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COSMIC ASYMMETRY, NEUTRINOS, AND THE SUN^{*)}

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A B S T R A C T

We consider the effects a cosmological asymmetry would have on various consequences of cold dark matter. To be specific, we suppose that stable Dirac neutrinos exist with masses of a few GeV. We then consider the contribution these neutrinos make towards a closure density for the Universe and the possibility of capturing neutrinos in the sun and observing their annihilation products. We concentrate on the role asymmetry plays in altering previous discussions. The arguments concerning the sun are only relevant if the neutrino mass is greater than the "evaporation" mass, m_{ev} . We evaluate $m_{ev} = 3.3$ GeV using a detailed balance technique.

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INTRODUCTION

Recently there has been a focus on the possibility that the dark matter comprising our galactic halo^[1] consists of particles with a mass of order a few GeV and interaction cross sections that are weak $\sigma \sim 10^{-36} - 10^{-40} \text{ cm}^2$. Such particles could naturally be left over from the big bang with an abundance necessary to close the universe. Popular candidate particles are photinos,^[2,3] sneutrinos,^[4] higgsinos^[3] from the supersymmetric theories and Dirac^[5] or Majorana^[6] neutrinos from various extensions of the standard $SU(3) \times SU(2) \times U(1)$ model of particle physics. The presence of such particles in the galaxy leads to the possibility of their detection through direct or indirect means. Following Goodman and Witten,^[7] several researchers^[8,9] have suggested that these particles may be directly observable here on Earth by a new series of detectors designed to detect small energy depositions ($\sim \text{KeV}$) from elastic collisions of dark matter with nuclei. Other authors have examined less direct signals. Press and Spergel^[10] have shown that such particles may be captured in the sun at interesting rates. One possible consequence is a solution to the solar neutrino problem,^[11] although existing candidate particles do not work.^[12] A more likely possibility is that dark matter particles captured in the sun may annihilate with each other, producing energetic ν 's that are observable in the proton decay experiments.^[13,14] Another indirect approach is to look for annihilation products produced in the galactic halo.^[15] Possibilities for a signal include γ -rays, positrons, and antiprotons. Antiprotons are a particularly interesting case, because the predicted rates are near the observation claimed by Buffington et al.^[16]

With the exception of an early paper by Hut and Olive^[17] all the above results have concentrated on dark matter candidates that do not possess a cosmological asymmetry, *i.e.*, the abundances of particles and anti-particles were assumed to be identical. For Majorana fermions such as photinos, higgsinos or Majorana neutrinos, such asymmetries cannot exist. However, for particles carrying a conserved quantum number, this assumption is not justified. Indeed, our

very existence owes itself to the conservation of baryon number, B , and a baryon asymmetry of order $\eta_B \sim 10^{-10}$. If an asymmetry is present, the observational consequences of dark matter are altered and it is these changes that we address in this paper. We concentrate on the questions of primordial abundance (Section II) and the detectability of dark matter through annihilations in the sun (Sections III, IV) although we also discuss some of the other issues listed above (Section V). We will illustrate our arguments by assuming the existence of a stable neutrino with Dirac mass $O(1 - 50 GeV)^*$ whose cosmic asymmetry is associated with the conservation of lepton number L . We take the asymmetry to be a free parameter and express our results as a function of it and the neutrino mass. Neutrino asymmetries of order the baryon asymmetry are especially interesting as they would allow the universe to be closed by a neutrino of mass $O(10 GeV)$ with no observational consequences other than their gravitational mass.

* Such a neutrino is not particularly attractive from a particle physics point of view. To be stable against decay into a light neutrino and a lepton-antilepton pair ($\nu_H \rightarrow \nu l \bar{l}$) we must prevent mixing of our heavy neutrino with the weak interaction eigenstates of the known neutrinos, to one part in 10^{16} for $m = 5 GeV$. Such models exist but are not particularly compelling.

II. PRIMORDIAL ABUNDANCE

Many authors^[3-6,18-21] have presented analytic and numerical primordial abundance calculations. We will generalize an analytic approximation reviewed by Steigman^[19] to the case of non-zero asymmetry. This approximation has been tested numerically by Turner and Scherrer^[20] who find it accurate to a few percent. We agree with their conclusion for zero asymmetry, although our analysis for non-zero asymmetry differs markedly from that in the appendix of Turner and Scherrer. When applied to the case of Dirac neutrinos our zero asymmetry results differ slightly with the results of Kolb and Olive.^[6] We have tested our analytic approximation numerically and find it accurate to 5% in the asymmetry range of interest.

For our purposes, the evolution of the primordial neutrino abundance is well described by the equation

$$\frac{dY_+}{dx} = \frac{dY_-}{dx} = \frac{c}{x^2} (Y_+^0 Y_-^0 - Y_+ Y_-) \quad (1)$$

In eq. (1) $Y_+ \equiv n_+/s$ is the abundance of neutrinos scaled to the entropy density, $n_+(n_-)$ is the number density of neutrinos (antineutrinos) and $s = (2\pi^2/45)g_*T^3$ is the entropy density. The quantity g_* is the effective number of degrees of freedom at temperature T , and is discussed in more detail below. A superscript ⁰ on the value Y indicates the value of Y in thermal and chemical equilibrium. We define the independent variable x by $x \equiv m/z$, with m the mass of the neutrino and $z \equiv s^{1/3} \equiv bT$ with $b = (2\pi^2 g_*/45)^{1/3}$. With this definition of x the function c is given by

$$c = \left(\frac{b}{2\pi}\right)^{1/2} \langle\sigma v\rangle_A m m_p \quad (2)$$

where $\langle\sigma v\rangle_A$ is the thermally averaged annihilation cross section, $m_p = 1.22 \times 10^{19}$ GeV, and m is the particle mass. For Dirac neutrinos, the annihilation cross

section at low velocities ($v \ll c$) is

$$\langle \sigma v \rangle_A = N_A \frac{G_F^2 m^2}{2\pi} P_A c = 2.5 \times 10^{-26} N_A P_A m_{10}^2 \text{ cm}^3/\text{sec} \quad (3)$$

where $N_A = \sum_i (1 - (m_i/m)^2)^{1/2} (B_i^2 + C_i^2 + (m_i^2/2m^2)(B_i^2 - C_i^2))$ is the effective number of annihilation channels, $P_A = m_Z^4 / ((4m_\nu^2 - m_Z^2)^2 + \Gamma_Z^2 m_Z^2)$ accounts for the Z_0 propagator and resonance, and $\Gamma_Z = 2.5 \text{ GeV}$ is the width of the Z_0 . We sum over the possible annihilation products of mass m_i , vector coupling B_i and axial coupling C_i . The sum includes e, μ, τ , their neutrinos and whichever of the five quarks have $m_i < m$. $N_A \sim 7.4$ when m is well above the bottom quark mass.

In writing down eq. (1) we have made some reasonable assumptions. First, we have assumed that the particles are non-relativistic, $x > 1$. This is implicitly assumed since the detailed balance argument that leads to eq. (1) ignores final state occupation number effects^[21] which is justified when $m > T$ but not if $T > m$. We will be interested in the regime $m \geq 20T$ so this assumption is quite good. Second, we have assumed that the universe expands adiabatically. Apart from possible entropy generation at the quark hadron transition (which we ignore) the adiabatic assumption is also valid.

In the non-relativistic approximation $\langle \sigma v \rangle_A$ for Dirac neutrinos (and many other particles) is independent of T , so the function c is almost constant, having only the weak temperature dependence $g_*^{1/6}$. If $\langle \sigma v \rangle_A$ depends on temperature then it is convenient to modify c/x^2 to $c/(x^{2+N})$ where the factor x^{-N} soaks up the principle temperature dependence in $\langle \sigma v \rangle_A$ and c is still nearly constant (see Ref. [20] for details). Except where specifically stated, for the rest of this paper, we consider only the Dirac neutrino case with $N = 0$.

As written, eq. (1) is two coupled equations for Y_+, Y_- , but it is easy to decouple them. Define $\alpha = (Y_+ - Y_-)/2$ as the asymmetry. Then $d\alpha/dx = 0$, since we are assuming lepton number conservation, and we can write $Y_+ = Y_- + 2\alpha$

and eq. (1) becomes

$$\frac{dY}{dx} = \frac{c}{x^2} [Y^0(Y^0 + 2\alpha) - Y(Y + 2\alpha)] \quad (4)$$

We have dropped the subscript (-). It will be understood that Y refers to the minority species (particle or anti-particle) and $\alpha \geq 0$ in eq. (4). We will solve for the minority abundance Y and then use the above relation to find Y_+ if necessary. We are more interested in the minority abundance for two reasons. First, at small x (large T) the solution to eq. (4) has $Y \approx Y^0$. As the universe expands, there comes a time when Y leaves thermal equilibrium by a significant amount. This happens first for the minority component. Secondly, in the late universe, annihilation will be controlled primarily by the abundance of the minority component, the majority species having an abundance fixed at roughly 2α .

