UPPER BOUND ON THE TOP QUARK MASS
FROM SUPERSTRING INSPIRED LEFT-RIGHT SYMMETRIC MODELS

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ABSTRACT

The recently proposed rank six left-right symmetric models inspired by the superstring are shown to imply an upper bound for the mass of the top quark of 46 GeV or less, depending on the exact value of the strange quark mass. The key argument for deriving this bound is based on the analysis of the magnitude of the lepton number violating couplings in the minimal superpotential and the mass mixing matrix of the exotic and down quarks.

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The theoretical successes of superstrings ^{1),2)} have recently motivated effort in trying to relate them to low energy phenomenology ^{3),4)}. The requirement of compactification from ten to four dimensions ⁵⁾ has been extensively analysed ⁶⁾, but finding the correct compactification scheme remains an open problem ⁷⁾. On the other hand superstring inspired model building ^{4),8)} with gauge group E_6 and a minimal set of fundamental representations ^{9),10)} although not closely connected to the superstring and the details of compactification, will necessarily provide the way of ultimately testing the theory. To this end, several subgroups of E_6 ⁹⁾ have been investigated including the standard model and models with extra $U(1)$ factors. Alternatively, left-right symmetric models provide one serious candidate for superstring unifications with a low energy rank six group being broken radiatively into the standard model. This is the group ¹⁰⁾ $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ where $SU(2)_R$ is the right handed group of the "conventional" right handed models which does not commute with electric charge. As explained in ref. 10), where the basic aspects of this model were investigated, since there are no mirror generations $\overline{27}$ left massless from E_6 flux breaking ³⁾ into left-right symmetric groups, the breaking of the intermediate group cannot be supersymmetric. Hence the intermediate scale M_I should be quite low, not exceeding 10^4 GeV. Under these conditions, the other possible left-right symmetry, that is $SU(3)_C \times SU(2)_L \times SU(2)_N \times U(1)_L \times U(1)_R$, where $SU(2)_N$ commutes with electric charge, can be shown ¹⁰⁾ as being inconsistent with the lowering of the group rank by two units into the standard

model. So this group need not be examined as a viable possibility.

In this paper we discuss a novel upper bound on the top quark mass implied by the $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_L \times U(1)_R$ model and not shared by any of the other superstring inspired models. The essential ingredients in the reasoning that leads to this upper bound are the mixing matrices between exotic (g) and down-type (d) quarks and the relative magnitude of the lepton number violating (LNV) to lepton number conserving (LNC) Yukawa couplings in the superpotential. Then, using the values of the quark masses normalized at 1 GeV we get $m_t(1\text{GeV}) \lesssim 81$ GeV which, after renormalization, leads into $m_t \lesssim 46$ GeV.

We start by recalling the decomposition of the fundamental 27 under this subgroup

$$\begin{aligned}
 27 = & A_1(\bar{3}, 1, 2, 0, -\frac{5}{6})_{x_2, u^c} + A_2(\bar{3}, 1, 1, 0, \frac{5}{3})_{x_1} + B_1(3, 2, 1, \frac{5}{6}, 0)_{u, d} + \\
 & + B_2(3, 1, 1, -\frac{5}{3}, 0)_g + C_1(1, 2, 2, -\frac{5}{6}, \frac{5}{6})_{y_2, H_u} + C_2(1, 1, 2, \frac{5}{3}, \frac{5}{6})_{e^+, S_1} + \\
 & + C_3(1, 2, 1, -\frac{5}{6}, -\frac{5}{3})_{y_1} + C_4(1, 1, 1, \frac{5}{3}, -\frac{5}{3})_{S_2}
 \end{aligned} \tag{1}$$

Here we have denoted by x_1, x_2 the right handed singlet and doublet which combine to form the d^c and g^c - type quarks. Similarly y_1, y_2 are the right handed singlet and doublet that transform to H_d, L , the Higgs giving mass to d-type quarks and the lepton doublet respectively. S_1 and S_2 denote the two singlets of the standard model that combine to form ν^c and N .

