^{ZZ} &= \frac{\alpha}{4\pi} (-\beta_R^e{}^2; \beta_L^e{}^2) (\beta_R^t{}^2 - \beta_L^t{}^2) F^{ZZ} \\ h_{R;L}^{Z\gamma} &= \frac{\alpha}{4\pi} (-\beta_R^e; \beta_L^e) (\beta_R^t - \beta_L^t) Q^e Q^t 2 F^{Z\gamma} \end{aligned} \quad (3.13)$$

where

$$\begin{aligned} F^{WW} &= \int \frac{d^4 q}{i\pi^2} \frac{q^2 + ql}{q^2(q+l)^2((q+\frac{P}{2})^2 - M_W^2)((q-\frac{P}{2})^2 - M_W^2)} \\ F^{ZZ} &= \int \frac{d^4 q}{i\pi^2} \frac{q^2 + 2ql + (qP)^2/P^2 + (ql)^2/l^2}{(q^2 - \frac{M^2}{4})(q+l)^2((q+\frac{P}{2})^2 - M_Z^2)((q-\frac{P}{2})^2 - M_Z^2)} \\ F^{Z\gamma} &= \int \frac{d^4 q}{i\pi^2} \frac{(q+\frac{P}{2}+l)^2 + (qP)^2/P^2 + (ql)^2/l^2}{q^2(q+\frac{P}{2}+l)^2((q+\frac{P}{2})^2 - \frac{M^2}{4})((q+P)^2 - M_Z^2)} \end{aligned} \quad (3.14)$$

Note that these integrals are symmetric under the substitutions $l \rightarrow -l$ and $P \rightarrow -P$ separately, since $Pl = 0$. The quantities q^2 , qp and ql can be expressed as a linear combination of the inverse propagators appearing in the integrand and, furthermore, these four inverse propagators are linearly dependent. It is therefore possible to decompose the integrand into a sum of two and three point functions. Then, making use of identities between various three point functions which are discussed in the appendix E of [15], one arrives at the following compact expressions for the box amplitudes:

$$\begin{aligned}
F^{WW} &= \frac{2M^2}{4M_W^2 - M^2} \left\{ B_0(P, M_W, M_W) - B_0\left(\frac{P}{2}, 0, M_W\right) \right. \\
&\quad \left. - M_W^2 C_0\left(\frac{P}{2} + l, -P, 0, M_W, M_W\right) + \frac{M^2}{4} C_0\left(l, \frac{P}{2} - l, 0, 0, M_W\right) \right\} \\
F^{ZZ} &= \frac{1}{\frac{1}{2} - u} \left\{ B_0\left(\frac{P}{2}, \frac{M}{2}, M_Z\right) - B_0(P, M_Z, M_Z) \right. \\
&\quad - \left(\frac{1}{2} - u\right) \left(B_0\left(l, 0, \frac{M}{2}\right) - B_0(P, M_Z, M_Z) \right) \\
&\quad + \left(-u^2 + u - \frac{1}{2}\right) M^2 C_0\left(l, \frac{P}{2}, 0, \frac{M}{2}, M_Z\right) \\
&\quad \left. + (1 - u) M_Z^2 C_0\left(\frac{P}{2} + l, -P, 0, M_Z, M_Z\right) \right\} \\
F^{Z\gamma} &= \frac{1}{u - 1} \left(B_0\left(\frac{P}{2}, \frac{M}{2}, 0\right) + B_0\left(\frac{P}{2}, \frac{M}{2}, M_Z\right) - 2B_0(P, 0, M_Z) \right) \\
&\quad + \left(B_0(P, 0, M_Z) - B_0\left(l, 0, \frac{M}{2}\right) \right) + (u - 1) M^2 C_0\left(l, \frac{P}{2}, 0, \frac{M}{2}, M_Z\right)
\end{aligned} \tag{3.15}$$

