RAL-86-075

Science and Engineering Research Council

Rutherford Appleton Laboratory

CHILTON, DIDCOT, OXON, OX11 0QX

RAL-86-075

The Cosmological Constant, Non-Compact Symmetries and Weyl Invariance

F. del Aguila and C.D. Coughlan

July 1986

THE COSMOLOGICAL CONSTANT, NON-COMPACT SYMMETRIES AND WEYL INVARIANCE

F. del Aguila

CERN, Geneva

and

Rutherford Appleton Laboratory

Chilton, Didcot, Oxon OX11 0QX, UK

and

G. D. Coughlan

Dept. of Theoretical Physics

1 Keble Road, Oxford, OX1 3NP, UK

and

St. John's College, Oxford OX1 3JP, UK

Abstract:

The vanishing of the cosmological constant is in principle unrelated to the presence of non-compact symmetries. Obvious exceptions appear to be unrealistic.

Compared to the Planck scale, the cosmological constant Λ is observed to be bounded by $|\Lambda/M_P^2| \lesssim 10^{-120}$ ^[1], where $M_P = 1.2 \times 10^{19}$ GeV is the Planck scale. Einstein's theory of general relativity^[2], successful as it is, offers no hint as to why this number should be so very small. Even if one accepts this as an accident of nature at the classical level, one would expect zero-point energies of quantum fluctuations to produce a value of order unity when the theory is quantised. However, lacking a consistent formulation of quantum gravity, one might suppose that these zero-point energies cancel or can be renormalised to zero. Nonetheless, there is still a change in the effective cosmological constant of order μ^4/M_P^2 whenever the universe undergoes a phase transition (μ is the scale relevant to the phase transition). It is difficult to understand why the bare value of Λ must be fine-tuned so precisely that, after all renormalisations and phase transitions, the effective cosmological constant is so small and indeed probably zero.

In the light of the above discussion, it is evident that there are two independent sectors which might contribute to a non-zero value of Λ : the gravitational sector and the matter sector.

Let us return to the classical problem. One ideally would like to explain the vanishing of the cosmological constant in terms of an exact symmetry that forbids $\Lambda \neq 0$. Since such a symmetry must constraint the cosmological term $\sqrt{-g} \Lambda$, its action on the metric $g_{\mu\nu}(x)$ should be non-trivial:

$$g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(x) + \delta g_{\mu\nu}(x) , \quad (1)$$

(as should its action on matter fields). Unfortunately, a symmetry of the appropriate type is not known to exist for Einstein's gravity. However, if one considers instead the unphysical theory of gravity due to Weyl^[3], a suitable symmetry is immediately obvious, viz, local conformal or Weyl invariance. A Weyl-invariant theory (even gravity + matter) is free from any dimensional parameters and consequently cannot have a non-vanishing cosmological constant.

Recently, it has been claimed that hitherto unconsidered non-compact symmetries could be implemented to banish the cosmological constant in Einstein's gravity^[4]. In this letter we argue that this cannot be the case for the reason that the physical metric as well as certain matter fields are invariant under the non-compact symmetry. We begin by considering the

simplest physical situation and observe that the non-compact symmetry cannot have any implications for sectors of the model which are inert under its action. Such singlet fields, it is argued, are always present in realistic models, in particular Weinberg-Salam Higgs doublets^[5]. Next, we re-examine a specific example of an SU(1,1) model in which it was claimed that the cosmological constant vanished by virtue of an unbroken non-compact symmetry^[4]. It will be obvious that no constraint on Λ follows from that analysis.

Consider first the simple case of gravity-plus-matter Lagrangian of the form

$$L = L_G(g) + L_{NC}(g, Z) + L_M(g, \chi) \quad (2)$$

where

$$L_G(g) = -\frac{1}{2} \sqrt{-g} R \quad (3)$$

is the Einstein Lagrangian^{*}, L_{NC} is the part of L containing fields Z transforming non-trivially under some non-compact symmetry, and L_M is the matter Lagrangian. The obvious point which needs to be stated, is that the non-compact symmetry involves Z only and places no restriction whatsoever on L_G or L_M : there may be a bare cosmological term $\sqrt{-g} \Lambda_{\text{bare}}$ in L_G and/or contributions to Λ coming from the potential $V(\chi)$ in L_M . Even if the latter vanished at $\chi = \langle \chi \rangle$, quantum corrections to V , à la Coleman-Weinberg [6] in general would generate a non-zero contribution. In any case, $\Lambda \neq 0$ despite the non-compact symmetry being exact.