For the analytic approximation we define the departure from equilibrium

$$\Delta = Y - Y^0$$

and then

$$\frac{d\Delta}{dx} = -\frac{c}{x^2} \Delta(\Delta + 2\alpha + 2Y^0) - \frac{dY^0}{dx} \quad (5)$$

As long as the departure from equilibrium is small, we can set $d\Delta/dx = 0$ and solve for Δ . To do this we need $Y^0 = e^{-y} Y^0(\alpha = 0)$ where $y \equiv \mu/T$, and μ is the chemical potential and

$$Y^0(\alpha = 0) \approx g \left(\frac{x}{2\pi b} \right)^{3/2} e^{-bx} \quad (6)$$

where $g = 2$ is the number of spin degrees of freedom for a Dirac neutrino and we have used the non-relativistic approximation ($x > 1$) for $Y^0(\alpha = 0)$. The value

of y is given by

$$\sinh y = \frac{\alpha}{Y^0(\alpha=0)}$$

so

$$\frac{dY^0}{dx} = Y^0 \left(-b + \frac{3}{2x} \right) \left(1 + \frac{\alpha}{(\alpha^2 + Y^0(\alpha=0)^2)^{1/2}} \right) \quad (7)$$

Now, we define “freeze out” as the temperature at which the departure from equilibrium abundance is about equal to the equilibrium abundance itself, *i.e.*, $\delta \equiv \Delta/Y^0 = O(1)^*$. Final results do not depend strongly on δ and we will later take $\delta = 1.5$ as a good fit to our numerical integration. Before freeze out, we set $d\Delta/dx = 0$, so at freeze out

$$\delta \left(2 + \frac{2\alpha}{Y_f^0} + \delta \right) = \frac{x_f^2}{cY_f^0} \left(b - \frac{3}{2x_f} \right) \left(1 + \frac{\alpha}{[\alpha^2 + Y_f^0(\alpha=0)^2]^{1/2}} \right) \quad (8)$$

where the subscripts f denote freeze out values. Since bx_f turns out to be about 20 the $3/2x_f$ term is small. Eq. (8) can be solved for x_f . For $\alpha/Y_f^0 \gg 1$ or $\alpha/Y_f^0 \ll 1$ we can approximate x_f as:

$$\alpha \ll Y_f^0 \quad x_f \approx \frac{1}{b} \left(\ln B - \frac{1}{2} \ln \left(\frac{1}{b} \ln B \right) \right); B = \frac{cg\delta(2+\delta)}{(2\pi)^{3/2}b^{5/2}} \quad (9)$$

$$\alpha \gg Y_f^0 \quad x_f \approx \left(\frac{\delta\alpha c}{b} \right)^{1/2} \quad (10)$$

The values of Y at freeze out are $Y_f \approx (1+\delta)Y_f^0(\alpha=0)$ for $\alpha \ll Y_f^0$ and $Y_f \approx ((1+\delta)/2\alpha)Y_f^0(\alpha=0)^2$ for $\alpha \gg Y_f^0$. After x_f we neglect the $Y^0(Y^0+2\alpha)$

* We use $\delta = 1.5$ because it seems to give a slightly better fit than $\delta = 1.0$ or $\delta(\delta+2) = 1$ as considered in Ref. [20]. The value $\delta = 1.5$ was determined empirically. The exact value of δ is not crucial as long as $\alpha < Y_f^0$.

term in eq. (4) , which drops exponentially, and solve

$$\frac{dY}{dx} = -\frac{c}{x^2}Y(Y + 2\alpha) \quad (11)$$

This equation is easily integrated giving

$$\frac{1}{2\alpha} \ln \left(\frac{(2\alpha + Y_\infty)}{Y_\infty} \frac{Y_F}{(2\alpha + Y_F)} \right) = \left(\frac{c}{x} \right)_f A_f \quad (12)$$

where Y_∞ is the value of Y today (*i.e.*, at $x = \infty$) and the function A_f corrects for the changing value of g_* ,

$$A_f = \int_0^1 \left(\frac{c}{c_f} \right) du \quad (13)$$

where $u = x_f/x = bT/(b_f/T_f)$ and c is a function of u since g_* depends on temperature.

A_f is a specific case ($N = 0$) of a more general integral A_f^N which applies when the cross section has some temperature dependence. In this case, the c/x^2 on the right side of eq. (11) is replaced by $c/(x^{2+N}b^N)$ and

$$A_f^N = (N + 1) \int_0^1 \frac{(c/c_f)u^N}{(b/b_f)^N} du \quad (14)$$

In fig. 1. we have plotted A_f^N as a function of T_f for $N = 0, 1, 2$ using the form for g_* described later in this section. In order to adapt the results from the rest of this section for $N \neq 0$ it is sufficient to replace the combination $C_f A_f/x$ by $c_f b_f^N A_f^N/(x_f^{N+1}(N + 1))$.

Eq. (12) may be rewritten as^[22]

$$Y_\infty = \frac{2\alpha e^{-2\alpha(\frac{cA}{x})_f}}{1 + \frac{2\alpha}{Y_f} - e^{-2\alpha(\frac{cA}{x})_f}} \quad (15)$$

which has three limiting behaviors. The first two arise from the case $\alpha \ll Y_f^0$. We distinguish the cases where $2\alpha(cA/x)_f$ is less than and greater than unity.

In the first case, $\alpha \ll (x/Ac)_f$

$$Y_\infty = \left(\frac{x}{Ac}\right)_f \quad (16)$$

In the latter case $\alpha \gg (x/Ac)_f$

$$Y_\infty = 2\alpha e^{-2\alpha(\frac{Ac}{x})_f} \quad (17)$$

Eq. (16) is just the usual zero asymmetry result with an $O(10\%)$ correction taking into account the changing value of g_* . On the other hand, if $\alpha > (x/Ac)_f$, then the number of antineutrinos that survive drops exponentially.

As α increases further we get to the regime where $\alpha > Y_f$. We then find

$$Y_\infty = Y_f e^{-2\alpha(\frac{Ac}{x})_f} \approx g^2 \frac{(1+\delta)}{2\alpha} \left(\frac{x_f}{2\pi b}\right)^3 e^{-2(\sqrt{\delta}+A/\sqrt{\delta})\sqrt{\alpha c_f b_f}} \quad (18)$$

This result is less certain than eq. (15) since the arbitrary definition of x_f (*i.e.*, $\delta = 1.5$) shows up in the exponential. On the other hand, in eq. (9) the error in x_f is logarithmic in δ so the true uncertainty is quite small. In actuality, the uncertainty for the $\alpha > Y_f$ case is unimportant. If $\alpha > Y_f$ then we necessarily have $\sqrt{\alpha bc} > (m/T)_{\alpha=0} \approx 20$ for our cases. Y_∞ is essentially zero. Although we may have a critical density of neutrinos, there will be so few antineutrinos that any uncertainty in the antineutrino density is unimportant.

We have compared the approximation of eqs. (15) and (8) with a numerical solution of eq. (4). We determine g_* in a manner similar to Olive, Schramm and Steigman^[23] (see their Figure 3). At temperatures above the QCD confinement transition ($T > 250 MeV$) we treat quarks, gluons, photons and leptons as free particles. Below confinement ($T < 150 MeV$) we use hadrons and mesons instead of quarks and gluons, specifically including the pion and rho nonets and the nucleon octet. These give the main contribution to g_* . We show our ignorance of the confinement transition ($T \approx 200 MeV$) by using a linear interpolation between

150 and 250 MeV. For quark masses we use $m_u = m_d = 0, m_s = 150\text{MeV}, m_c = 1250\text{MeV}$ and $m_b = 4500\text{MeV}$. We find that for the region of interest ($\alpha < Y_f^0$) the numerical results and the analytic approximation agree to better than 5%. Using Y_∞ we can find the contribution that neutrinos and antineutrinos make towards closure density. Defining $\Omega_i = \rho_i/\rho_{crit}$ where $\rho_{crit} = 3H_0^2/8\pi G_N$ is the critical density for closure, the Hubble parameter is $H_0 = h100\text{km/sec/Mpc}$, and G_N is Newton's constant, we find

$$\frac{h^2(\Omega_\nu + \Omega_{\bar{\nu}})}{T_{2.7}^3} = 5.32 \times 10^8 (Y + \alpha)m \quad (19)$$

where the microwave background temperature is $T = T_{2.7}(2.7^\circ\text{K})$ and m is the mass of the neutrino in GeV. In Fig. 2, we have plotted two curves in the α, m plane corresponding to $(\Omega_\nu + \Omega_{\bar{\nu}})h^2/T_{2.7}^3 = 1, .25$. So, for example, if one believes $\Omega h^2 = .25$, then the only acceptable values for α lie below the curve marked $\Omega h^2 = .25$.