The first term in $F^{\gamma Z}$ has its origin in

$$\int \frac{d^4 q}{i\pi^2} \{ q^2 (q^2 + qP) ((q + P)^2 - M_Z^2) \}^{-1} \tag{3.16}$$

and after decomposition in terms of B 's it seems to be enhanced by a factor reminiscent of the Z -propagator. However, it is multiplied by a sum of B -functions which vanishes for $M \rightarrow M_Z$ and is therefore less important than the leading terms which are amplified by the Z -propagators. The next term in the expansion in $M^2 - M_Z^2$ is logarithmically divergent. However, even for this rather special case the divergence can be regulated if the toponium binding energy is reintroduced in the quark propagator and the nonvanishing width of the Z is taken care of. The enhancement proportional to the logarithm of Γ_Z/M_Z and E_{Binding}/M_Z cannot compete with the resonance enhancement of the leading amplitude.

The three point functions C which appear in this context are all finite for arbitrary mass values. They are of a particularly simple form owing to the fact that at least one internal mass is zero and that they depend on one mass ratio only. A collection of formulas which are useful in the evaluation of the C 's is listed in appendix A. One finds

$$\begin{aligned}
M^2 C_0\left(\frac{P}{2} + l, -P, 0, M_Z, M_Z\right) &= -\ln^2\left(\frac{\sqrt{1-4u}+1}{\sqrt{1-4u}-1} - i\epsilon\right) \\
M^2 C_0\left(l, \frac{P}{2} - l, 0, 0, M_Z\right) &= 2(\pi^2/4 - Sp(1 - \frac{1}{2u}) + Sp(\frac{1}{2u} - 1 + i\epsilon)) \\
M^2 C_0\left(l, \frac{P}{2}, 0, \frac{M}{2}, M_Z\right) &= \pi^2/6 - Sp(1 - \frac{1}{u}) \\
u &\equiv M_Z^2/M^2; \quad l \equiv (p_{e^-} - p_{e^+})/2.
\end{aligned} \tag{3.17}$$

With these ingredients and together with the expressions for lepton vertex corrections, $Z - A$ mixing and self energies listed in appendix B the helicity amplitudes are given by

$$\begin{aligned}
h_R &= Q^e \left(1 - \frac{\Pi_R^{AA}}{M^2}\right) Q^t + \beta_R^e \frac{M^2}{M^2 - M_Z^2 + \Pi_R^{ZZ}} \frac{\beta_R^t + \beta_L^t}{2} \\
&\quad - \left(Q^e \frac{\beta_R^t + \beta_L^t}{2} + \beta_R^e Q^t\right) \Pi_R^{ZA} \frac{1}{M^2 - M_Z^2 + \Pi_R^{ZZ}} \\
&\quad + F_R^A Q^t + F_R^Z \frac{M^2}{M^2 - M_Z^2 + \Pi_R^{ZZ}} \frac{\beta_R^t + \beta_L^t}{2} \\
&\quad + Q^e \frac{\alpha}{4\pi} \sum a^A + \beta_R^e \frac{M^2}{M^2 - M_Z^2 + \Pi_R^{ZZ}} \frac{\alpha}{4\pi} \sum a^Z \\
&\quad + h_R^{WW} + h_R^{ZZ} + h_R^{Z\gamma}
\end{aligned} \tag{3.18}$$

and similarly for h_L .

4. Discussion of results

Compared to the lowest order formulas of equ.2.9 the individual helicity amplitudes are changed substantially by the one loop corrections, since both electromagnetic and neutral current amplitude receive there dominant correction from the light fermion vacuum polarization. This leads for both to an increase by a factor $(1 - \Delta r)^{-1} \approx 1.07$. The change is far less pronounced for their ratio and thus for the asymmetries. This has been anticipated already in [4] on the basis of leading log arguments and is borne out by this full calculation.

