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ANTISYMMETRIC STRING ACTIONS

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A B S T R A C T

We present an action for the free bosonic string on external flat space in terms of an antisymmetric second-rank string background tensor which is classically equivalent to the Nambu-Goto action. Both action and field equations are entirely described in terms of 2-D world-sheet forms, without any reference to a 2-D metric tensor background. The analysis of its canonical formulation shows how the quadratic Virasoro constraints are generated in this case and what their connection with the Bianchi identities are. Since in the orthonormal gauge the reduced action coincides with the standard one, it has the same critical dimension $D = 26$. The existence of an interaction term of a purely geometric structure stemming in the extrinsic curvature is pointed out. Its action and the new string field equations are then derived. This polynomial antisymmetric string action is uniformly generalized in order to describe $d < D$ -dimensional extended objects in D -dimensional flat space.

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In spite of its apparent simplicity, the Nambu-Goto¹⁾ action for the bosonic string has to be represented in a polynomial form in order to understand both its functional quantization²⁾ and its locally supersymmetric completion³⁾. The well-known solution found by Brink, di Vecchia, Howe and by Deser and Zumino was to make the internal metric "a priori" independent of the metric tensor defined by the string and then to recover, as a field equation, that the conformal invariant part of the (initially) independent metric coincides with the conformal invariant part of the string metric.

String theory is a gauge theory. It is presently believed that the physics of gauge theories is better understood whenever it is possible to stress their geometrical structure⁴⁾, in the sense of being able to analyse their properties using the geometric-form language of Cartan. The string action we present here leads to field equations which can be written entirely in terms of forms defined on the string world sheet. It was pointed out by Kalb and Ramond a long time ago⁵⁾ that strings talk naturally with a second rank Lorentz antisymmetric tensor B_{rs} . This lends support to the convenience of emphasizing, whenever possible, the presence of second-rank antisymmetric quantities in the formulation of the string action.

The antisymmetric action we present here, instead of having an internal symmetric metric background, "a priori" independent of the string variables, has an external antisymmetric background, also "a priori" independent of the string variables, as we shall see. The action we want to consider is

$$S(b_{rs}, x^r, \rho) \equiv \frac{1}{\pi\alpha'^2} \left\{ \frac{1}{2} \int d^2\sigma b_{rs} \epsilon^{ab} x^r_{,a} x^s_{,b} + \frac{1}{2} \int d^2\sigma \rho \left(\frac{1}{2} b_{rs}^2 + 1 \right) \right\} \quad (1)$$

where $b_{rs} = -b_{sr}$ is an antisymmetric second-rank $O(D-1,1)$ Lorentz tensor and ρ is a world-sheet density. Note that the last term contains the external D -dimensional flat background metric tensor $\eta^{rs} = (-, +, \dots, +)$ which induces the metric structure of the string world sheet. The string fields are $x^r = x^r(\sigma^a)$, $r = (0, 1, \dots, D-1)$, $\sigma^a = (\sigma^0, \sigma^1) = (\tau, \sigma)$, $\epsilon^{01} = +1$, $x^r_{,a} \equiv \partial_a x^r$, $\eta^{ab} = (-1, +1)$.

S is a first-order action in the sense that each term has, at most, only one time derivative in it. It is 2-D invariant under the world sheet reparametrization group $\sigma \rightarrow \sigma' = f(\sigma)$.

Independent variations of ρ , b_{rs} and x^r yield, respectively

$$B \equiv \frac{1}{2} b_{rs}^2 + 1 = 0 ; \quad (2a)$$

$$B^{rs} \equiv \rho b^{rs} + \epsilon^{ab} x_{,a}^r x_{,b}^s = 0, \quad (2b)$$

$$S_r \equiv \partial_a [b_{rs} \epsilon^{ab} x_{,b}^s] = 0. \quad (2c)$$

Equation (2a) is the typical extended object algebraic constraint; it contains the metric aspect, i.e. the induction of the external metric on the lower dimensional extended object. Eq. (2b) is a kind of stringy torsion equation, where b^{rs} plays a role reminiscent of the affinities in curved space and Eq. (2c), the dynamical field equation, if regarded as $\sim \partial \omega = R$, might be considered as the string Einstein equation.