So in this notation we have

$$\begin{pmatrix} g^c \\ d^c \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ s_\theta & -c_\theta \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \begin{pmatrix} H_d \\ L \end{pmatrix} = \begin{pmatrix} c_\phi & s_\phi \\ s_\phi & -c_\phi \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \quad (2)$$

where c and s denote $\cos$ and $\sin$ respectively.

In the most appealing superstring theory only the trilinear coupling of the 27 appears in the superpotential which is therefore

$$\begin{aligned} W = & \lambda_1 C_2 A_1 B_2 + \lambda_2 C_4 A_2 B_2 + \lambda_3 A_1 B_1 C_1 + \lambda_4 A_2 B_1 C_3 + \\ & + \lambda_5 C_4 C_1 C_1 + \lambda_6 C_2 C_1 C_3 + \lambda_7 A_1 A_1 A_2 + \lambda_8 B_1 B_1 B_2 \end{aligned} \quad (3).$$

As explained in ref.10), the suppression of proton decay requires the elimination of two of the three couplings: $\lambda_1, \lambda_7, \lambda_8$. Since λ_1 plays an essential role in the fittings, we chose to eliminate λ_7, λ_8 . Furthermore, defining

$$\begin{pmatrix} S_1 \\ S_2 \end{pmatrix} = \begin{pmatrix} c_S & s_S \\ -s_S & c_S \end{pmatrix} \begin{pmatrix} N \\ \nu^c \end{pmatrix} \quad (4)$$

we have, expanding the terms proportional to λ_5, λ_6

$$\lambda_5 C_4 C_1 C_1 = \lambda_5 H_u (-s_S N + c_S \nu^c) (s_\phi H_d - c_\phi L) + \dots \quad (5)$$

$$\lambda_6 C_2 C_1 C_3 = \lambda_6 H_u (c_S N + s_S \nu^c) (c_\phi H_d + s_\phi L) + \dots$$

Therefore, vanishing neutrino masses require necessarily that $\lambda_5 = \lambda_6 = 0$, unless unnatural fine tunings are made. Moreover,

from the analysis of the neutralino mass matrix¹⁰⁾, it turns out that condition $\lambda_5 = \lambda_6 = 0$ is also sufficient to ensure $m_\nu \approx 0$ in the case of the "true" left-right symmetric model, that is, the one for which all left and right couplings are strictly equal. It is then straightforward to guarantee the elimination of $\lambda_5, \lambda_6, \lambda_7, \lambda_8$ by means of a discrete symmetry¹⁰⁾. The superpotential reduces thus simply to the following terms

$$\begin{aligned}\lambda_1 C_2 A_1 B_2 &= \lambda_1 S_1 g(s_\theta g^c - c_\theta d^c) + \lambda_1 e^+ u^c \\ \lambda_2 C_4 A_2 B_2 &= \lambda_2 S_2 g(c_\theta g^c + s_\theta d^c) \\ \lambda_3 A_1 B_1 C_1 &= \lambda_3 Q(s_\theta g^c - c_\theta d^c)(s_\phi H_d - c_\phi L) + \lambda_3 H_u Q u^c \\ \lambda_4 A_2 B_1 C_3 &= \lambda_4 Q(c_\theta g^c + s_\theta d^c)(c_\phi H_d + s_\phi L)\end{aligned}\tag{6}.$$

We have therefore for each matrix displaying the masses and mixings of the g , d -type quarks¹⁰⁾

$$M = \begin{pmatrix} M_{gg}^- & \mu_{gd}^- \\ \mu_{dg}^- & m_{dd}^- \end{pmatrix} = \begin{pmatrix} c_\theta \lambda_2 s_2 + s_\theta \lambda_1 s_1 & c_\theta \lambda_4 c_\phi \bar{v} + s_\theta \lambda_3 s_\phi \bar{v} \\ s_\theta \lambda_2 s_2 - c_\theta \lambda_1 s_1 & s_\theta \lambda_4 c_\phi \bar{v} - c_\theta \lambda_3 s_\phi \bar{v} \end{pmatrix}\tag{7}$$