One may wonder what would happen if $L_M = 0$, that is, all matter fields transform non-trivially under the non-compact symmetry. For definiteness, consider just one complex field in the non-compact sector and take non-compact symmetry to be SU(1,1)^{[7]**}. The Lagrangian can be written as

$$L_{NC}(g, Z) = \sqrt{-g} g^{\mu\nu} \frac{1}{2} \frac{\partial_\mu Z \partial_\nu Z^*}{(Z + Z^*)^2}, \quad (4)$$

and the action of SU(1,1) on Z is realised non-linearly:

* From now onwards, we adopt units in which $M_P / \sqrt{8\pi} = 1$.

** For a review of "no-scale" models see ref [8].

$$Z \rightarrow \frac{\alpha Z + i\beta}{i\gamma Z + \delta} \quad ; \quad \begin{aligned} \alpha\delta + \beta\gamma &= 1, \\ \alpha, \beta, \gamma, \delta &\text{ real} \end{aligned} \quad (5)$$

In this example, the non-compact symmetry clearly leads to a vanishing cosmological constant, but in an entirely trivial way. Two special cases of the $SU(1,1)$ transformations are

$$Z \rightarrow Z + i\beta \quad (\alpha = \delta = 1, \gamma = 0) \quad (6)$$

$$Z \rightarrow \alpha^2 Z \quad (\alpha = 1/\delta, \beta = \gamma = 0) \quad (7)$$

Eq (6) implies that the potential $V(Z)$ is independent of $\text{Im } Z$ and there is a flat direction with no contribution to the cosmological constant^{*}. Similarly, Eq (7) implies that there is also a flat direction along $|Z|$ and, taking Z to be neither pure imaginary nor zero, then this second flat direction is seen to be independent of the first and so the potential is totally flat and hence, identically zero. Consequently, as long as $SU(1,1)$ is unbroken, no potential can ever be generated.

The same result appears to be true in the more general case in which there are several fields Z_i ($i = 1, \dots, n+1$) in the non-compact sector. The $2n+2$ real fields transform non-trivially under the non-compact symmetry, giving rise to $2n+2$ flat directions which are in general independent and lead to $V(Z_i) \equiv 0$. An example of such a symmetry is $SU(1,n)$ [7,8], which is the obvious extension of $SU(1,1)$ and has arisen recently in the context of superstrings [9], but as an approximate symmetry with no apparent implication for the vanishing of Λ .

This situation is clearly intolerable in any realistic theory and the non-compact symmetry is expected to be broken, probably explicitly. This allows non-zero potentials to be generated for the low-energy matter fields, and we may extract from L_{NC} (as in Eq (2)) a piece containing fields χ which are singlets under any remaining non-compact symmetry.

Hence, it appears that it is not possible in general to have an unbroken non-compact symmetry under which all fields transform non-trivially. There will always be singlet fields whose Lagrangian is unconstrained by the

* If the potential is required to be analytic, this is enough to make it independent of Z , i.e. flat.

symmetry.

We now turn to discuss a specific example [4] in which it was claimed that the vanishing of the cosmological constant is a direct consequence of a non-compact $U_{NC}(1)$ symmetry (Eq (6)) in $SU(1,1)$. We note that no relevant constraint on Λ could be derived from that analysis and explain where the confusion arose. This example was presented in the framework of a no-scale $N=1$ Poincare supergravity model; however, the supersymmetry is of no relevance to the issue in question, and we shall ignore it. Let us briefly recount the argument in ref [4].

The model consisted of $L_G + L_{NC}$ in Eqs (3) and (4). However, for their analysis the authors preferred to linearise the $SU(1,1)$ symmetry. This was achieved by introducing two extra degrees of freedom and two associated gauge symmetries with which they can be removed by gauge fixing. Replacing the complex field Z by two complex fields ϕ_0 and ϕ_1 :

$$Z = \frac{1}{2} \frac{\phi_0 + \phi_1}{\phi_0 - \phi_1}, \quad (8)$$

two "compensating" degrees of freedom were introduced. Then, a Weyl rescaling of the metric

$$g_{\mu\nu} \rightarrow g'_{\mu\nu} = g_{\mu\nu} \frac{3}{(|\phi_0|^2 - |\phi_1|^2)}, \quad (9)$$

allowed one to elucidate the associated "compensating" gauge symmetries from the transformed Lagrangian:

$$\begin{aligned} L(g, Z) &\rightarrow L'(g', \phi_0, \phi_1) \\ &= \sqrt{-g'} \left\{ -\frac{1}{6} (|\phi_0|^2 - |\phi_1|^2) R' \right. \\ &\quad \left. - g'^{\mu\nu} [(D_\mu \phi_0)^\dagger D_\nu \phi_0 - (D_\mu \phi_1)^\dagger D_\nu \phi_1] \right\} \end{aligned} \quad (10)$$

where $D_\mu \equiv \partial_\mu + iA_\mu$. Here, A_μ is an auxiliary field which can be eliminated algebraically through its equation of motion:

$$2iA_\mu = \frac{\phi_1^* \partial_\mu \phi_1 - \phi_0^* \partial_\mu \phi_0}{|\phi_0|^2 - |\phi_1|^2} - \text{h.c.} \quad (11)$$