The first two equations can be solved in terms of the string field variables x^r . One obtains:

$$\rho = (-g)^{\frac{1}{2}} = (-\frac{1}{2} x_{rs}^2)^{\frac{1}{2}}, \quad (3a)$$

$$b_{rs} = -\frac{1}{\rho} x_{rs} \quad , \quad (3b)$$

where $g \equiv \det g_{ab}$, $x_{rs} \equiv \epsilon^{ab} x_{,r,a} x_{,s,b}$, and we are using the standard notation for the 2-D metric tensor $g_{ab} \equiv x_{,a}^r x_{,b}^r$. The initially unknown quantities ρ , b_{rs} have a clear geometrical meaning. ρ is the 2-D area element and b_{rs} is the complex unitary 2-D tangent density which is the natural generalization of the unitary tangent vector to a curve.

After introducing the values (3) into the original action (1), one regains the Nambu-Goto action

$$S_{NG}(x) = \frac{1}{\pi \lambda^2} \int (-g)^{\frac{1}{2}} d^2 \sigma = \frac{1}{2\pi \lambda^2} \int d^2 \sigma b_{rs} x^{rs} \quad (4)$$

showing their equivalence at the classical level.

Observe that the fact that the polynomial action (1) contains as background variables b_{rs} and ρ instead of the usual 2-D metric tensor g_{ab} will make a

substantial difference in the treatment of the associated partition function $Z(b_{rs}, x^r, \rho)$. Now the functional integration has to be carried out on the set of all possible configurations of the antisymmetric second rank tensors $b_{rs}(\sigma)$, instead of having to functionally integrate on the set of the conformally equivalent 2-D metrics, as was recently done by Polchinski⁶⁾ for the one-loop case. "A priori" it does not seem to have two equivalent functional spaces. However, the map which associates to each point of a 2-D metric surface its tangent plane b_{rs} is a known mathematical object⁷⁾.

Action (1) can be written in a totally geometrical fashion, taking into account that b_{rs} are 2-D zero-forms (scalars), ρ can be associated to a two-form $\rho d\sigma^0 \sigma^1$ (which we continue to call ρ) and that the one-forms $dx^r \equiv d\sigma^a x_a^r$ pack the two independent tangent vectors of the world sheet. It takes the form:

$$S(b_{rs}, dx^r, \rho) = \frac{1}{4\lambda^2} \left\{ \frac{1}{2} \int dx^r dx^s b_{rs} + \frac{1}{2} \int \left(\frac{1}{2} b_{rs}^2 + 1 \right) \rho \right\}, \quad (1b)$$

while the string field equations (2) become

$$B \equiv \frac{1}{2} b_{rs}^2 + 1 = 0, \quad B^{rs} \equiv b_1^{rs} \rho + dx^r dx^s = 0 \quad (5a, b)$$

$$S_r \equiv d[b_{rs} dx^s] = 0. \quad (5c)$$

This last expression is locally equivalent to the existence of a set of D-dual zero-forms $y_r(\sigma)$ such that

$$b_{rs} dx^s = -dy_r \quad (5d)$$

Equivalently one can say that the standard covariant string moments $p_r^b = \delta I / \delta x_{r,b} = (-g)^{\frac{1}{2}} g^{bc} x_{rc}$ are the duals of these D zero-forms, i.e.,

$$(-g)^{\frac{1}{2}} g^{ac} x_{rc} \equiv p_r^a = \epsilon^{ab} y_{r,b}. \quad (5e)$$

The existence of such first integrals is not a usual feature of Hamiltonian systems. One may check that, making use of the on-shell value (3b) of b_{rs} , after multiplying (5d) times b^{pr} one gets

$$b^{pr}_{,a} = -x^p_{,a} . \quad (5f)$$

Equations (2) are not all independent as is expected of a gauge theory. The gauge group of transformations is the two-dimensional diffeomorphism group of the world sheet. Under an infinitesimal transformation of this group defined in the contravariant vector $\xi^a(\sigma)$, ρ , b_{rs} , x^r and x^r_a transform according to:

$$\delta_\xi \rho = \partial_a [\xi^a \rho] , \quad \delta_\xi b_{rs} = \xi^a \partial_a b_{rs} , \quad (6a,b)$$

$$\delta_\xi x^r = \xi^a \partial_a x^r , \quad \delta_\xi x^r_a = \xi^b D_b x^r_a + (D_a \xi^b) x^r_{,b} . \quad (6c,d)$$