where we introduced the notation $\langle S_{1,2} \rangle = s_{1,2}$ and $\langle H_d \rangle = \bar{v}$. Starting our analysis by the third family, we can readily derive a lower bound for λ_3 at low energy, the coupling responsible for the top mass. Denoting $\langle H_u \rangle = v$, we have for the W mass

$$m_W = g_2(m_W) \sqrt{\frac{v^2 + \bar{v}^2}{2}}\tag{8}.$$

Inserting $m_W = 82$ GeV, $g_2(m_W) = 0.685$ (corresponding to

$\alpha = 0.00784$, $\sin^2 \theta_w = 0.21$) we have the upper limit

$$v < 169 \text{ GeV} \quad (9).$$

Using the lower experimental bound on the top quark mass $m_t > 30 \text{ GeV}^{12)}$ we can infer from $m_t = \lambda_3 v$

$$\lambda_3 > 0.177 \quad (10)$$

at the low energy scale.

As referred to above, the discussion of the LNV sector of the model is essential for our main argument. The corresponding relevant couplings are λ_3 and λ_4 , as seen from equations (6). The ratio δ of LNV/LNC couplings in the superpotential has been analysed in a somewhat similar context by Hall and Suzuki¹³⁾. In a model such as ours, where m_ν is guaranteed to vanish, there is no straightforward constraint on this ratio¹³⁾ for the second and third generations so we will just require $\delta < 1$ for these. From the orthogonality of the ϕ matrix (2), two cases can be distinguished which we now consider separately (moduli of the couplings are understood throughout),

a) $s_\phi < \frac{\sqrt{2}}{2}$. The suppression of LNV requires in this case

$$\frac{\lambda_3}{\lambda_4} \leq \delta < 1 \quad (11)$$

which upon insertion of the lower bound (10) implies

$$0.177 < \lambda_3 \leq \lambda_4 \quad (12).$$

Furthermore, since we have $c_\phi > \frac{\sqrt{2}}{2}$, the quantity $\lambda_4 c_\phi \bar{v}$ which appears in the matrix (7) has a lower limit

$$\lambda_4 c_\phi \bar{\nu} > 0.177 \cdot \frac{\sqrt{2}}{2} \cdot 5 \text{ GeV} = 0.625 \text{ GeV} \quad (13).$$

The mixing mass $\mu_{\bar{g}d}$ (in fact $\mu_{\bar{g}b}$ for the third family) is strongly constrained. Typically one should require $\mu_{\bar{g}d} < 10^{-2} m_{\bar{d}d}$ ¹⁴⁾ which implies $c_\theta < 10^{-1}$ and $s_\theta \approx 1$. Then, from the second term of $\mu_{\bar{g}d}$ in the matrix (7), we see that one must have $\lambda_3 s_\phi < 10^{-2}$ which means that the coefficient s_ϕ is also rather small: $s_\phi < 0.05$, corresponding to $c_\phi \approx 1$. In conclusion for the case $s_\phi < \sqrt{2}/2$, the first term in the expansion of $\lambda_3 A_1 B_1 C_1$ in the superpotential (6) reduces essentially to:

$$\lambda_3 Q g^c L \quad (14).$$

The possible magnitude of this Yukawa coupling has already been investigated¹¹⁾ and the bound $\lambda_3 < 10^{-3}$ was found within the first and second family^{*)}. At the tree level the main contribution to this bound comes from the ratio $\Gamma(K^+ \rightarrow \pi^+ \nu \bar{\nu}) / \Gamma(K^+ \rightarrow \pi^0 e^+ \nu)$ which is of the order of 10^{-5} . At the one loop level it comes from the branching ratio of $\mu \rightarrow e \gamma$, of order 10^{-10} . For the third family, which is our present concern, no upper limit exists for the corresponding decay of the B meson but there is an upper limit for the corresponding branching ratio of $\tau \rightarrow \mu \gamma$ ¹⁵⁾ at the order of 10^{-4} . Replacing m_τ for m_μ in the expression for the branching ratio

*) The authors of ref.11) do not consider any suppression due to family mixing. In case a suppression factor exists the bound becomes slightly milder.

