THE INVERSE GYROTRON - A POSSIBLE MECHANISM
FOR PARTICLE ACCELERATION WITH TEM WAVES

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In a gyrotron, the kinetic energy of slightly relativistic electrons (50 keV to about 1 MeV) is converted with good efficiency (~40%) into microwave radiation. The typical frequency range for gyrotrons is between 30 GHz to 200 GHz. A beam of relativistic electrons passes a static, slightly tapered magnetic field parallel to the axis (z-direction). With some initial transverse momentum, the electrons carry out a spiral movement due to Lorentz forces by deflection in the $B_z$ field. Due to coherent interaction in a resonator, synchrotron radiation like microwave emission takes place.

We propose to consider the inverse effect, namely accelerating electrons spiralling in a $B_z$ field by means of TEM microwaves. The idea is that a spiralling electron in a $B_z$ field is no longer a free particle as it is bound to the $B$-field. For microwave radiation with the proper frequency (electron cyclotron resonance frequency $= 28 \frac{GHz}{Tesla}$), the scattering cross-section should be much bigger than for a free electron, which only sees radiation pressure in a TEM field in free space. This avoids the problem of very low efficiency when using electromagnetic TEM waves to accelerate particles in vacuum directly. The movement of the electrons would be similar to a IFEL with helical wiggler, except that the wiggler wavelength is no longer limited by the scale of an alternating magnetic structure and its strength is adjustable through the microwave intensity.

Let us first consider a possible Gedanken-experiment at rest. Suppose an electron is in circular movement in a static magnetic field, like the well-known thread ray discharge launched from a small electron gun into a circular orbit around an axial field. Now assume that the thread ray discharge is not continuous but has short pulses (some ns). With an external coil and 2 capacitor plates, one may artificially generate the field of an electro-magnetic wave ($E/H = 377 \Omega$, $E, H$ in phase, travelling in z direction with the appropriate cyclotron resonance frequency. The electron on the circular orbit sees an accelerating tangential $E$ field and a radial $H$ field, which deflects it due to $v_{tangential} \times B_{radial}$ in z-direction. This must of course be properly-timed with reference to the RF provided.
With some more effort (superposition of two linear polarized TEM fields 90 deg off phase), one may generate a circular polarized wave. Here the electron would experience a constant tangential accelerating force due to $E_t$ and at the same time a constant forward deflection (due to $B_{radial}$). However, not all of the momentum gain due to $E_{tangential}$ may be converted into movement in $z$ direction. Thus the ratio of $E$ and $H$ has to be properly adjusted in order to keep the radius of the circle constant. To provide additional $B_{radial}$ a standing wave pattern may be added, which has the same frequency, and with the node of $E$ in the plane of the circle described by the electron. Now, for this static case, it should be possible to convert all energy gain due to $E_{transversal} = E_{tangential}$ into $z$-momentum. Another way to arrive at an invariant radius is to use a frequency slightly different from $\omega_e$. For a highly relativistic electron in $z$ direction, one may consider the motion in a reference frame in the centre of the circular (spiral) movement and Lorentz transform, solution back into the laboratory.

If one provides a TEM wave and a (moving standing wave pattern in this new reference frame, the electron should be accelerated. Of course, one has to take into account the Doppler shift of the TEM wave emitted by a source in the laboratory and the moving electron ($10^3 < \gamma < 10^6$). For microwaves or RF in the electron frame, one would thus need a (pulsed) infrared or visible light laser in the laboratory. We must also be careful to ensure that losses due to radiation by the spiral movement do not make the whole mechanism impractical.

For a circular polarized wave ($\omega = \omega_e$) travelling in $z$-direction, an electron on cyclotron frequency ($\omega_e, r = r_0$) in the $xy$ plane (Fig. 1) will see an $E$ field as

$$E_y = E_0 \cdot \cos \omega t, \quad E_x = E_0 \cdot (-\sin \omega t)$$

$$E_{tangential} = E_x^2 + E_y^2 = E_0 = \text{constant}$$
The corresponding H-field would be

\[ H_x = -\frac{E_0}{Z_0} \cos \omega t \quad H_y = \frac{E_0}{Z_0} \sin \omega t \]

\[ Z_0 = 377 \quad H_0 = \frac{E_0}{Z_0} \quad B_0 = \mu_0 H_0 \]

\[ H_{\text{radial}} = \sqrt{H_x^2 + H_y^2} = \frac{E_0}{Z_0} \text{ (towards centre of the circle)} \]

Thus the force in z-direction is

\[ f_z = (v_{\text{tangential}} \times \mu_0 H_0) \cdot \mathbf{e} \text{ or in general } f_z = v_t \times (B_0 + B_1) \cdot e \]

If the diameter of the circle is supposed to stay constant on average, we require that over one turn the energy gained by the tangential E field fully translates into longitudinal acceleration

\[ 2\pi r_0 \cdot E_0 \cdot e = \frac{1}{2} m_0 \cdot v_z^2 \quad m_0 = 0.51 \text{ MeV} \]

\[ v_z = \int_{0}^{T} \frac{1}{B_0} dt \quad v_z = 0 \text{ at } t = 0 \]

\[ \omega_c = \frac{2\pi}{T} \]

The term \( \mu_0 H_1 = B_1 \) may be generated in free space at the E node of a standing wave pattern and may be required to keep the radius of the circular movement constant.

The synchrotron radiation loss in the rest frame for a single electron is per orbit

\[ \Delta E = \frac{4\pi e^2}{3 \mu_0^2} \beta^3 \gamma^4 \]

and the power radiated

\[ p = \frac{2 e^2 c}{3 \mu_0^2} \beta^4 \gamma^4 \]

assuming \( \beta = 0.5 \) for \( v_t \) this is in the order of 10^{-15} Watt for a single electron describing a 1 mm radius orbit in a field of 1T.

We also suggest that this type of interaction might explain the existence of very high energy particles from pulsar stars.
References