In (6d) we introduced Riemannian covariant derivatives in order to simplify the calculations. (They are not necessary to calculate the physical variations since all the physical variations are independent of the Riemannian affinities.)

From the identical vanishing of S under the infinitesimal variations (6), we have:

$$0 \equiv \pi \lambda^2 \delta_\xi S \equiv \int d^2\sigma \xi^a \mathcal{B}_a \Rightarrow 0 \equiv \mathcal{B}_a \equiv x^r_{,a} S_r - \frac{1}{2} (\partial_a b_{rs}) B^{rs} - \frac{1}{2} g \partial_a B . \quad (7a)$$

These identities say that the 2-D projection of the field equations $S_r = 0$ along the world-sheet tangent plane is irrelevant. Only the (D-2)-dimensional projection of S_r contained in the transverse plane to the world sheet constitutes a set of (D-2) independent field equations. Making use of the identity:

$$(D_{ac} x^r) b_{rs} \epsilon^{ab} x^s_{,b} \equiv \frac{1}{2} D_c (x^r_a b_{rs} \epsilon^{ab} x^s_{,b}) - \frac{1}{2} x^{rs} D_c b_{rs}$$

the gauge variation of the action can be put in the form:

$$0 \equiv \delta_\xi S = - \int \xi^a D_b \left\{ x^r_a p_r^b - \frac{1}{2} \delta_a^b x^r_c p_r^c \right\} d^2\sigma \equiv - \int \xi^a D_b T_a^b(d^2\sigma) \quad (7b)$$

where p_r^b is a short notation for $b_{rs} \epsilon^{bc} x^s_{,c}$, and $T_a^b \equiv x^r_a p_r^b - \frac{1}{2} \delta_a^b x^r_c p_r^c$. This is equivalent to stating that locally $D_b T_a^b \equiv 0$. Note that T_a^b is traceless. In this representation one can check that $T_a^a \equiv 0$. Therefore, the two "a priori" likely constraints T_a^0 disappear since they vanish identically. (Recall that the Virasoro-Goto constraints become identities in the co-ordinate representation.) We shall see that this is not the case when we go to the phase-space Hamiltonian

representations. Before going to the Hamiltonian representation, it is worth pointing out that the minimal coupling of the string to an external second-rank antisymmetric tensor B_{rs} discovered by Kalb and Ramond⁵⁾ can be obtained by the substitution in (1) of $b_{rs} \rightarrow b_{rs} - B_{rs}$. The action (1) becomes

$$S(b_{rs}, x^r, \rho, B_{rs}) = \frac{1}{\pi\alpha^2} \left\{ \frac{1}{2} \int d^2\sigma b_{rs} \epsilon^{ab} x^r_{,a} x^s_{,b} + \frac{1}{2} \int d^2\sigma \rho \left(\frac{1}{2} (b_{rs} - B_{rs})^2 + 1 \right) \right\} \quad (8a)$$

from which one obtains that $b_{rs} = -1/\rho x^r_{,s} + B_{rs}$, $\rho = (-\frac{1}{2} x^2_{,rs})^{\frac{1}{2}}$. Then, introducing this value of b_{rs} into (8a), one regains the Nambu-Goto action minimally coupled to the string, as was first proposed by Kalb and Ramond⁵⁾:

$$S_{NG}(x^r, B_{rs}) = \frac{1}{\pi\alpha^2} \left\{ \int (-g)^{\frac{1}{2}} d^2\sigma + \frac{1}{2} \int dx^r dx^s B_{rs} \right\}. \quad (8b)$$

The Hamiltonian formulation arises when one establishes a distinction between the time co-ordinate $\tau = \sigma^0$ and the spacelike co-ordinate $\sigma = \sigma^1$ in the string world sheet. It is a standard notation to denote with dot time derivatives $\partial x^r / \partial \sigma^0 \equiv \dot{x}^r$ and to use a prime for spacelike derivatives $\partial x^r / \partial \sigma^1 \equiv x'^r$.