of $\mu \rightarrow e\gamma$ 11)

$$\text{BR}(\mu \rightarrow e\gamma) \leq \frac{3\alpha}{\pi} \frac{\lambda_3^4}{G_F^2 m_g^2} \left(\frac{m_b}{m_\mu}\right)^2 \quad (15)$$

and requiring the upper limit 10^{-4} we obtain $\lambda_3 < 0.13$ for the third family, which conflicts with (12). So we must discard the possibility $s_\phi < \sqrt{2}/2$.

b) $s_\phi > \frac{\sqrt{2}}{2}$. The suppression of LNV requires now

$$\frac{\lambda_4}{\lambda_3} \leq \delta < 1 \quad (16).$$

Analogously to the previous case there is a lower limit on the quantity $\lambda_3 s_\phi \bar{v}$ from the coefficient μ_{gd}^- in the matrix (7). This is

$$\lambda_3 s_\phi \bar{v} > 0.177 \cdot \frac{\sqrt{2}}{2} \cdot 5 \text{ GeV} = 0.625 \text{ GeV} \quad (17)$$

which implies now $s_\theta < 10^{-1}$, $c_\theta \simeq 1$. So, from the first term of μ_{gd}^- we have $\lambda_4 c_\phi < 10^{-2}$. Immediately one realizes that the expression of the diagonal mass $m_{\bar{d}d}$ is dominated by the term $c_\theta \lambda_3 s_\phi \bar{v}$, the coefficient of $\bar{v}$ being at least 10^3 times the coefficient $s_\theta \lambda_4 c_\phi$ in the first term. Therefore, since we are considering the third family, using $m_b = \lambda_3 s_\phi \bar{v}$ we obtain the mass relation

$$m_t = \frac{m_b}{s_\phi \eta} \quad (18)$$

which will be our working formula to derive an upper bound on the top quark mass. To this end we obviously need information on the quantity η , the ratio between the $\bar{v}$ and v vacuum

expectation values: $\eta = \bar{v}/v$. We will henceforth use the values of the quark masses normalized at 1 GeV¹⁶⁾

$$m_S(1\text{ GeV}) = 180 \pm 50 \text{ MeV}, \quad m_C(1\text{ GeV}) = 1.34 \pm 0.05 \text{ GeV}, \quad m_b(1\text{ GeV}) = 5.30 \pm 0.10 \text{ GeV} \quad (19)$$

and obtain the physical mass of each quark identified as the solution of the corresponding renormalization group equation $m_q(\mu) = \mu$. With the numbers (19) and equation (18) we can derive upper and lower bounds on the parameter η from the data on the second fermion generation. In analogy with the previous case, the tree level expression for the charm quark mass, $m_C = \lambda_3 v$, provides the lower bound

$$\lambda_3 > \frac{1.29}{169} = 7.6 \times 10^{-3} \quad (20)$$

As before, we consider separately two cases;

a) $s_\phi < \frac{\sqrt{2}}{2}$. We have from the requirement of LNV suppression and corresponding to eq.(11)

$$\frac{\lambda_3}{\lambda_4} \leq \delta < 1 \quad (21)$$

Requiring again $\mu_{\bar{g}s} < 10^{-2} m_{\bar{S}S}$, inserting the upper limit on the strange quark mass (19), the lower limit on $\bar{v}$ (=5 GeV), the fact that $\lambda_4 \geq \lambda_3 > 7.6 \times 10^{-3}$ (20) and that in this case

$c_\phi > \frac{\sqrt{2}}{2}$, we find

$$c_\theta \lambda_4 c_\phi \bar{v} \leq 10^{-2} m_S \quad \text{or} \quad c_\theta \leq \frac{2.3 \times 10^{-3} \text{ GeV}}{7.6 \times 10^{-3} \times \frac{\sqrt{2}}{2} \times 5 \text{ GeV}} \quad (22)$$

giving similar bounds for θ as in the third family, namely $c_\theta \leq 10^{-1}$ and $s_\theta \approx 1$. This in turn implies $\lambda_3 s_\phi < 0.5 \times 10^{-3}$, from which one realizes that $m_{\bar{s}s}$ in the matrix (7) is dominated by the term $s_\theta \lambda_4 c_\phi \bar{v}$ (note $s_\theta \lambda_4 c_\phi \approx \lambda_4 c_\phi > 7.6 \times 10^{-3} \frac{\sqrt{2}}{2}$, whereas $c_\theta \lambda_3 s_\phi \leq 5 \times 10^{-5}$). Dividing m_S by m_C one then obtains