We note that with zero asymmetry, to get $\Omega h^2 = .25$ requires a neutrino mass of 4.1GeV . This differs from the value determined by Kolb and Olive^[6] of 3.7GeV . As the analytic and numerical results in our paper agree to better than 5%, we believe this is a real difference and is due to at least two factors. First, we used a different prescription for g_* during the quark-hadron transition. Second, and more fundamentally, Kolb and Olive use independent variable $x = m/T$ whereas we use $x = m/z$. As a result, their eq. 4 (equivalent to our eq. (1)) should have a correction involving $\dot{g}_*/g_*$. Presumably this introduces an error of order A_f .

III. ANNIHILATION IN THE SUN

One of the most likely observational consequences of neutrino halo matter is the production of events in proton decay experiments by light neutrino products of heavy neutrino annihilations in the sun. This problem or its analog for photinos, etc. has been studied by several authors.^[13,14]

The first step in this calculation is to evaluate the rate of annihilations in the sun. The equations describing the number of neutrinos and antineutrinos in the sun are

$$\begin{aligned}\dot{N} &= C_\nu - C_A N \bar{N} - C_E N \\ \dot{\bar{N}} &= C_{\bar{\nu}} - C_A N \bar{N} - C_E \bar{N}\end{aligned}\tag{20}$$

The constants $C_\nu(C_{\bar{\nu}})$ give the rate for capture of neutrinos (antineutrinos) from the halo, C_A gives the rate of annihilation in the sun, and C_E is the evaporation rate from the sun.

If there is no asymmetry then $C_\nu = C_{\bar{\nu}}$ and eq. (20) becomes

$$\dot{N} = C_\nu - C_E N - C_A N^2\tag{21}$$

The solution to eq. (21) is

$$N = N_0 \frac{\tanh(At/\tau_A)}{A + \frac{(C_E \tau_A)}{2} \tanh(At/\tau_A)}\tag{22}$$

where $\tau_A \equiv (C_A C_V)^{-1/2}$ is the time scale to reach the equilibrium number $N_0 \equiv (C_\nu/C_A)^{1/2}$ in the absence of evaporation, and $A = (1 + C_E^2 \tau_A^2/4)^{1/2}$. If evaporation is much more important than annihilation in reaching equilibrium, then the equilibrium number becomes $N_E = N_0/C_E \tau_A = C_\nu/C_E$. In the next section we will evaluate C_E , but for now, we take $C_E = 0$.

When some asymmetry is turned on, we do not know an explicit form for N ; however, its behavior is not difficult to understand. We take anti-neutrinos

to be the minority species. The time scale τ_A is now the time at which the annihilation rate is comparable to antineutrino capture rate. After that time neutrino abundance increases as $\dot{N} \simeq C_\nu - C_{\bar{\nu}}$ and anti-neutrino abundance decreases to zero as $1/t$. That is, for $t > \tau_A$

$$\begin{aligned} N &\simeq (C_\nu - C_{\bar{\nu}})t + \bar{N} \\ \bar{N} &\simeq \frac{C_{\bar{\nu}}}{C_A(C_\nu - C_{\bar{\nu}})t} \end{aligned} \quad (23)$$

and the annihilation rate is just about $C_{\bar{\nu}}$.

Press and Spergel^[10] have calculated the capture rate by the sun for weakly interacting halo matter. Their results may be expressed as

$$C_{\bar{\nu}} = a \left(\frac{3\pi}{2} \right)^{1/2} N_\odot \frac{v_\odot^2}{\bar{v}} n_{\bar{\nu}} \sum_i x_i \sigma_i \quad (24)$$

In this equation $N_\odot \equiv M_\odot/\mu$ is the number of nuclei in the sun, μ is the mean molecular weight and $M_\odot$ is the sun's mass. Also, $v_\odot = 617 \text{ km/sec}$ is the escape velocity from the surface of the sun, $\bar{v} \equiv 300 \bar{v}_{300} \text{ km/sec}$ is the local velocity dispersion in the halo, $n_{\bar{\nu}}$ is the local number density of antineutrinos in the halo, x_i is the fractional solar abundance by number of nuclear species i , and σ_i is the elastic scattering cross section for neutrinos off nuclear species i . The factor a embodies details of the sun's structure. Press and Spergel^[10] have calculated $a = .89$ for a nuclear species whose number density traces the mass density of the sun. We will use this value, ignoring the changing H/He ratio as one penetrates to the core.

The reason for including all nuclear species is that heavier elements, although much less abundant, have much larger elastic cross sections for Dirac neutrinos. Indeed, for neutrinos of mass 10 GeV, we find that the O, Si, Ne, etc. group and Fe together contribute 40% of the total capture rate and for $m_\nu = 50 \text{ GeV}$ Fe accounts for roughly 40% the capture rate by itself. We use the abundances

given by Ross and Aller^[24] for most elements. For H and He we use $x_H = .86$ and $x_{He} = .14$ for a sun of age $\tau_\odot = 4.7 \times 10^9$ yrs. The elastic cross section is

$$\sigma_i = 2.1 \times 10^{-39} \left[\frac{m_i m_\nu}{m_i + m_\nu} Q_i \right]^2 \text{ cm}^2 \quad (25)$$

where $Q_i^2 = (N - (1 - 4 \sin^2 \theta_w)Z)^2$ for heavy nuclei and $Q_H^2 \approx 3g_A^2$ for hydrogen. We use $(1 - 4 \sin^2 \theta_w) = .124$ and an axial vector coupling for the proton of $g_A = 1.25$. All masses are in GeV. In Figure 3 we show the total effective cross section $\bar{\sigma} \equiv \sum_i x_i \sigma_i$ as a function of neutrino mass. We also show the partial cross sections $x_i \sigma_i$ for H, He, C + N + O + Ne + Si + S + Mg, Fe, and everything else.

With these results we rewrite eq. (24) as

$$C_{\bar{\nu}} = 9.0 \times 10^{25} \frac{\bar{\sigma}}{\bar{\sigma}_{10}} \frac{\rho_{\bar{\nu}2}^{\bar{\nu}}}{m_{10}} \frac{1}{\bar{\nu}_{300}} \text{ sec}^{-1} \quad (26)$$

where we have found it convenient to scale our result to a neutrino mass of 10 GeV, and the number density in the halo has been expressed in terms of $\rho_{\bar{\nu}2}^{\bar{\nu}}$, a mass density of $.2 \text{ GeV/cm}^3$. $\bar{\sigma}/\bar{\sigma}_{10}$ is the summed cross section divided by the summed cross section at 10 GeV. As the neutrino mass increases the effective cross section increases in such a way that apart from the density dependence, $\rho^{\bar{\nu}}$, $C_{\bar{\nu}}$ varies by a factor of only 1.4 over a neutrino mass range from 4 to 50 GeV. Note, that if there is no asymmetry and the halo is made entirely of neutrinos we expect $\rho_{\bar{\nu}2}^{\bar{\nu}} = \rho_{\bar{\nu}2}^{\bar{\nu}} \approx 1$, while if there is a large asymmetry $\rho_{\bar{\nu}2}^{\bar{\nu}} \approx 2$, but $\rho_{\bar{\nu}2}^{\bar{\nu}} \ll 1$.