Inspection of action (1) shows that, since $\frac{1}{2} b_{rs} x'^rs = b_{rs} x'^r \dot{x}^r$, the natural definition of momentum is

$$\delta S / \delta \dot{x}^r \equiv p_r = \frac{1}{\pi} b_{rs} x'^s. \quad (9)$$

The Hamiltonian form of action (1) can then be written as:

$$S_H(b_{rs}, p_r, \rho^r, x^r, \rho) \equiv \int d^2\sigma \left\{ p_r \dot{x}^r - \rho^r \left(p_r - \frac{1}{\pi} b_{rs} x'^s \right) + \frac{1}{2\pi} \rho \left(\frac{1}{2} b_{rs}^2 + 1 \right) \right\} \quad (10a)$$

where the Lagrange multipliers ρ^r are needed in order to recover the right definition (9) of the associated momenta p_r . In this representation of the action, the two possible constraints that emerge from the Bianchi identities T_a^0 take the form:

$$T_0^0 = \dot{x}^r (p_r - b_{rs} x'^s) \equiv \dot{x}^r C_r(x, p) = 0; \quad T_1^0 = x'^r p_r = 0, \quad (11a)$$

where we denote by $C_r(x, p) \equiv p_r - b_{rs} x'^s$ the D-differential constraints needed in order to introduce the canonical momenta p_r . It is interesting to obtain their value on the field equations (15) and (16a,b) in order to get some insight into the connection of the Virasoro constraints to the primary ones (9). They become

$$T_0^0 = \dot{x}^r C_r \Big|_{(16)(17ab)} = -\frac{2e}{x'^2} \left\{ \frac{1}{2} p_r^2 - \frac{1}{2} b_{rs} b_{rt} x'^s x'^t \right\} \equiv -\frac{2e}{x'^2} G_2(x, p) \equiv \quad (11b)$$

$$\equiv -\frac{e}{x'^2} (2p_r - C_r) C_r,$$

$$T_1^0 = x'^r p_r \equiv G_1(x, p) \equiv x'^r C_r, \quad (11c)$$

showing that the two quadratic differential constraints stemming in the 2-D gauge reparametrization invariance of action (10) can be algebraically generated from the linear differential constraints $C_r = 0$. Note that in its canonical first-order form (10), our string action is composed by its dynamical generator $\sim d\sigma p_r \dot{x}^r$ plus the Lagrangian multipliers ρ_s times the linear differential constraints plus the algebraic constraint term. There is no non-negative Hamiltonian $H(x, p)$ contributing to the action, similarly to what happens with the Nambu-Goto action in its first-order form⁸⁾.

This action (10) can be taken as the initial step in order to obtain its associated phase-space path integral formulation⁹⁾ using the FVB method¹⁰⁾ or for calculating its functional measure¹¹⁾. Since it is basic for the understanding of these points we do analyse now, at a fixed time $\sigma^0 \equiv \tau$ (which we omit), the structure of the set of constraints $\{C_r\}$. Observe that from the phase-space point of view, i.e. for computing Poisson brackets (PB), since there are no dynamical evolutions either for b_{rs} or for ρ_r, ρ , these variables behave like a "constant" background.