$$\frac{m_S}{m_C} = \frac{\lambda_4 c_\phi}{\lambda_3} \frac{\bar{v}}{v} \geq \frac{\sqrt{2}}{2\delta} \eta \quad (23)$$

and finally,

$$\eta \leq 0.252\delta \quad \text{or} \quad \eta < 0.252 \quad (24).$$

Alternatively one can derive this upper bound from

$$\bar{v} = \frac{m_S}{\lambda_4 c_\phi} \quad (25)$$

giving $\bar{v} < 43$ GeV, from which using eq.(8), $\eta < 0.264$.

b) $s_\phi > \frac{\sqrt{2}}{2}$. This case provides a lower bound on η . We have now

$$\frac{\lambda_4}{\lambda_3} \leq \delta < 1 \quad (26)$$

implying $\lambda_3 s_\phi > 7.6 \times 10^{-3} \times \frac{\sqrt{2}}{2}$ (using (20)) and hence via an entirely analogous argument to the one used in the previous case we are led to $s_\theta \leq 10^{-1}$, $c_\theta \approx 1$. So now $\lambda_4 c_\phi \leq 0.5 \times 10^{-3}$ and $m_{\bar{s}s}$ is dominated by $c_\theta \lambda_3 s_\phi \bar{v} = \lambda_3 s_\phi \bar{v}$. Hence, dividing m_S by m_C we obtain

$$\frac{m_S}{m_C} = \frac{\lambda_3 s_\phi \bar{v}}{\lambda_3 v} = s_\phi \eta \quad (27a).$$

Inserting now the upper limit on s_ϕ ($s_\phi = 1$) and the lower and upper limits on m_S and m_C (see eq.(19)) we finally get the

lower bound

$$\eta > 0.094 \quad (27b).$$

The two ranges $\eta > 0.094$, $\eta < 0.252$ correspond to two independent situations, $s_\phi < \sqrt{2}/2$ and $s_\phi > \sqrt{2}/2$, so they cannot be combined. Therefore, if we do not know which case to select, we will be unable to use the upper and lower bounds on η . However, as in the third family, case a) for which $s_\phi > \sqrt{2}/2$ can now be excluded using a similar argument. In fact on one hand we require in case a) $\lambda_3 > 7.6 \times 10^{-3}$ (eq.(20)) and on the other, using directly the result of ref.11) since we are referring to the second family, $\lambda_3 < 10^{-3}$. Thus we are also left with the single possibility, $s_\phi > \sqrt{2}/2$ for the second generation, corresponding to the effective lower limit (27b). Using now formula (18) and the value of m_b (19) we can obtain the value of the top mass normalized at 1 GeV for s_ϕ in the interval $\sqrt{2}/2 < s_\phi < 1$ and several values of η (see Table I). The actual prediction of m_t will be obtained by renormalizing m_t (1 GeV) to $m_t(\mu)$ with $\mu = m_t$. To this end we use the one loop renormalization group equation for quark masses given by ¹⁶⁾

$$m_i(\mu) = \bar{m}_i \left(\frac{1}{2} \ln \frac{\mu^2}{\Lambda^2} \right) - \frac{2\gamma_0}{\beta_0} \quad (28)$$

for quark i. Here $\gamma_0 = 2$ and

$$\beta_0 = \frac{11}{3} N - \frac{2}{3} \sum_f T_f(R) - \frac{1}{6} \sum_S T_S(R) \quad (29)$$

for SU(N), where the second and third terms denote the two component fermion and real scalar contribution respectively for

representation R and $T(R)$ is defined in the usual way. Obviously $N=3$ and recalling from the decomposition (1) of the 27 that there are four fermion triplets in this mass range (a triplet and anti-triplet corresponding to g and g^c decouple), we have, for three generations

$$\beta_0 = 7 \quad (30).$$

Here we are assuming that all squarks and gluinos are heavier so that no supersymmetric effects are present at this scale.