The annihilation rate in the sun is given by $C_A \equiv \Gamma_A / N \bar{N}$ with

$$\Gamma_A = \int d^3x n(x) \bar{n}(x) \langle \sigma v \rangle_A \quad (27)$$

where $n(\bar{n})$ is the spatial density of neutrinos (anti-neutrinos) within the sun and $\langle \sigma v \rangle_A$ is the thermally averaged annihilation cross section. The elastic cross

section is small enough that neutrinos make many orbits, $O(100)$, between interactions. We will therefore approximate the neutrino phase space distribution by a global temperature and a density variation depending on the local gravitational potential. With this assumption

$$n(r) = n_0 e^{-\frac{m\phi}{T}} \quad (28)$$

where n_0 is the density at the core and

$$\phi(r) = \int_0^r \frac{GM(r')}{r'^2} dr' \quad (29)$$

is the gravitational potential with respect to the core and $M(r) = 4\pi \int_0^r r'^2 \rho(r') dr'$ is the mass within radius r . We define the effective volumes

$$\begin{aligned} V_j &= 4\pi \int_0^{R_\odot} r^2 e^{-j\frac{m\phi}{T}} dr \\ &= 6.5 \times 10^{28} \left(\frac{T_{1.4}}{jm_{10}} \right)^{3/2} \text{ cm}^3; \quad j \geq 1 \end{aligned} \quad (30)$$

where $T = 1.4 \times 10^7 T_{1.4}^\circ K$ is the neutrino temperature and m_{10} is the neutrino mass in units of 10 GeV, and we assumed a constant density of 150 gm/cm^3 for the volume of interest. Then

$$\begin{aligned} C_A &= \frac{\langle \sigma v \rangle_A V_2}{V_1^2} \\ &= 1.0 \times 10^{-54} \frac{m_{10}^{7/2}}{T_{1.4}^{3/2}} \left(\frac{N_A}{7.4} \right) \text{ sec}^{-1} \end{aligned} \quad (31)$$

Knowing C_A, C_ν we give

$$\tau_A = (C_A C_\nu)^{-1/2} = 1.0 \times 10^{14} \left[\frac{\bar{\sigma}_{10}}{\bar{\sigma}} \frac{T_{1.4}^{3/2}}{m_{10}^{5/2}} \left(\frac{7.4}{N_A} \right) \frac{\bar{v}_{300}}{\rho_{\nu,2}^\nu} \right]^{1/2} \text{ sec} \quad (32)$$

This is much less than the age of the sun $\tau_\odot = 1.5 \times 10^{17} \text{ sec}$. In the absence of

asymmetry the equilibrium number is

$$N_0 = (C_\nu/C_A)^{1/2} = 9.2 \times 10^{39} \left[\frac{\bar{\sigma}}{\bar{\sigma}_{10}} \frac{T_{1.4}^{3/2}}{m_{10}^{9/2}} \left(\frac{7.4}{N_A} \right) \frac{\rho_{.2}^\nu}{\bar{v}_{300}} \right]^{1/2} \quad (33)$$

If there are no antineutrinos then the current solar abundance is

$$N = C_\nu \tau_\odot = 2.4 \times 10^{43} \frac{\bar{\sigma}}{\bar{\sigma}_{10}} \frac{(.5\rho_{.2}^\nu)}{m_{10}} \frac{1}{\bar{v}_{300}} \quad (34)$$

This is about 2.1×10^{-14} times the number of baryons in the sun for $\rho_{.2}^\nu = 2$. The value of C_ν used in eq. (34) is slightly smaller than that used in eq. (26) because we have used the average chemical abundances over the lifetime of the sun, rather than the current abundances. We will use eq. (34) later, when we discuss the solar neutrino problem.

We now turn to the problem of detecting these annihilations on the Earth through the flux of light neutrinos they produce. We begin with the raw neutrino fluxes, then define different types of observable events, and finally give the event rates.

Since $\tau_A \ll \tau_\odot$ it is an excellent approximation to take the annihilation rate equal to the capture rate of antineutrinos, $\Gamma_A = C_{\bar{\nu}}$. The flux of annihilation products in light neutrinos at the Earth is then

$$\phi_i = \frac{2f_i C_{\bar{\nu}}}{4\pi R_{es}^2} = 4.5 \times 10^{-3} \frac{\bar{\sigma}}{\bar{\sigma}_{10}} \frac{\rho_{.2}^{\bar{\nu}}}{m_{10}} \frac{1}{\bar{v}_{300}} \text{sec}^{-1} \text{cm}^{-2} \quad (35)$$

where the earth sun distance is $R_{es} = 1.5 \times 10^{13} \text{cm}$, $f_i \approx .07$ is the branching ratio into light neutrino species i , and the 2 counts the neutrino and the antineutrino produced. In our discussion of detection events, we consider only the direct light neutrinos produced by $\nu_H \bar{\nu}_H \rightarrow \nu_i \bar{\nu}_i$ and do not concern ourselves with the secondaries produced by, say, $\nu_H \bar{\nu}_H \rightarrow q\bar{q}$ with one or both of the quarks subsequently decaying leptonically. Although these secondaries are about half

the total flux, they are harder to detect because their mean energy is roughly $m_\nu/3$ compared to m_ν for the prompt neutrinos. Their energy distribution is much broader as well.

The background flux is due primarily to neutrinos produced in cosmic ray air showers.^[25,26] The differential flux of muon neutrinos is given approximately by $d\phi/dE = 4 \times 10^{-2} E_\nu^{-3} \text{cm}^{-2} \text{sec}^{-1} \text{sr}^{-1} \text{GeV}^{-1}$, where E_ν is given in GeV. The background flux of electron neutrinos is a slowly decreasing fraction of the muon neutrino background. For our interests, we will take $\phi_{\nu_e} \lesssim 1/3 \phi_{\nu_\mu}$ for $E > 2 \text{GeV}$. We will not consider tau neutrinos at all, mostly because their conversion to tau leptons is kinematically suppressed by the tau mass until neutrino energies are many tens of GeV. In addition to this the signals for tau events will be harder to interpret as they depend on the subsequent decay of the tau.

It is not sufficient to merely compare the signal and background fluxes. The actual signal is a lepton produced in a charged current interaction with a baryon, so we must compare the rates for the observation of a lepton of energy E_l due to signal and background. We will consider four event types; V_e, V_μ, S_μ and T_μ . $V_e(V_\mu)$ events are those where the electron(muon) is produced *within* the detector. In S_μ and T_μ events the muon is produced in the rock outside the detector, enters into the detector and either stops in (S_μ) or passes through (T_μ) the detector.

For V_μ it is not possible to get very good energy resolution for the lepton, other than to say the muon had enough energy to escape the detector. We therefore define the event rate to be

$$R = \int_{E_T}^{\infty} \frac{dR}{dE_\mu} dE_\mu \quad (36)$$

where E_T is the threshold energy to escape the detector and

$$\frac{dR}{dE_\mu} = \int_{E_\mu}^{\infty} N \frac{d\sigma(E_\mu, E_\nu)}{dE_\mu} \frac{d\phi}{dE_\nu} dE_\nu \quad (37)$$

is the differential production rate. N is the number of nucleons in the detector

and we will assume the detector is half neutrons and half protons in giving average cross sections. Following the notation in Gaisser and Stanev^[25] the differential cross section is

$$\frac{d\sigma}{dE_\mu}(E_\mu, E_\nu) = \sigma(1 + A \frac{E_\mu^2}{E_\nu^2}) \times 10^{-38} \text{cm}^2 \text{GeV}^{-1} \quad (38)$$

where for the signal we have an equal number of neutrinos and antineutrinos, and we find $\sigma = .41, A = .93$; but for the background, neutrinos are more abundant by a factor of 1.2, so $\sigma = .43, A = .80$. For $d\phi/dE_\nu$, we use either the background flux or eq. (35) multiplied by a delta function $\delta(E_\nu - m_\nu)$ for the signal.

In Table 1 we give the event rates for signal and background. We scale the results for a water detector of size $[20 \text{ meter}]^3$ running for one year. The energy threshold is assumed constant over the detector volume, which is clearly a simplification. The background rate is for one steradian solid angle, corresponding to an opening angle of 33° . One might think that a substantial improvement in signal to noise may be possible since the angular size of the sun's core is about 2×10^{-4} radians. However, other factors dominate the angular resolution such as the resolution of the detector (about 30° for water detectors) or the opening angle of the event vertex. ($\sim 10^\circ$ at 10 GeV).^[25]

The considerations for V_e events are similar, except that the background is lower and the energy resolution of detectors is much better since the electrons are contained within the detector. If one wants to plan a long range experiment to detect low signal rates, then V_e events are the most promising because of the possibility of using energy resolution to further lower the background.

Turning to S_μ and T_μ events, we follow the techniques laid out by Gaisser and Stanev.^[25] They calculate dR/dE_μ , the differential rate at which muons enter the detector with energy E_μ , assuming the differential production cross section in eq. (38) and an energy loss rate for the muons, $dE_\mu/dx = -\alpha(1 + E_\mu/\epsilon)$, with $\alpha = 2 \times 10^{-3} \text{GeV}/(\text{gm}/\text{cm}^2)$ and $\epsilon = 510 \text{GeV}$. The rate of S and T

events is then $R_s = \int_{E_{min}}^{E_T} (dR/dE_\mu) dE_\mu$ and $R_T = \int_{E_T}^{\infty} (dR/dE_\mu) dE_\mu$, where E_T is the energy necessary for the muon to pass through the detector and E_{min} is the minimum energy necessary to identify the muon. In Table 1 we present their backgrounds for values of $E_T = 4\text{GeV}$, $E_{min} = 2\text{GeV}$ appropriate to a $(20\text{meter})^3$ water detector collecting data for one year. Using the same technique we have evaluated the rate for signal events. We express our results leaving E_T variable and set $E_{min} = E_T/2$ to get a simple closed form. As with the V events, the signal to background ratio may be slightly enhanced by angular cuts. This will be especially attractive for higher mass cosmic neutrinos where T events will dominate the signal but must contend with a high background. In this case an ionization detector with good angular resolution would be more efficient than a water Čerenkov detector. The last relevant remark for S and T events is that they cannot be detected during the day unless the detector is very deep because the background will then be dominated by air shower muons that punch through from the surface. In determining rates for S and T events we will use an on time efficiency of .3.