It is immediate to see that, owing to the antisymmetry of b_{rs} , one has

$$\{C_n(\sigma'), C_r(\sigma'')\} \equiv \sum_{(m, \sigma)} \frac{\partial C_n(\sigma')}{\partial x^m(\sigma)} \frac{\partial C_r(\sigma'')}{\partial p_m(\sigma)} - \frac{\partial C_n(\sigma')}{\partial p_m(\sigma)} \frac{\partial C_r(\sigma'')}{\partial x^m(\sigma)} = 0, \quad (12a)$$

saying that the basic set $\{C_r\}$ is a set of first-class linear differential constraints. However, one would like to know whether this set of constraints contains or generates the two constraints stemming in the reparametrization invariance (11b,c). The last expressions of these equations indicate how they can be constructed out of the initial set C_r . Their PB turn out to be

$$\{G_1(\sigma'), G_1(\sigma'')\} = 2 G_1(\sigma') \partial_{\sigma'} \delta(\sigma' - \sigma''), \quad (13a)$$

$$\{G_2(\sigma'), G_2(\sigma'')\} = 2 b_{rs} p_s C_r(\sigma') \partial_{\sigma'} \delta(\sigma' - \sigma''), \quad (13b)$$

$$\{G_1(\sigma'), G_2(\sigma'')\} = 2 G_2(\sigma') \partial_{\sigma'} \delta(\sigma' - \sigma'') \quad . \quad (13c)$$

They vanish on the initial set $C_r = 0$ and since their PB with the C_r are linear combinations of this basic set

$$\{G_1(\sigma'), C_r(\sigma'')\} = C_r(\sigma') \partial_{\sigma'} \delta(\sigma' - \sigma''), \quad (14a)$$

$$\{G_2(\sigma'), C_r(\sigma'')\} = -b_{rs} C_s(\sigma') \partial_{\sigma'} \delta(\sigma' - \sigma''), \quad (14b)$$

they are first class. Note that Eqs. (14) imply that $\dot{G}_1 = 0 = \dot{G}_2$ along the evolution of the system, independently of the knowledge of the existence of the Bianchi identities. The unusual feature of the PB (13) is that at this level of generality, without having solved any constraint of those present in action (10), $G_1(\sigma')$, $G_2(\sigma'')$ do not form an algebra by themselves, even if they do form a set of first-class constraints.

We will see in a moment that after eliminating (ρ, b_{rs}) as independent variables from the action, the new quadratic constraints $\tilde{G}_1, \tilde{G}_2$ become the standard Virasoro constraints and the initial set of constraints $\tilde{C}_r$ has, as one expects, rank D-2. [One can also check that at the level of action (10) the set $G_1(\sigma'), G_3(\sigma'') \equiv 2^{-1} C_r^2(\sigma'')$ is both closed and first class.] One can obtain the values of b_{rs} and ρ determined by action (10). They turn out to be:

$$g b_{rs} = -g_r x'_s + g_s x'_r; \quad (15a)$$

$$g = [(g_r x'_r)^2 - g_r^2 x_s'^2]^{\frac{1}{2}}, \quad (15b)$$

which, after being introduced in action (10), lead to the reduced and more symmetrical form:

$$S_{H, red}(p_r, g^r, x^r) = \int d\tau \left\{ p_r \dot{x}^r - g^r p_r + \frac{1}{\pi} [(g_r x'_r)^2 - g_r^2 x_s'^2]^{\frac{1}{2}} \right\} \equiv$$

$$\equiv \int d\sigma \{ p_r \dot{x}^r - \mathcal{H} \} \quad (10b)$$

where we are using dimensionless variables $x^r \sim \lambda^{-1} x^r$ for convenience.

Observe that no choice of co-ordinates has been done up to now. Independent variations of p_r , ρ^r and x^r yield the field equations governing the string dynamical evolution. They are, respectively:

$$\dot{x}^r = \mathcal{H}^r \quad , \quad (16a)$$

$$\tilde{C}_r \equiv p_r - \frac{1}{\pi \mathcal{H}} [(\mathcal{H}_s x_s') x_r' - x_s'^2 \mathcal{H}_r] = 0, \quad (16b)$$

$$\dot{p}_r = \frac{1}{\pi} \{ \mathcal{H}^{-1} [\mathcal{H}_s^2 x_r' - (\mathcal{H}_s x_s') \mathcal{H}_r] \}' . \quad (16c)$$

The two constraints (11) have now the form

$$-T_0 = -\mathcal{H} p_r + [(\mathcal{H}_r x_r')^2 - \mathcal{H}_r^2 x_s'^2]^{\frac{1}{2}} = 0, \quad T_1 = x_r' p_r = 0. \quad (11d)$$

They are contained in the larger set of differential constraints (16b) as one can check directly. One can also check that Eqs. (16b) imply the other standard Virasoro constraint equation $\pi^2 p_r^2 + x_r'^2 = 0$.