In Fig. 3a we display the result of the full calculation for the asymmetry and the size of the first order correction. We have chosen as representative values

$M_Z = 94\text{GeV}$ and $M_H = 100\text{GeV}$. Since we require to reproduce with our set of input parameters correctly the muon life time, a change of $m_t \equiv M/2$ leads to a slight change of M_W and of the Weinberg angle as shown in Fig. 3b. We should stress that the same input parameters α , M_W and M_Z have been used in lowest and first order. Instead of α , M_W and M_Z one may also take directly α , G_μ , and M_Z as input parameters. Employing then tree and one-loop relations between these two sets of input parameters in the evaluation of $\alpha_{RL}^{(0)}$ and $\alpha_{RL}^{(1)}$, respectively, the size of the corrections is increased significantly, $\alpha_{RL}^{(1)}$, however, remains unchanged up to terms of order α^2 [4].

One of the motivations for this work was the hope to gain information on the Higgs coupling to fermions through the amplitude depicted in Fig. 1d. However, quite similar to the “screening” of Higgs effects in other instances [16] also here the Higgs mass dependence turns out to be rather weak. In fact, the contribution from the vertex correction $a(Z, H)$ vanishes for $M_H \rightarrow \infty$. If we drop $a(Z, H)$ altogether to test the sensitivity of our calculation on the Higgs fermion coupling the asymmetry is changed by less than 0.001 for toponium masses below 160 GeV. There remains, however, the contribution from the Higgs particle to the Z self energy and in addition the indirect dependence on M_H through the variation of the Weinberg angle. To display these effects, we show in Fig. 4a the variation of the asymmetry for different Higgs masses if we include both these effects. In Fig. 4b the Weinberg angle has been fixed to the value calculated for $M_H = 100\text{GeV}$ and M_H varied to 10GeV and 1000GeV . The difference between the extreme choices of the Higgs mass may be up to 0.025.

The size of the box diagrams is illustrated in Fig. 5 which gives the change in the asymmetry if h^{WW} or $h^{Z\gamma}$ is dropped. For toponium in the low mass region this change is small, but also in the high mass region it is not particularly important. Only the contribution from the WW intermediate state, which is enhanced by $\sin^2\theta_W$, amounts up to 0.02 in the high mass region. The corresponding effects from h^{ZZ} are below 0.001. As is evident from these figures, the truly weak corrections are quite small and with a characteristic magnitude of 0.01-0.03.

Finally we should comment on the role of QED and QCD corrections. It is well known that QED corrections to the asymmetry in the continuum are ex-

tremely important and partly acceptance dependent. In our case the situation is far simpler, as discussed in more detail in appendix *C*. Since the bound state is neutral, final state radiation can be ignored and the box diagram is infrared finite. The corrections to the photon propagator from the vacuum polarization Π_{AA} has to be and was easily accommodated in our numerical evaluation. The contribution from light quarks is taken into account in the standard manner with the help of dispersion relations [11]. The top contribution has been evaluated without QCD corrections. We have checked, that this is a valid approximation despite the close proximity of toponium and open top threshold. The momentum which flows through the photon and Z propagators is nearly fixed by the resonance mass since the small variation due to the tiny toponium width can safely be ignored and the helicity amplitudes can be treated just as constants. Corrections from initial state bremsstrahlung together with those from the photonic virtual corrections to the electron vertex thus affect only the overall normalization and not the relative importance of h_R versus h_L . This is of course quite different from the situation in the continuum where the invariant mass e. g. of the muon pair is allowed to vary.

QCD corrections can be safely ignored to order α . The helicity amplitudes are given as a power series in α and α_s . The correction of order α_s affects only the overall normalization and can be absorbed in f_ν . Corrections of order $\alpha\alpha_s$ have two qualitatively different sources. Some originate from QCD corrections at the toponium vertex together with electroweak corrections at the lepton vertex or at the boson propagators. These can be essentially taken care of by the definition of f_ν and are irrelevant for the asymmetry. The others originate from simultaneous QCD and electroweak corrections at the toponium vertex and have not been calculated. However, these corrections, are not enhanced by large logarithms and thus truly of order α_s/π compared to those evaluated in this work.

5. Application to other asymmetries on toponium

As mentioned already in chapter 2, our results for the polarization asymmetry are directly applicable for final states which do not interfere with the continuum. They are valid before and after smearing with the beam energy resolution. The same holds true for the polarization of toponium produced with unpolarized beams which is given through equ.2.3 . However, leptons from SQD's are used to analyse this polarization and the resulting spectra are subject of QCD and electroweak corrections. It remains to be seen, whether the simple result for the analysing power of leptons, $f = 3/4$, remains valid after these corrections have been calculated.