Now, the $SU(1,1)$ symmetry is linearly realised, and the "compensating" gauge symmetries are seen to be a local Weyl invariance and a local (compact) $U(1)$ invariance:

$$\delta_W g'_{\mu\nu} = 2\omega(x) g'_{\mu\nu} , \quad \delta_W \phi_{0,1} = -\omega(x) \phi_{0,1} , \quad (12)$$

$$\delta_{U(1)} g'_{\mu\nu} = 0, \quad \delta_{U(1)} \phi_{0,1} = i\theta(x) \phi_{0,1} \quad (\delta A_\mu = -i\partial_\mu \theta(x)) . \quad (13)$$

The original non-linearly-realised formulation can be recovered by fixing the (compensating) gauge^{*}

$$\phi_0 = \sqrt{3} \quad (14)$$

(or by performing an appropriate compensating-gauge transformation) and inverting Eq (8). For completeness, the transformations of the fields under the non-compact $U_{NC}(1)$ contained in $SU(1,1)$ are:

$$\delta_{NC} g'_{\mu\nu} = 0, \quad \delta_{NC} \phi_{0,1} = i\rho(\phi_0 - \phi_1) \quad (15)$$

The Ward identities for the three above symmetries can now be written down. In particular, for the tadpoles, there are three equations:

$$\delta g'_{\mu\nu} \frac{\delta \Gamma}{\delta g'_{\mu\nu}} + \delta \phi_i \frac{\delta \Gamma}{\delta \phi_i} = 0 . \quad (16)$$

It was then shown that these Ward identities, when evaluated in the vacuum state

$$(\langle g'_{\mu\nu} \rangle = \eta'_{\mu\nu} , \quad g'_{\mu\nu} = \eta'_{\mu\nu} + h'_{\mu\nu} , \quad \langle \phi_{0,1} \rangle = v_{0,1}) , \text{ implied that}$$

$$\eta'_{\mu\nu} \frac{\delta \Gamma}{\delta h'_{\mu\nu}} = 0 , \quad (17)$$

and this was interpreted as a vanishing cosmological constant.

The key point is, however, that $g'_{\mu\nu}$ is not the physical metric. The physical metric is the $g_{\mu\nu}$ with which we started, and one may verify explicitly from Eqs (9), (12) and (13) that (in contrast to $g'_{\mu\nu}$) it is invariant under the compensating symmetries $\delta_W g_{\mu\nu} = 0$ and $\delta_{U(1)} g_{\mu\nu} = 0$.

^{*} Since ϕ_0 is complex, Eq (14) corresponds to two gauge conditions for the two compensating symmetries. - 6 -

(Both metrics are invariant under $SU(1,1)$). The three Ward identities in Eq (16) merely state that

$$\delta g_{\mu\nu} \frac{\delta \Gamma}{\delta g_{\mu\nu}} = 0, \quad (18)$$

not for any deep reason such as the vanishing of the cosmological constant, but simply because the three symmetries act trivially on $g_{\mu\nu}$, i.e., $\delta g_{\mu\nu} = 0$. There are absolutely no implications for the physical cosmological constant which is proportional to $\eta_{\mu\nu} \delta \Gamma / \delta h_{\mu\nu}$.

We may conclude from this model that non-compact symmetries, under which the physical metric does not transform, cannot constrain Λ . The theory in which the symmetry is realised linearly (with compensating fields and compensating symmetries) appears to be equivalent to the original theory in which the symmetry is non-linearly realised, and where Λ is clearly not constrained.