From the results in Table 1 we construct a total signal rate $R_{tot} = V_e + V_\mu + .3(S_\mu + T_\mu)$. For the V events we use $E_T = 2\text{GeV}$ and for the S and T events $E_T = 4\text{GeV}$. At low neutrino masses ($< 10\text{ GeV}$) the signal is mostly V events, while for higher neutrino masses the signal is mostly T events. From the table we conclude that a signal of $R_{tot} \lesssim 1$ will be difficult to detect over background (not to mention the arguments over last year's event), but a signal of $R_{tot} = 10$ could be detectable against a background of $R_{tot}(\text{background}) = 22$. R_{tot} depends on the local halo density through the parameter D . We can take two attitudes towards the local halo density of antineutrinos. The first, and simplest, is that antineutrinos contribute to halo dark matter in proportion to their contribution to Ω , $\rho_{halo}^{\bar{\nu}} = \Omega_{\bar{\nu}} \rho_{halo}$. With this assumption we have drawn the solid curves in Figure 2 labeled by $R_{tot} = \# / h^2$ where R_{tot} is found using a $(20\text{meter})^3$ detector, $\bar{\nu}_{300} = 1$, and $\rho_{halo} = .4\text{GeV}$. It is also possible to assume that neutrinos and antineutrinos are more concentrated than other forms of dark matter which may

not cluster in galaxies. With this in mind we have drawn the dashed curve which represents an event rate of $R_{tot} = 10$, assuming $\rho_{halo}^\nu + \rho_{halo}^{\bar{\nu}} = \rho_{halo}$ for $\Omega_\nu + \Omega_{\bar{\nu}} > .1$ and $\rho_{halo}^\nu + \rho_{halo}^{\bar{\nu}} = 10(\Omega_\nu + \Omega_{\bar{\nu}})\rho_{halo}$ for $\Omega_\nu + \Omega_{\bar{\nu}} < .1$.

It is not difficult to understand the shape of the curves in Figure 2 in terms of the neutrino asymmetry. As long as the asymmetry is small the value of $\Omega_{\bar{\nu}}$ (and hence the annihilation signal) depends only on m . However, as soon as the asymmetry becomes large ($\alpha > Y_\infty$), $\Omega_{\bar{\nu}}$ decreases dramatically. So the lines of equal $\Omega_{\bar{\nu}}$ (i.e. equal event rates) bend to lower m , following the line $\alpha = Y_\infty$. Roughly speaking, if $\Omega_\nu + \Omega_{\bar{\nu}}$ is determined by annihilation there is a signal in the proton decay detectors, but if $\Omega_\nu + \Omega_{\bar{\nu}}$ is determined by asymmetry then $\Omega_{\bar{\nu}}$ is suppressed and there is no signal. The cross over between these regimes is given roughly by $\alpha = \alpha_{LW}(m_{LW}/m)^3$ where m_{LW} is the ‘‘Lee Weinberg’’ mass which gives $\Omega_\nu + \Omega_{\bar{\nu}} = 1$ with zero asymmetry and α_{LW} is the value of α required to give $\Omega_\nu - \Omega_{\bar{\nu}} = 1$ at $m = m_{LW}$.

For a given scenario, the area below the appropriate curve is not allowed because too much light neutrino flux would be observed from the sun. One can readily see that it is quite possible to have a Dirac neutrino provide the missing mass and still have escaped detection through the sun. On the other hand, if there is no asymmetry a Dirac neutrino of mass less than $O(10)GeV$ should be detectable.

IV. EVAPORATION

The results of the previous section were derived assuming evaporation was unimportant; however, this is not the case. There exists an “evaporation mass” m_{ev} such that if $m < m_{ev}$ equilibrium in the sun is reached by evaporation and not by annihilation. On the other hand if $m > m_{ev}$, evaporation has almost no effect on our previous results. Several authors^[10–14,29] have argued that $m_{ev} = O(5\text{GeV})$, but they disagree upon exact values and use different techniques to arrive at this result. Now, $m_{ev} = 5\text{GeV}$ lies just in the most interesting regime for the primordial abundance of Dirac neutrinos so we feel it is worth some effort to obtain a more accurate result.

The evaporation rate per neutrino is defined in eq. (20) and is equal to

$$C_E = \frac{4\pi}{N} \int_0^{R_0} r^2 \Gamma(r, T) dr \quad (39)$$

where Γ is the evaporation rate per unit volume, N is the total number of heavy neutrinos in the sun, and T is the global temperature of the neutrino distribution. Γ depends on the radial coordinate, r , through the nuclear temperature, T_N , the local abundance of neutrinos, n , given by eq. (28), the local escape velocity, $v_e(r)$ and the density of the different nuclear species, n_i . The local escape velocity is $v_e = (2\phi_e)^{1/2}$ with

$$\phi_e(r') = \int_{r'}^{\infty} \frac{GM(r)}{r^2} dr \quad (40)$$

For future reference we define the core potential

$$\phi_c = \int_0^{\infty} \frac{GM(r)}{r^2} dr \quad (41)$$

and note that $\phi_c = \phi(r) + \phi_e(r)$ independent of r .

The local evaporation rate is given by

$$\Gamma = \sum_i n(r) n_i(r) \left(\frac{m}{2\pi T} \right)^{3/2} \left(\frac{m_i}{2\pi T_N} \right)^{3/2} \int_0^{v_e} d^3 v \int_0^\infty d^3 v_i e^{-\frac{mv^2}{2T}} e^{-\frac{m_i v_i^2}{2T_N}} \sigma_e^i(v, v_i) |v - v_i| \quad (42)$$

where we sum over nuclear species i . We integrate over all of phase space for the nucleons but truncate the neutrino phase space distribution at v_e since neutrinos of energy greater than ϕ_e will have already escaped. It should be a good approximation to use a neutrino phase space distribution below v_e that is exactly thermal because the scattering rate to redistribute the captured neutrinos (to make up for escapees) is much faster than the escape rate itself. The cross section for evaporation in a collision between a neutrino of mass m , velocity v , and a nucleus of mass m_i , velocity v_i is

$$\sigma_e^i(v, v_i) = \int_{v_e}^\infty d^3 v_f \int_0^\infty d^3 v_{if} d\sigma^i(v, v_i, v_f, v_{if}) \quad (43)$$

where we have integrated over the differential cross section that leaves the final neutrino velocity $v_f > v_e$.

Now, in general, the evaporation rate must be calculated numerically; however, for $mv_e^2/2T \gg 1$ it is possible to employ detailed balance to get an analytic result accurate to leading order in $(mv_e^2/2T)^{-1}$. We have checked this technique by carrying through the integral in eq. (42) for the simpler case where $m = m_i, T = T_N$ and find that it agrees with our detailed balance result to this accuracy.

The detailed balance argument proceeds as follows. Assume for the moment that $T = T_N$. Then energy and momentum conservation imply

$$e^{\frac{mv^2}{2T}} e^{-\frac{m_i v_i^2}{2T_N}} |v - v_i| = e^{-\frac{mv_f^2}{2T}} e^{-\frac{m_i v_{if}^2}{2T_N}} |v_f - v_{if}| \quad (44)$$

This allows us to relabel initial and final states while keeping thermal distributions. The real evaporation rate is then equal to the fictitious capture rate from

the $v > v_e$ high velocity tail of a thermal neutrino distribution at temperature T , i.e., , define

$$\sigma_{cap} = \int_0^{v_e} d^3 v_f \int_0^\infty d^3 v_i d\sigma(v, v_i, v_f, v_{if}) \quad (45)$$

and then

$$\Gamma = \sum_i n n_i \left(\frac{m}{2\pi T}\right)^{3/2} \left(\frac{m_i}{2\pi T}\right)^{3/2} \int_{v_e}^\infty d^3 v \int d^3 v_i e^{-\frac{mv^2}{2T}} e^{-\frac{m_i v_i^2}{2T}} \sigma_{cap} |v - v_i| \quad (46)$$

The advantage of this step is twofold. First, virtually any neutrino in this fictitious tail that scatters will be captured, $\sigma_{cap} \approx \sigma_{tot}$. This is because the typical neutrino energy is $E \approx m\phi_e + T$, the typical energy loss is of order E , and since $T \ll m\phi_e$ the final neutrino energy is almost certainly less than $m\phi_e$ and so the neutrino is captured. Second, it is a good approximation to take $v_N = 0$. This follows since for neutrinos of escape velocity their energy and momenta are much greater than those for almost any nucleon they might scatter from. The evaporation rate when $T = T_N$ is then easily found to be

$$\Gamma_{eq} = n \sum_i n_i \sigma_i \sqrt{\frac{8T}{\pi m}} \frac{m\phi_e}{T} e^{-m\frac{\phi_e}{T}} \quad (47)$$