For lepton or quark pairs in the final state one has to incorporate interference with the continuum. With the formulas given in [6] for cross sections before and after folding with the beam energy resolution these effects are easily calculated. For toponium away from the Z and after folding, the interference effects are typically smaller than the corrections discussed in this paper and one may approximate the cross section by an incoherent sum of resonance and continuum.

The situation becomes slightly more involved for the forward backward asymmetry of μ^- or τ^- pairs. As far as purely weak corrections are concerned, equ.2.4 remains valid under the corresponding assumptions. Initial state radiation of photons can still be ignored, since the arguments for the polarization asymmetry hold true also in this case. Final state radiation, however, will affect the asymmetry. The size of this correction depends on the experimental cuts and is best taken into account though Monte Carlo calculations.

The experimental prospects for these measurements depend of course on the presently unknown mass of toponium, on the question whether longitudinally polarized beams will be available and on possible reductions of the center of mass energy spread [9]. Given an integrated luminosity of $100pb^{-1}$, and using semileptonic single quark decays (μ^- and τ^- pairs) to analyse toponium polarization, an accuracy better than 0.05 (between 0.025 and 0.10 depending on the mass) can be achieved with LEP 1 even with unpolarized beams. With longitudinal polarization and assuming the same luminosity, the statistical error could be reduced to less than 0.02. A reduction of the energy spread below the present design value would lead to a further improvement of these numbers. The big corrections from

the light fermion vacuum polarization are thus easily accessible experimentally, the remaining ones will require some effort.

6. Summary and conclusions

The complete one loop electroweak radiative corrections have been calculated to polarization asymmetries on the toponium resonance. It has been shown that these results are not affected by QCD corrections and, furthermore, that also initial state bremsstrahlung and QED vertex corrections do not modify our results. In a renormalization scheme which uses α , M_W and M_Z as input parameters, the corrections to the asymmetries are small. This is in contrast to a scheme based on α , M_W and G_μ where the difference between Born term and corrected result may amount up to 0.20. The asymmetry depends not only on the coupling between the Higgs and the gauge bosons but also on the Higgs-fermion coupling. This makes this measurement potentially interesting - in the standard model, however, this latter dependence turns out to be rather weak.

These results are of immediate relevance also for other asymmetries, notably the forward backward asymmetry of μ -pairs on a toponium resonance and the asymmetry of leptons from the single quark decay. The former is given by the square of α_{RL} plus detector and acceptance dependent QED corrections. Since polarized beams may not be available in the first round of experiments these will serve as an equally suited test of radiative corrections. The latter, however, depends on as yet uncalculated QCD corrections to the single quark decay and the interpretation of precision measurements will in this case require some caution.

Appendix A.

For clarity and convenience we shall list some properties of the two and three point form factors B and C , following [17,15] using, however, the metric $diag(g_{\mu\nu}) = (+, -, -, -)$. The B 's relevant for this paper are defined through

$$\{B_0, p_\mu B_1, p_\mu p_\nu B_{21} - g_{\mu\nu} B_{22}\} \equiv \int \frac{d^n k}{i\pi^2} \frac{\{1, k_\mu, k_\mu k_\nu\}}{(k^2 - m_1^2 + i\epsilon)((k+p)^2 - m_2^2 + i\epsilon)} \quad (A.1)$$

They are given by

$$\begin{aligned} & \{B_0, B_1, B_{21}\} \\ & = \{1, -\frac{1}{2}, \frac{1}{3}\} \Delta - \int_0^1 dx \{1, -x, x^2\} \ln(m_1^2(1-x) + (m_2^2 - p^2)x + p^2x^2 - i\epsilon) \end{aligned} \quad (A.2)$$

where

$$\Delta = \pi^{n/2-2} \Gamma(2 - \frac{n}{2}) \longrightarrow (\frac{2}{4-n} - \gamma - \ln \pi) \quad (A.3)$$