The structure of the D differential constraints $\tilde{C}_r = 0$ is very much different from what was previously found for action (10). The fact that one has now specific values for b_{rs} and ρ shifts the behaviour of the previous C_r to the physical behaviour one would have expected on physical grounds, as we shall see.

From this definition (16b), one can generate the two standard first-class string constraints:

$$x_r' \tilde{C}_r \equiv x_r' p_r \equiv G_1 \equiv \tilde{G}_1, \quad (17a)$$

$$\frac{1}{2}(\tilde{C}_r - p_r)^2 = -\frac{1}{2}\pi^{-2}x'^2 \sim \frac{1}{2}\tilde{C}_r(2p_r - \tilde{C}_r) = \frac{1}{2}p_r^2 + \frac{1}{2}\pi^{-2}x'^2 \equiv \tilde{G}_2 \quad (17b)$$

which constitute an algebra under the Poisson bracket multiplication. Relations (14a) and (14b) hold for them, and instead of Eq. (13b) which prevented the closure of this subalgebra, one now has

$$\{\tilde{G}_2(\sigma'), \tilde{G}_2(\sigma'')\} = 2\tilde{G}_1(\sigma') \partial_{\sigma'} \delta(\sigma' - \sigma''). \quad (14d)$$

The Poisson brackets between the $\tilde{C}_r$ now have the value

$$\begin{aligned} \{\tilde{C}_n(\sigma'), \tilde{C}_r(\sigma'')\} &= -2\bar{g}^4(p_s x'_s) \left\{ \eta_{nr} + \bar{g}^2 x'^2 p_n p_r + \bar{g}^2 p_s^2 x'_n x'_r - \right. \\ &\quad \left. - \bar{g}^2 (p_s x'_s) (p_n x'_r + p_r x'_n) \right\} \partial_{\sigma'} \delta(\sigma' - \sigma'') = \\ &= -2\bar{g}^4(p_s x'_s) \left\{ \eta_{nr} - \frac{p_n^\perp p_r^\perp}{(p_r^\perp)^2} - \frac{x'_n x'_r}{x'^2} \right\} \partial_{\sigma'} \delta(\sigma' - \sigma''), \end{aligned} \quad (12b)$$

where $p_n^\perp$ is the part of p_n orthogonal to x'_r , $p_n^\perp \equiv p_r - (x'^2)^{-1} p_s x'_s x'_r$.

The last expression clearly shows that this set of D constraints, in the process of bringing it to have the maximal number of first-class constraints⁹⁾, can be reduced down to (D-2) second-class constraints since the $\det \{\tilde{C}_n \tilde{C}_r\}$ has rank (D-2). One maximal subset of first-class constraints that can be taken is $\tilde{G}_1, \tilde{G}_2$ and then we can complete it with (D-2) linear independent combinations of the $\tilde{C}_r$. In fact we also have that:

$$\{\tilde{G}_1(\sigma'), \tilde{C}_n(\sigma'')\} = \tilde{C}_n(\sigma') \partial_{\sigma'} \delta(\sigma' - \sigma''), \quad (15c)$$

$$\{\tilde{G}_2(\sigma'), \tilde{C}_r(\sigma'')\} = \alpha_{nr} \tilde{C}_r(\sigma') \partial_{\sigma'} \delta(\sigma' - \sigma'') \quad (15d)$$

(where α_{nr} are complicated expressions depending upon p_r, x'_s) showing that $\tilde{G}_{1,2}$ are first-class constraints which, jointly with (13a,c) and (14d), ensure of their vanishing time evolution. From the basic Poisson brackets' value (12') it is immediate to check that $\dot{\tilde{C}}_r = 0$, i.e. the initial system $\{\tilde{C}_r\}$ of constraints does not require the inclusion of new secondary constraints.