Now, the reader has surely noticed that $T \neq T_N$ in general and we must correct for this. Because T is assumed constant and T_N decreases with radius, we expect that in the core T_N is slightly larger than T and in the outer regions of the sun T_N is significantly smaller than T . Loosely speaking, the major effect will be that where $T_N < T$, there are not as many energetic nucleons available for neutrinos to scatter off and escape. In determining a numerical estimate of this effect we must be careful to integrate the suppression over phase space. We can do this conveniently by using the detailed balance picture and so define the

suppression factor A , $\Gamma_{T \neq T_N} = A\Gamma_{T=T_N}$,

$$A = \frac{\int_{m\phi}^{\infty} n(E) \int_{E_{min}}^{E_{max}} n_i^0(E_N) \frac{d\sigma_i}{dE_N} \left(\frac{T}{T_N}\right)^{3/2} e^{-E_N \frac{T-T_N}{TT_N}} dE_N dE}{\int_{m\phi}^{\infty} n(E) \int_{E_{min}}^{E_{max}} n_i^0(E_N) \frac{d\sigma_i}{dE_N} dE_N dE}. \quad (48)$$

In this expression the neutrino energy is the initial energy of the fictitious neutrino to be captured and E_N is the final energy of the nucleon (i.e., the initial nucleon energy from which the escaping neutrino scatters). The differential cross section is that which leaves the nucleon with final energy E_N , and $n_i^0(E_N) \propto T^{-3/2} e^{-E_N/T}$ is what the density of nucleons would be if $T_N = T$. The denominator is proportional to $\Gamma_{T=T_N}$, eq. (46). The numerator contains the suppression factor $(T/T_N)^{3/2} e^{-E_N(T-T_N)/TT_N}$ which is the reduction in nuclei available to promote escape in that element of phase space. The limits on the E_N integration are

$$\begin{aligned} E_{min} &= E - m\phi \\ E_{max} &= \frac{4\mu^2\phi}{m_N} \end{aligned} \quad (49)$$

where $\mu = mm_N/(m + m_N)$ is the reduced mass. These are just the kinematic limits to the nucleon energy if the energy of the escaping neutrino is E .

Now, since neutrinos scatter isotropically, a convenient thing happens, $d\sigma/dE_N$ is a constant and drops out of the integrals. Setting $\Delta = E - m\phi$ and changing variables we can define

$$B(T) = \frac{1}{T^{3/2}} \int_0^{\infty} (m\phi + \Delta)^{1/2} e^{-\Delta/T} \int_{\Delta}^{E_{max}} E_N^{1/2} e^{-\frac{E_N}{T_N}} dE_N d\Delta \quad (50)$$

we find

$$A = \frac{B(T_N)}{B(T)} \approx \frac{1 - \left(\frac{T}{T+T_N}\right)^{3/2}}{1 - (.5)^{3/2}} \quad (51)$$

where in the last relation we have again used the fact $m\phi/T \gg 1$ to do the integral in eq. (50). For $T_N \ll T$, A has the limiting behavior, $A \simeq 2.32T_N/T$

but for $T_N \sim T$ $A \simeq 1$. In the sun, 90% of the mass of the sun is inside the radius where $T_N = (1/4)T$ (assuming T equals the core temperature) for which $A = .44$. The suppression of evaporation due to $T_N < T$ is not at all large.

Putting these results together we express the evaporation rate as

$$C_E = \epsilon \frac{v_\odot^2 \bar{\sigma}}{V_1} \frac{M_\odot}{\mu_\odot} \left(\frac{2m}{\pi T} \right)^{1/2} e^{-\frac{m\phi_c}{T}} \quad (52)$$

where $\bar{\sigma}$ is the effective cross section given in eq. (25) and figure 3, V_1 is the effective volume of the neutrinos defined in eq. (30), $M_\odot/\mu_\odot$ is the number of nuclei in the sun, $\epsilon = \int_0^1 A \tilde{\phi}_e df_m$ is an efficiency integral expressed over the mass coordinate $f_m = M(r)/M_\odot$. In the relation for ϵ we have expressed the gravitation potential in terms of the gravitational potential at the surface of the sun, $\phi_e = \tilde{\phi}_e v_\odot^2/2$. The most important thing about this result is that the factor in the exponential was independent of radius ($\phi + \phi_e \equiv \phi_c$) and came outside the integral. Apart from the efficiency factor every nucleon in the sun contributes equally to evaporation regardless of the local neutrino density. Those towards the core benefit from high density, those outside the core benefit from low escape velocity. We use solar models from Bahcall^[27] and Faulkner^[28] to numerically evaluate $\epsilon = 2.4$ and $\phi_c = 5.0$ and give

$$C_E = 1.3 \times 10^{-2} x^2 e^{-32.9x} \frac{\bar{\sigma}}{\bar{\sigma}_{3.75}} \text{sec}^{-1} \quad (53)$$

where $x = (m/3.75 \text{GeV})(1.4 \times 10^7 \text{K}/T)(\tilde{\phi}_i/5.0)$ and $\bar{\sigma}_{3.75}$ is the value of $\bar{\sigma}$ when $m = 3.75 \text{GeV}$, a convenient fiducial.

Throughout our derivation we have consistently made the approximation $m\phi/T \gg 1$. Relevant values are $m\phi/T \simeq 30$. Since we use this approximation several times, the accumulated errors may be as much as a factor of 2. However, this hardly makes any difference to the result which is dominated by the exponential. The major uncertainty in eq. (52) lies in the value of the global temperature of the neutrino distribution, and in the assumption of a global temperature to begin with.

To determine whether or not evaporation affects the results of the previous section we must compare C_E to the other time scales. First of all, if $C_E \tau_\odot < 1$ then evaporation has no effect during the life of the sun. Solving for x in the relation $C_E \tau_\odot = 1$ we find

$$x = (1/32.9)(\ln a + 2 \ln \frac{\ln a}{32.9}) \quad (54)$$

with $a = 1.9 \times 10^{15} \bar{\sigma} / \bar{\sigma}_{3.75}$, or $x = 1.07$. The evaporation mass for a solar age is $m_{ev} = 4.0 T_{1.4} (5.0 / \tilde{\phi}_c) GeV$.

A more relevant time scale to consider is τ_A , the time it takes the neutrino number to reach equilibrium through annihilation. Using eq. (32) for τ_A we find a in eq. (54), $a = 5.0 \times 10^{12} \left[(\bar{\sigma} / \bar{\sigma}_{3.75}) (T_{1.4}^{3/2} / m_{3.75}^{5/2}) (7.4 / N_A) (\bar{v}_{300} / \rho_{.2}^{\bar{v}}) \right]^{1/2}$, so in this case $x = .88$ and $m_{ev} = 3.3 T_{1.4} (5.0 / \tilde{\phi}) GeV$. Since the evaporation rate depends exponentially on the mass, for $m < m_{ev}$ the equilibrium number of particles in the sun will be determined solely by evaporation: $N_\nu = C_\nu / C_E$. For the case of no asymmetry $N_{\bar{\nu}} = N_\nu$ and the annihilation rate is $\Gamma_A = C_A C_\nu^2 / C_E^2$. So, as soon as $m < m_{ev}$ the annihilation rate decreases exponentially and the observed signal vanishes. Note that a change in mass of $\Delta m = .26 GeV$ makes a factor of 10 change in C_E and factor of 100 reduction in any annihilation signal at the Earth.

If there is some asymmetry the evaporation mass does not change. If $m > 3.3 GeV$ then the equilibrium number of neutrinos is always greater than N_0 , the zero asymmetry value. Then we have $N_\nu C_A > N_0 C_A = \tau_A^{-1} > C_E$. But $N_\nu C_A > C_E$ is the condition for annihilation to control the antineutrino abundance and we have $\Gamma_A = C_\nu$. On the other hand if $m < 3.3 GeV$ then the abundance of neutrinos cannot be greater than $C_\nu / C_E < N_0$. Then, the abundance of antineutrinos must be controlled by evaporation since $N_\nu C_A < N_0 C_A < C_E$. The annihilation rate is therefore $\Gamma_A = C_A C_\nu C_{\bar{\nu}} / C_E^2$.

There is therefore a very sharp transition at $m \simeq 3.3 GeV$ where for $m > 3.3 GeV$ the full annihilation signal is obtained and for $m < 3.3 GeV$ almost no

annihilations occur. The width of the transition is about $.2\text{GeV}$. In Figure 2 we have demonstrated the effect of evaporation by truncating the event rate curves at $m_\nu = m_{ev}$.