The following identities are used:

$$\begin{aligned} 2p^2 B_1(p, m_1, m_2) &= (m_1^2 - m_2^2) B_0(0, m_1, m_2) - (p^2 + m_1^2 - m_2^2) B_0(p, m_1, m_2) \\ 4p^4(n-1) B_{21} &= 4p^2 m_2^2 B_0(0, m_2, m_2) - n(p^2 + m_1^2 - m_2^2)(m_1^2 - m_2^2) B_0(0, m_1, m_2) \\ &\quad + (n(p^2 + m_1^2 - m_2^2)^2 - 4p^2 m_1^2) B_0(p, m_1, m_2) \end{aligned} \quad (A.4)$$

The C 's relevant for our paper are defined through

$$\begin{aligned} & \{C_0, p_\mu C_{11} + q_\mu C_{12}\}(p, q, m_1, m_2, m_3) \\ & \equiv - \int \frac{d^4 k}{i\pi^2} \frac{\{1, k_\mu\}}{(k^2 - m_1^2 + i\epsilon)((k+p)^2 - m_2^2 + i\epsilon)((k+p+q)^2 - m_3^2 + i\epsilon)} \end{aligned} \quad (A.5)$$

C_{11} and C_{12} can be expressed in terms of C_0 and B_0 as discussed in the Appendix of ref [15].

$$\begin{pmatrix} C_{11} \\ C_{12} \end{pmatrix} = -\frac{1}{2} \begin{pmatrix} p^2 & pq \\ pq & q^2 \end{pmatrix}^{-1} \begin{pmatrix} f_1 C_0 + B_0(p+q, m_1, m_3) - B_0(q, m_2, m_3) \\ f_2 C_0 + B_0(p, m_1, m_2) - B_0(p+q, m_1, m_3) \end{pmatrix} \quad (A.6)$$

$$f_1 \equiv m_1^2 - m_2^2 + p^2; \quad f_2 \equiv m_2^2 - m_3^2 + q^2 + 2pq.$$

At least one mass vanishes in all our C 's, which leads to a considerably simplified form. For $m_3 = 0$

$$C_0 = \int_0^1 dy \frac{\ln(a+b) - \ln(b)}{a} \quad (A.7)$$

$$a = (q + yp)^2; \quad b = (m_2^2 - q^2) + y(m_1^2 - m_2^2 - p^2 - 2pq)$$

To arrive at the simple form for $C_0(l, \frac{P}{2}, 0, \frac{M}{2}, M_Z)$, we applied Hills formula. For complex argument it reads

$$\begin{aligned} Sp(xy) &= Sp(x) + Sp(y) + Sp\left(\frac{xy-x}{1-x}\right) + Sp\left(\frac{xy-y}{1-y}\right) \\ &+ \frac{1}{2} \left[\ln\left(\frac{1-x}{1-y}\right) - \eta\left(\frac{1-x}{1-xy}, \frac{1-xy}{1-y}\right) \right]^2 \\ &+ \ln(x) \eta\left(\frac{1}{1-x}, 1-xy\right) + \ln(y) \eta\left(\frac{1}{1-y}, 1-xy\right) \\ \eta(x, y) &\equiv \ln(xy) - \ln(x) - \ln(y) \end{aligned}$$

and has been used for $x = \frac{1}{2}$ and $y = \frac{1-2u}{1-u-i\epsilon}$.

Appendix B.

As mentioned in the introduction the purely weak part of the corrections to the fermion- Z and -photon vertex for massless fermions and the boson self energies and mixings are given in the literature [15, 18, 11, 19]. However, for completeness and clarity we specify our input, based on Ref. 11.