All these properties of the constraint algebras provide the expected picture of the role of the constraints: the initial set $\{\tilde{C}_r\}$ contains a subsystem of two

first-class constraints $\tilde{G}_1, \tilde{G}_2$ which are exactly the Virasoro ones; a complete representation with a maximal subset of first-class ones would contain $\tilde{G}_1, \tilde{G}_2$ and (D-2) additional independent linear combinations of the $\tilde{C}_r$, which are second class. Then one should have to fix two non-singular gauge conditions^{6),9)} $F_{1,2} = 0$ [in the sense that $\det(F_i, \tilde{G}_j) = 0$] in order to quantize the system. There are two natural gauge choices convenient to deal with this system. Both reduce action (10b) and its associated field equations (16) to a very simple linear uncoupled system which can be taken as the starting point in order to carry out its quantization by imposing the closure of the light-front (LF) Lorentz algebra¹²⁾.

The LF gauge¹³⁾ consists in assuming $\rho_r^2 = 0 = x_r'^2$. Action (10b) becomes:

$$S_H|_{LF} = \int d^2\sigma \left\{ p_r \dot{x}^r - g^r p_r - \frac{1}{\pi} g_r x^{r'} \right\} \quad (18a)$$

showing a remarkable symmetry under the exchange of $(\pi p_r, -\rho_r, \partial_\tau, \partial_\sigma)$ by $(-\rho_r; +\pi p_r, \partial_\sigma, \partial_\tau)$.

The more widely used gauge is the orthonormal one (ON) which can be defined by imposing co-ordinates defined by $\rho_r x_r' = 0 = \rho_r^2 + x_r'^2$. In this case, (10b) reduces to the linearized Klein-Gordon form:

$$S_H|_{ON} = \int d^2\sigma \left\{ p_r \dot{x}^r - \frac{1}{\pi} \left[\frac{1}{2} x_r'^2 + \frac{1}{2} \pi^2 p_r^2 \right] \right\} \quad (18b)$$

(where we have shifted from $p_r \rightarrow -p_r$). In this gauge choice, $\rho^r = \pi p^r$ and the two constraints (11') become the two-standard constraints^{14),1)} of bosonic strings:

$$-T^0_0|_{ON} = p_r^2 + x_r'^2 = 0, \quad T^0_1 = x_r' p_r = 0. \quad (11e)$$

At this point, one can follow the GGRT procedure and will reach the $D = 26$ critical dimension.

Up to this point we have been dealing with the free string. Recently, Polyakov¹⁵⁾ has argued that one has the possibility of adding to the free string action a term containing extrinsic curvatures which might play the role of an Einstein term (compared to the cosmological behaviour of the Nambu-Goto action) and determine many qualitative properties of the strings. He found a generalization of the Nambu-Goto action where the new "extrinsic" contribution is about $\alpha_0^{-1} \langle \sqrt{-g} g^{ab} \partial_a b_{rs} \partial_b b_{rs} \rangle$, which is the modulus of the extrinsic curvature object

$\partial_a b_{rs}$. Its effect is to produce a fourth-order contribution to the physical action $\sim \rho^{-1} (\dot{x}^r)^2$. Note that this action has a metric structure.

Here we present an alternative driving term, which is reminiscent of the topological structure of the Euler densities or of the Chern-Simons terms. Its origin is strictly geometrical: it does not depend upon the metric tensor. Similarly to those geometrical or topological terms, it does not affect as deeply as the Polyakov action the propagating behaviour of the string; it is at most, of third order in the time derivatives of the string field $x^r(\sigma)$. This geometrical extrinsic curvature generated self-interaction can be written

$$S_{geom} \equiv \frac{1}{\beta_0 3!} \int b^{rs} db_{nr} db_{ns} = \frac{1}{\beta_0 3!} \int d\tau b^{rs} \epsilon^{ab} (\partial_a b_{nr})_{,s} (\partial_b b_{ns}) \quad (19a)$$

where $b_{rs} = -1/\rho \epsilon^{ab} x_{r,a} x_{s,b}$ as in Eqs. (3a,b), and β_0 is a dimensionless constant. The full action is now

$$S_{full}(x) \equiv S_{NG} + S_{geom}. \quad (19b)$$

Here one is working in "second-order" formalism, i.e., b_{rs} given by Eq. (3b), and this is also what usually happens in field theory: Chern-Simons terms or Euler densities have their real meaning in second-order formalism otherwise affinities propagate.