We emphasize that it is interesting to get m_{ev} correct since it is in this mass range that $\Omega_\nu + \Omega_{\bar{\nu}} = 1$ is allowed for zero asymmetry. Although the factor in the exponential dominates, one still does not want to make a large error in the prefactor, *i.e.*, by several orders of magnitude. To this extent we emphasize two points. From the detailed balance calculation it may be seen that the initial nucleus and heavy neutrino come from a region of phase space where $E_N + E_\nu \simeq m\phi_{es} + T$. However, there is no preference for taking $E_N \simeq m\phi_{es}$, $E_\nu \simeq T$ or vice versa $E_N \simeq T$, $E_\nu \simeq m\phi_{es}$. We find that calculations based on such schemes underestimate the evaporation rate by a factor of $m\phi_{es}/T \simeq 30$. Second, it is not a good approximation to treat the neutrinos as if they are all at the core. Because of the balance between escape velocity and number density the evaporation rate is proportional only to the normal matter density and not to the neutrino density. Therefore, including only the neutrinos in the core underestimates the evaporation rate by a factor of 10 or more. The corrections for the nucleon temperature being lower than the neutrino temperature are not large. However, we have assumed that the neutrino temperature is constant. If the neutrinos are cooler or non-thermal in the outer parts of the sun then that volume may not be so effective.

Lastly we remark that our technique is not so different from the “planetary atmosphere” technique used by Krauss, Srednicki, and Wilczek.^[29,30] However, their calculation implicitly assumes that evaporation takes place from a last scattering surface, *i.e.*, the “top of the atmosphere”, which they take to be the surface of the astronomical body in question (the earth). Unfortunately, the sun and the earth are thin objects for scattering neutrinos, there is roughly one interaction per 100 orbits depending on σ , so these authors do not calculate the prefactor

correctly although their technique yields the same factor in the exponential.*

* Actually, Krauss et al. report $m_{\nu} = 6\text{GeV}$ for the sun, a value a little less than twice ours. This discrepancy cannot be explained by the prefactor. In addition, the planetary atmosphere technique should have resulted in a value of m_{ν} slightly smaller than ours.

V. DISCUSSION AND SUMMARY

In this paper we have considered the effect a cosmological asymmetry may have on various arguments concerning cold dark matter. We began by considering the question of the primordial abundance of particles and anti-particles. We find that there are three regimes in which useful analytic approximations may be made. Two useful fiducial quantities are the final abundance $Y_\infty(\alpha = 0)$ and the freeze out abundance $Y_f(\alpha = 0)$. If the asymmetry is small compared to the final abundance then the number of particles and antiparticles is not greatly changed from the zero asymmetry case. Experimental signals that depend on annihilation are not modified. If the asymmetry is greater than $Y_\infty(\alpha = 0)$ but less than $Y_f(\alpha = 0)$ then a simple extension of existing techniques may be used to estimate (to $\sim 5\%$ accuracy) the final abundance of particles and antiparticles. The minority species is exponentially suppressed. The total contribution to Ω is given by the asymmetry. Any signals due to annihilation become unobservable as soon as the asymmetry is significant. For cases where the asymmetry is greater than Y_f then we give a different approximation where the suppression of the minority species is even stronger.

We have applied the results of the abundance calculations to the case of Dirac neutrinos. Besides deriving conditions for $\Omega_\nu + \Omega_{\bar{\nu}} = 1$ we have considered the capture and subsequent annihilation of Dirac neutrinos in the sun. Possible annihilation products are light neutrinos which may subsequently be detected in proton decay experiments on Earth. We have made estimates of the signal rates and background rates due to cosmic ray air showers for various event configurations. It is important to remember that events consist of observed leptons so it is not sufficient to consider just incident neutrino flux when comparing signal and background. For Dirac neutrinos with no asymmetry we find observable signals in the mass range $m_{ev} < m_\nu < (10\text{GeV})$, where m_{ev} is the evaporation mass. If the cosmic asymmetry is large enough then the annihilation signal disappears and it is possible to have $\Omega_\nu \simeq 1$ for $O(1\text{TeV}) > m_\nu > m_{LW}$. The upper limit of

order 1 TeV arises since for $m_\nu > m_Z$ the annihilation cross section drops and Ω_ν increases again.

We have considered the problem of evaporation from the sun in some detail. We used a detailed balance technique simplified by the fact that $m\phi/T \gg 1$. We have assumed that the neutrinos have a phase space distribution that is thermal with a global temperature T and a chemical potential given by the gravitational potential relative to the core of the sun. We treat the nuclei more generally, taking account of their density and temperature changes at different radii. Our result takes the form $\Gamma_{ev} \sim x^2 e^{-x}$, $x = m\phi_c/T$. For the sun $x \approx 30$ defines where evaporation becomes important. So the evaporation mass m_{ev} depends most strongly on the temperature of the massive neutrino distribution and the gravitational potential at the core of the sun. It depends only logarithmically on the age of the sun, the scattering cross section, etc. To preclude observing the annihilation signal requires m less than $m_{ev} = 3.3 GeV$.

We would now like to discuss the relevance of our paper for other results concerning cold dark matter. We begin by considering the capture and annihilation of Dirac neutrinos in the Earth.^[29-31] At first thought this may appear to be a more efficient method of generating a signal than capture and annihilation in the sun. First, because the Earth is so much closer, and second because the Earth is made of iron while the sun is predominantly Hydrogen and Helium. For “coherently” interacting particles the elastic cross section of iron is much larger. In fact, even though it is a small fraction of the sun, there is far more iron in the sun than in the Earth. Also, although the Earth is closer, the escape velocity from the Earth is so low that capture from the halo is greatly inhibited. A comparison of the relative light neutrino fluxes at the Earth’s surface finds that, even if we assume the Earth is all iron, count only the iron in the sun for capture, and ignore the smaller angular size of the sun, the flux from the sun is greater than that from the Earth by a factor of 3. This result holds for $m_\nu \approx 50 GeV$. For m_ν greater or less than $50 GeV$ the ratio of fluxes increases.

The next issue is that of solving the solar neutrino problem.^[11,12] Since we may assume a cosmic asymmetry there need be almost no annihilation. As a result, for a heavy Dirac neutrino galactic halo, the abundance in the sun could be about 2×10^{-14} (see eq. (34)). The question arises, can this abundance of Dirac neutrinos provide enough heat conductivity to lower the core temperature sufficiently to solve the solar neutrino problem? Almost certainly not. If we convert our elastic cross section for a 5 GeV neutrino, $\bar{\sigma}_5 \simeq 1.2 \times 10^{-38} \text{cm}^2$ to the units of Gilliland^[11] et al. we obtain $\sigma_{xp} = 4.5 \times 10^{-39}$. With these values for σ_{xp} and N_x , we can see from their graphical results that a reduction in the solar neutrino rate by even 10^{-2} solar neutrino units (SNU's) is unlikely, whereas a 3.6 SNU reduction is necessary to solve the problem. This is too bad, because the possibility of having $\Omega = 1$ and solving the solar neutrino problem within a recognizable particle theory was a motivating factor for this research.

Another possible signal of cold dark matter annihilations is $\bar{p}, e^+$, and γ 's produced from annihilation in the halo of the galaxy.^[15,32] The existing papers consider the photino and higgsino as candidates for this process; however Dirac neutrinos will give similar signals. In fact, Dirac neutrinos would probably give larger signals. Photinos and higgsinos suffer from "p-wave" suppression, so their annihilation cross sections at velocity dispersions expected in the galactic halo are suppressed from those that are relevant at freeze out.^[32] Dirac neutrinos do not have this feature. As a result, for a given value of Ω , neutrinos will exhibit greater annihilation in the late universe than photinos or higgsinos. As for production of $\bar{p}$'s, e^+ 's or γ 's the efficiency for Dirac neutrinos should be similar. Only a small branching ratio ($\sim .2$) goes directly into neutrinos. This leaves us with a cosmic ray signal for dark matter about 10 times larger than for photinos or higgsinos. Of course, as we turn on asymmetry this signal goes away.