The renormalized vertices are

$$\Gamma = ie\gamma_\mu \left[\begin{pmatrix} Q \\ \beta_R \gamma_R + \beta_L \gamma_L \end{pmatrix} + \gamma_R F_R + \gamma_L F_L \right] \quad (B.1)$$

with

$$\begin{aligned} \gamma_R F_R + \gamma_L F_L &= \frac{\alpha}{4\pi} \left[\gamma_R \begin{pmatrix} Q \\ \beta_R \end{pmatrix} \beta_R^2 \rho(q, M_Z) + \gamma_L \begin{pmatrix} Q \\ \beta_L \end{pmatrix} \beta_L^2 \rho(q, M_Z) \right. \\ &\left. + \gamma_L \begin{pmatrix} Q - 2t_3 \\ \beta_L - 2t_3 \frac{c}{s} \end{pmatrix} \frac{1}{2s^2} \rho(q, M_W) - \gamma_L \begin{pmatrix} 1 \\ c/s \end{pmatrix} \frac{t_3}{s^2} \Lambda(q, M_W) \right] \end{aligned}$$

Upper and lower entries refer to A and Z respectively. ρ and Λ are given by

$$\begin{aligned} \rho(q, M) &= B_0(q, 0, 0) - B_0(0, M, M) \\ &+ 2q^2 C_0(P, q, M, 0, 0) + (M^2 + 2q^2) C_{11}(P, q, M, 0, 0) - \frac{3}{2} \\ \Lambda(q, M) &= 3(B_0(0, M, M) - B_0(q, M, M)) \\ &- (M^2 + 2q^2) C_{11}(p, q, 0, M, M) - \frac{1}{2} \end{aligned} \quad (B.2)$$

and can be written in explicit form as

$$\begin{aligned}
\rho(q, M) &= \frac{1}{2} - \ln(-u - i\epsilon) + 4\left(1 + \frac{1}{2u}\right)(\ln(-u - i\epsilon) - 1) \\
&\quad - 2\left(1 + \frac{1}{u}\right)^2 \left(\frac{\pi^2}{6} - Sp(1 + u + i\epsilon)\right) \\
\Lambda(q, M) &= -\frac{5}{2} + \frac{2}{u} + \left(1 + \frac{2}{u}\right)l\sqrt{1 - \frac{4}{u}} + \left(1 + \frac{1}{2u}\right)\frac{4}{u}l^2 \\
u &\equiv q^2/M^2; \quad l \equiv \ln \frac{(\sqrt{1 - \frac{4}{u}} + 1 + i\epsilon)}{(\sqrt{1 - \frac{4}{u}} - 1 + i\epsilon)}
\end{aligned} \tag{B.3}$$

This form holds for positive and negative arguments of the square root and independent of its sign. The transverse parts of the one-particle irreducible vector boson self energies, as taken from [11], expressed in terms of B_0 and separated according to boson and fermion intermediate states are as follows:

$$\begin{aligned}
\Pi_b^{AA} &= \frac{\alpha}{4\pi} [-(3q^2 + 4M_W^2)B_0(q, M_W, M_W) + 4M_W^2 B_0(0, M_W, M_W)] \\
\Pi_f^{AA} &= \frac{\alpha}{4\pi} \sum_f Q_f^2 \frac{4}{3} [(q^2 + 2m_f^2)B_0(q, m_f, m_f) - 2m_f^2 B_0(0, m_f, m_f) - \frac{q^2}{3}] \\
\Pi_b^{ZZ} &= \frac{\alpha}{4\pi s^2 c^2} \frac{1}{12q^2} \left\{ [-(9c^4 + c^2 - \frac{1}{4})q^2 - (12c^4 + 8c^2 - 5)M_W^2] 4q^2 B_0(q, M_W, M_W) \right. \\
&\quad + (12c^4 - 4c^2 + 1)4q^2 M_W^2 B_0(0, M_W, M_W) - \frac{4}{3}q^4(c^2 - s^2) \\
&\quad + [(q^2 + M_Z^2 - M_H^2)^2 + 8q^2 M_Z^2] B_0(q, M_Z, M_H) \\
&\quad + [2q^2 - M_Z^2 + M_H^2] M_Z^2 B_0(0, M_Z, M_Z) \\
&\quad + [2q^2 - M_H^2 + M_Z^2] M_H^2 B_0(0, M_H, M_H) \\
&\quad \left. - (M_Z^2 - M_H^2)^2 \right\} \\
\Pi_f^{ZZ} &= \frac{\alpha}{4\pi} \sum_f \left\{ \frac{\beta_R^{f^2} + \beta_L^{f^2}}{2} \frac{4}{3} [(q^2 + 2m_f^2)B_0(q, m_f, m_f) - 2m_f^2 B_0(0, m_f, m_f) - \frac{q^2}{3}] \right. \\
&\quad \left. - \frac{1}{2s^2 c^2} m_f^2 B_0(q, m_f, m_f) \right\} \\
\Pi_b^{ZA} &= \frac{\alpha}{4\pi s c} \frac{1}{3} \left\{ [-(9c^2 + \frac{1}{2})q^2 - 4M_W^2(3c^2 + 1)] B_0(q, M_W, M_W) \right. \\
&\quad \left. + 2(6c^2 - 1)M_W^2 B_0(0, M_W, M_W) - \frac{q^2}{3} \right\} \\
\Pi_f^{ZA} &= \frac{\alpha}{4\pi} \sum_f \frac{\beta_R^f + \beta_L^f}{2} Q_f^2 \frac{4}{3} [(q^2 + 2m_f^2)B_0(q, m_f, m_f) - 2m_f^2 B_0(0, m_f, m_f) - \frac{q^2}{3}]
\end{aligned} \tag{B.4}$$