Variations of the string field variables $x^r(\sigma)$ lead to the field equations which can be written in a rather compact form, using Cartan's geometric notation:

$$d \left\{ -\frac{1}{\pi \alpha'} b_{rs} dx^s + \frac{1}{\beta_0} \cdot \frac{1}{3} \left(\frac{1}{2} \delta_{rs}^{pq} + \frac{1}{2} b_{rs} b^{pq} \right) (db_{np}, db_{nq}) dx^s \right\} = 0 \quad (20)$$

where (db_{np}, db_{nq}) is a short notation in 2-D for $\epsilon^{ab} \partial_a b_{np} \partial_b b_{nq}$ and the operator it has in front is the projection transverse to the string tangent plane.

Finally we want to point out that action (1) can be uniformly generalized in order to describe the evolution of a d-dimensional extended object lying in a $D > d$ -dimensional space-time. For such a case, one defines the generalized tangent tensor density

$$x^{r_1 \dots r_d} \equiv \epsilon^{a_1 \dots a_d} x^{r_1}_{,a_1} \dots x^{r_d}_{,a_d}. \quad (21)$$

The action will depend upon an antisymmetric tensor of rank d , $b_r, \dots, r_d$ and of a d -density ρ . It has the form:

$$S_d(b_{r_1 \dots r_d}, x^r, \rho) \equiv \frac{1}{\lambda^d} \int (d\sigma) b_{r_1 \dots r_d} x^{r_1 \dots r_d} + \frac{1}{2} \int (d\sigma) \rho \left[\frac{1}{d!} b_{r_1 \dots r_d}^2 + 1 \right] \quad (22a)$$

which, after independent variations of $b_r, \dots, r_d$, ρ , x^r becomes the corresponding d -dimensional Nambu-Goto action

$$S_{NG,d}(x^r) = \frac{1}{\lambda^d} \int (d\sigma) \left[-\frac{1}{d!} x_{r_1 \dots r_d}^2 \right]^{\frac{1}{2}}. \quad (22b)$$

Field equations will have a form aspect, similar to that of Eqs. (5).

In summary, we have presented an action for the bosonic string which has a D -dimensional second-rank antisymmetric tensor as the background field which allows us to polynomialize the Nambu-Goto action. In this representation, all the string field equations have been written as 2-D-world-sheet-form geometrical equations. The corresponding Bianchi identities show that there are only $(D-2)$ independent field equations and indicate what are the two Virasoro constraints at each level of the formulation. The canonical associated action contains D -linear differential constraints; none of them coincides with the Virasoro ones (which are quadratic in the phase space). However, they (the former) algebraically generate the latter ones which are essentially the unique set of independent quadratic D -invariants one can build out of them. It is only after the value of the antisymmetric background has been expressed in terms of the phase-space variables that the structure of the D -linear differential constraints takes the aspect expected on physical grounds. This set of constraints can be represented by the two Virasoro constraints as a maximal subset of first-class constraints, jointly with a subset of $(D-2)$ linearly independent ones taken from the initial set, these last $(D-2)$ being second class. As the usual ON gauge choice makes the action to coincide with the standard forms of the bosonic string, we noted that the light-front GGRT procedure will lead to the critical dimension 26. We have then pointed out that there exists a geometrical higher-order contribution to the free string action stemming in the extrinsic curvature; this will introduce an alternative type of dynamics, whose effects are likely to be milder (this has to be proved through explicit calculations) than those due to the fourth-order metric modification studied by Polyakov. We have also seen that there exists a natural uniform generalization of this string action to represent any $d < D$ extended object evolving on flat space. Since the Nambu-Goto action can be supersymmetrized³⁾, we believe (we have not proved it) that there will likely exist a supersymmetric version of the antisymmetric string actions presented here. This point is currently under investigation.

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