As far as the upper bound for the top quark mass is concerned, it is seen from (18) that this will be obtained by minimizing η , s_ϕ , maximizing m_b and performing the evolution to $m_t(m_t)$. This is done in three stages by renormalizing from $m_t(1\text{GeV})$ to $m_t(m_c)$ (3 flavours), then to $m_t(m_b)$ (4 flavours) and finally, switching on the bottom quark, to $m_t(m_t)$ (5 flavours). The result is shown in Table I and Fig. 1. Using $\Lambda_3 = 150 \text{ MeV}$, $\Lambda_4 = 120 \text{ MeV}$, $\Lambda_5 = 78 \text{ MeV}^{16)}$ and a strange quark mass close to its lower limit ($m_s = 130 \text{ MeV}$), we obtain

$$m_{t_{\text{max}}} = 45.9 \text{ GeV} \quad (31)$$

This bound can be slightly lowered if we allow for greater values of m_s . For instance, with $m_s = 150 \text{ MeV}^{17)}$ we get $m_{t_{\text{max}}} = 40.5 \text{ GeV}$. In table I we also show the near stability of the upper bound on m_t against Λ_{N_f} . Let us finally add that with $s_\phi = s_{\phi\text{min}} = \sqrt{2}/2$ and $m_t \geq 30 \text{ GeV}$ we can also

get an absolute upper bound for η in this model (normalized at 1 GeV): it is $\eta = 0.15$. So, combining the bound (27b) with this one, we get

$$0.094 < \eta < 0.15 \quad (32)$$

We now add a word concerning this order of magnitude range and the radiative breaking of the standard model via the VEV's $\langle H_u \rangle = v$ and $\langle H_d \rangle = \bar{v}$. Both these fields have Yukawa couplings: they are λ_3 for H_u and λ_3, λ_4 for H_d . Owing to their positive contribution to the mass evolution equations, these couplings can generate negative masses for the H_u, H_d scalars which means¹⁸⁾ gauge symmetry breaking. The inequalities (32), which tell us that $\bar{v} < v$, imply therefore that the mass equation for H_u must receive at hierarchically small scales a larger positive contribution than the corresponding equation for H_d . In view of the uncertainties in the evaluation of some of the quantities in these equations, we will only use them in practice as consistency conditions. We have

$$\begin{aligned} \frac{dm_{H_u}^2}{d(\ln Q)} &= \frac{1}{8\pi^2} (-3M_{2L}^2 g_{2L}^2 - 3M_{2R}^2 g_{2R}^2 - \frac{5}{3} M_{1L}^2 g_{1L}^2 - \frac{5}{3} M_{1R}^2 g_{1R}^2 + \delta_{a3} 3\lambda_3^2 X_{H_u}) \\ \frac{dm_{H_d}^2}{d(\ln Q)} &= \frac{1}{8\pi^2} (-3M_{2L}^2 g_{2L}^2 - 3M_{2R}^2 s_\phi^2 g_{2R}^2 - \frac{5}{3} M_{1L}^2 g_{1L}^2 - \frac{5}{3} M_{1R}^2 s_\phi^2 g_{1R}^2 - \frac{20}{3} M_{1R}^2 c_\phi^2 g_{1R}^2 + \\ &\quad + \delta_{a3} 3\lambda_3^2 s_\phi^2 X_{H_d} + \delta_{a3} 3\lambda_4^2 c_\phi^2 Y_{H_d}) \end{aligned} \quad (33)$$

$$\begin{aligned}
 \text{with } X_{H_u} &= m_{H_u}^2 + m_{\tilde{Q}}^2 + m_{\tilde{u}^c}^2 + A_{\lambda_3}^2 \\
 X_{H_d} &= m_{H_d}^2 + m_{\tilde{Q}}^2 + m_{\tilde{d}^c}^2 + A_{\lambda_3}^2 s_\phi \\
 Y_{H_d} &= m_{H_d}^2 + m_{\tilde{Q}}^2 + m_{\tilde{g}^c}^2 + A_{\lambda_4}^2 \bar{c}_\phi
 \end{aligned} \tag{34}$$