The situation is completely analogous to that of the conformal scalar-metric theory where a compensating scalar field ϕ is added to the Einstein action in the following way [10,11]:

$$S[g] \rightarrow S'[g', \phi] = S[\phi^{\frac{4}{n-2}} g'], \quad (19)$$

where n is the dimension of space-time. The associated compensating symmetry is again Weyl invariance. The theories described by S and S' are equivalent [11], and the physical metric is not $g'_{\mu\nu}$ but $g_{\mu\nu} = \phi^{4/(n-2)} g'_{\mu\nu}$. Once again, the physical metric is inert under the compensating Weyl invariance and no conclusions can be drawn regarding the value of Λ .*

Finally, it is interesting to ask if some remnant of a (scale) symmetry could have implications for Λ . In particular, an obvious possibility is a scaling behaviour in some field direction

$$V(\alpha Z_k, \chi_i) = \alpha^p V(Z_k, \chi_i), \quad (20)$$

* However, a contrary claim has been made recently^[12]. There, an attempt was made to banish Λ by minimizing the potential containing the compensator assuming that it was an independent field. In any case, the compensator-independent potential would have to be fine-tuned to zero.

two examples being (i) a quartic potential for which $p = 4$, and (ii) lowest-order terms of the superstring potential [9]. The behaviour of such potentials is given in Fig. 1, for $p > 0$. Either $V(Z)$ is unbounded from below (which is clearly unphysical), has a flat direction ($V(Z) = 0$), or has a minimum at the origin ($V(0) = 0$). Such behaviour cannot be usefully applied to the standard model [5], where Λ must be zero after spontaneous symmetry breaking.

We wish to acknowledge discussions with M. Daniel, M. Duff, L.E. Ibáñez, H.P. Nilles, M. Quiros and G.G. Ross. One of us (F.A.) is grateful to the Theory Group at the Rutherford Appleton Laboratory for the hospitality extended to him during the writing of this work.

References

- [1] A. Sandage, Observatory 88 (1968) 91.
- [2] S. Weinberg, Gravitation and Cosmology (Wiley, New York, 1972).
- [3] H. Weyl, Raum, Zeit, Materie, vierte erweiterte Auflage, Berlin (1921).
- [4] I. Antoniadis, C. Kounnas and D.V. Nanopoulos, Phys. Lett. 162B (1985) 309.
- [5] S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
A. Salam, in Elementary particle theory: relativistic groups and analayticity, ed. N. Swartholm (Almqvist and Wiksell, Stockholm, 1968) p. 367;
S.L. Glashow, Nucl. Phys. 22 (1961) 579.
- [6] S. Coleman and E. Weinberg, Phys. Rev. D7 (1973) 1888.
- [7] E. Cremmer, S. Ferrara, C. Kounnas and D.V. Nanopoulos, Phys. Lett. 133B (1983) 61.
J. Ellis, C. Kounnas and D.V. Nanopoulos, Nucl. Phys. B241 (1984) 406; Nucl. Phys. B247 (1984) 373; Phys. Lett. 143B (1984) 410;
S. Ferrara and A. van Proeyen, Phys. Lett. 138B (1984) 77.
J. Ellis, K. Enqvist and D.V. Nanopoulos, Phys. Lett. 147B (1984) 99.
- [8] A.B. Lahanas and D.V. Nanopoulos, "The Road to no-scale Supergravity", to appear in Phys. Repts.

- [9] E. Witten, Phys. Lett. 155B (1985) 151;
 J.P. Derendinger, L.E. Ibañez and H.P. Nilles, Nucl. Phys. B267 (1986) 365;
 J. Ellis, C. Gomez and D.V. Nanopoulos, CERN preprint 4328/85;
 C. Burgess, A. Font and F. Quevedo, Texas preprint UTTG-31-85 (1985);
 Y.J. Ahn, J.D. Breit and G. Segre, Pennsylvania preprint (1985);
 M. Quiros, Phys. Lett. B173 (1986) 265;
 J. Ellis, C. Gomez, D.V. Nanopoulos and M. Quiros, Phys. Lett. B173 (1986) 59;
 M. Dine, N. Seiberg, X.G. Wen and E. Witten, Princeton preprint, April 1986.
- [10] S. Deser, Ann. of Phys. (N.Y.) 59 (1970) 248;
 S. Deser, M.T. Grisaru, P. van Nieuwenhuizen and C.C. Wu, Phys. Lett. B58 (1975) 355;
 F. Englert, C. Truffin and R. Gastmans, Nucl. Phys. B117 (1976) 407;
 M.J. Duff, Nucl. Phys. B125 (1977) 334.
- [11] N.C. Tsamis and R.P. Woodard, Ann. of Phys. 168 (1986) 457.
- [12] C.Z. Zha, Phys. Lett. 171B (1986) 366.

Figure Caption

Possible forms for a potential with the scaling symmetry $V(\alpha Z) = \alpha^p V(Z)$, with $p > 0$. (For $p < 0$, replace Z by $1/Z$).

Fig. 1