$$\begin{aligned}
\Pi_b^{WW} &= \frac{\alpha}{4\pi s^2} \frac{1}{12q^2} \left\{ [(1 - 40c^2)q^4 + (-16c^2 - 54 + \frac{10}{c^2})q^2 M_W^2 \right. \\
&\quad + s^4(8c^2 + 1)M_Z^4] B_0(q, M_W, M_Z) \\
&\quad + [(q^2 + M_W^2 - M_H^2)^2 + 8q^2 M_W^2] B_0(q, M_W, M_H) \\
&\quad + 8s^2(-5q^4 - 2q^2 M_W^2 + M_W^4) B_0(q, M_W, 0) \\
&\quad + (20q^2 - 2M_W^2 + M_Z^2 + M_H^2) M_W^2 B_0(0, M_W, M_W) \\
&\quad + (8c^2 + 1)(2q^2 - s^2 M_Z^2) M_Z^2 B_0(0, M_Z, M_Z) \\
&\quad + (M_W^2 - M_H^2 + 2q^2) M_H^2 B_0(0, M_H, M_H) \\
&\quad \left. + [-\frac{4}{3}q^4 - s^2(8c^2 + s^2)M_Z^4 - (M_W^2 - M_H^2)^2] \right\} \\
\Pi_f^{WW} &= \frac{\alpha}{4\pi s^2} \frac{1}{3} \sum_{\text{doublets}} \left[(q^2 - \frac{m_1^2 + m_2^2}{2} - \frac{(m_1^2 - m_2^2)^2}{2q^2}) B_0(q, m_1, m_2) \right. \\
&\quad + (-1 + \frac{m_1^2 - m_2^2}{2q^2}) m_1^2 B_0(0, m_1, m_1) \\
&\quad \left. + (-1 + \frac{m_2^2 - m_1^2}{2q^2}) m_2^2 B_0(0, m_2, m_2) + \frac{(m_1^2 - m_2^2)^2}{2q^2} - \frac{q^2}{3} \right]
\end{aligned}$$

They are rendered finite by counterterms as follows:

$$\begin{aligned}
\Pi_R^{AA}(q^2) &= \Pi^{AA}(q^2) - q^2 \Pi^{AA'}(0) \\
\Pi_R^{ZZ}(q^2) &= \Pi^{ZZ}(q^2) - \text{Re}(\Pi^{ZZ}(M_Z^2)) + \delta Z_Z(q^2 - M_Z^2) \\
\Pi_R^{ZA}(q^2) &= \Pi^{ZA}(q^2) - \Pi^{ZA}(0) + \delta Z_{ZA} q^2
\end{aligned} \tag{B.5}$$

$$\begin{aligned}
\delta Z_{ZA} &= -4sc\delta_- + \frac{2}{M_Z^2} \Pi^{ZA}(0) \\
\delta Z_Z &= \Pi^{AA'}(0) - \frac{s^2 - c^2}{sc} \delta Z_{ZA} \\
\delta_- &= \frac{1}{4s^2} \text{Re} \left[\frac{\Pi^{WW}(M_W^2)}{M_W^2} - \frac{\Pi^{ZZ}(M_Z^2)}{M_Z^2} \right]
\end{aligned}$$