To get a feel for the magnitude of the signal one may expect we have considered a model with no asymmetry, $\Omega_\nu = 1, h = .5, m_\nu = 4.1 \text{GeV}$, and $\langle \sigma v \rangle_A = 2.6 \times 10^{-26} \text{cm}^3 \text{s}^{-1}$. The branching ratios to up type quarks, down type quarks, charged leptons and neutrinos are $u : d : e : \nu = 1.80 : 2.28 : .77 : 1.50$. Assuming

production efficiencies for $p, \bar{p}$ of .2 for quarks and 0 for leptons, we get a cosmic ray flux

$$\phi_{\bar{p}} = 7.6 \times 10^{-5} cm^{-2} sec^{-1} sr^{-1}. \quad (55)$$

This is 2.2 times that estimated by Kane and Hagelin^[32] for their case 1, a light photino with large branching ratios to $c\bar{c}$ and relatively high $\bar{p}$ production. It is also 25 times the observation claimed by Buffington, et al.,^[16] although the predicted flux would not fall totally within the energy window of their experiment. We note that eq. (55) assumes that $\bar{p}$'s are trapped in the galaxy for roughly 10^8 years and that any observations of this signal presupposes that low energy $\bar{p}$'s may negotiate the magnetic fields of the solar wind and the Earth to reach the detector. Neither of these assumptions seems compelling to these authors.

The most direct evidence for Dirac neutrinos in the halo would probably come from low temperature devices designed to detect the small ($\sim 1KeV$) energy transfer in elastic collisions between nuclei and the heavy neutrinos comprising the halo. Already, an analysis of the Homestake Mine Double Beta Decay experiment yields the limits $m_\nu < 16GeV$.^[9] This limit may improve down to $8GeV$.^[3a] More sensitive experiments being planned should be able to eliminate the Dirac neutrino as a cold dark matter candidate within the decade. These experiments will not be affected by a cosmological asymmetry.

Finally, we consider the possibility that sneutrinos make up the dark matter and carry some asymmetry. It has been pointed out^[4] that lepton number is not conserved for $\tilde{\nu}$ since there exists the possibility of $\tilde{\nu}\tilde{\nu} \rightarrow \nu\nu$ via zino exchange. In general, this annihilation mechanism dominates over Z^0 exchange ($\tilde{\nu}\tilde{\nu} \rightarrow \nu\bar{\nu}$) and little asymmetry can be preserved in the sneutrino sector. However, it is possible to suppress the zino exchange mechanism so that it is subdominant to the Z^0 exchange annihilation. In that case, it may be possible to preserve asymmetry in the sneutrino sector.

How much suppression is necessary for the asymmetry to be interesting? As long as zino exchange is proceeding fast enough to keep the sneutrinos in

chemical equilibrium the chemical potential for the sneutrinos will be equal to that of the neutrinos into which they annihilate. If we take the point of view that the lepton number of the universe should be comparable to the baryon number then the total lepton asymmetry is $\eta_L = (n_L - n_{\bar{L}})/s \simeq 10^{-9}$. The chemical potential will be of order 10^{-7} , the difference coming from the ratio of the number of relativistic degrees of freedom that carry lepton number to the total g_* . The asymmetry in sneutrinos is then 10^{-7} times the no asymmetry neutrino abundance, $\alpha_{\tilde{\nu}} \approx Y_0 10^{-7}$. As Y_0 decreases with falling temperature the asymmetry decreases as well, until the zino exchange processes freeze out. This happens, crudely, when $\Gamma = H$ (Γ is the annihilation rate per sneutrino via zino exchange) or $Y_0 \simeq 10^{-9} \langle \sigma v \rangle_Z / \langle \sigma v \rangle_{\tilde{Z}}$ where $\langle \sigma v \rangle_{Z(\tilde{Z})}$ are the annihilation cross sections via $Z(\tilde{Z})$ exchange. So, in order to achieve an interesting asymmetry ($\alpha_{\tilde{\nu}} \sim 10^{-9}$) we require $\langle \sigma v \rangle_{\tilde{Z}} \sim 10^{-7} \langle \sigma v \rangle_Z$. Since this is a crude estimate we will require $\langle \sigma v \rangle_{\tilde{Z}} \sim 10^{-6} \langle \sigma v \rangle_Z$ for sneutrino asymmetry to be interesting cosmologically.

It is possible to get around this limit by supposing $\eta_L \gg \eta_B$. The lepton asymmetry is only constrained to be $\eta_L \lesssim 1$ by considerations of big bang nucleosynthesis.^[34] Then we may ask how our results for the sun change for $\tilde{\nu}$ dark matter with an asymmetry. The anti-sneutrinos captured in the sun will annihilate quickly through Z exchange leaving an abundance of sneutrinos that grows as $N_{\tilde{\nu}} = (C_{\tilde{\nu}} - C_{\bar{\tilde{\nu}}})t$. Now, even though zino annihilation is suppressed it still occurs at a reduced level. We must compute τ_A for the process $\tilde{\nu}\tilde{\nu} \rightarrow \nu\nu$. If we scale our results to the Dirac neutrino case we find roughly $\tau_A \sim 10^{14} \text{sec} (\langle \sigma v \rangle_Z / \langle \sigma v \rangle_{\tilde{Z}})^{1/2}$. To make $\tau_A > \tau_{\odot}$ requires $\langle \sigma v \rangle_{\tilde{Z}} \lesssim 10^{-6} \langle \sigma v \rangle_Z$. If $\tau_A < \tau_{\odot}$ then the asymmetry has no effect on the detectable signal from the sun.

In the light of these arguments it is interesting to understand if a suppression by 10^{-6} is possible for zino exchange. Kane and Quiros^[35] have shown that $\langle \sigma v \rangle_{\tilde{Z}}$ may be set to zero at tree level. Further, the radiative corrections are proportional to $\langle \sigma v \rangle_{\tilde{Z}}$ so this value is stable. In that case the results obtained for neutrinos also apply to sneutrinos except for details of the cross sections.

Specifically, because elastic cross sections are $\lesssim 4$ times larger for sneutrinos, the capture rate in the sun is larger for the same halo abundance. With no zino annihilation the branching ratios are the same as for neutrinos so proton decay experiments give $\lesssim 4$ times the signal they give for neutrinos. With the enhanced cross section can one solve the solar neutrino problem? No, $\bar{\sigma}_{eff}$ is still about 20 times too small.

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Table 1. Rates^{a)} for Different Event Types

	Signal ^{b)}	Background ^{c)}	x
V_e	$2.8m [1.31 - x - .31x^3] DF_1$	$6.1/E_T$	E_T/m
V_μ	$2.8m [1.31 - x - .31x^3] DF_1$	$18.3/E_T$	E_T/m
$S_\mu^{d)}$	$.70mE_T [.65 - .38x - .07x^3] DF_2$	$4.8^e)$	$E_T/m, (x_{min} = x/2)$
$T_\mu^{d)}$	$.70m^2 [.73 - 1.31x + .5x^2 + .08x^4] DF_2$	$27.2^e)$	E_T/m

$$D = \frac{\bar{\sigma}}{\bar{\sigma}_{10}} \frac{\rho_{\bar{\nu}_2}}{m_{10}} \frac{1}{\bar{v}_{300}} \left(\frac{f_i}{.07} \right) (t/yr), \quad F_1 = \frac{V}{8 \times 10^9 cm^3}, \quad F_2 = \frac{A}{4 \times 10^6 cm^2}.$$

- a) Assuming a (20 meter)³ water detector collecting data for one year.
- b) All masses and energies in GeV.
- c) Background calculated assuming an angular aperture of 1 steradian.
- d) Note: several years of running are necessary to get one year's worth of data.
- e) Background use $E_T = 4\text{GeV}$ and average over zenith angles of -90° to 0 .
From Gaisser and Stanev.^[25]

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Figure Captions

- Fig. 1* The correction factor A_f^N for $N = 0, 1, 2$ as a function of the freeze-out temperature.
- Fig. 2* Cosmological and astrophysical constraints as a function of neutrino asymmetry α and m_ν . Bold curves are iso-abundances for $\Omega h^2 = .25, 1$. The universe is overclosed in the areas above and to the left of these curves. Dotted curves are iso-event rates for observation of annihilation in the sun. These curves assume a $(20 \text{ meter})^3$ water detector and that $\rho_{halo}^{\bar{\nu}} = \Omega_{\bar{\nu}} \rho_{halo}$. The dashed curve is for an event rate of 10 per year assuming $h = .5$ and $\rho_{halo}^{\bar{\nu}} + \rho_{halo}^{\nu} = \rho_{halo}$ for $m_\nu < 17$, $\rho_{halo}^{\bar{\nu}} = 10\Omega_{\bar{\nu}}\rho_{halo}$ for $m_\nu > 17$. The event rates drop dramatically if the asymmetry is significant or if $m_\nu < m_{evap}$, where $m_{evap} = 3.3\text{GeV}$ is the evaporation mass. For comparison the cosmic baryon asymmetry is constrained to be $2-7 \times 10^{-11}$.^[36]
- Fig. 3* Effective elastic scattering cross sections in the sun as a function of the neutrino's mass. The relative importance of various elements is shown.

- Figure 1 -

- Figure 2 -

- Figure 3 -