the A's being the trilinear couplings in the scalar potential. From equations (33) it is clear that if for instance $s_\phi = 1$ (corresponding to $c_\phi = 0$) one must have $X_{H_u} > X_{H_d}$ in order to satisfy (32). Since $g_{2R}^2 \approx 10 g_{1R}^2$ in the model we are discussing¹⁰⁾, having s_ϕ in the allowed range, that is $\sqrt{2}/2 < s_\phi < 1$, the fact that $\eta < 1$ would imply $\lambda_4 < \lambda_3$ if we assume the X,Y masses (34) of the same order. But $\lambda_4 < \lambda_3$ is precisely what we want. So the weak symmetry breaking can proceed radiatively in this model in a consistent manner.

To summarize, we have investigated a novel aspect which stems from the left-right symmetric models as inspired by the superstring, namely an upper bound on the top quark mass. Such models have already been analysed in some detail^{8),10)} as far as their general aspects were concerned. The reasoning presented in this paper results essentially from the combination of two suppression requirements: that of lepton number violating relative to lepton number conserving couplings and the suppression of the off diagonal mass $\mu_{\tilde{g}d}^-$ relative to the diagonal one $m_{\tilde{d}d}$ in the g^c, d^c mass matrix. The orthogonality of this matrix together with those two requirements leads into two different physical situations which are distinguished by the two ranges of the parameter s_ϕ : $0 < s_\phi < \sqrt{2}/2$ and $\sqrt{2}/2 < s_\phi < 1$.

Accordingly, for the first range where the H_d Higgs field is defined to be predominantly an $SU(2)_R$ singlet (y_1) and the lepton doublet L predominantly an $SU(2)_R$ doublet (y_2), the g^c and d^c quarks will correspond almost entirely to an $SU(2)_R$ doublet (x_2) and to an $SU(2)_R$ singlet (x_1) respectively. For the other, $\sqrt{2}/2 < s_\phi < 1$, the roles are of course interchanged. In principle both situations are equally acceptable. Experimental (and theoretical) data then enable us to tell which case is self consistent and which is self contradictory. It turns out that the self consistent case, namely $\sqrt{2}/2 < s_\phi < 1$ leads us to an upper bound on the top mass at 46 GeV. The expected range of the ratio $\eta = \bar{v}/v$ is $0.094 < \eta < 0.15$.

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m_s (1 GeV)	$\eta_{\min}$	m_t (1 GeV)	$m_t(m_t)$		
			$\Lambda_5=0.047$	$\Lambda_5=0.078$	$\Lambda_5=0.111$
0.13	0.094	81.2	48.9	45.9	43.4
0.15	0.108	70.8	43.1	40.5	38.3
0.17	0.122	62.4	38.4	36.1	34.1
0.19	0.137	55.9	34.6	32.6	30.9
0.21	0.151	50.6	31.6	29.7	
0.23	0.165	46.2	29.1		
			$\Lambda_4=0.076$	$\Lambda_4=0.12$	$\Lambda_4=0.165$
			$\Lambda_3=0.10$	$\Lambda_3=0.15$	$\Lambda_3=0.20$

Table I - The values of $\eta_{\min}$ (given by $\eta = \frac{1}{s_\phi} \frac{m_s}{m_c}$ for $s_\phi = 1$) and the upper bounds on m_t (normalized at 1 GeV) and $m_t(m_t)$ taken as the solution of the one loop renormalization group equation of the top mass $m_t(\mu)$ for $\mu = m_t$. The smooth dependence on Λ is apparent upon comparison of the last three columns, each for a set of values of Λ_{N_f} . Taking the central one which corresponds to $\Lambda_6 \approx 0.025$ GeV (for $\mu > m_t$) we expect an upper bound on the top mass close to 46 GeV. See also fig. 1.

Fig. 1 - The upper bound of the top quark mass as a function of the strange quark mass. We note the absolute upper bound of $m_t = 46$ GeV corresponding to the minimum assumed value $m_s = 0.13$ GeV. See also Table I.

Fig. 1