REFERENCES

- [1] For a recent review see e. g.: L. Maiani, preprint CERN-TH 4226. Proceedings of the HEP-85 Conference, Bari, Italy, July 1985.
- [2] J. Jersák, E. Laermann and P. M. Zerwas, *Phys. Rev. D* **25** (1982) 1218.
- [3] J. M Jowett, preprints CERN LEP TH 85-04(1985), CERN LEP Note 544(1985).
- [4] B. Grzadkowski, P. Krawczyk, J. H. Kühn and R. G. Stuart, *Phys. Lett.* **163 B** (1985) 247.
- [5] B. Grzadkowski, P. Krawczyk, J. H. Kühn and R. G. Stuart, preprint MPI-PAE/PTh 07/86, to be published in *Phys. Lett.*
- [6] S. Güsken, J. H. Kühn and P. Zerwas, *Nucl. Phys. B* **262** (1985) 393 and references therein.
- [7] J. H. Kühn and K. H. Streng, *Nucl. Phys. B* **198** (1982) 71.
- [8] A. Martin, unpublished
- [9] LEP Study Group, W. Buchmüller et al., "Toponium Physics at LEP", preprint MPI-PAE/PTh 85/85.
- [10] A. Sirlin, *Phys. Rev. D* **22** (1980) 93.
- [11] B. W. Lynn, R. G. Stuart, *Nucl. Phys. B* **253** (1985) 216;
R. G. Stuart, Oxford University, D. Phil. Thesis (1985); Rutherford and Appleton Lab. Report, RAL T 008.
- [12] M. Böhm, W. Hollik and H. Spiesberger, *Z. Phys. C* **27** (1985) 523.
- [13] R. Karplus and A. Klein, *Phys. Rev.* **87** (1952) 848,
R. Barbieri et al., *Phys. Lett.* **57 B** (1975) 455.
- [14] J. H. Kühn, J. Kaplan and E. G. O. Safiani, *Nucl. Phys. B* **157** (1979) 125.

- [15] G. Passarino and M. Veltman, *Nucl. Phys. B* **160** (1979) 151.
- [16] M. Veltman, *Acta Physica Polonica B* **8** (1977) 475.
- [17] G. t'Hooft and M. Veltman, *Nucl. Phys. B* **153** (1979) 365.
- [18] W. Wetzel, *Nucl. Phys. B* **227** (1983) 1.
- [19] M. Böhm, W. Hollik and H. Spiesberger, preprint DESY84-027

FIGURE CAPTIONS

1. Feynman diagrams relevant for the evaluation of one loop corrections. Permutations have been omitted.
2. Feynman diagrams relevant for the corrections to the top quark propagator.
3. a) Prediction for the polarization asymmetry as a function of the toponium mass for $M_H = 100\text{GeV}$ and $M_Z = 94\text{GeV}$. Solid line: Full one loop correction. Dashed line: difference between first order and lowest order prediction.
 b) Value of $\sin^2 \theta_W$ used both for lowest order and one loop corrected prediction for different values of M_H .
4. Change in the asymmetry $\alpha_{RL}(M_H) - \alpha_{RL}(100\text{GeV})$ for a Higgs mass of 1000GeV (solid line) and 10GeV (dashed line),
 a) including the variation of the Weinberg angle,
 b) fixing the Weinberg angle to the value calculated for $M_H = 100\text{GeV}$.
5. Change in the asymmetry if the WW or the $Z\gamma$ box diagram is dropped. The corresponding change for the ZZ box is below .001.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5